Stability of magnetically driven relativistic jets

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Outline

- normal mode analysis
- unperturbed jet solutions
- dispersion relation results

Unperturbed flow

Cylindrical jet

helical, axisymmetric, cylindrically symmetric and steady flow

$$\boldsymbol{V}_0 = V_{0z}(\varpi)\hat{z} + V_{0\phi}(\varpi)\hat{\phi}\,, \quad \gamma_0 = \gamma_0(\varpi) = (1 - V_{0z}^2 - V_{0\phi}^2)^{-1/2}\,,$$

$$\rho_{00} = \rho_{00}(\varpi)\,, \quad \xi_0 = \xi_0(\varpi)\,, \quad \boldsymbol{B}_0 = B_{0z}(\varpi)\hat{z} + B_{0\phi}(\varpi)\hat{\phi}\,,$$

$$\boldsymbol{E}_0 = (V_{0z}B_{0\phi} - V_{0\phi}B_{0z})\hat{\varpi}\,,$$

$$\Pi_0 = \frac{\Gamma - 1}{\Gamma}(\xi_0 - 1)\,\rho_{00} + \frac{B_0^2 - E_0^2}{2}\,.$$

Equilibrium condition

$$\frac{B_{0\phi}^2 - E_0^2}{\varpi} - \xi_0\rho_{00}\frac{\gamma_0^2 V_{0\phi}^2}{\varpi} + \frac{d\Pi_0}{d\varpi} = 0\,.$$

Linearized equations

$$Q(\varpi\,, z\,, \phi\,, t) = Q_0(\varpi) + Q_1(\varpi)\exp\left[i(m\phi + kz - \omega t)\right]$$

$$\left(\begin{array}{c} 10 \times 12 \text{ array} \\ \text{function of } \varpi\,, \omega\,, k \end{array} \right) \begin{pmatrix} \gamma_1 \\ \rho_{01} \\ B_{1z} \\ B_{1\phi} \\ iB_{1\varpi} \\ \xi_1 \\ V_{1z} \\ V_{1\phi} \\ d\left(i\varpi V_{1\varpi}\right)/d\varpi \\ d\Pi_1/d\varpi \\ i\varpi V_{1\varpi} \\ \Pi_1 \end{pmatrix} = 0$$

reduces to (4 equations in real space)

$$\frac{d}{d\varpi}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \frac{1}{\mathcal{D}}\begin{pmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{21} & \mathcal{F}_{22} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0\,,$$

where the (complex) unknowns are

$$y_1 = i\frac{\varpi V_{1\varpi}}{\omega_0}\,, \qquad y_2 = \Pi_1 + \frac{y_1}{\varpi}\frac{d\Pi_0}{d\varpi}$$

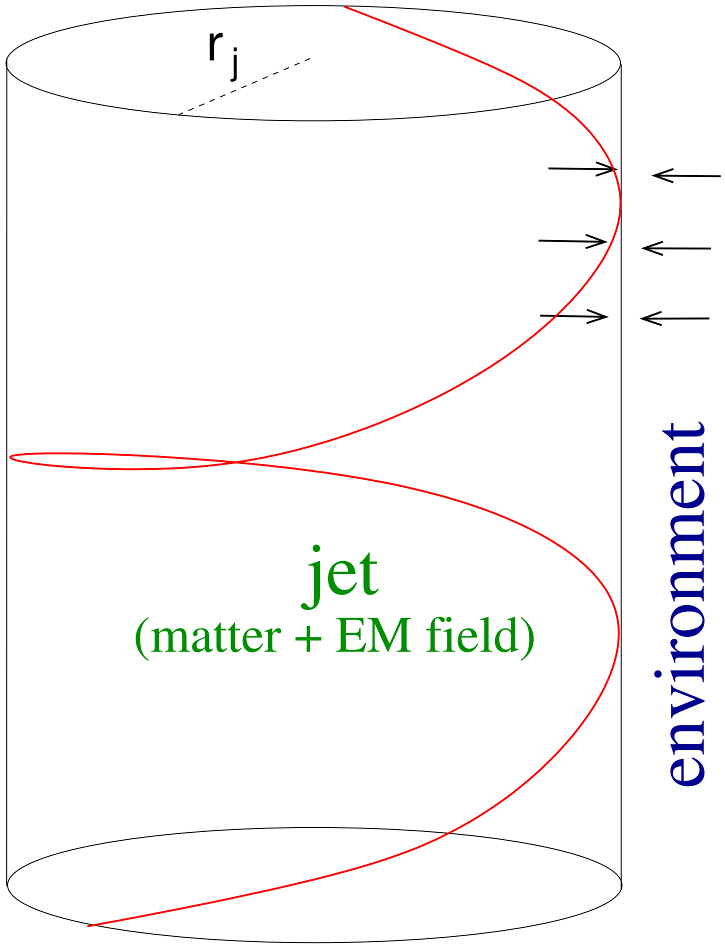
($\mathcal{D}\,,\mathcal{F}_{ij}$ are determinants of 10×10 arrays).

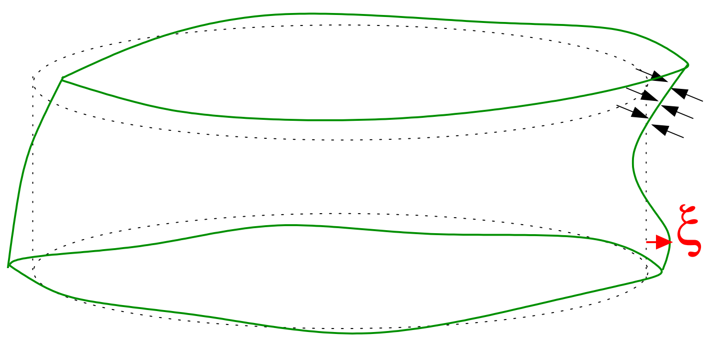
Equivalently

$$y_2'' + \left[\frac{\mathcal{F}_{11}+\mathcal{F}_{22}}{\mathcal{D}} + \frac{\mathcal{F}_{21}}{\mathcal{D}}\left(\frac{\mathcal{D}}{\mathcal{F}_{21}}\right)'\right] y_2' + \left[\frac{\mathcal{F}_{11}\mathcal{F}_{22}-\mathcal{F}_{12}\mathcal{F}_{21}}{\mathcal{D}^2} + \frac{\mathcal{F}_{21}}{\mathcal{D}}\left(\frac{\mathcal{F}_{22}}{\mathcal{F}_{21}}\right)'\right] y_2 = 0\,,$$

which for uniform flows with $V_{0\phi} = 0$, $B_{0\phi} = 0$, reduces to Bessel.

Eigenvalue problem





- solve the problem inside the jet (attention to regularity condition on the axis)

- similarly in the environment (solution vanishes at ∞)

- Match the solutions at r_j:
$[\![y_1]\!] = 0\,,\, [\![y_2]\!] = 0 \longrightarrow$
dispersion relation
$\star$ spatial approach: $\omega = \Re\omega$ and
$\Re k = \Re k(\omega), \Im k = \Im k(\omega)$
$Q = Q_0(\varpi) + Q_1(\varpi) e^{-\Im k z} e^{i(m\phi + \Re k z - \omega t)}$
$\star$ temporal approach: $k = \Re k$ and
$\Re\omega = \Re\omega(k), \Im\omega = \Im\omega(k)$
$Q = Q_0(\varpi) + Q_1(\varpi) e^{\Im\omega t} e^{i(m\phi + kz - \Re\omega t)}$

Unperturbed jet solutions

Try to mimic the Komissarov et al simulation results (for AGN and GRB jets)

- cold, nonrotating jet

$$\boldsymbol{V}_0 = V_0(\varpi)\hat{z}\,, \quad \gamma_0 = \gamma_0(\varpi) = (1 - V_0^2)^{-1/2}\,,$$

$$\rho_{00} = \rho_{00}(\varpi)\,, \quad \xi_0 = 1\,, \quad \boldsymbol{B}_0 = B_{0z}(\varpi)\hat{z} + B_{0\phi}(\varpi)\hat{\phi}\,,$$
$$\boldsymbol{E}_0 = V_0 B_{0\phi}\hat{\varpi}\,.$$

- Equilibrium condition

$$\frac{B_{0\phi}^2/\gamma_0^2}{\varpi} + \frac{d}{d\varpi}\left(\frac{B_{0z}^2 + B_{0\phi}^2/\gamma_0^2}{2}\right) = 0\,,$$

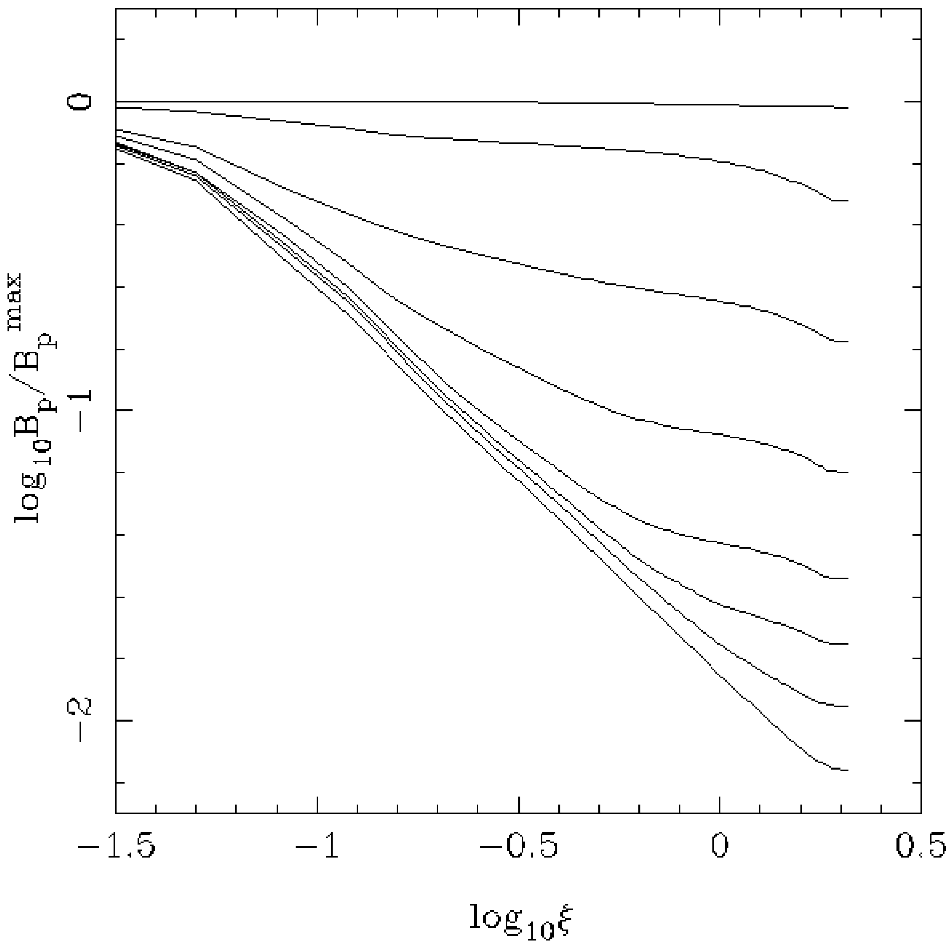
relates B_{0z} with $B_{0\phi}/\gamma_0$.

A cold, nonrotating solution:

$$B_{0z} = \frac{B_j}{\left[1+(\varpi/\varpi_0)^2\right]^{\zeta}}, \quad B_{0\phi} = -\gamma_0 B_{0z}\sqrt{\frac{\left[1+(\varpi/\varpi_0)^2\right]^{2\zeta}-1-2\zeta(\varpi/\varpi_0)^2}{(2\zeta-1)(\varpi/\varpi_0)^2}}.$$

ϖ_0, ζ free parameters, γ_0, ρ_{00} free functions.

- choice of ζ:



$B_{0z} \propto \varpi^{-1.2}$

$\zeta = 0.6$

Formation of core crucial for the acceleration.

The bunching function $\mathcal{S} \equiv \dfrac{\overbrace{\pi\varpi^2}^{S} B_{0z}}{\int_0^{\varpi} B_{0z} \underbrace{2\pi\varpi d\varpi}_{dS}}$ is related to the acceleration efficiency $\sigma = \dfrac{1}{\frac{\mathcal{S}_f}{\mathcal{S}} - 1}$, where $\mathcal{S}_f$ integral of motion ~ 0.9.

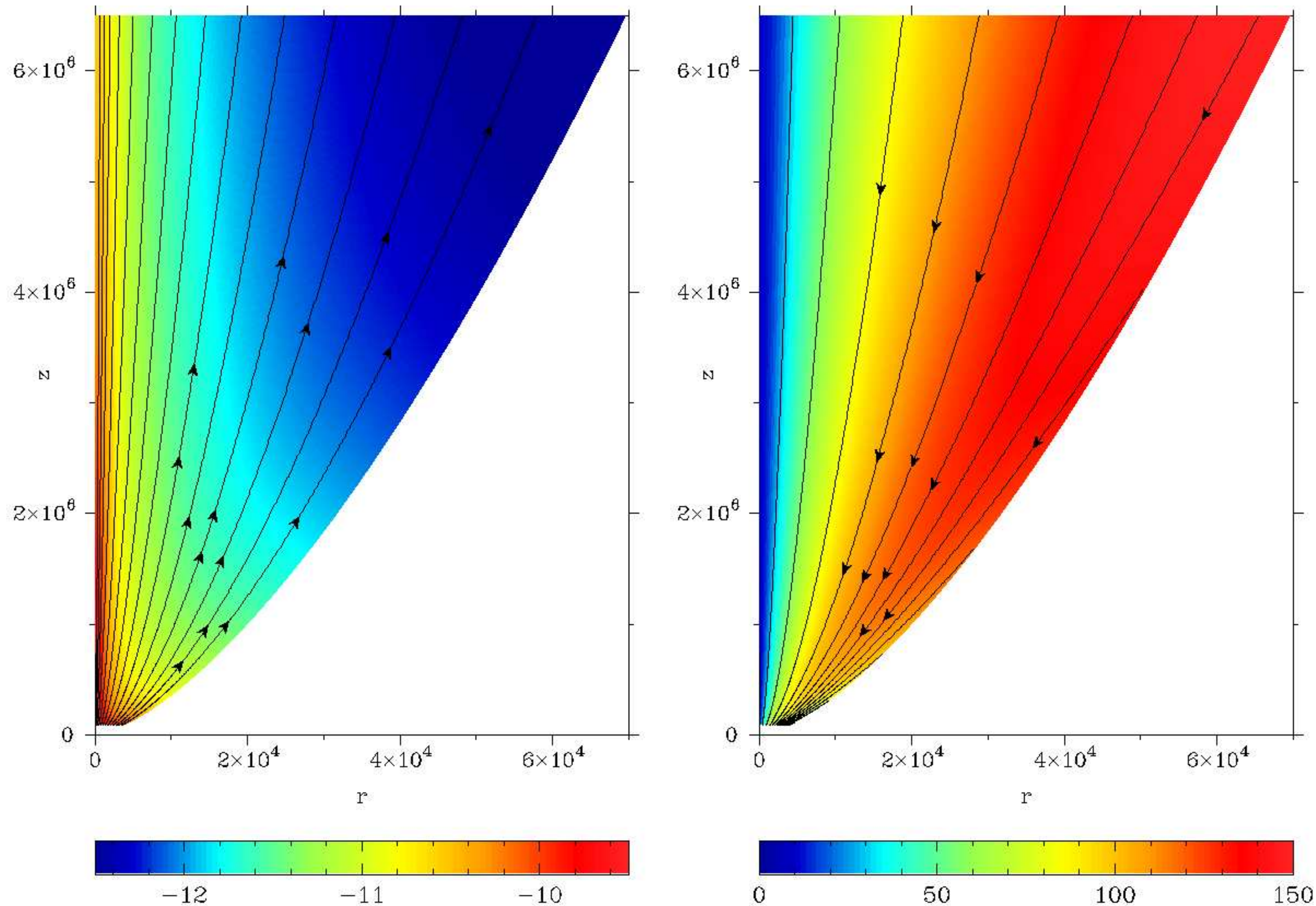
Since $\mathcal{S} \approx 1 - \zeta$ we get $\sigma = \dfrac{1-\zeta}{\zeta - 0.1}$.

- choice of $\gamma_0(\varpi)$:

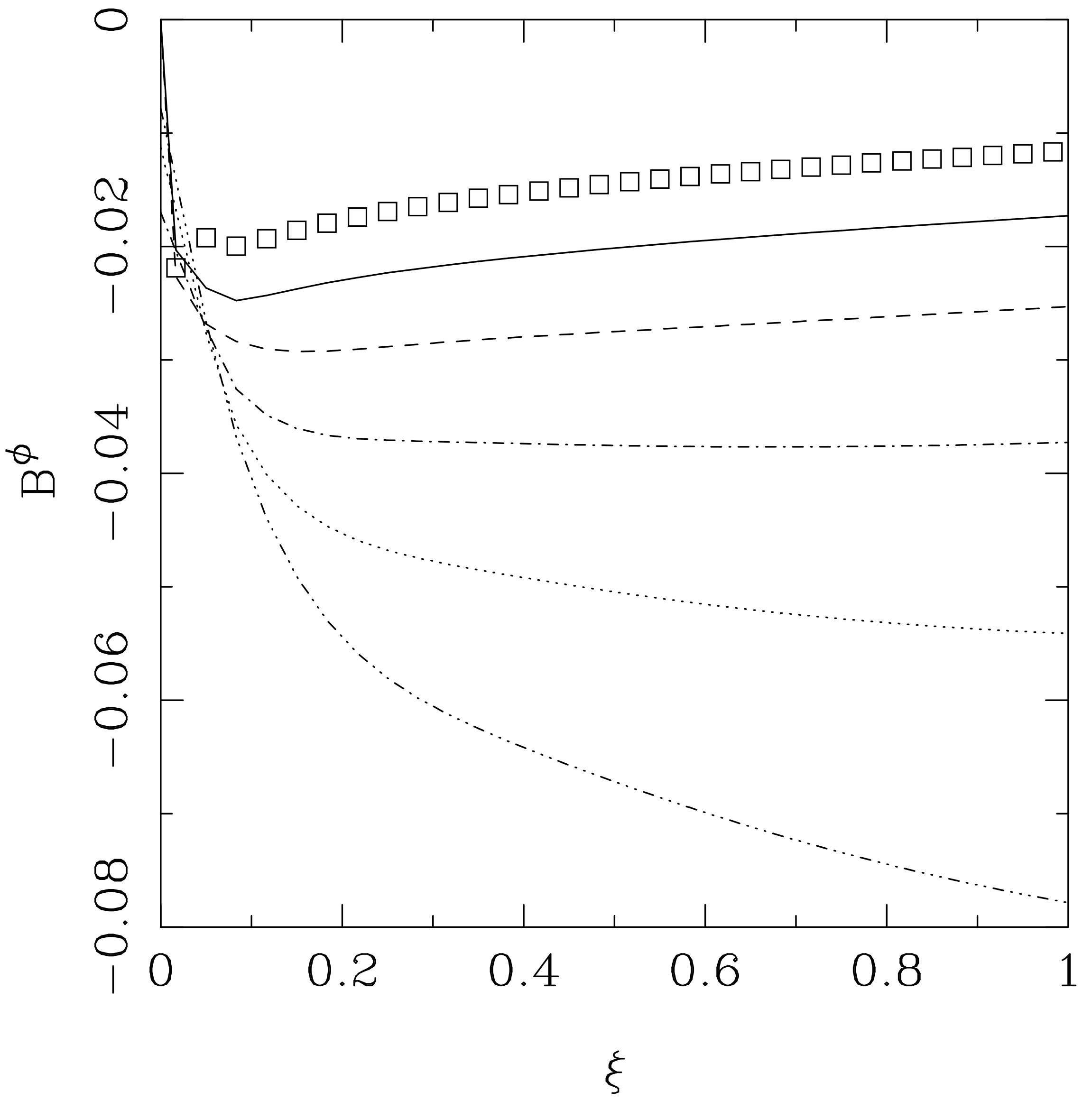
From Ferraro's law $V_{0\phi} = \varpi\Omega + V_{0z}B_{0\phi}/B_{0z}$, where Ω integral of motion, we get $-B_{0\phi}/B_{0z} \approx \varpi\Omega$, or,

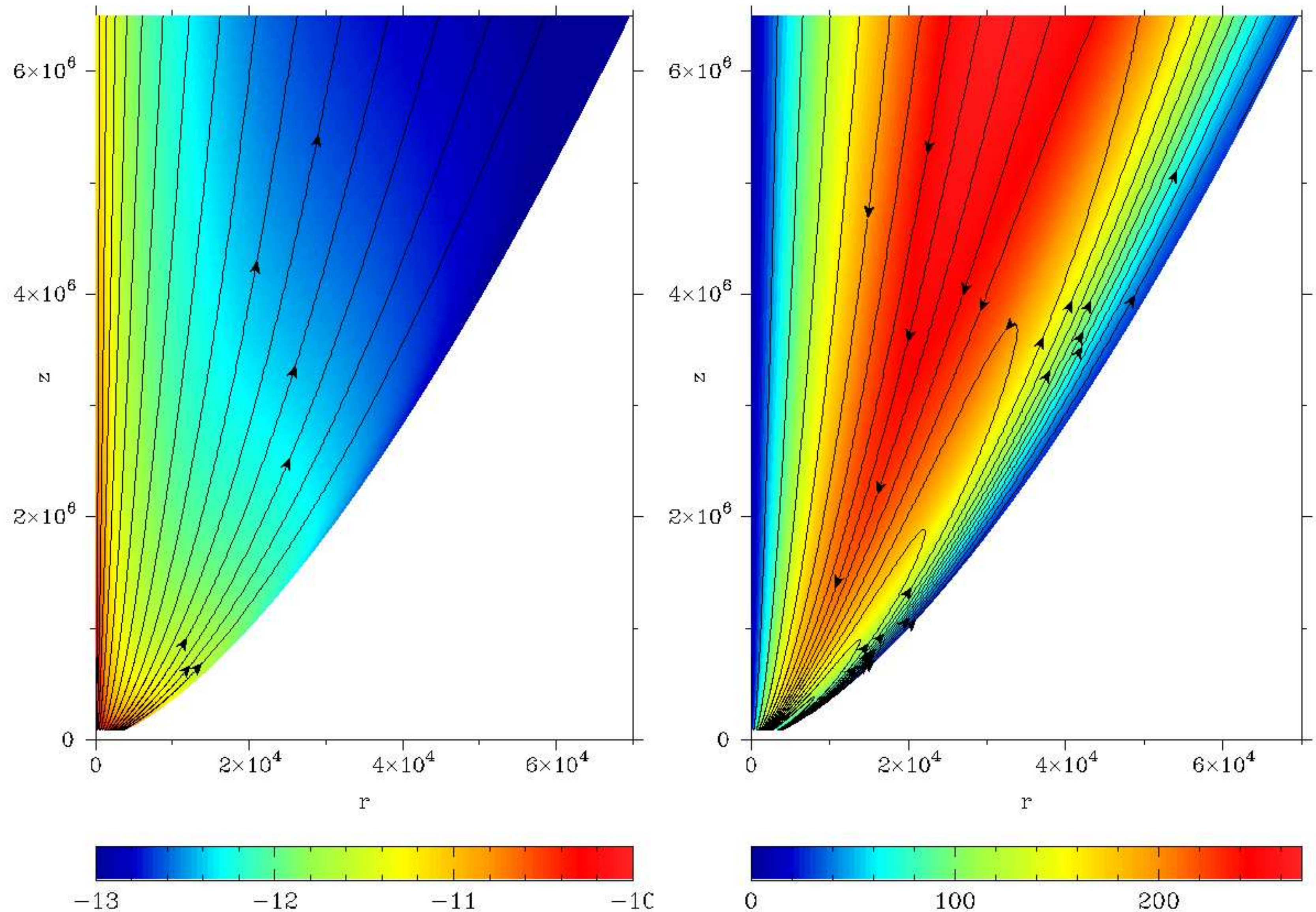
$$\gamma_0 \approx \varpi\Omega\sqrt{\frac{(2\zeta-1)(\varpi/\varpi_0)^2}{\left[1+(\varpi/\varpi_0)^2\right]^{2\zeta}-1-2\zeta(\varpi/\varpi_0)^2}}.$$

The choice of ϖ_0 controls the value of γ_0 on the axis and the jet surface.



left: density/field lines, right: Lorentz factor/current lines (jet boundary $z \propto r^{1.5}$)
Uniform rotation $\rightarrow \gamma$ increases with r


0
−0.02
−0.04
−0.06
−0.08
B^ϕ
0
0.2
0.4
0.6
0.8
1
ξ



Differential rotation → slow envelope and faster decrease of B_ϕ

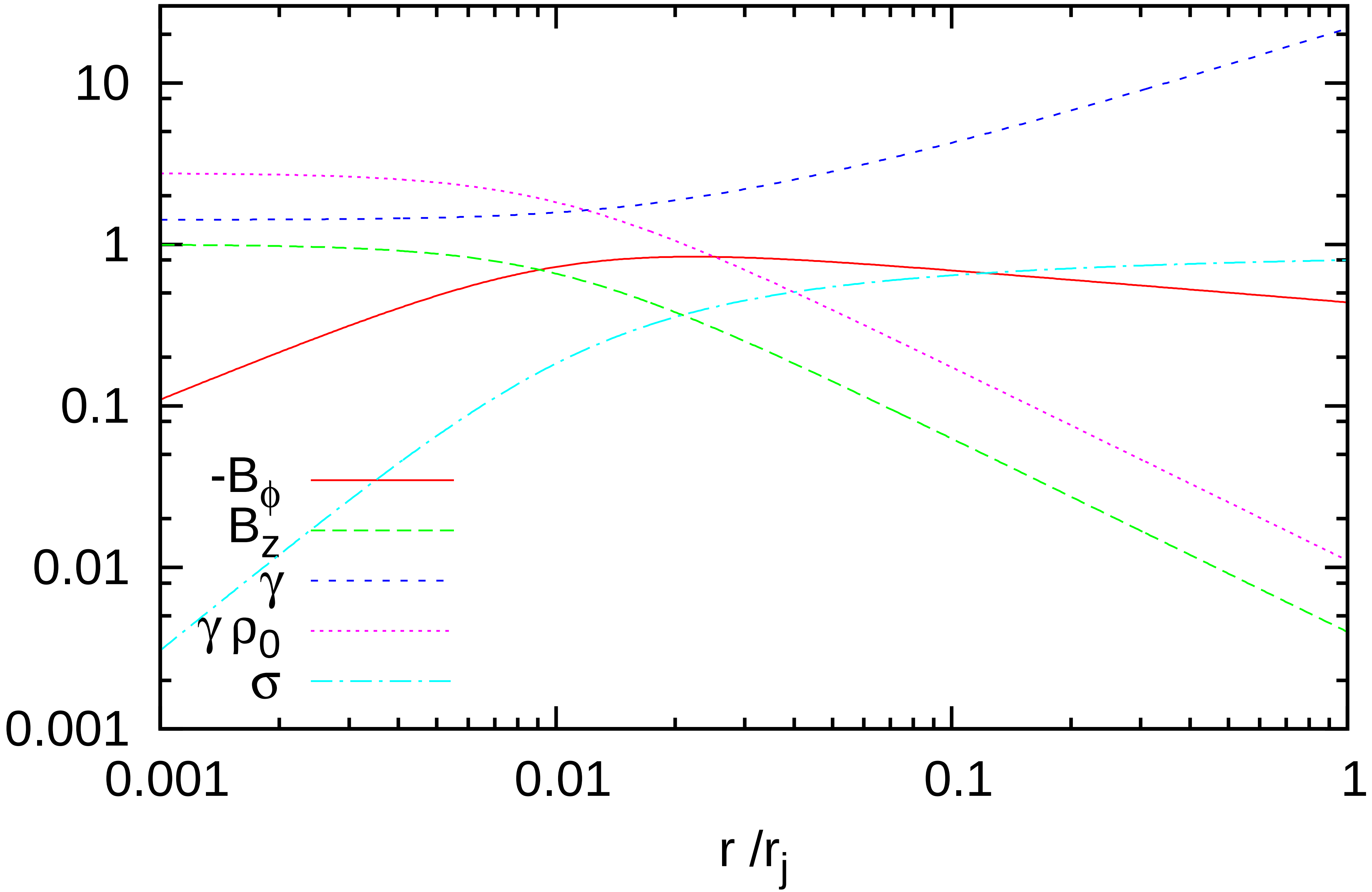
- choice of $\rho_{00}(\varpi)$:

This comes from the mass-to-magnetic flux ratio integral $\frac{\gamma_0 \rho_{00} V_0}{B_{0z}}$, which is assumed constant in the simulations. So $\rho_{00} \propto B_{0z}/\gamma_0$.
The constant of proportionality from the value of $\sigma = \frac{B_{0\phi}^2/\gamma_0^2}{\rho_{00}}$.

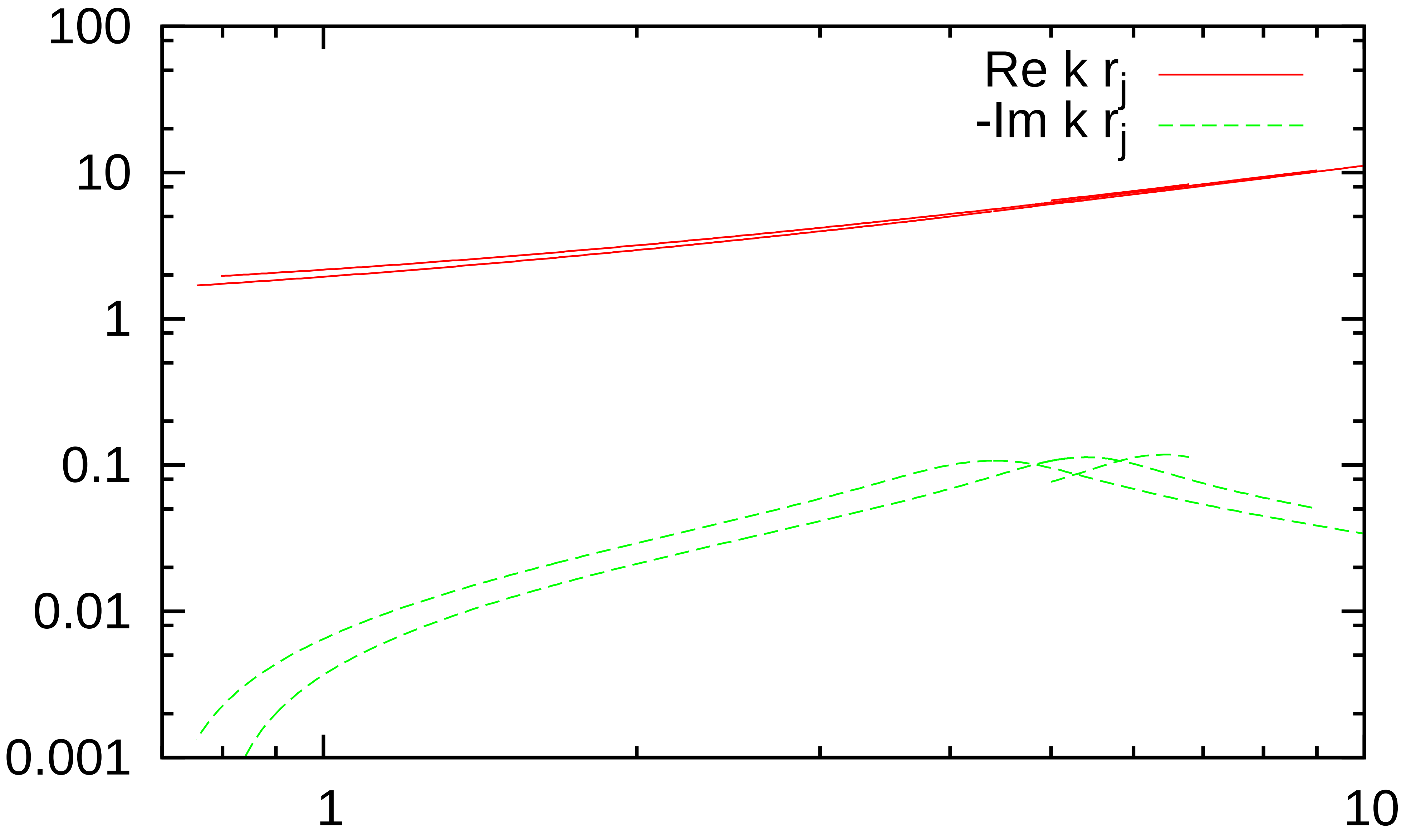
- external medium:

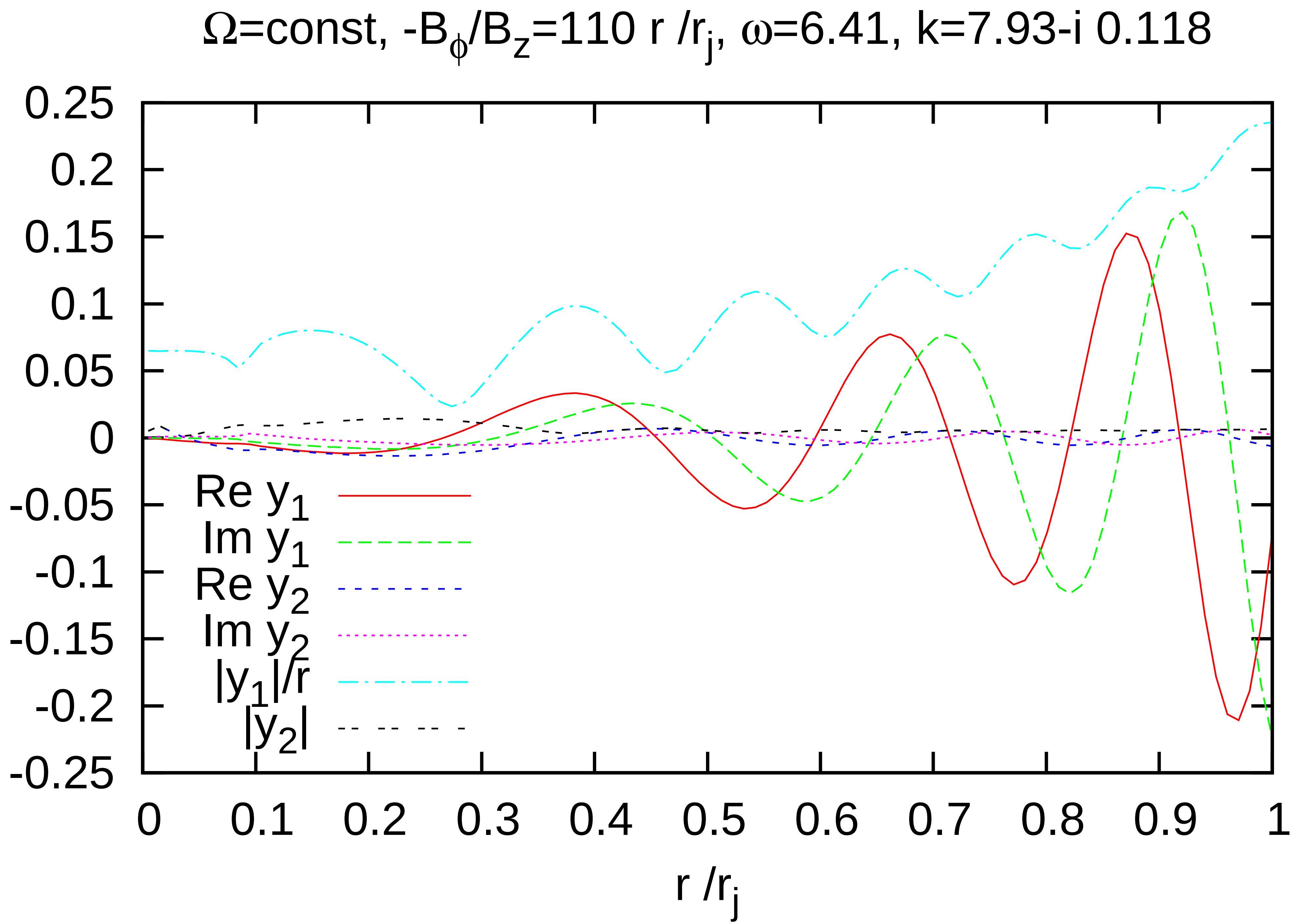
uniform, with zero $B_{0\phi}$ and $V_{0\phi} \rightarrow$ Bessel.
In all the following a thermal pressure is assumed, $\xi_e = 1.01$.
A cold, magnetized environment gives approximatelly same results.

Ω=const, $-B_\phi/B_z$=110 r /r_j



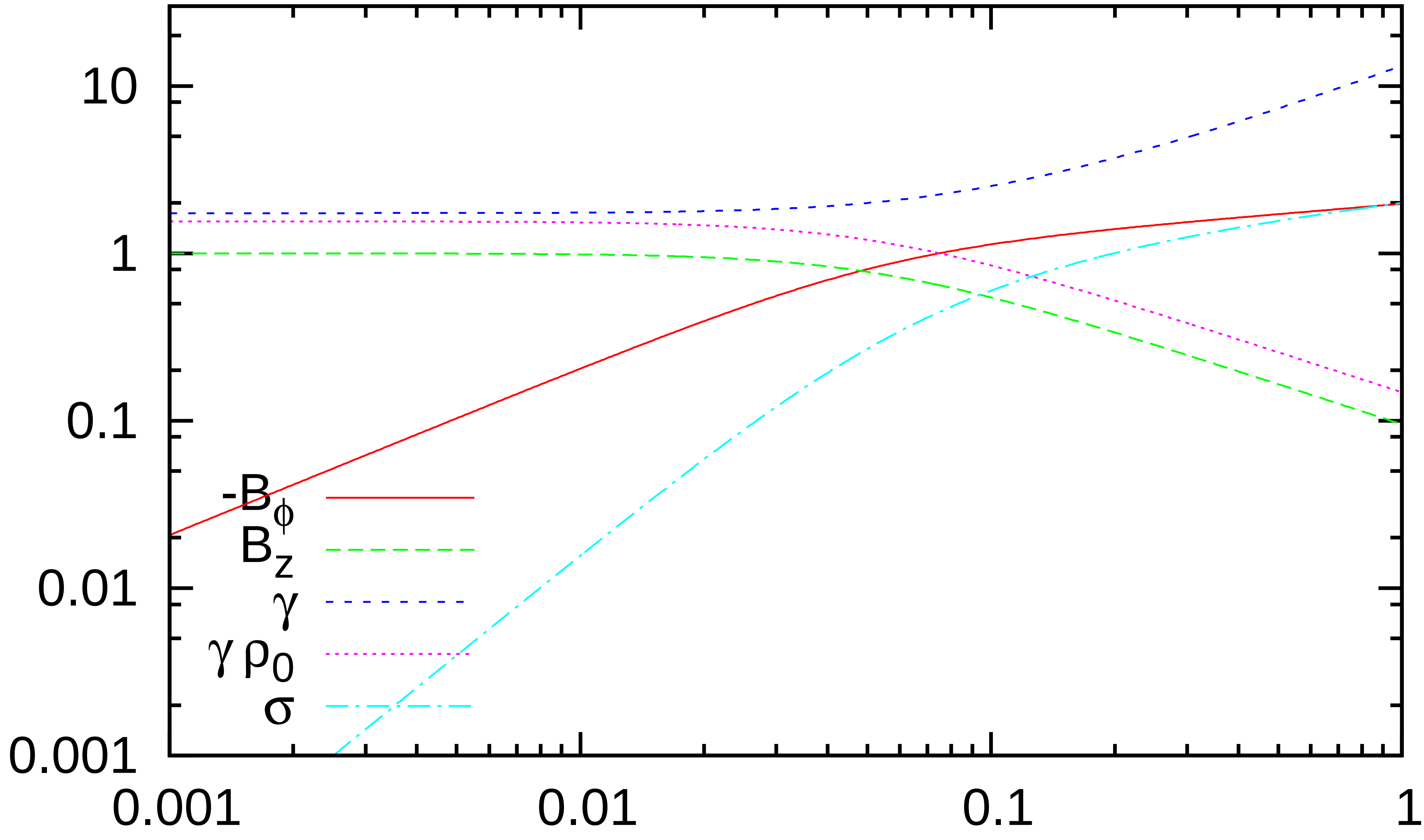
m=1, Ω=const

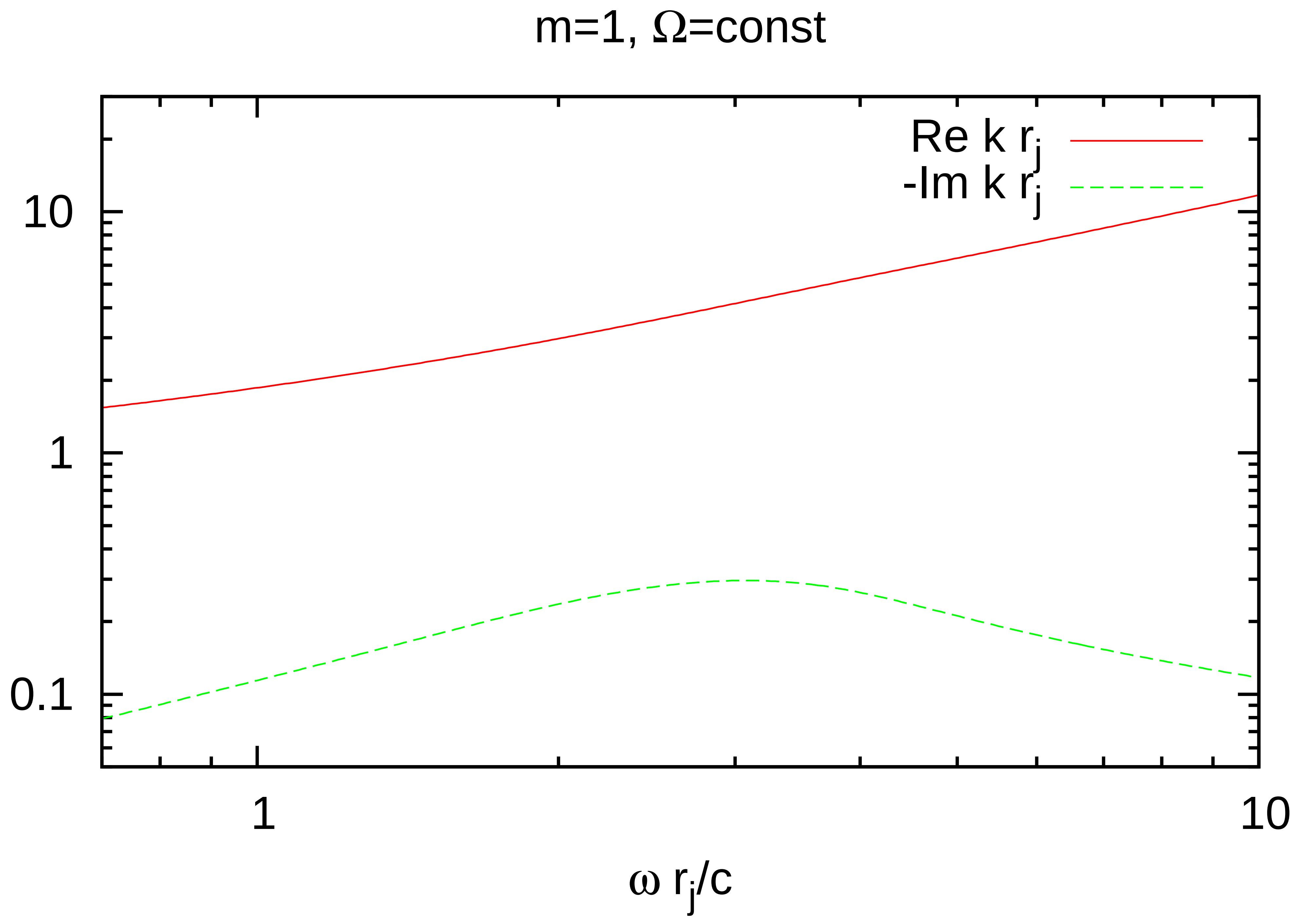




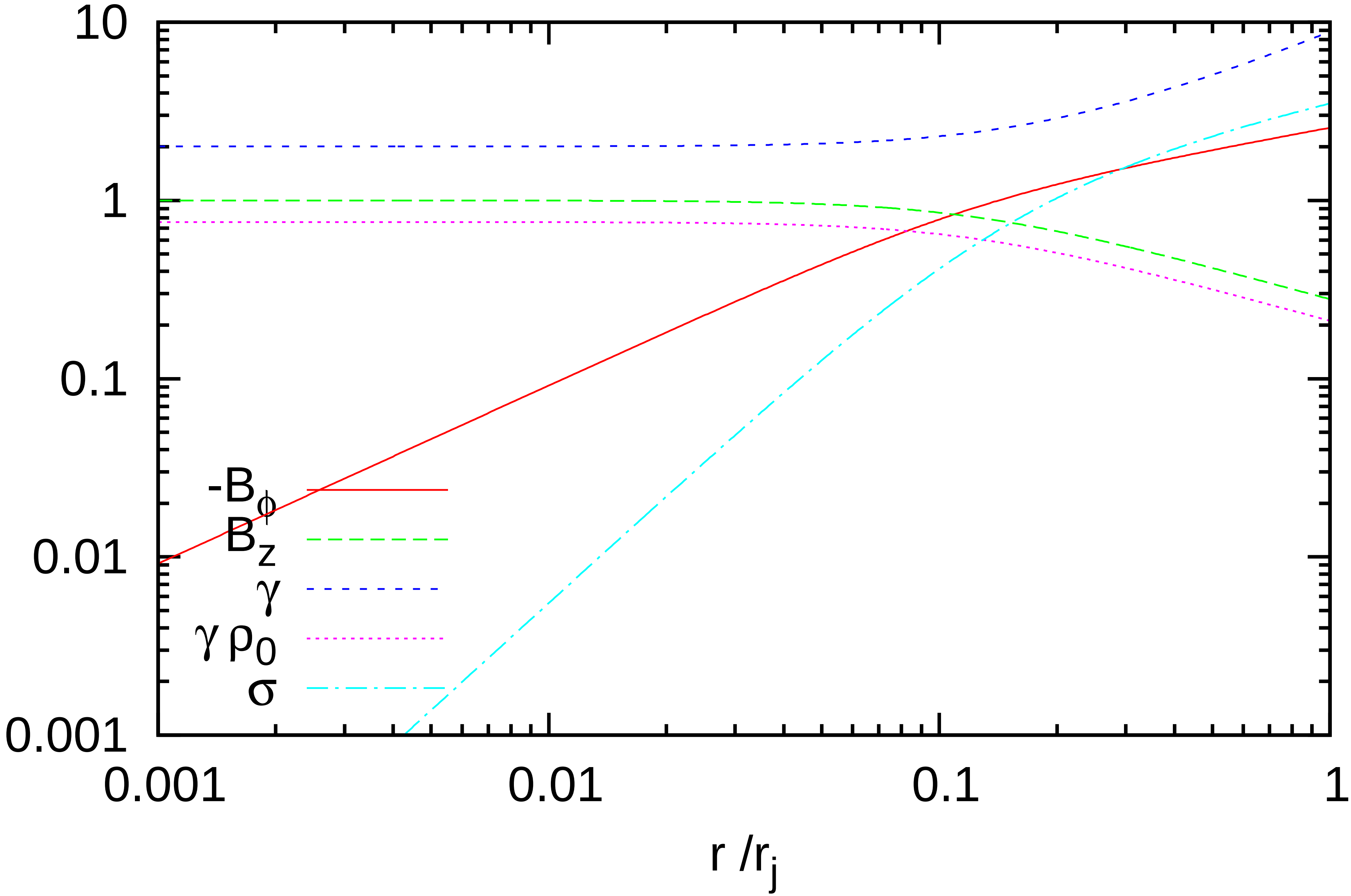
Ω=const, -B_φ/B_z=110 r /r_j, ω=6.41, k=7.93-i 0.118
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r /r_j
Re y_1
Im y_1
Re y_2
Im y_2
|y_1|/r
|y_2|

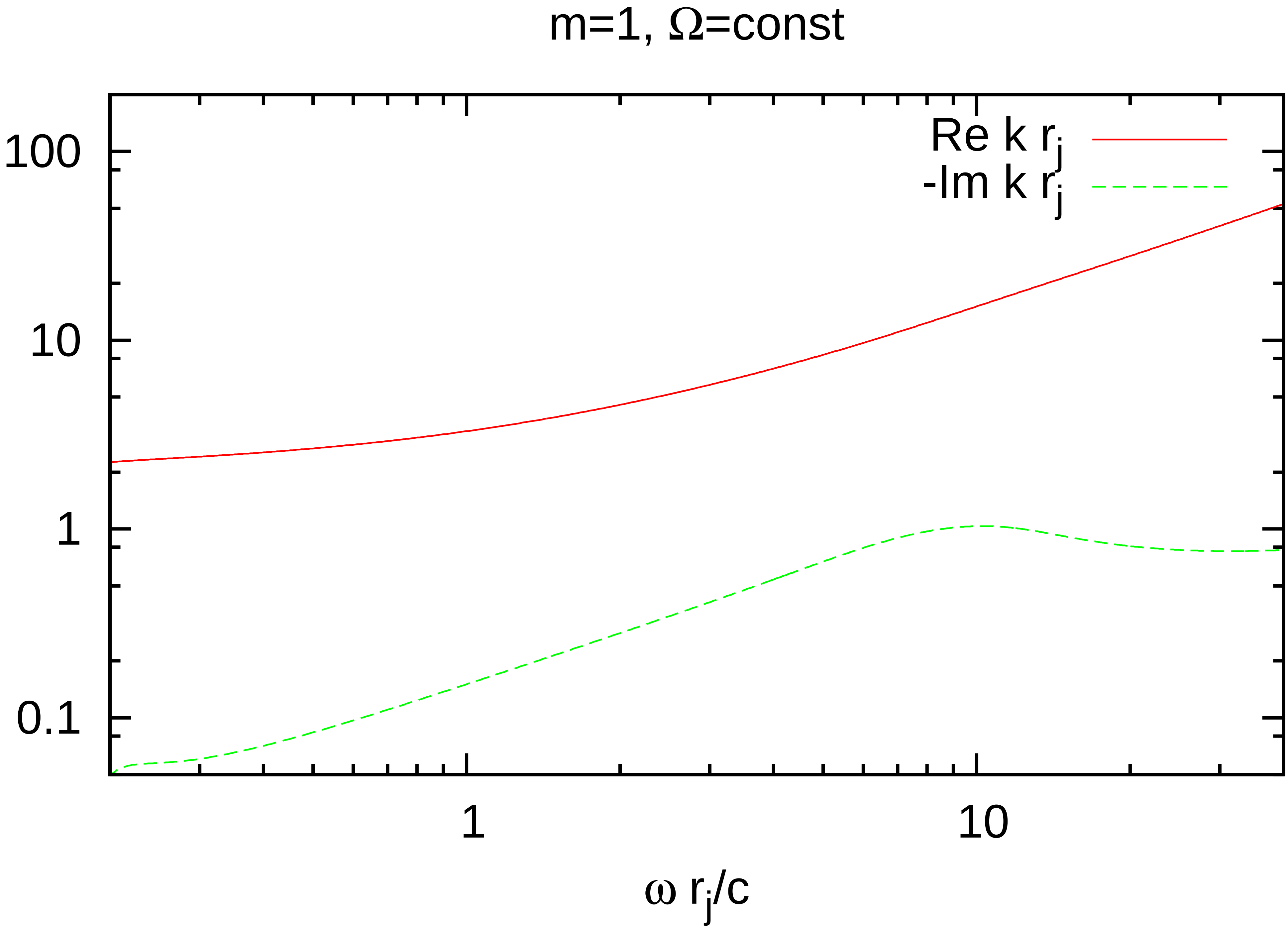
Ω=const, $-B_\phi/B_z$=20.75 r /r_j



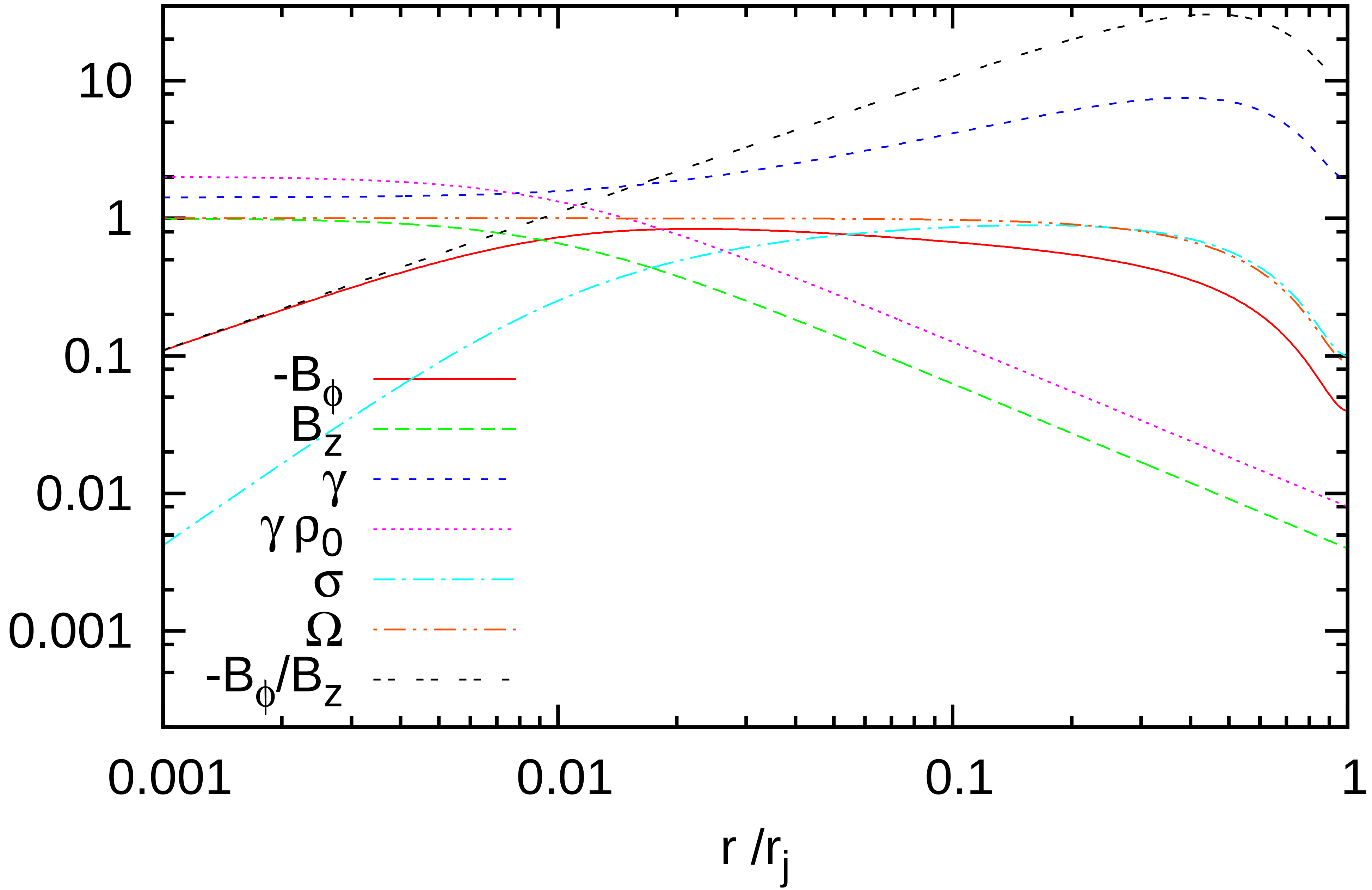

m=1, Ω=const
Re k r_j
-Im k r_j
10
1
0.1
1
10
ω r_j/c

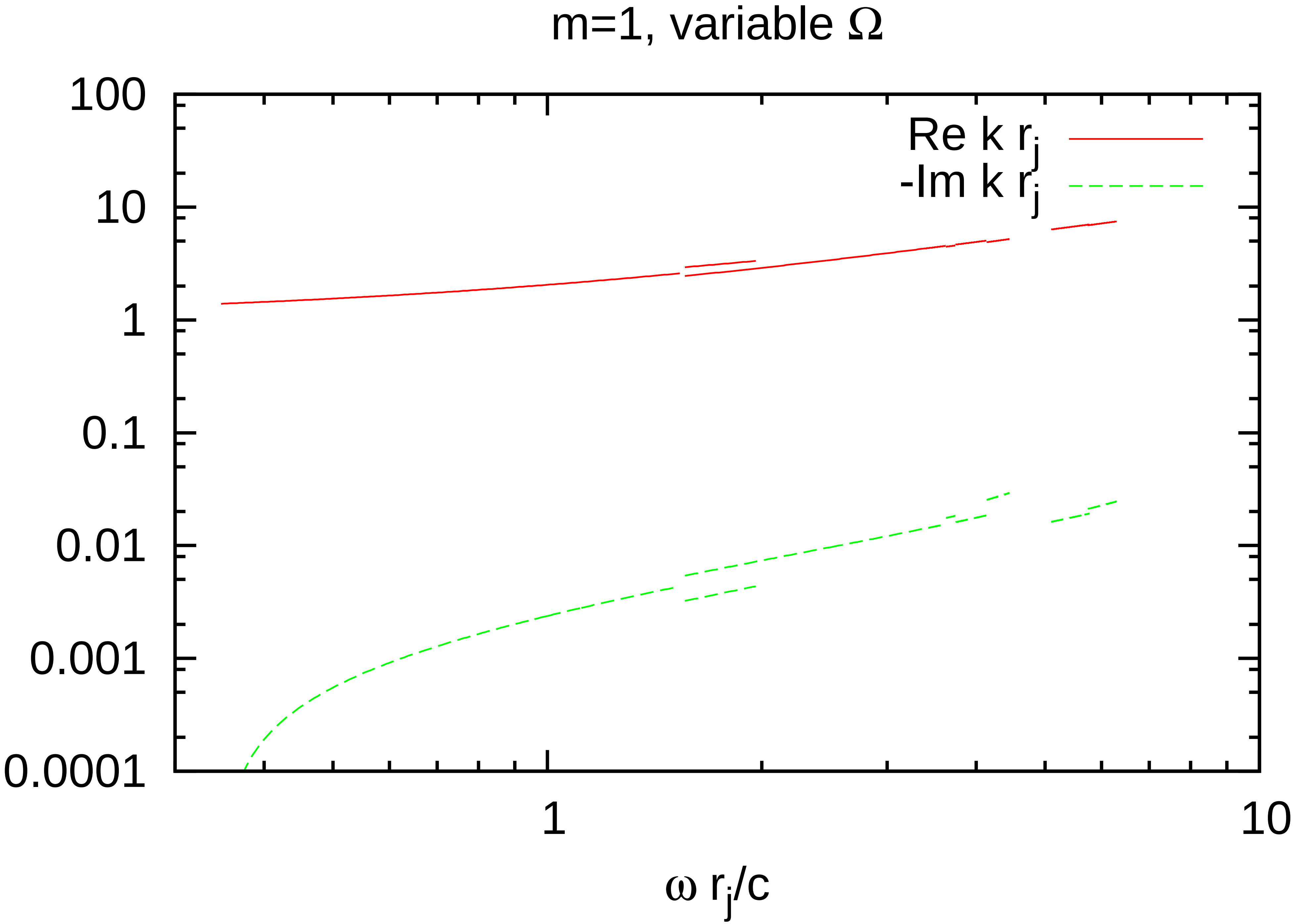
Ω=const, $-B_\phi/B_z$=9.17 r /r$_j$




m=1, Ω=const
Re k r_j
-Im k r_j
100
10
1
0.1
1
10
ω r_j/c

variable Ω




m=1, variable Ω
100
10
1
0.1
0.01
0.001
0.0001
1
10
ω r_j/c
Re k r_j
-Im k r_j

Summary – Next steps

- ⋆ Kink instability in principle is in action.
- ⋆ High γ stabilize.
- ⋆ High $|B_\phi|/B_z$ and high σ destibilize.
- ⋆ During the acceleration, growth time vs dynamical timescale?
- ⋆ Probably kink instability not so important during the acceleration phase and for a few tens jet radii after its end.
- ⋆ Jets from accretion disks more stable?

- Explore the parameter space for kink and other modes
- colder/moving environment? other jet equilibrium models?
- comparison with numerical studies.