Magnetic Fireball II

Hydromagnetic Outflows in GRB Sources

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Outline

- relativistic MHD model – results
- the acceleration efficiency in general
- neutron-rich outflows

GRB + afterglow

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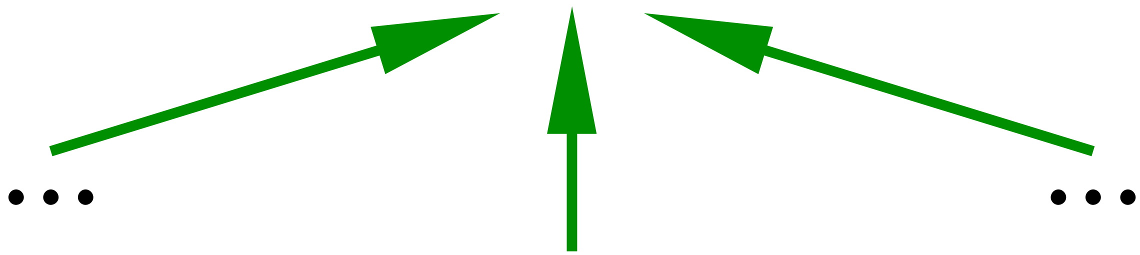
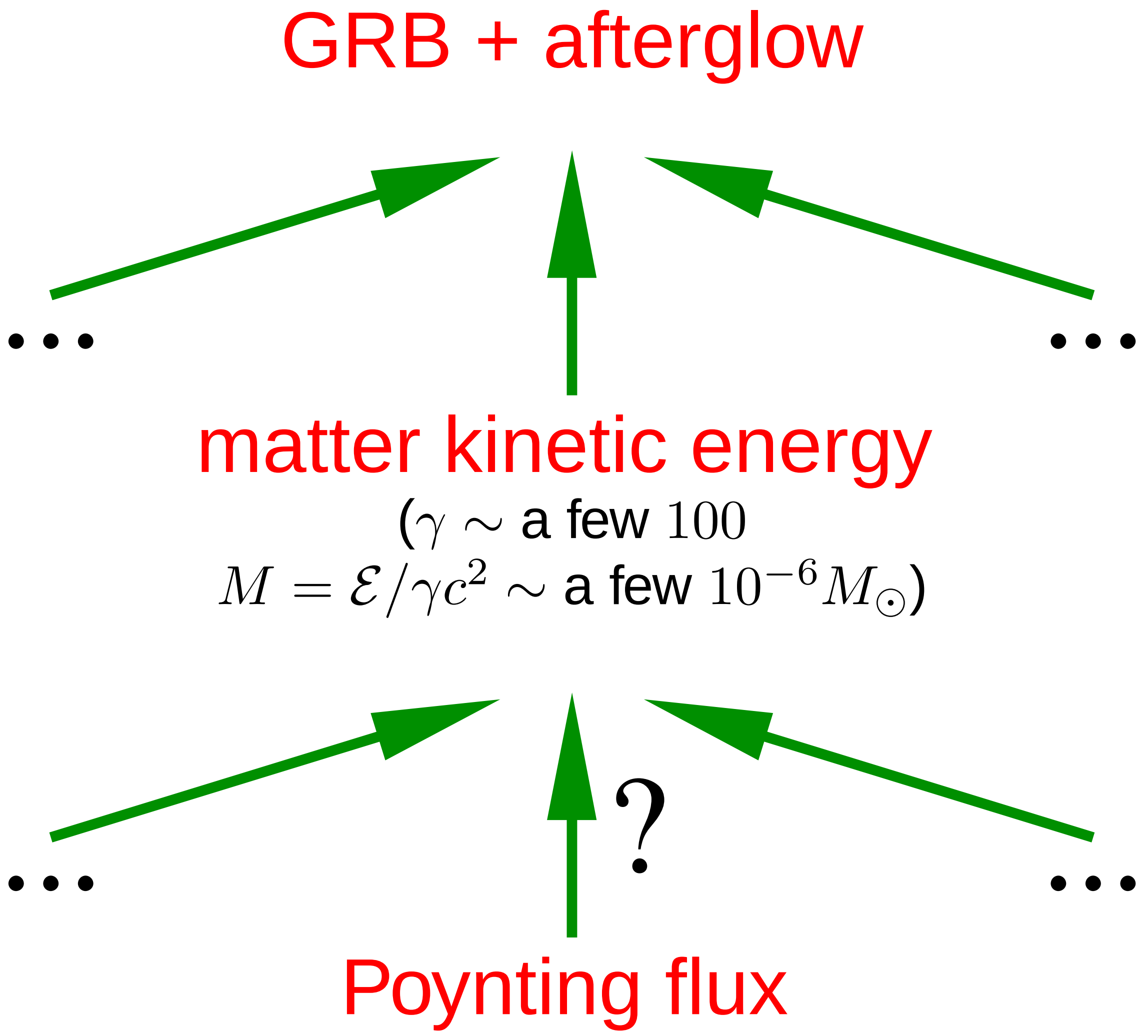


matter kinetic energy

($\gamma \sim$ a few 100

$M = \mathcal{E}/\gamma c^2 \sim$ a few $10^{-6} M_{\odot}$)


GRB + afterglow
...
...
matter kinetic energy
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?
...
...
Poynting flux

Progenitor models $\Rightarrow$ NS or BH or BH + debris disk

At the ejection surface $\mathcal{E} = \frac{c}{4\pi} \underbrace{\frac{\varpi\Omega}{c} B_p}_{E} B_\phi \times$ area $\times$ duration $\Rightarrow$

$$\frac{B_p B_\phi}{(10^{14}\mathrm{G})^2} = \left[\frac{\mathcal{E}}{10^{51}\mathrm{ergs}}\right] \left[\frac{\mathrm{area}}{4\pi \times 10^{12}\mathrm{cm}^2}\right]^{-1} \left[\frac{\varpi\Omega}{10^{10}\mathrm{cm\ s}^{-1}}\right]^{-1} \left[\frac{\mathrm{duration}}{10\mathrm{s}}\right]^{-1}$$

- from the BH: $B_p \gtrsim 10^{15}$G (small B_ϕ, small area)
- from the disk: smaller magnetic field required $\sim 10^{14}$G
  - If initially $B_p/B_\phi > 1$, a **trans-Alfvénic** outflow is produced.
  - If initially $B_p/B_\phi < 1$, the outflow is **super-Alfvénic** from the start.

Relativistic Magneto-Hydro-Dynamics

- Outflowing matter:
  - baryons
  - ambient electrons (neutralize the protons)
  - $e^{\pm}$ pairs (Maxwellian distribution)

- photons (blackbody distribution)

- large scale electromagnetic field

We need to integrate:

- Maxwell + Ohm equations
- mass + entropy conservation
- momentum equation

Assumptions

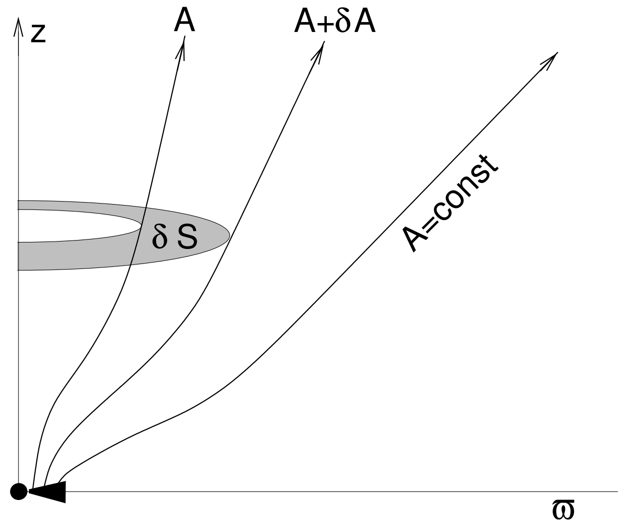
- special relativity
- steady-state is a safe assumption!
  - $\Omega \sim 10^4$rad s^{-1} $\Rightarrow$ many rotations during the engine's activity (~ 10s)
  - the outflow is faster than the fastest signals propagating inside the flow $\Rightarrow$ different parts of the flow (shells) are causally disconnected (frozen pulse)

    (proof can be found in Vlahakis & Königl 2003, ApJ, 596, 1080)
- axisymmetry
- ideal MHD (infinite conductivity)

The problem reduces to two coupled equations: the Bernoulli (or wind equation) and the transfield component of the momentum equation. The unknowns are γ and A (the later is the magnetic flux function $A = \int \boldsymbol{B}_p \cdot \boldsymbol{dS}$ that determines the field-, stream-line shape).

The problem remains difficult (even numerically) and many tried to solve it by simply ignoring the transfield equation (e.g., Michel 1969, Fendt & Ouyed 2004).

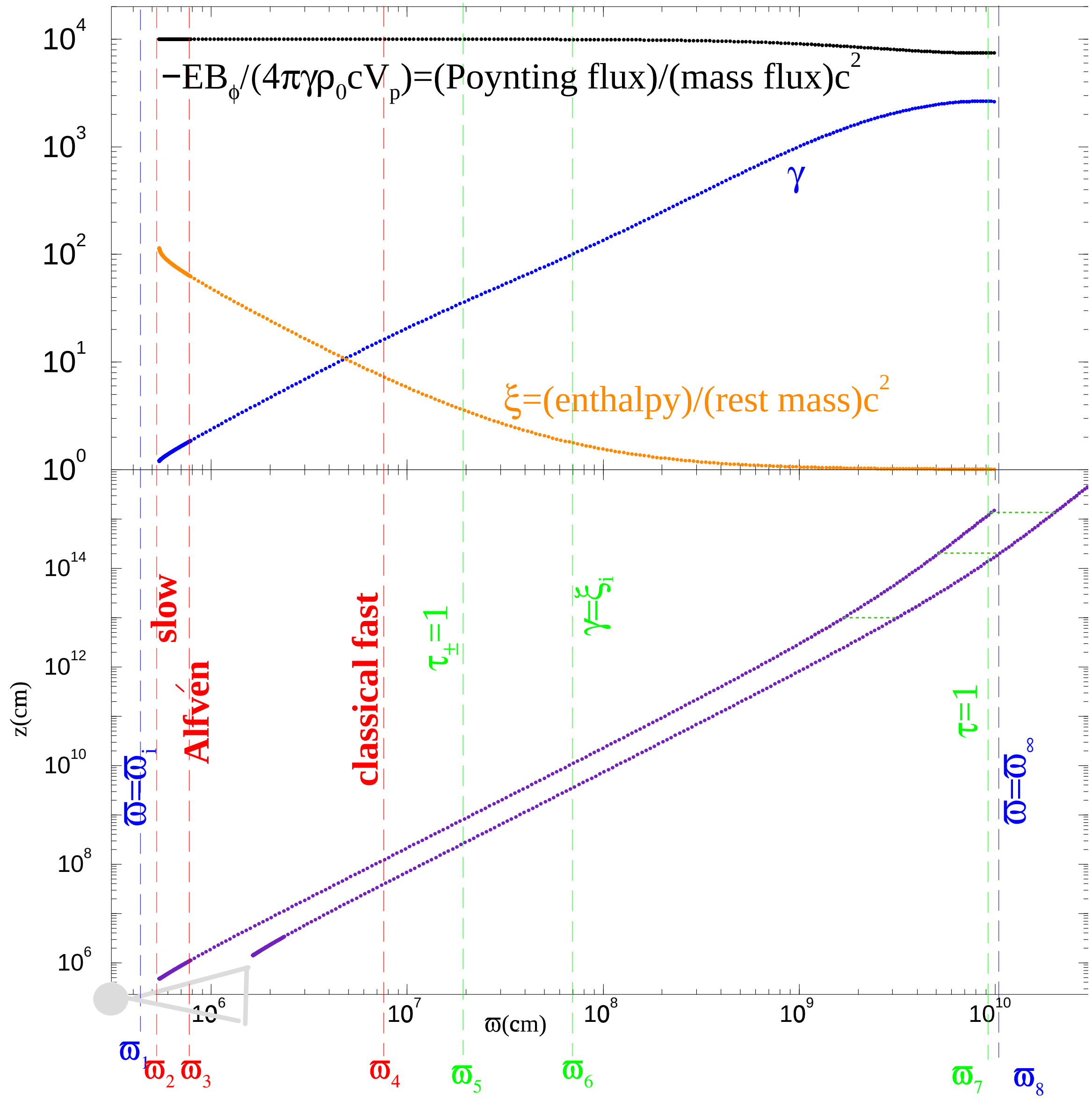
Necessary to solve the transfield because the line shape controls the acceleration:

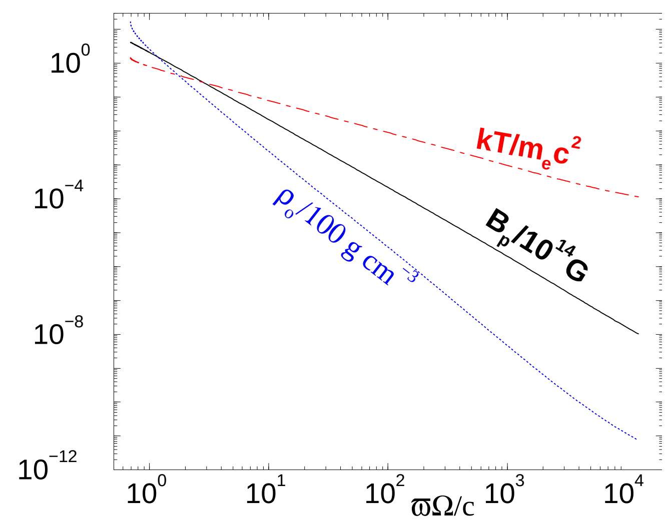


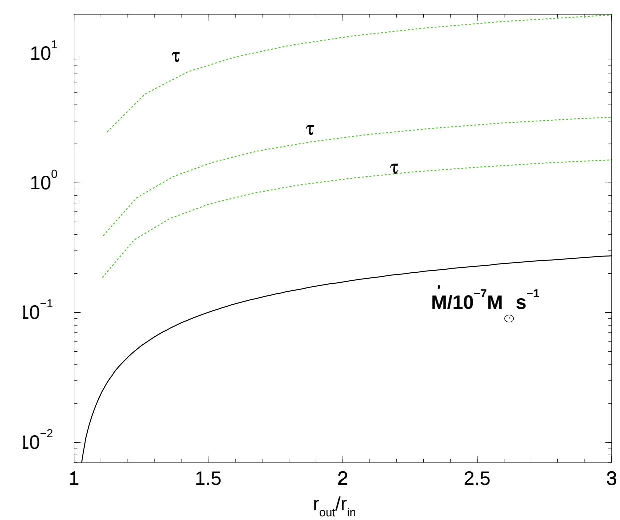
Poynting-to-mass flux ratio $\propto \varpi |B_\phi| \rightarrow$
$\gamma \uparrow$ when $\varpi |B_\phi| \downarrow$
$E = |\boldsymbol{V}/c \times \boldsymbol{B}| \approx |B_\phi|$, $E = (\varpi \Omega / c) B_p$
So, $\varpi |B_\phi| \propto \varpi^2 B_p = (\varpi^2 / \delta S) \delta A$.

A way out (the only one at present): choose a special form of boundary conditions that lead to separation of variables $\longrightarrow$
r self-similarity (all quantities on the conical disk surface are power laws in r)
(details of the model can be found in Vlahakis & Königl 2003, ApJ, 596, 1080).

Trans-Alfvénic Jets (NV & Königl 2001, 2003a)

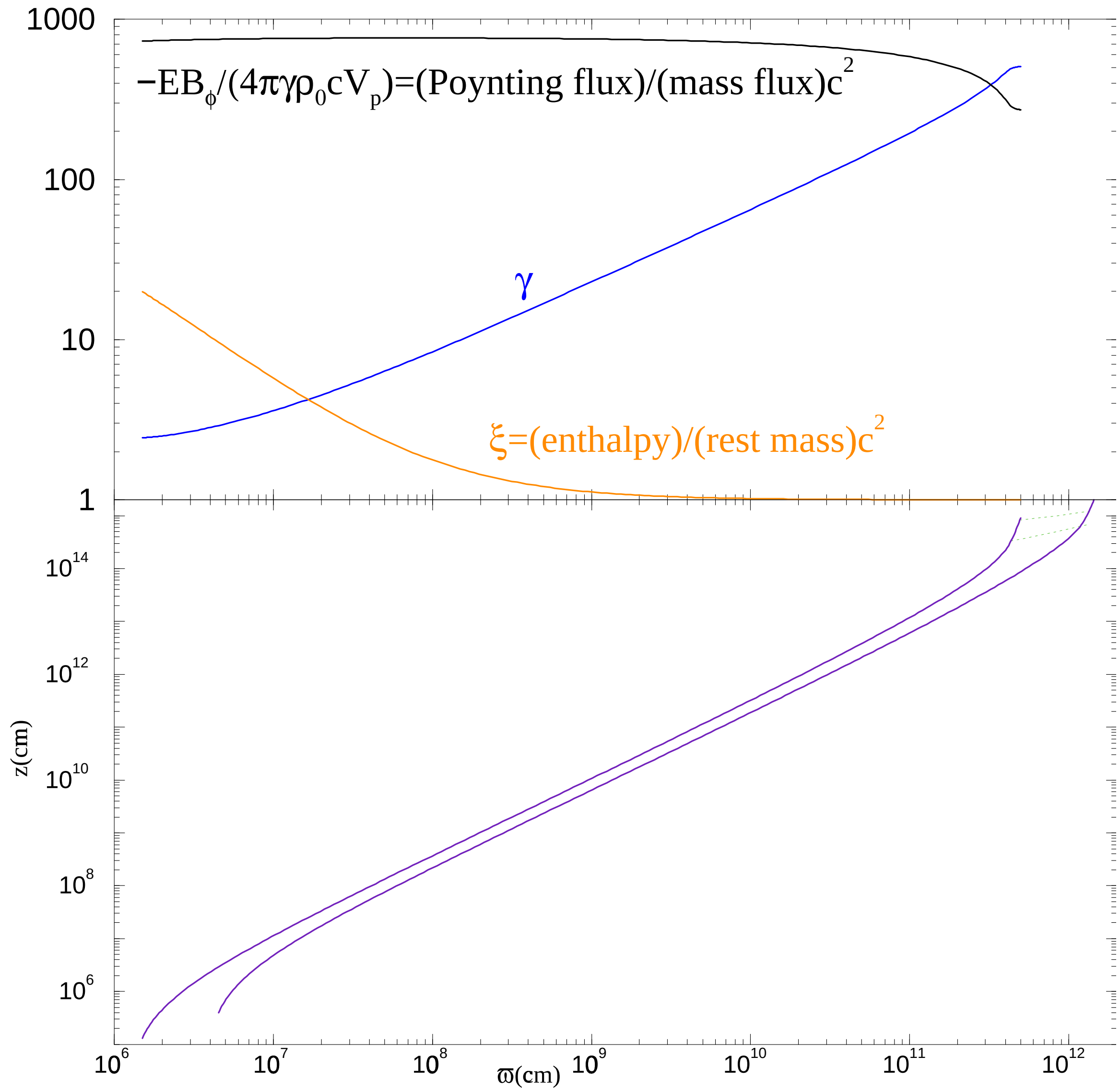


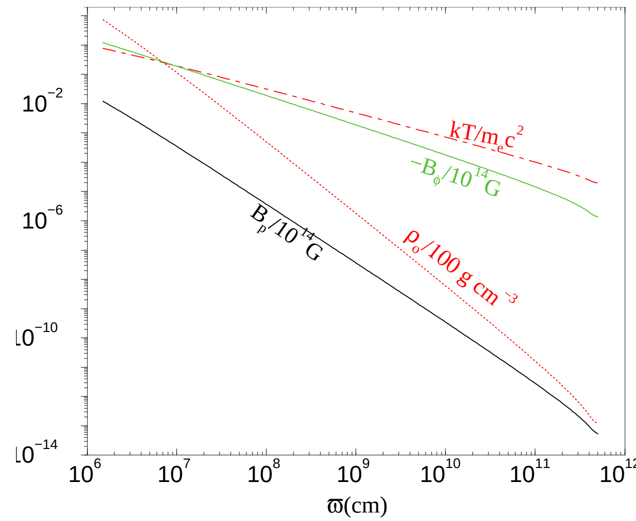


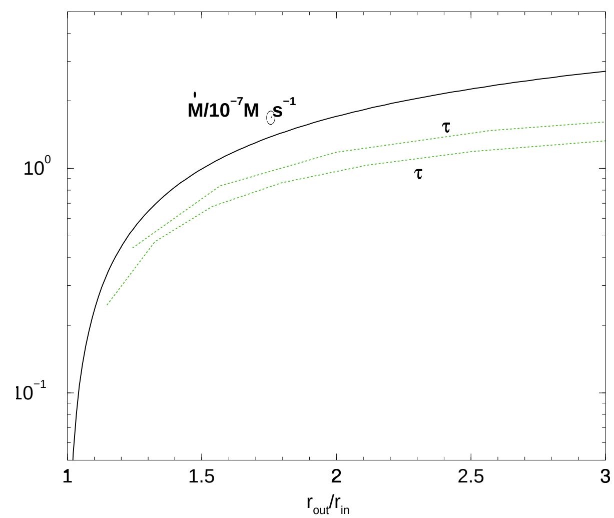


- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

Super-Alfvénic Jets (NV & Königl 2003b)







- Thermal acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$, $T \propto \varpi^{-0.8}$, $B_\phi \propto \varpi^{-1}$, $z \propto \varpi^{1.5}$)
- Magnetic acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$)
- cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

The acceleration efficiency in general

We proved that Poynting-to-mass flux ratio $\propto \varpi|B_\phi| \propto \varpi^2 B_p$.

If $\mu c^2 = \dfrac{\text{total energy flux}}{\text{mass flux}}$ (μ is the maximum possible γ) then $\varpi|B_\phi| \propto (\mu - \gamma)$, and

$$\frac{\mu - \gamma_\infty}{\mu - \gamma_i} = \frac{(\varpi^2 B_p)_\infty}{(\varpi^2 B_p)_i}.$$

Since $(\varpi^2 B_p)_\infty \approx A$ we find that

$$\boxed{\gamma_\infty \approx \mu - \mu \frac{A}{(\varpi^2 B_p)_i}}$$

- The more bunched the fieldlines near the origin the higher the acceleration efficiency.

- In the previous numerical results it happened that $(\varpi^2 B_p)_i \approx 2A$, resulting in equipartition $\gamma_\infty \approx \mu/2$. Efficiencies higher that 50% have been found, corresponding to $(\varpi^2 B_p)_i \gg A$.

Neutron-rich hydromagnetic flows

(Vlahakis, Peng, & Königl 2003 ApJL)

A possible resolution to the baryon loading problem (Fuller et al. 2000):

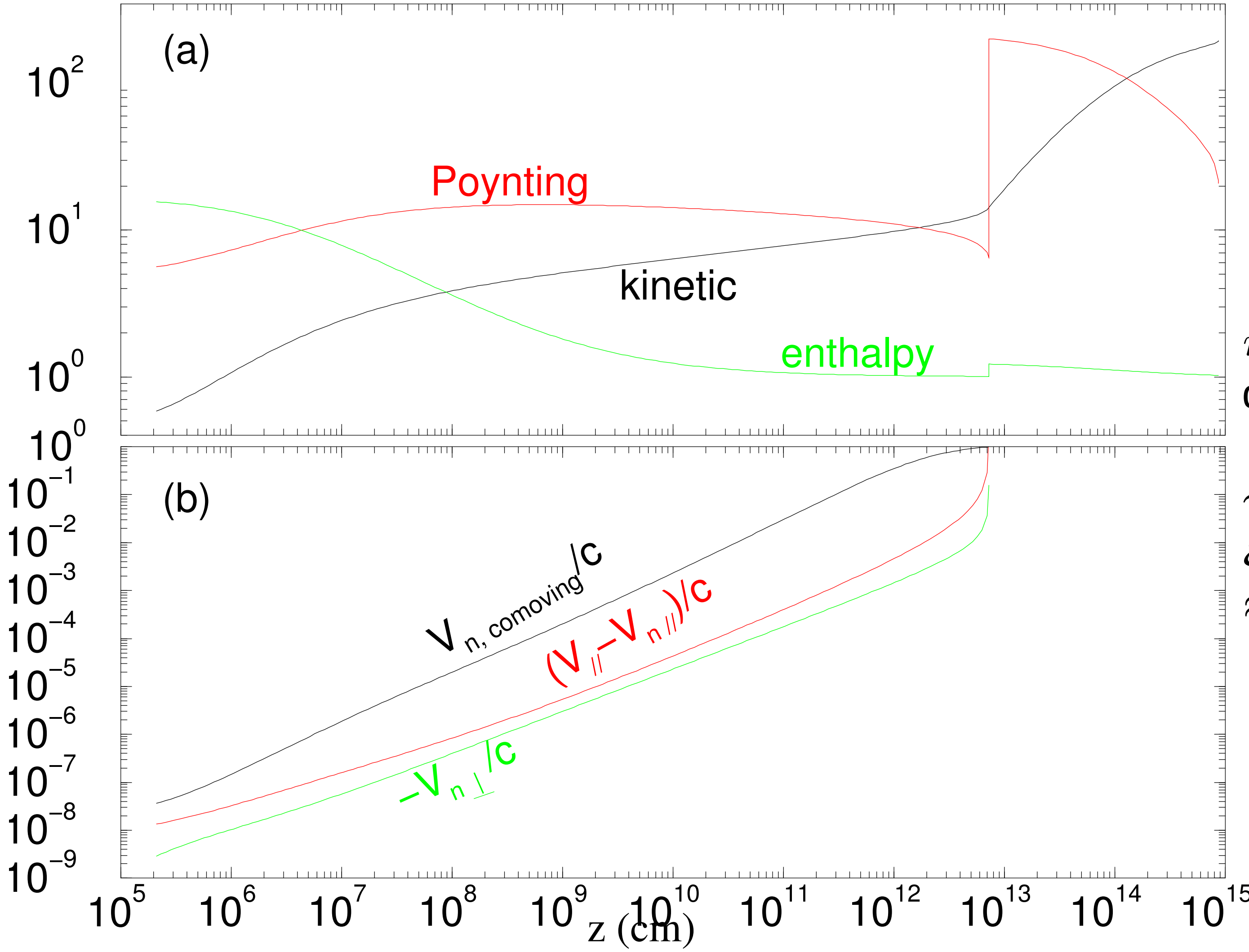
- If the source is neutron-rich, then the neutrons could decouple from the flow before the protons attain their terminal Lorentz factor.
- Disk-fed GRB outflows are expected to be neutron-rich, with $\sim 20 - 30$ neutrons per proton (Pruet et al. 2003; Beloborodov 2003; Vlahakis et al. 2003).

However, it turns out that the decoupling Lorentz factor $\gamma_{\rm d}$ in a thermally driven, purely hydrodynamic outflow is of the order of γ_∞ (e.g., Derishev et al. 1999; Beloborodov 2003), which has so far limited the practical implications of the Fuller at al. (2000) proposal.

In a magnetized outflow:

- Part of the thermal energy could be converted to electromagnetic (with the remainder transfered to baryon kinetic).
- The Lorentz factor increases with lower rate compared to the hydrodynamic case. This makes it possible to attain $\gamma_{\rm d} \ll \gamma_\infty$, as it is shown in the following solution.
- The energy deposited into the Poynting flux is returned to the matter beyond the decoupling point.

- In the pre-decoupling phase:
  - The momentum equation for the whole system (protons/neutrons/$e^{\pm}$/photons/electromagnetic field) yields the flow velocity.
  - The momentum equation for the neutrons alone yields the neutron-proton collisional drag-force, and the drift velocity.
  - When $V_{\rm proton} - V_{\rm neutron} \sim c$ the neutrons decouple.
- In the post-decoupling phase:
  - We solve for the protons alone ($+$ electromagnetic field).



$n/p = 30$
decoupling at $\gamma_d = 15$

$\gamma_\infty = 200$
$\mathcal{E}_{\rm proton} \approx 10^{51}$ergs
$\approx 0.5\, \mathcal{E}_{\rm neutron}$

(a) The three components of the total energy flux, normalized by the mass flux $\times\, c^2$.

(b) Proton–neutron drift velocity.

Due to the magnetic collimation $V_{\mathrm{neutron},\perp} \sim 0.1c$ at decoupling.

Thus, a two component outflow is naturally created:

- An inner jet consisting of the protons (with $\gamma = 200$ and $\mathcal{E}_p = 10^{51}$ ergs).
- The decoupled neutrons, after undergoing β decay at a distance $\sim 4 \times 10^{14}(\gamma_d/15)$cm, form a wider proton component (with $\gamma = 15$ and $\mathcal{E}_p = 2 \times 10^{51}$ ergs).

(See Peng, Königl, & Granot, poster in this meeting, for implications.)

Discussion

- Magnetic fields provide a viable mechanism to accelerate and collimate GRB outflows.

- Dissipation enhances the acceleration (Drenkhahn & Spruit 2002). However, a more detailed calculation is needed.

- Neutron-rich magnetized outflows significanty alleviate the baryon-loading problem. They also provide a way to create a two-component outflow, consisting of a narrow and a wider jet. It is important to examine the characteristics of these two-component jets (e.g., find the relation between their opening angles), and for that we need to solve the two-fluid equations near the decoupling point.

The frozen-pulse approximation

- The arclength along a poloidal fieldline

$$\ell = \int_{\frac{s}{c}}^{t} V_p dt \approx ct - s \Rightarrow s = ct - \ell$$

- s is constant for each ejected shell. Moreover, the distance between two different shells $\ell_2 - \ell_1 = s_1 - s_2$ remains the same (even if they move with $\gamma_1 \neq \gamma_2$).
- Eliminating t in terms of s: we show that all terms with $\partial / \partial s$ are $\mathcal{O}(1/\gamma) \times$ remaining terms (generalizing the HD case examined by Piran, Shemi, & Narayan 1993). Thus we may examine the motion of each shell using steady-state equations.
$\left(\text{e.g., } \frac{d}{dt} = (c - V_p)\frac{\partial}{\partial s} + \mathbf{V} \cdot \nabla_s \approx \mathbf{V} \cdot \nabla_s \right)$
- All physical quantities are functions of $\boldsymbol{r}$ and s.

The ideal MHD equations

Maxwell:
$$\nabla\cdot\mathbf{B}=0=\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{c\partial t}\,,\nabla\times\mathbf{B}=\frac{\partial\mathbf{E}}{c\partial t}+\frac{4\pi}{c}\mathbf{J}\,,\nabla\cdot\mathbf{E}=\frac{4\pi}{c}J^0$$

Ohm: $\mathbf{E}=\mathbf{B}\times\mathbf{V}/c$

baryon mass conservation (continuity):
$$\frac{d(\gamma\rho_0)}{dt}+\gamma\rho_0\nabla\cdot\mathbf{V}=0\,,\quad \text{where}\quad \frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla$$

energy $U_\mu T^{\mu\nu}_{,\nu}=0$ (or specific entropy conservation, or first law for thermodynamics): $\dfrac{d\left(P/\rho_0^{4/3}\right)}{dt}=0$

momentum $T^{\nu i}_{,\nu}=0$: $\gamma\rho_o\dfrac{d\left(\xi\gamma\mathbf{V}\right)}{dt}=-\nabla P+\dfrac{J^0\mathbf{E}+\mathbf{J}\times\mathbf{B}}{c}$

Eliminating t in terms of s: $\left(\mathbf{V}\cdot\nabla_s\right)\left(\xi\gamma\mathbf{V}\right)-\dfrac{\left(\nabla_s\cdot\mathbf{E}\right)\mathbf{E}+\left(\nabla_s\times\mathbf{B}\right)\times\mathbf{B}}{4\pi\gamma\rho_0}+\dfrac{\nabla P}{\gamma\rho_0}=$
$$(V_p-c)\frac{\partial\left(\xi\gamma\mathbf{V}\right)}{\partial s}+\frac{\partial(E+B_\phi)}{4\pi\gamma\rho_0\partial s}\frac{\nabla_s A}{|\nabla_s A|}\times\mathbf{B}-\nabla_s A\frac{\nabla_s\ell\cdot\nabla_s A}{|\nabla_s A|^2}\frac{\partial(E^2-B_\phi^2)}{8\pi\gamma\rho_0\partial s}$$