Jets in the MHD context

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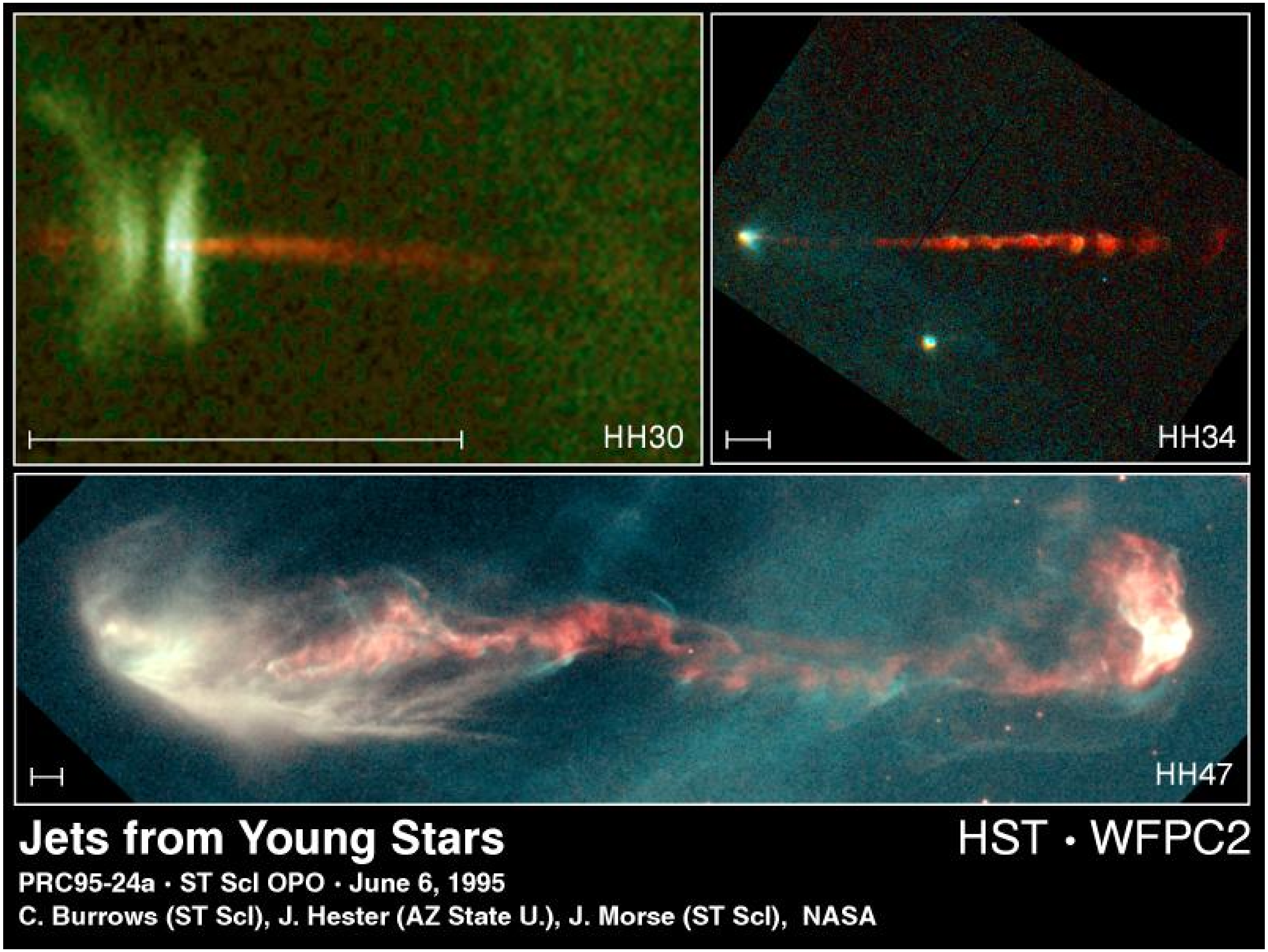
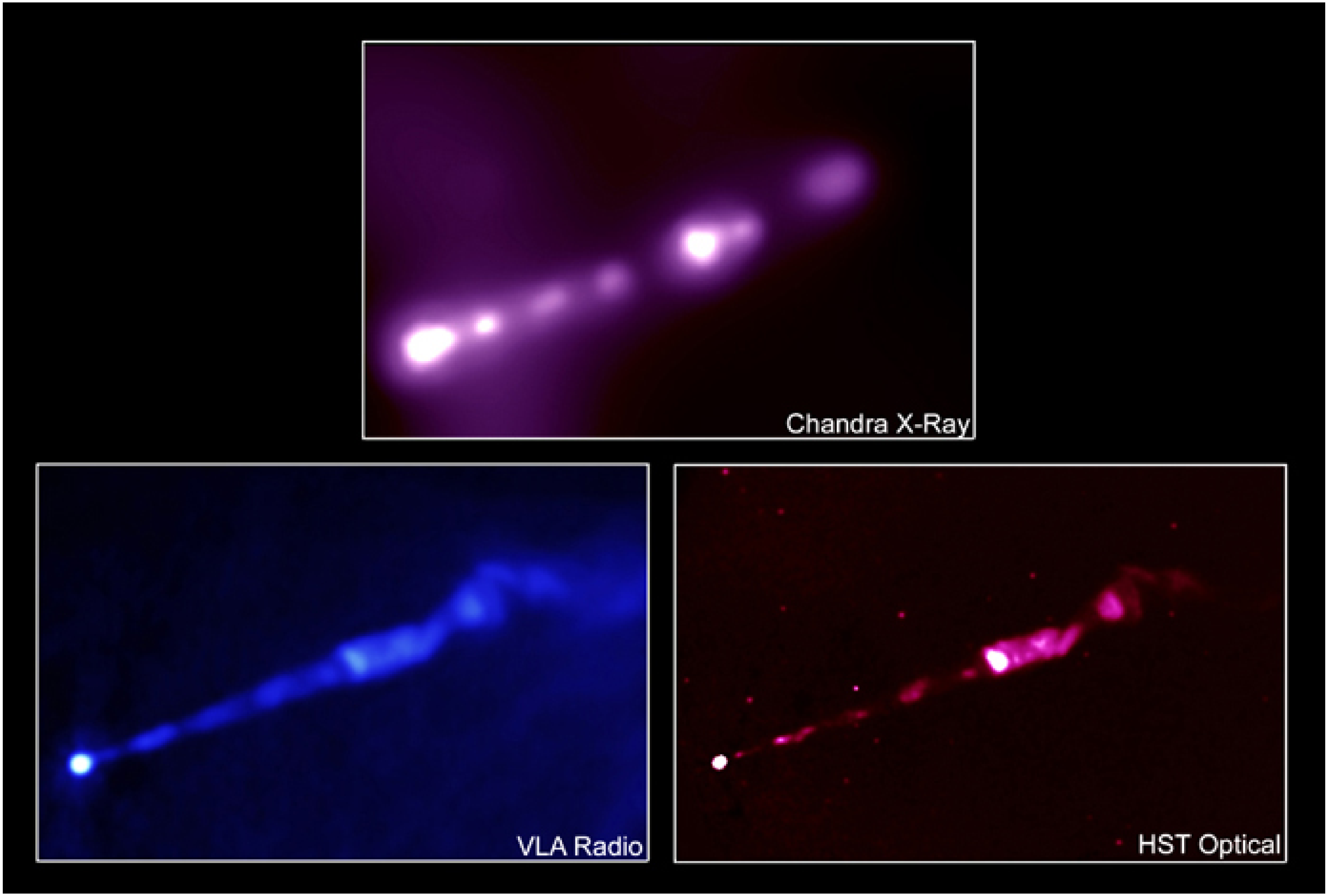
Outline

- MHD formalism
  analytical insight into the Grad-Shafranov equation
- example solutions

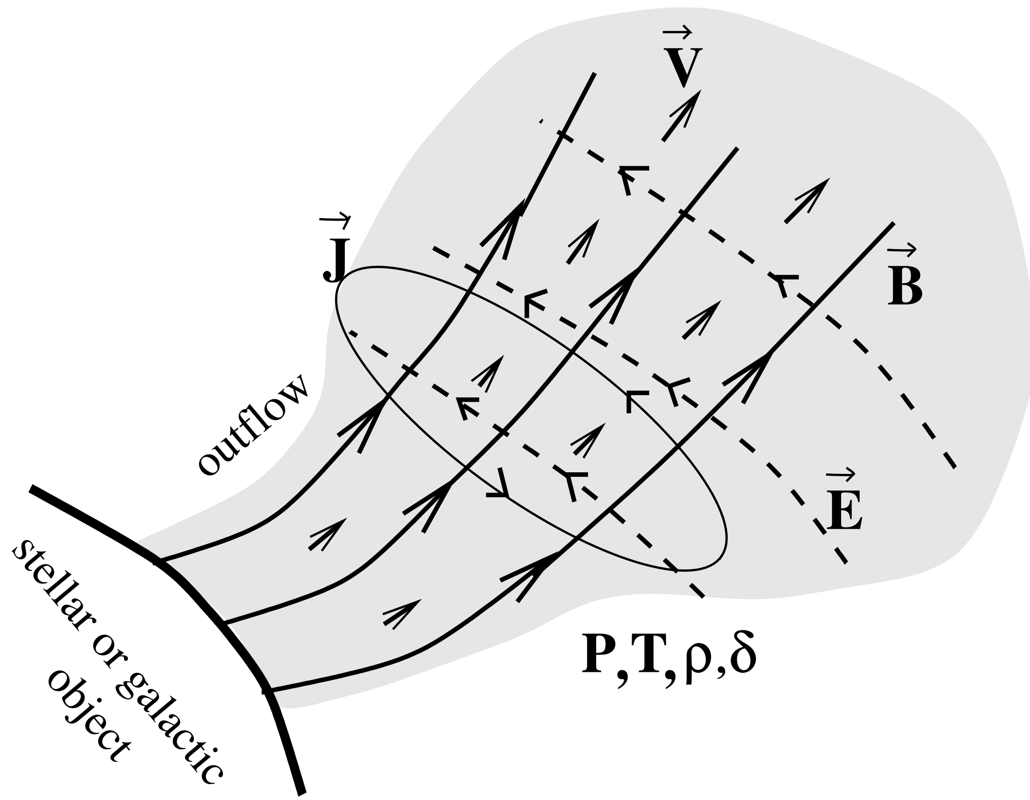


(scale =1000 AU, $V_\infty = a few 100$km/s)



collimation at ~100 Schwarzschild radii, $\gamma_\infty \sim 10$

MHD (Magneto-Hydro-Dynamics)



Equations: Maxwell, Ohm, continuity, momentum, entropy

Their solutions describe the jet dynamics (acceleration–collimation)

Partial Integration

- assumptions
  - zero resistivity (ideal MHD)
  - axisymmetry $(\partial/\partial\phi = 0)$
  - steady state $(\partial/\partial t = 0)$

- introduce the magnetic flux function $A(\varpi\,, z) = (1/2\pi)\iint \boldsymbol{B}_p \cdot d\boldsymbol{S}$
  The equation for a poloidal field-streamline is $A(\varpi\,, z) =$const

- the full set of ideal MHD equations can be partially integrated to yield five constants of motion:
  ① the mass-to-magnetic flux ratio Ψ_A
  ② the field angular velocity Ω
  ③ the specific angular momentum L
  ④ the total energy-to-mass flux ratio μc^2
  ⑤ the adiabat Q

  The corresponding expressions give $\boldsymbol{B}\,, \boldsymbol{V}\,, \rho\,, P$ as functions of A.

- one equation remains to be solved, the transfield force-balance, or Grad-Shafranov equation

The Grad-Shafranov equation

$$a\frac{\partial^2 A}{\partial \varpi^2} + 2b\frac{\partial^2 A}{\partial \varpi \partial z} + c\frac{\partial^2 A}{\partial z^2} + d = 0\,,$$

where $a\,, b\,, c\,, d$ are functions of A and its 1st order derivatives.

[variants: nonrelativistic, relativistic, force-free (pulsar equation), etc]

Nonlinear — Mixed type (elliptic-hyperbolic)

50 years after its derivation we only have:

- self-similar solutions

  R self-similar if $A =$ function of $\dfrac{\varpi}{G(\theta)}$ [with $\theta = \arctan(\varpi/z)$]

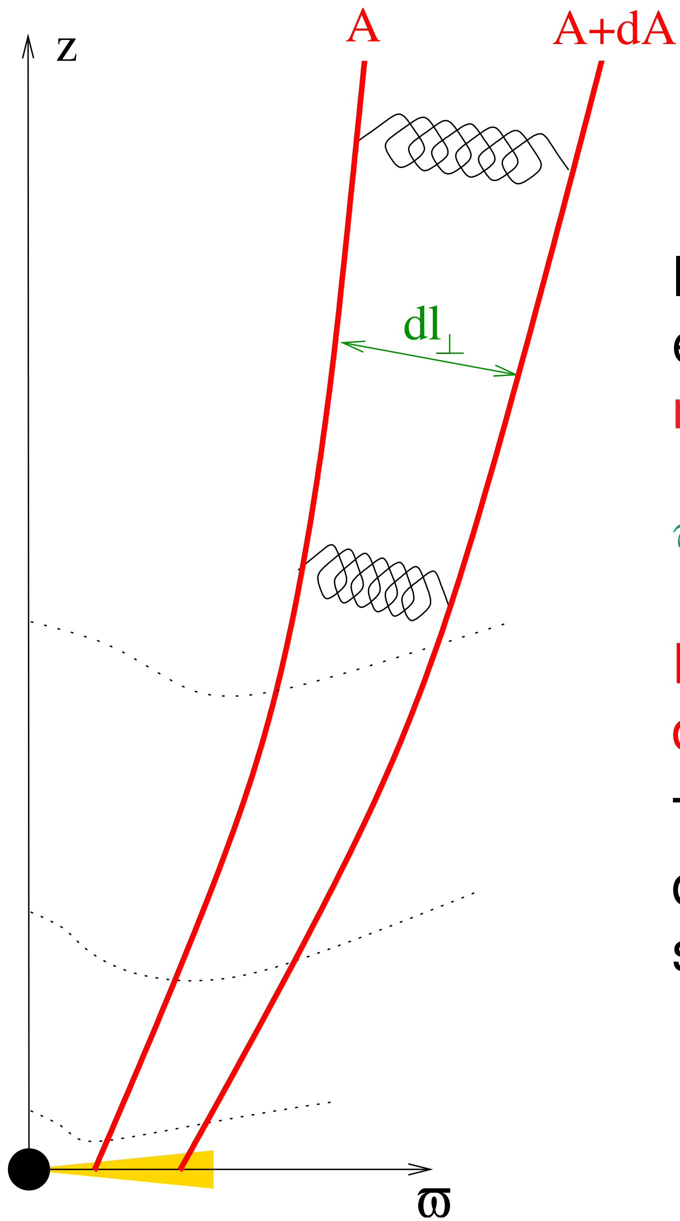
  θ self-similar if $A =$ function of $\dfrac{\varpi}{G(R)}$ (with $R = \sqrt{\varpi^2 + z^2}$)

- asymptotic analysis

- works where this equation is simply ignored! (prescribed flow-shape)

- simulations ending in a steady-state

Solution characteristics



By expansion the magnetic field minimizes its energy under the condition of keeping the magnetic flux constant.

$$\varpi^2 B_p = \frac{\varpi^2}{2\pi\varpi dl_\perp}\underbrace{(B_p dS)}_{dA} \propto \frac{\varpi}{dl_\perp}.$$

Expansion with increasing $dl_\perp/\varpi$ leads to decreasing Poynting flux.

The expansion ends in a more-or-less uniform distribution $\varpi^2 B_p \approx A$ (in a quasi-monopolar shape).

The function $\mathcal{S} = \varpi^2 B_p / 2A$

The expansion is controled by the decline of the function

$$\mathcal{S} = \frac{\varpi |\nabla A|}{2A} = \frac{\varpi^2 B_p}{2A} = \frac{\pi \varpi^2 B_p}{\iint \boldsymbol{B}_p \cdot d\boldsymbol{S}}$$

$\mathcal{S}$ can be seen as $\dfrac{B_p}{< B_z >}$, or as $\dfrac{1}{2}\varpi |\nabla \ln A|$

Examples:

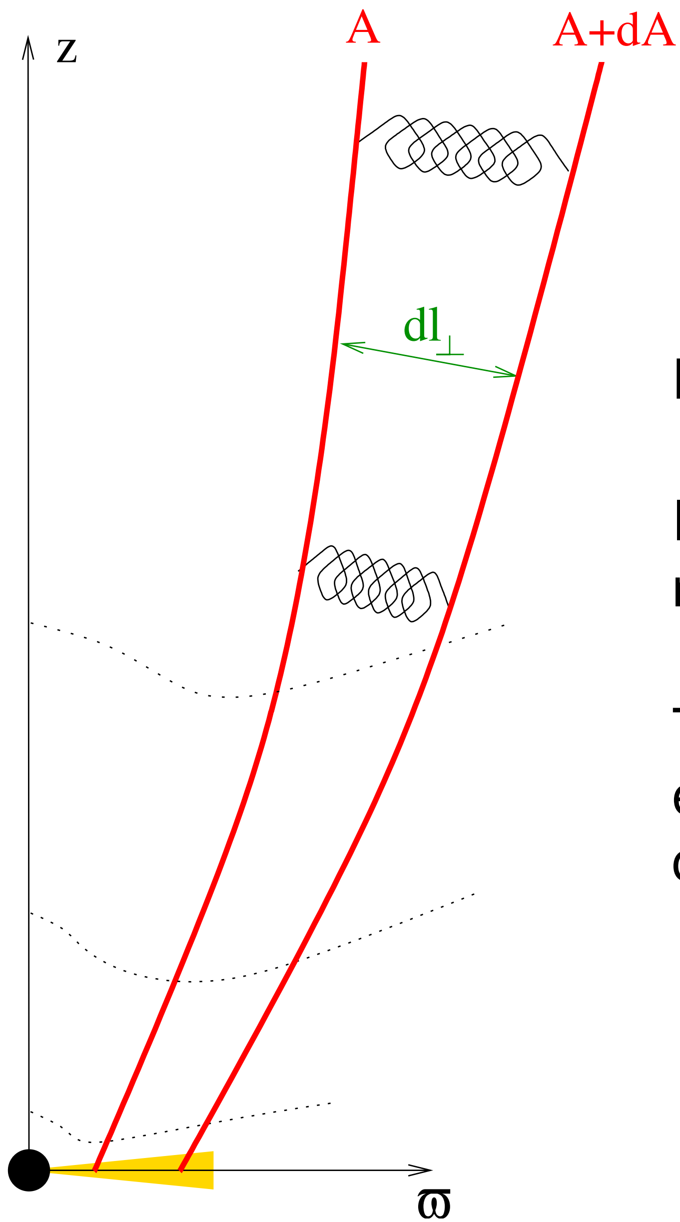
A monopolar field $A \propto 1 - \cos\theta$ has $\mathcal{S} = (1 + \cos\theta)/2$.
A dipolar field $A \propto \sin^2\theta / r$ has $\mathcal{S} = (\cos^2\theta + \sin^2\theta/4)^{1/2}$.
A field with parabolic lines $z \propto \varpi^b$ has $\mathcal{S} = d \ln \Psi / d \ln(\varpi^2 / z^{2/b})$.

Near the axis $\mathcal{S} \approx 1$, since the magnetic flux enclosed by the circle $z =$const, $\varpi =$const scales as $A \propto \varpi^2$.

As the flow expands $\mathcal{S} \downarrow$ and $\mathcal{S}_\infty \approx 1/2$.

A transition from $\mathcal{S} \approx 1$ to $\mathcal{S}_\infty \approx 1/2$ means that $A \propto \varpi^2$ changes to $A \propto \varpi$.

Collimation



Expansion $\longleftrightarrow$ collimation:

Inner field lines become better aligned with the rotation axis compared with outer ones.

This self-collimation goes along with the expansion and the formation of a cylindrical core.

Acceleration

$\mathcal{S}$ is proportional to the Poynting flux.

Defining the constant of motion

$$\sigma_m = \frac{A\Omega^2}{\Psi_A \mathcal{E} V_{\max}} = \frac{A\Omega^2(1+\mathcal{E}/c^2)}{\Psi_A \mathcal{E}^{3/2}\sqrt{2+\mathcal{E}/c^2}}$$

we find (by combining the integral relations)

$$\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi\Omega}\right) \frac{V_{\max}}{V_p} \frac{B_p \varpi^2}{A} \propto \frac{B_p \varpi^2}{A}$$

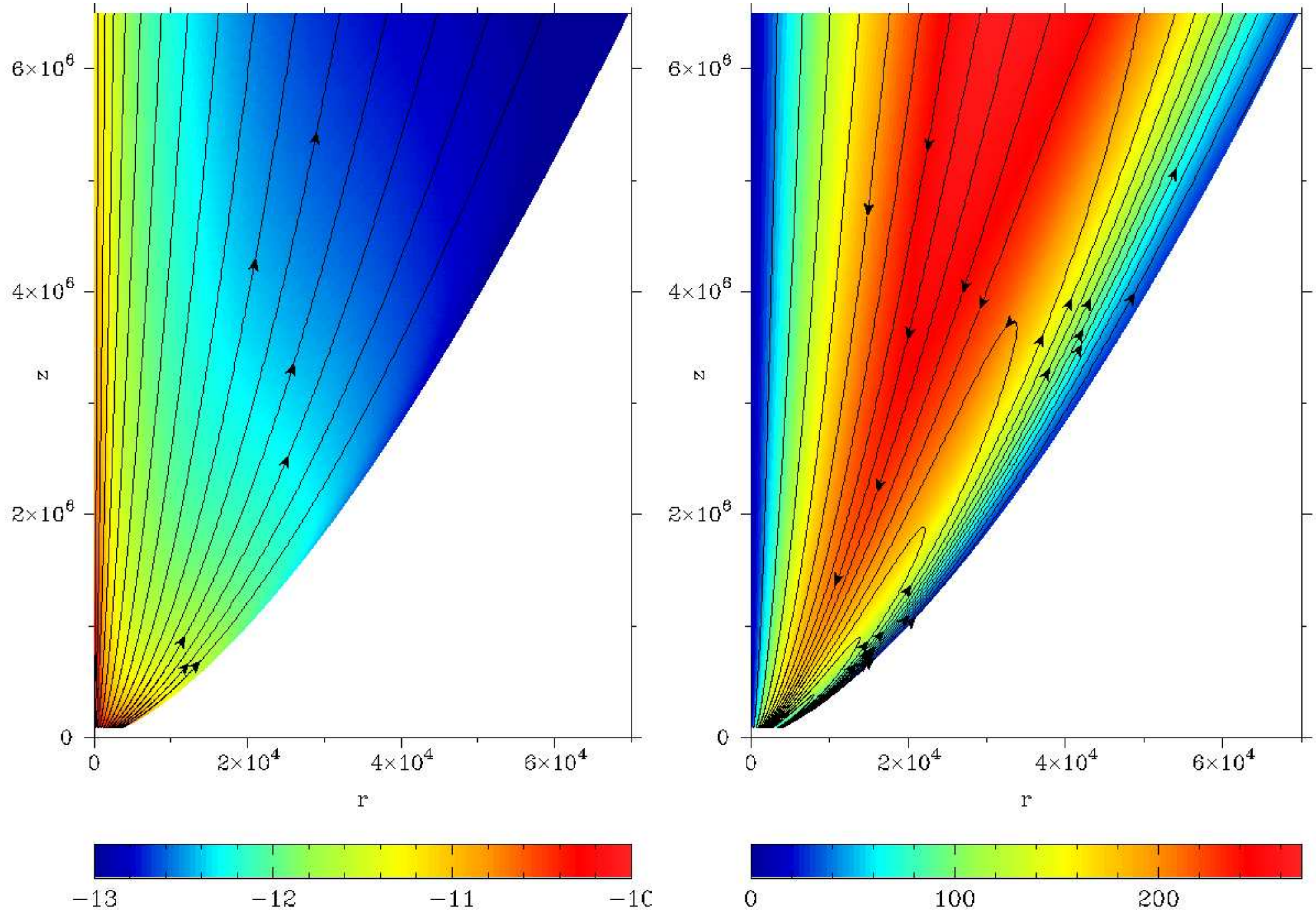
So, $\mathcal{S} \downarrow$ means bulk acceleration.

E.g., a transition from $\mathcal{S} \sim 1$ to $\mathcal{S} \sim 0.5$ means that half of the energy flux (initially in the electromagnetic field) is transfered to kinetic energy flux.

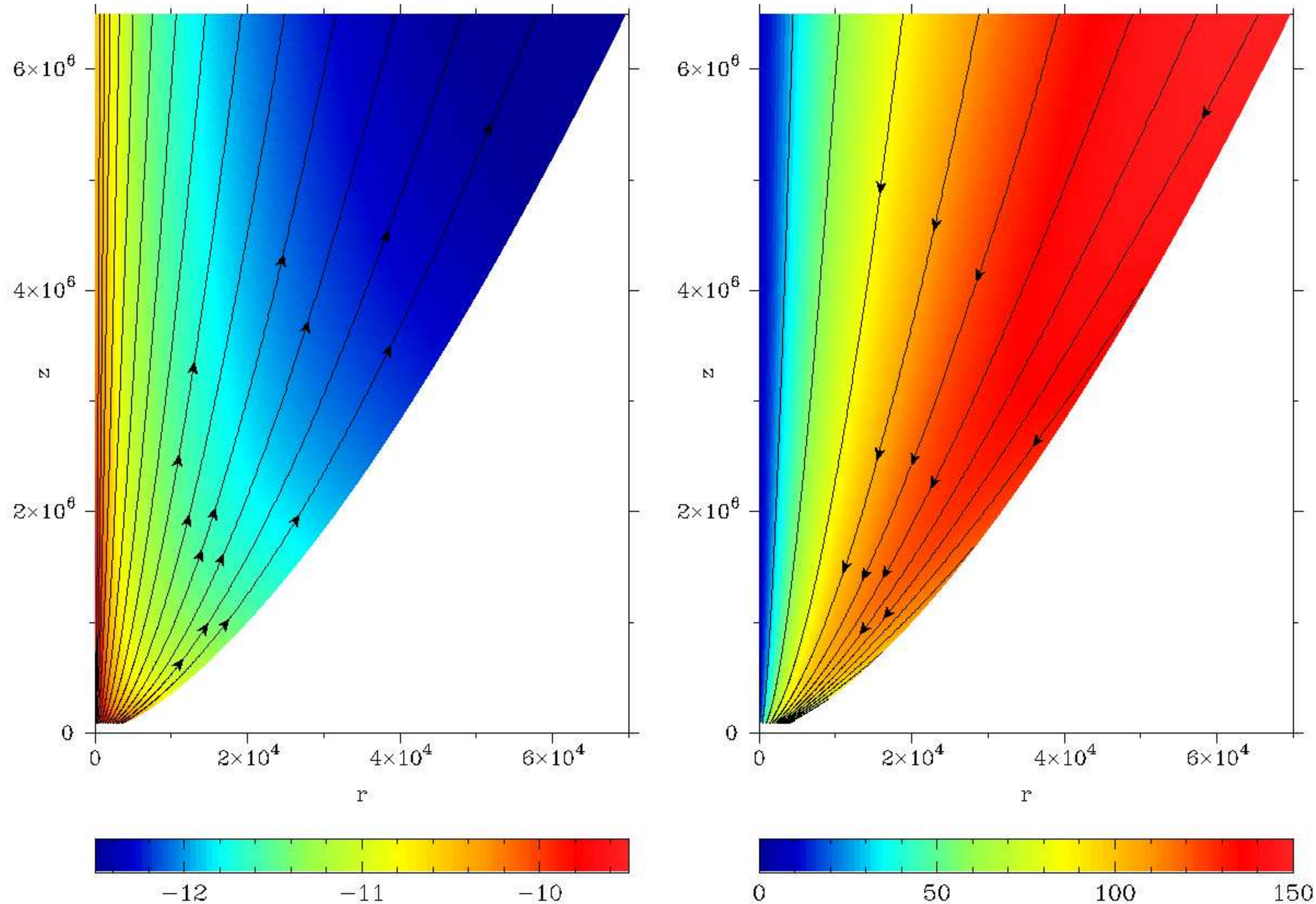
Acceleration mechanisms

- thermal (due to ∇P) $\rightarrow$ velocities up to C_s
- magnetocentrifugal (beads on wire)
  - initial half-opening angle $\vartheta > 30^o$ (only for cold flows)
  - velocities up to $\lesssim \varpi_i \Omega$
- relativistic thermal (thermal fireball – works for relativistic temperatures) gives $\gamma \sim \xi_i$, where $\xi = \dfrac{\text{enthalpy}}{\text{mass} \times c^2}$
- magnetic due to $\boldsymbol{J} \times \boldsymbol{B}/c \propto \nabla(\varpi B_\phi)$ — connected to $\mathcal{S}$ — it can give $\mathcal{E}_{\rm kinetic}$ up to the total $\mathcal{E}$

Komissarov, Vlahakis, Königl, & Barkov, in preparation



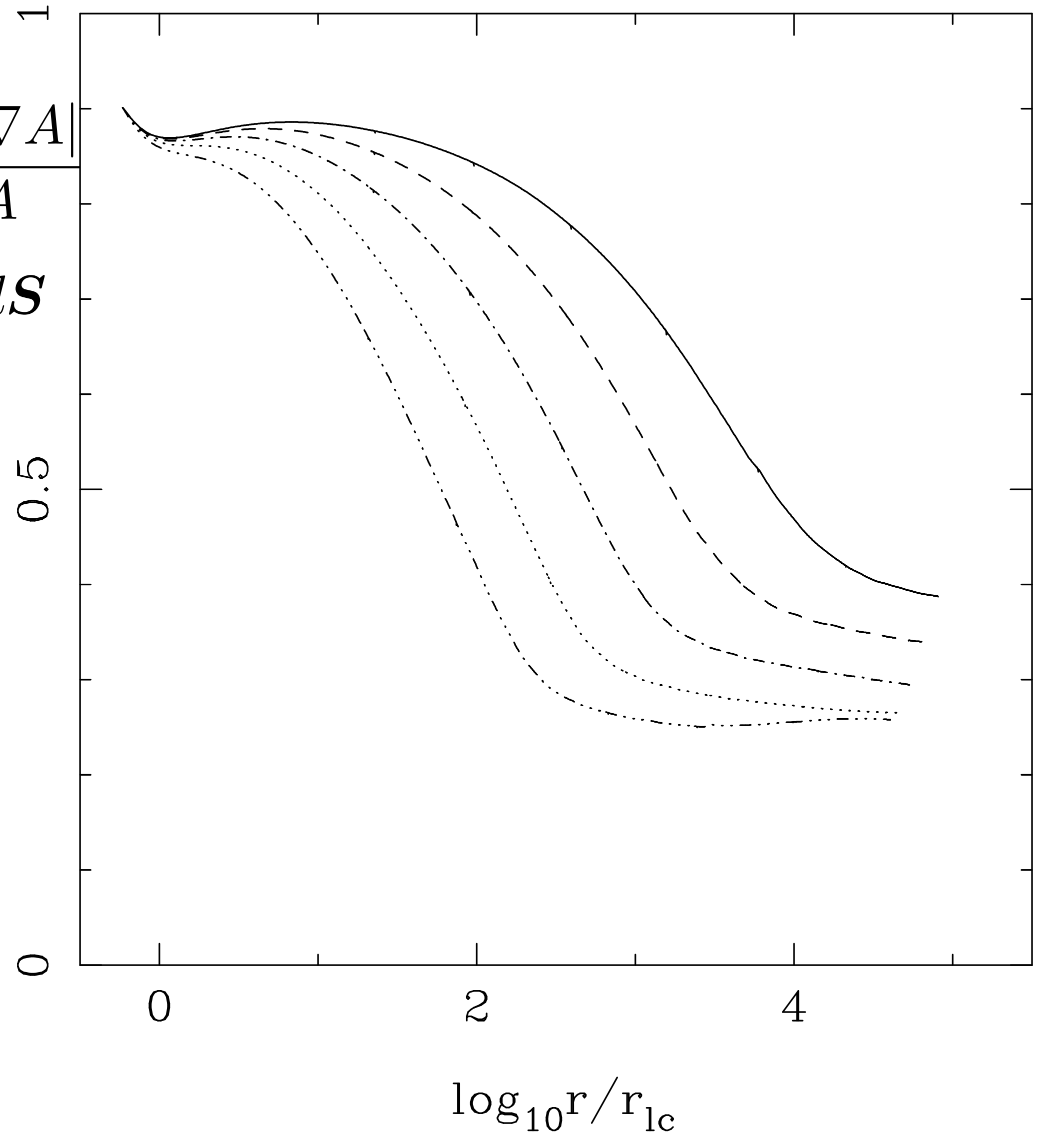
left: density/field lines, right: Lorentz factor/current lines (wall shape $z \propto \varpi^{1.5}$)
Differential rotation $\rightarrow$ slow envelope

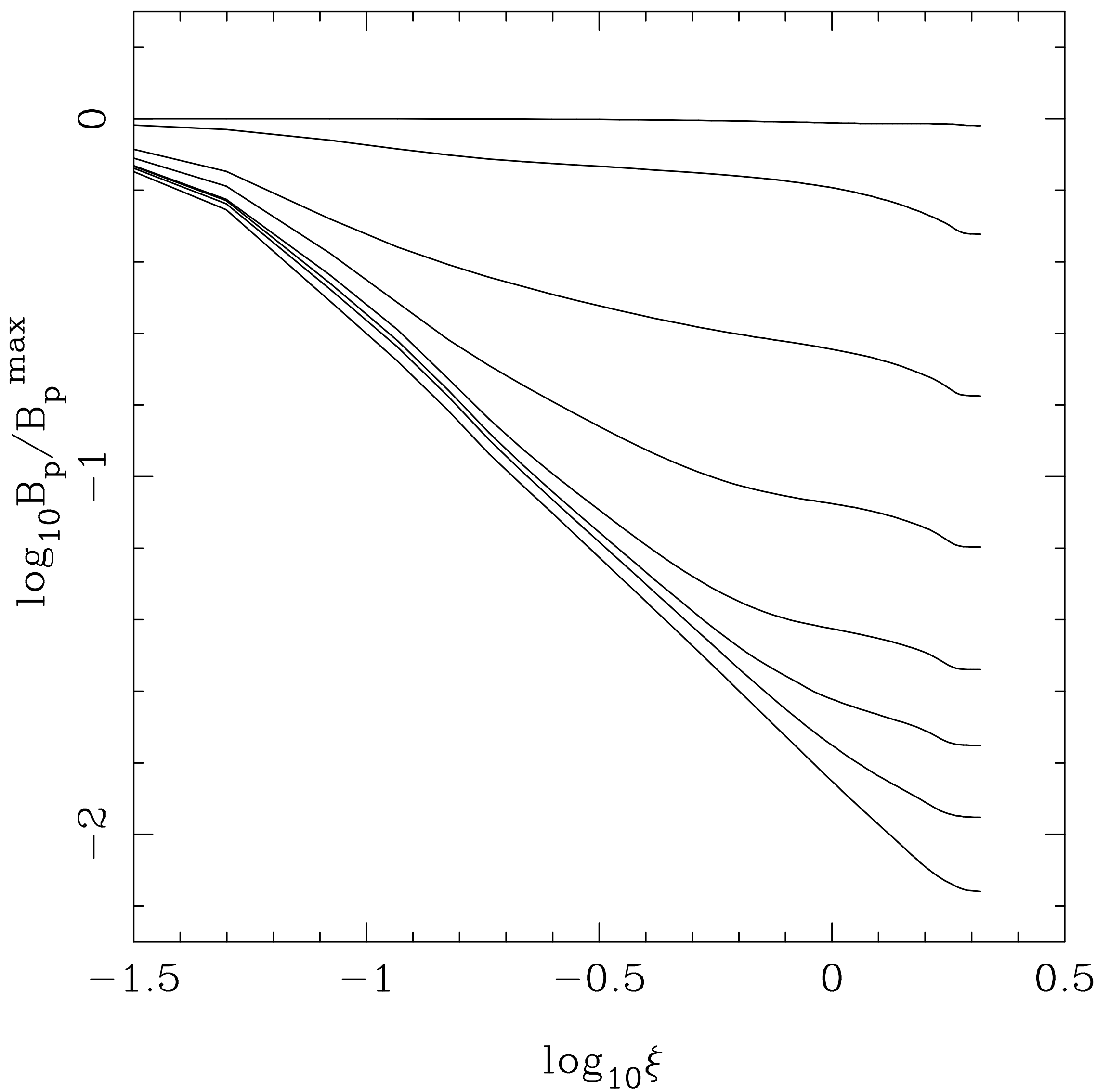


Uniform rotation $\rightarrow \gamma$ increases with ϖ

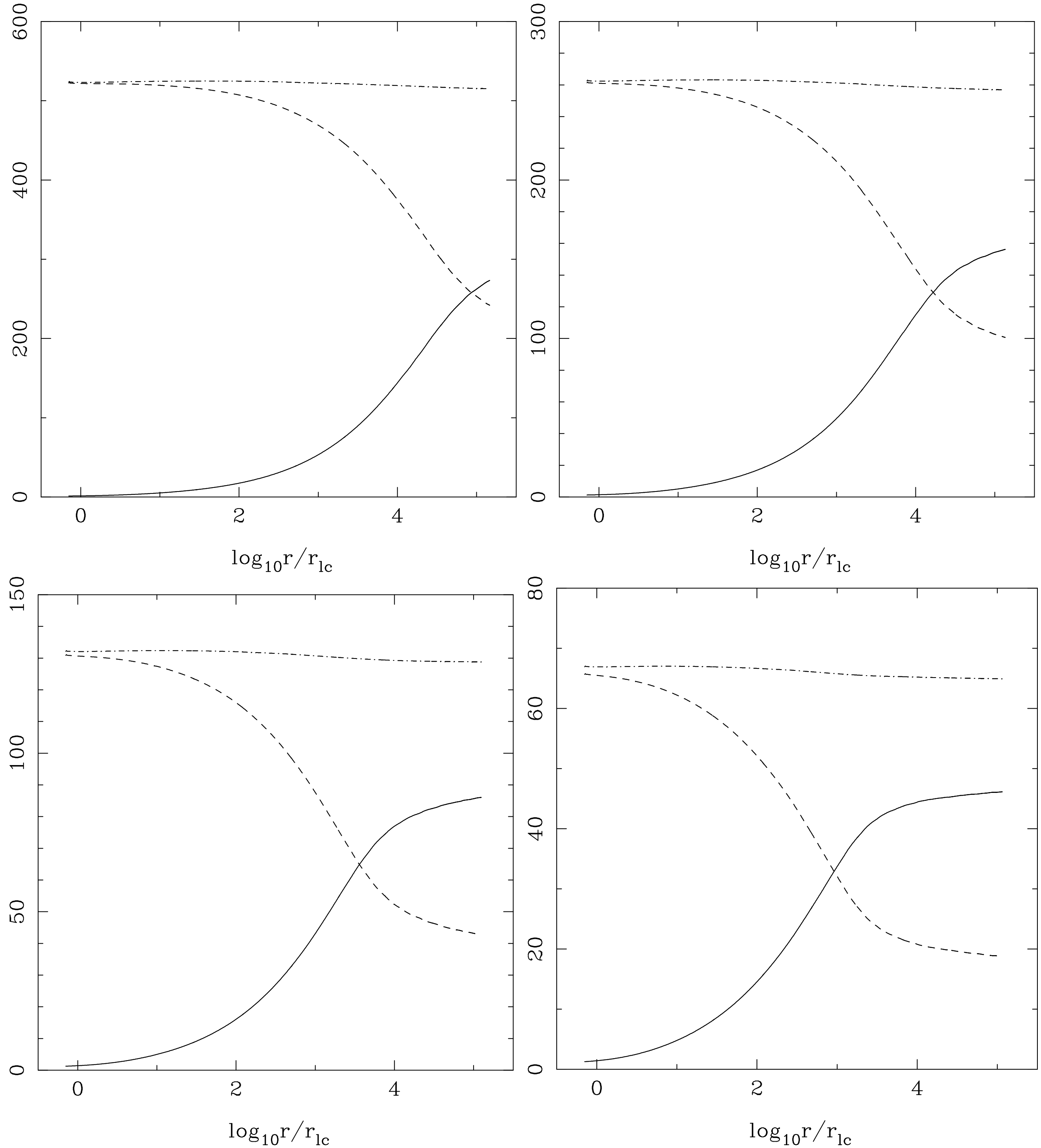
$$\mathcal{S} = \frac{\pi \varpi^2 B_p}{\int \boldsymbol{B} \cdot d\boldsymbol{S}} = \frac{1}{2}\,\frac{\varpi |\nabla A|}{A}$$

where $A = \dfrac{1}{2\pi} \displaystyle\int \boldsymbol{B} \cdot d\boldsymbol{S}$





Distribution of the poloidal magnetic field lines across the jet


600
400
200
0
0
2
4
$\log_{10} r/r_{lc}$
300
200
100
0
0
2
4
$\log_{10} r/r_{lc}$
150
100
50
0
0
2
4
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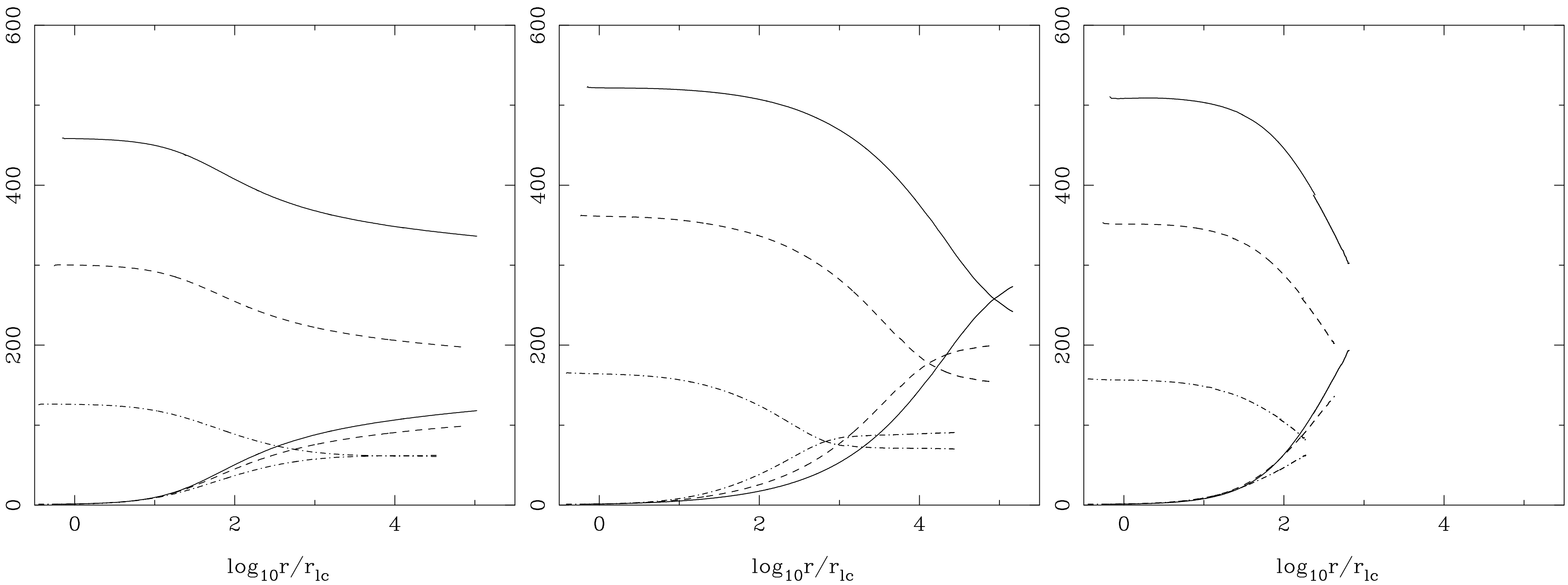
80
60
40
20
0
0
2
4
$\log_{10} r/r_{lc}$



γ and $\gamma\sigma$ for wall-shapes:
$z \propto \varpi$ (left), $z \propto \varpi^{1.5}$ (middle), $z \propto \varpi^2$ (right)

In the conical $\gamma \sim \varpi\Omega/c$, but small efficiency

In parabolic, Lorentz factor $\gamma \sim z/\varpi \propto \varpi^{1/2} \propto R^{1/3}$ (middle)
and $\gamma \sim z/\varpi \propto \varpi \propto R^{1/2}$ (right)
efficiency $\sim 50\%$

Summary

⋆ magnetic fields provide a viable explanation of the dynamics of jets

[they extract energy and angular momentum (transfer them to matter) – they collimate outflows and produce jets – in AGN jets they could explain relatively large-scale acceleration and polarization/RM maps]

⋆ the paradigm of MHD jets works in a similar way in all astrophysical jets

more than half of the Poynting flux is trasfered to kinetic energy flux

- if $\mathcal{E}/Mc^2 \gg 1 \rightarrow$ relativistic flow with $\gamma_\infty \sim 0.5\frac{\mathcal{E}}{Mc^2}$

  if $\mathcal{E}/Mc^2 \ll 1 \rightarrow$ nonrelativistic flow with $V_\infty \sim \sqrt{\frac{\mathcal{E}}{M}}$
- collimation goes along with the decrease of the Poynting flux