Dynamics of astrophysical magnetized jets

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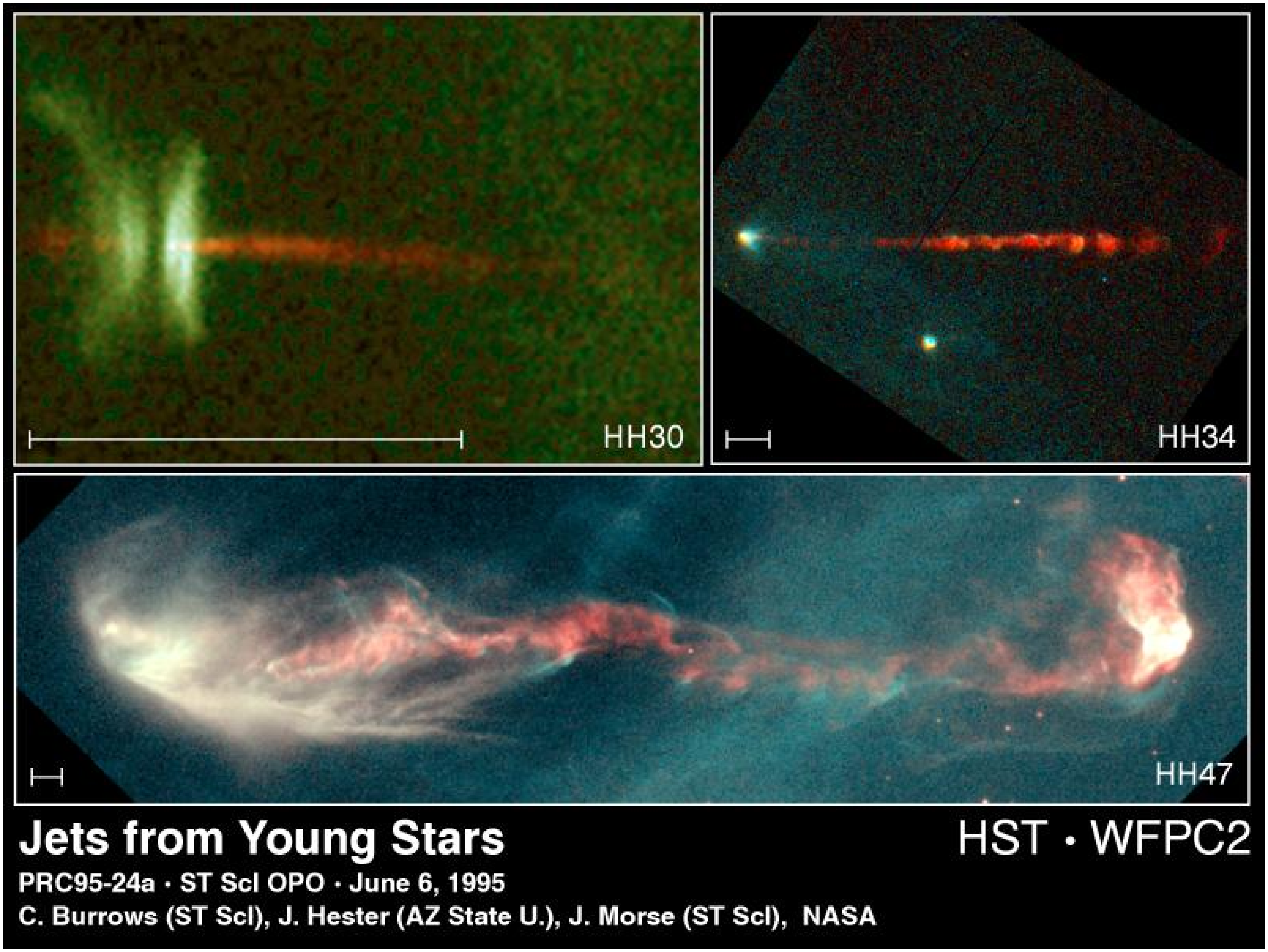
Outline

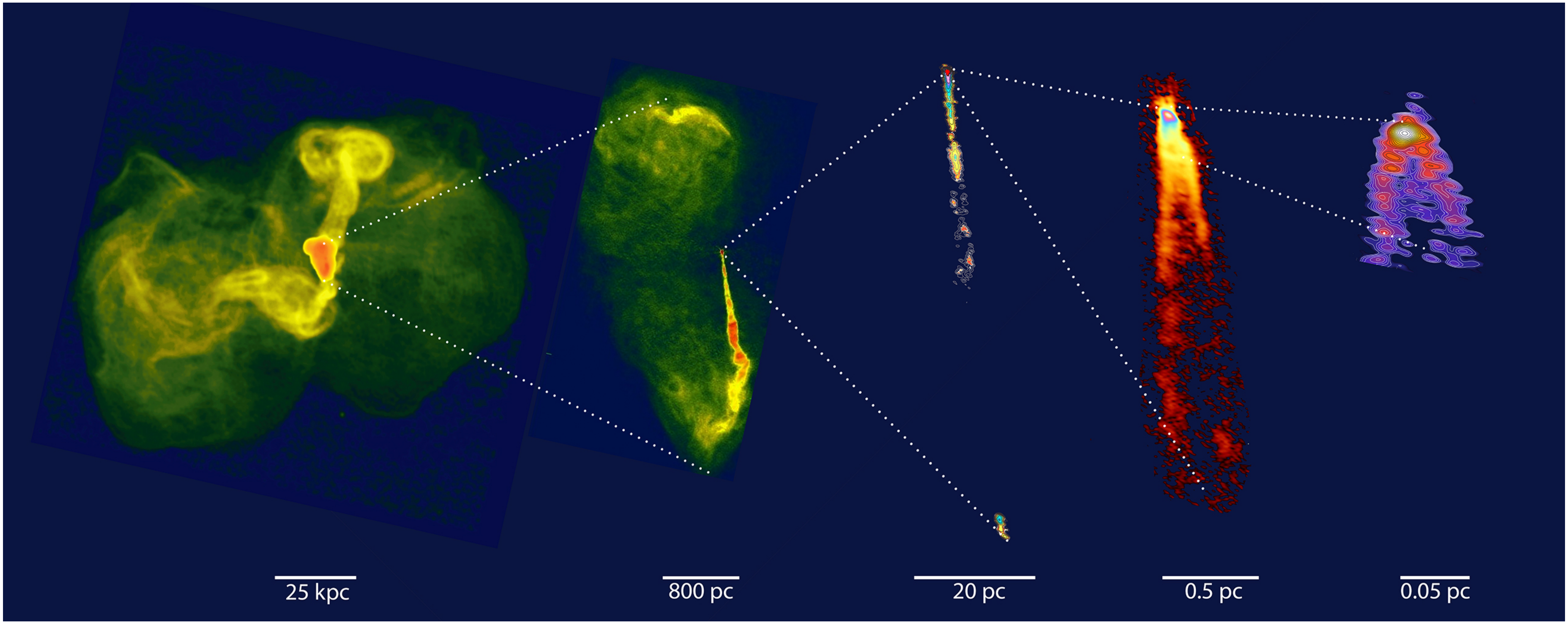
- introduction (observed jet characteristics)
- magnetohydrodynamics (collimation-acceleration)
- linear jet stability (resulting growth rates)

Examples of astrophysical jets



(scale =1000 AU, $V_\infty = a few 100$km/s)

The jet from the M87 galaxy



(from Blandford+2018)

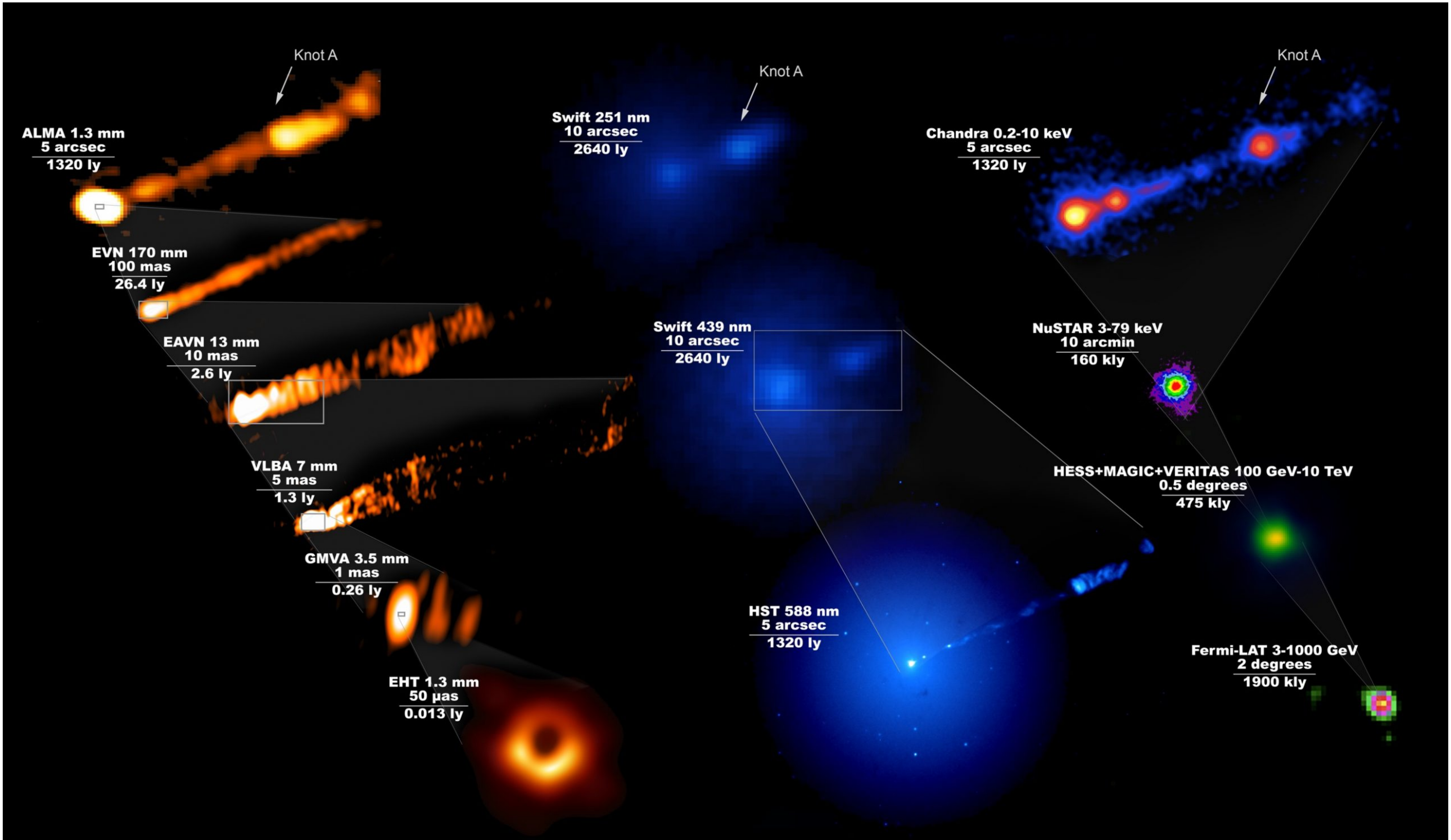
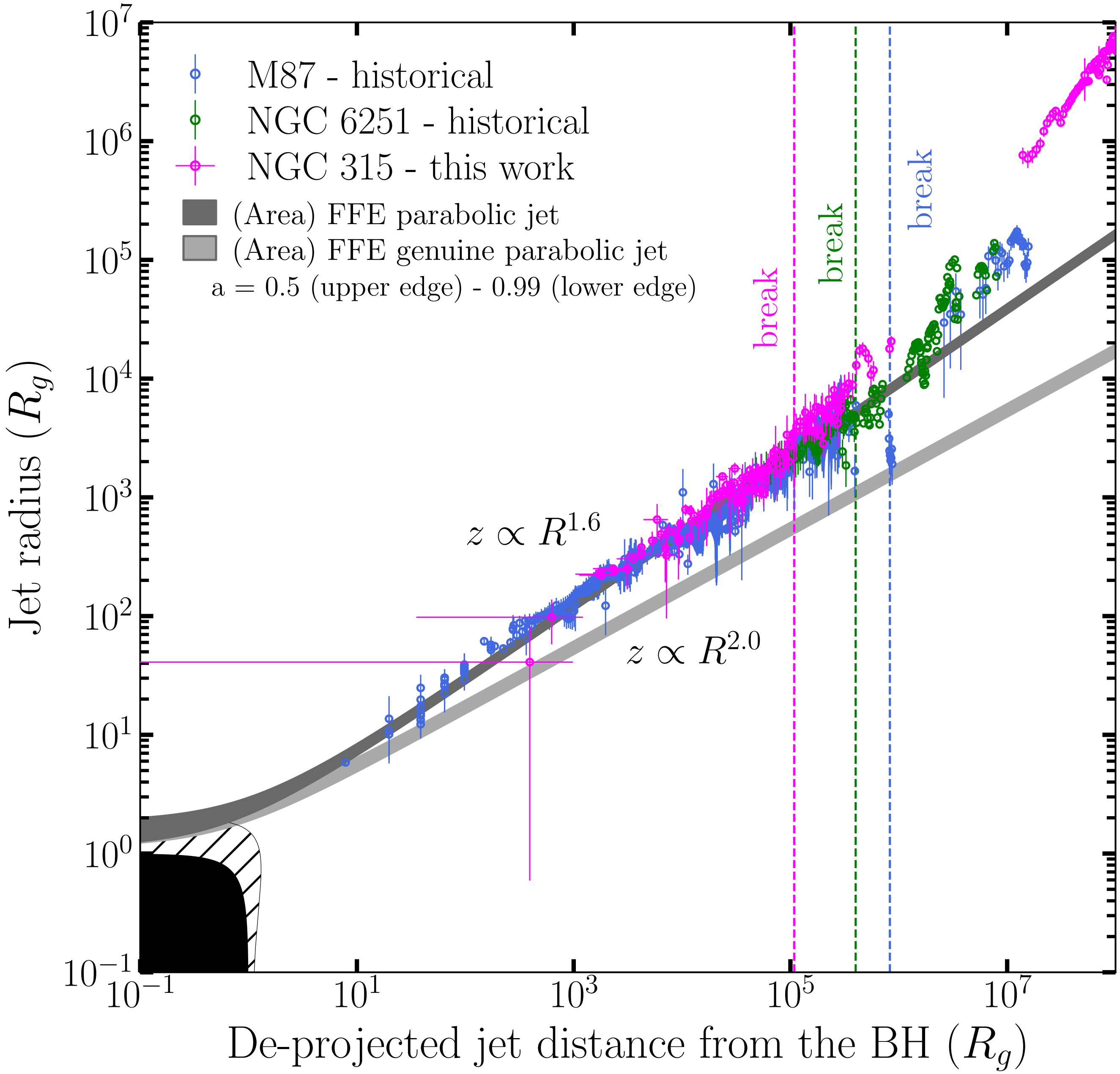
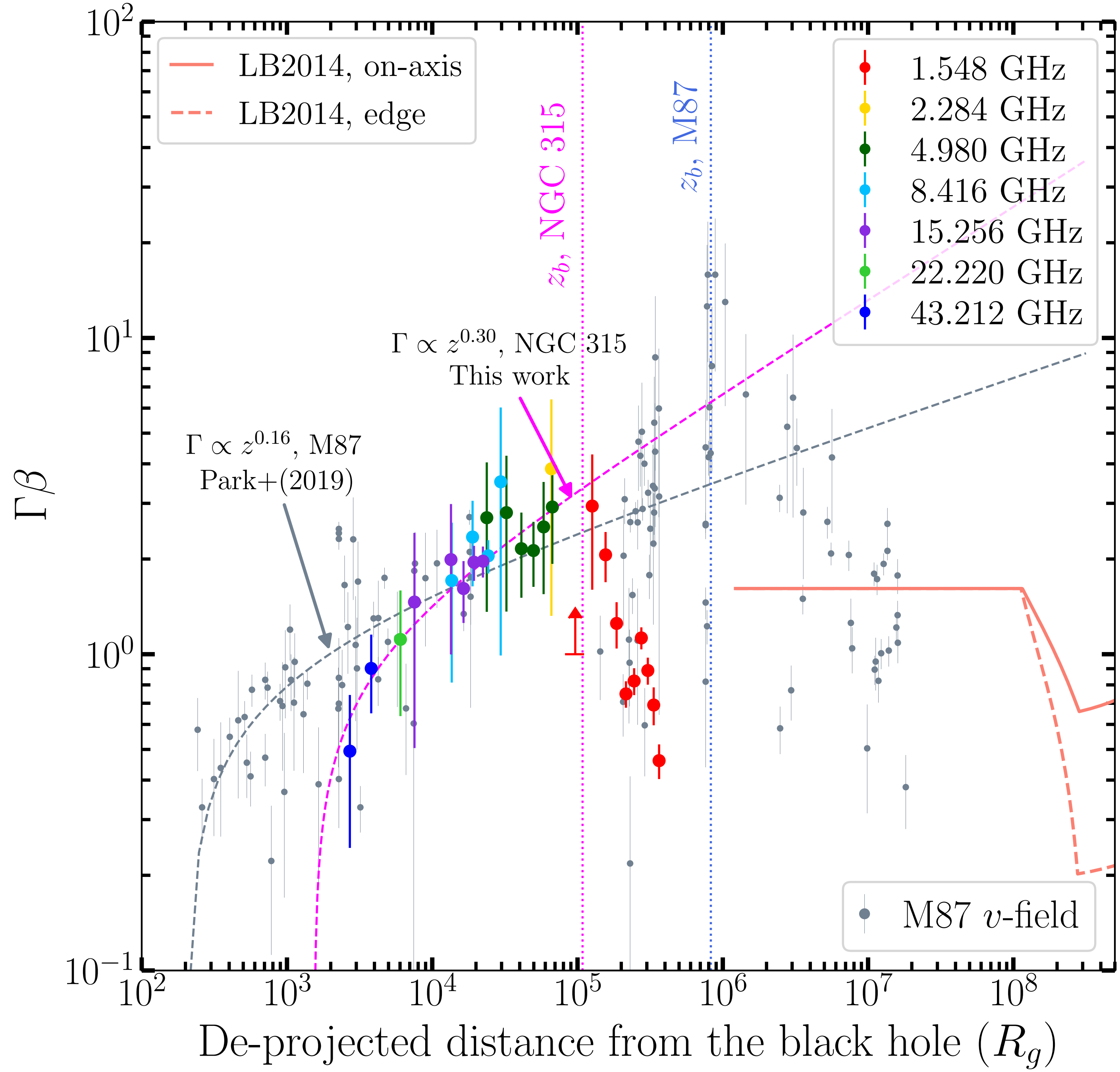


Image Credit: The EHT Multi-wavelength Science Working Group; the EHT Collaboration; ALMA (ESO/NAOJ/NRAO); the EVN; the EAVN Collaboration; VLBA (NRAO); the GMVA; the Hubble Space Telescope; the Neil Gehrels Swift Observatory; the Chandra X-ray Observatory; the Nuclear Spectroscopic Telescope Array; the Fermi-LAT Collaboration; the H.E.S.S collaboration; the MAGIC collaboration; the VERITAS collaboration; NASA and ESA. Composition by J. C. Algaba

(Park+2021)



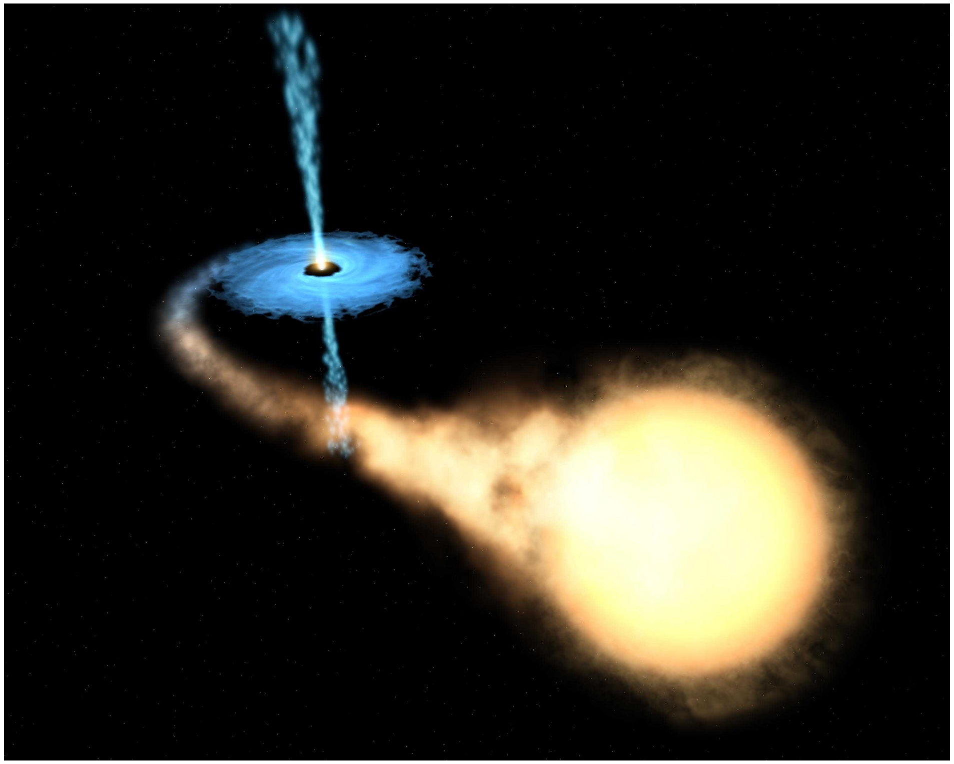
M87 - historical
NGC 6251 - historical
NGC 315 - this work
(Area) FFE parabolic jet
(Area) FFE genuine parabolic jet
a = 0.5 (upper edge) - 0.99 (lower edge)
break
break
break
$z \propto R^{1.6}$
$z \propto R^{2.0}$
Jet radius (R_g)
De-projected jet distance from the BH (R_g)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{-1}
10^{1}
10^{3}
10^{5}
10^{7}

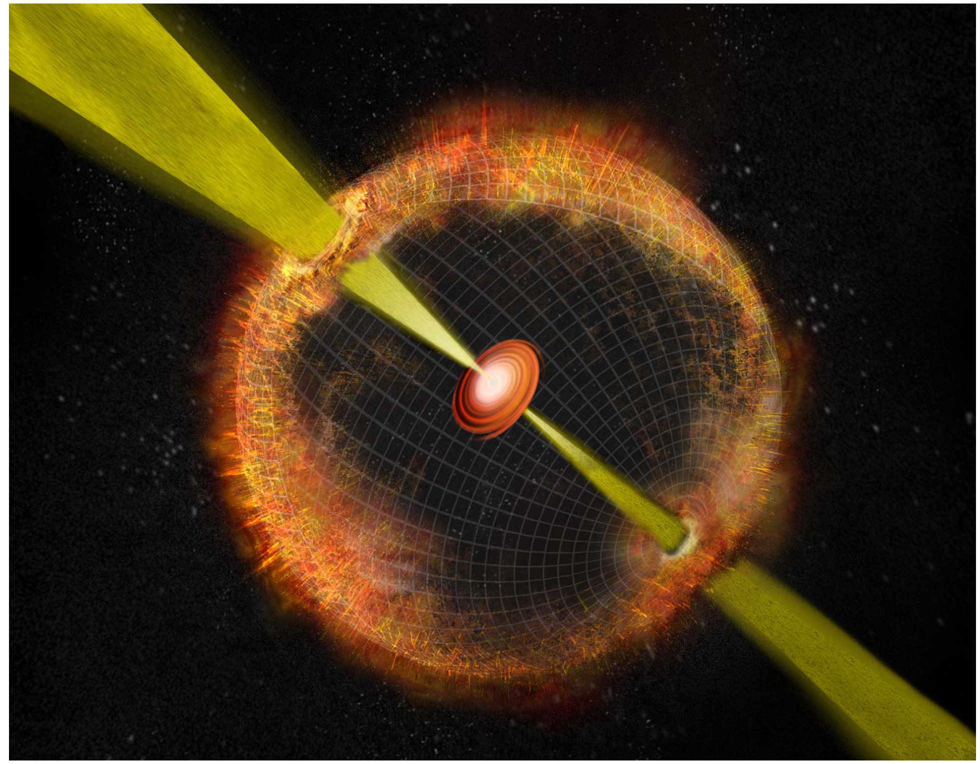


LB2014, on-axis
LB2014, edge
1.548 GHz
2.284 GHz
4.980 GHz
8.416 GHz
15.256 GHz
22.220 GHz
43.212 GHz
z_b, NGC 315
z_b, M87
$\Gamma \propto z^{0.30}$, NGC 315
This work
$\Gamma \propto z^{0.16}$, M87
Park+(2019)
M87 v-field
$\Gamma\beta$
De-projected distance from the black hole (R_g)

X-ray binaries

γ-ray bursts

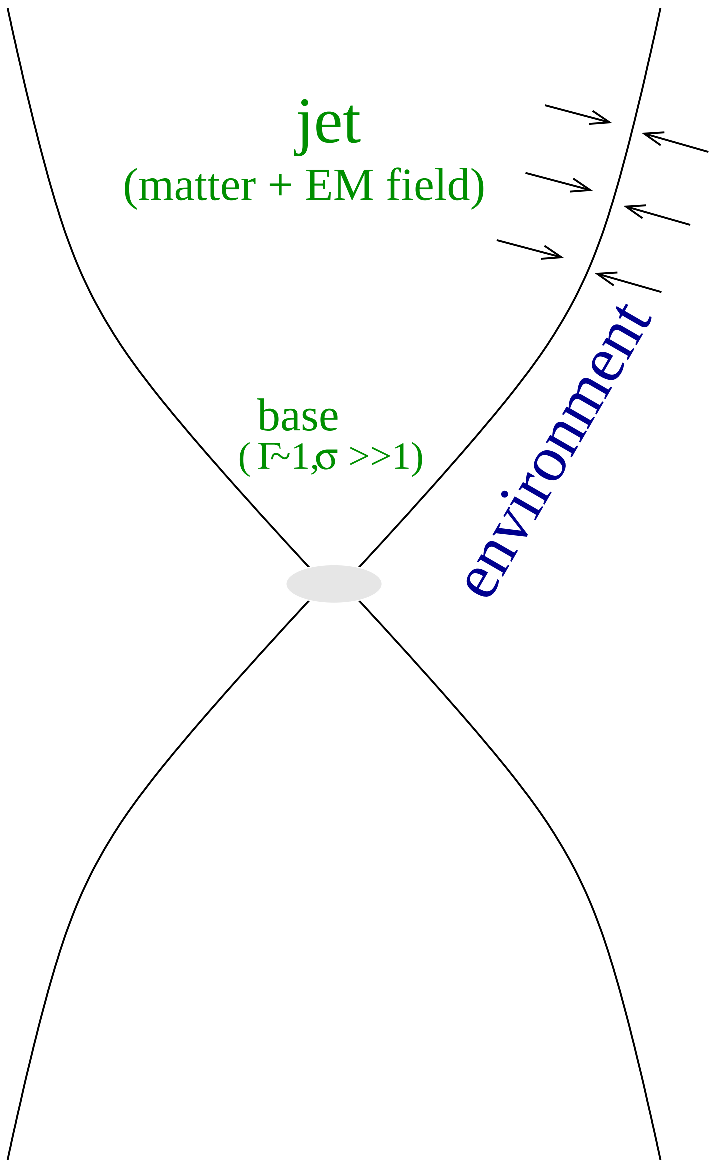




mildly relativistic

$\gamma =$ a few 100

Basic questions



- source of matter/energy?
- bulk acceleration?
- collimation?
- role of environment?

Theoretical modeling

- ☞ if energy source = thermal energy:

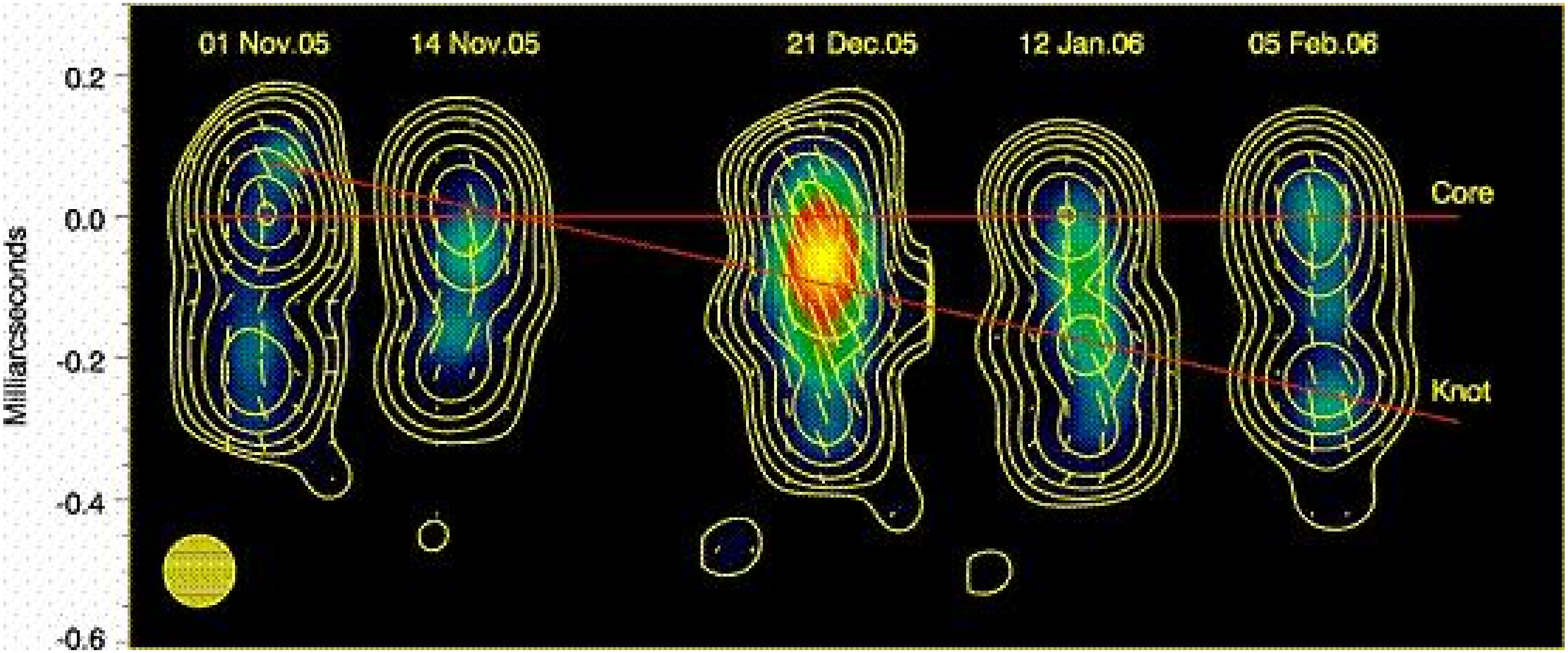
thermal acceleration is an efficient mechanism

gives terminal speed $\frac{m_p V_\infty^2}{2} \sim k_{\rm B} T_i$ for YSO jets
or terminal Lorentz factors $\gamma_\infty m_p c^2 \sim k_{\rm B} T_i$ for relativistic jets

in both cases needs high initial temperatures T_i to explain the observed motions

- ☞ magnetic acceleration more likely

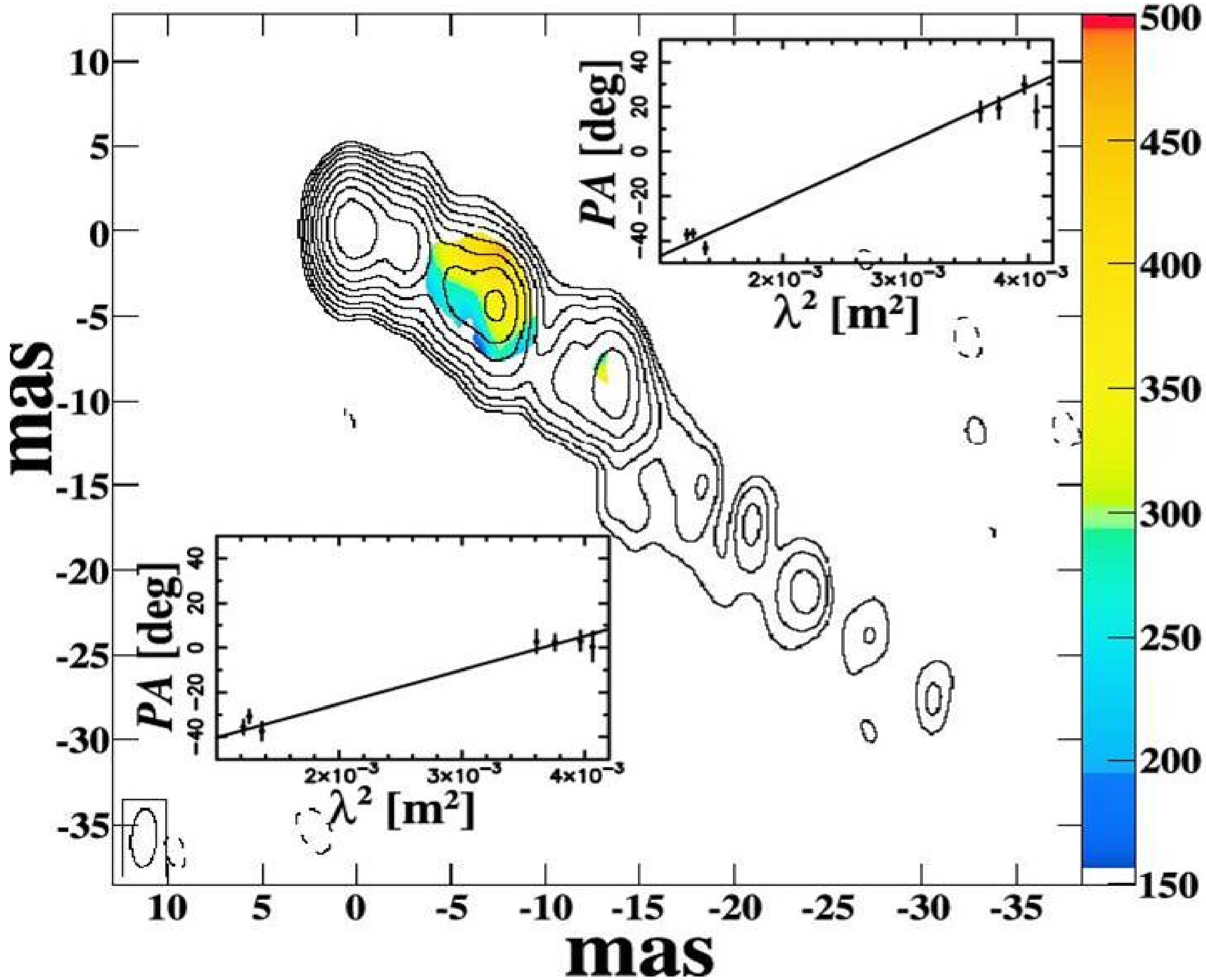
Polarization

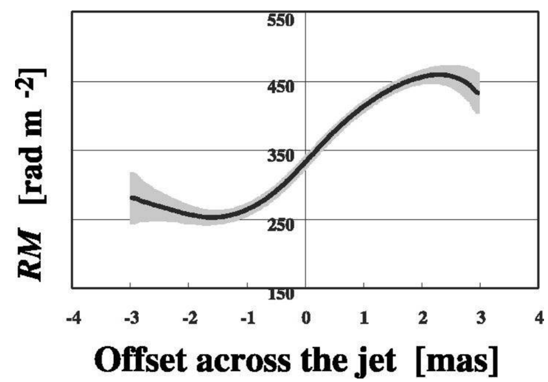


(Marscher et al 2008, Nature)

observed $\boldsymbol{E}_{\rm rad} \perp \boldsymbol{B}_{\perp \rm los}$
(modified by Faraday rotation and relativistic effects)

Faraday RM gradients across the jet





(Asada et al)

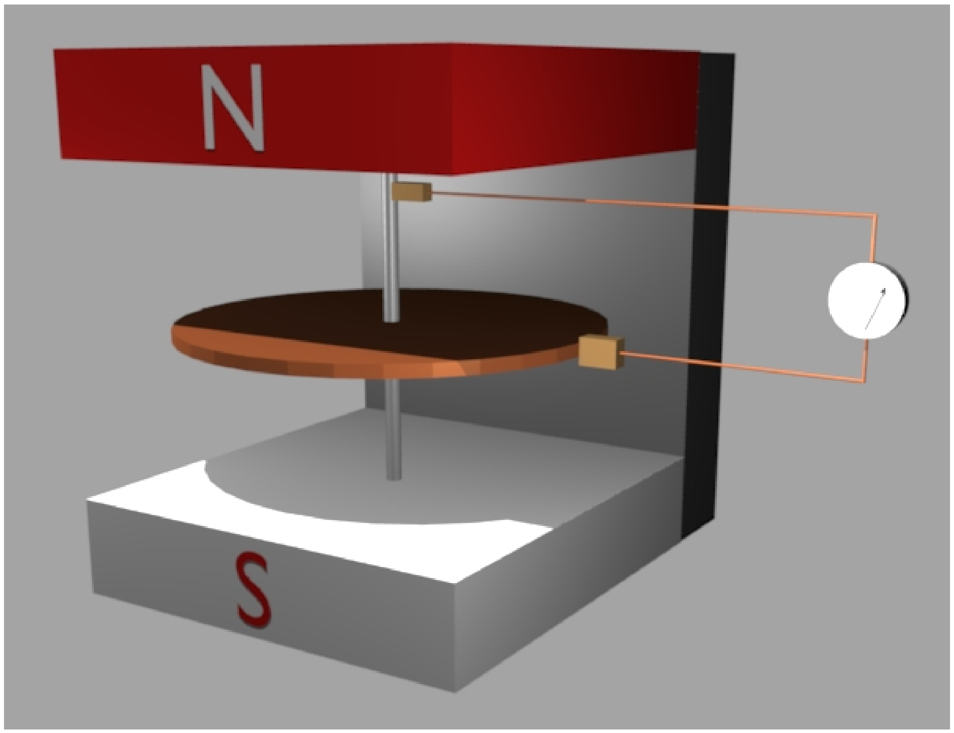
helical field surrounding the emitting region (Gabuzda)

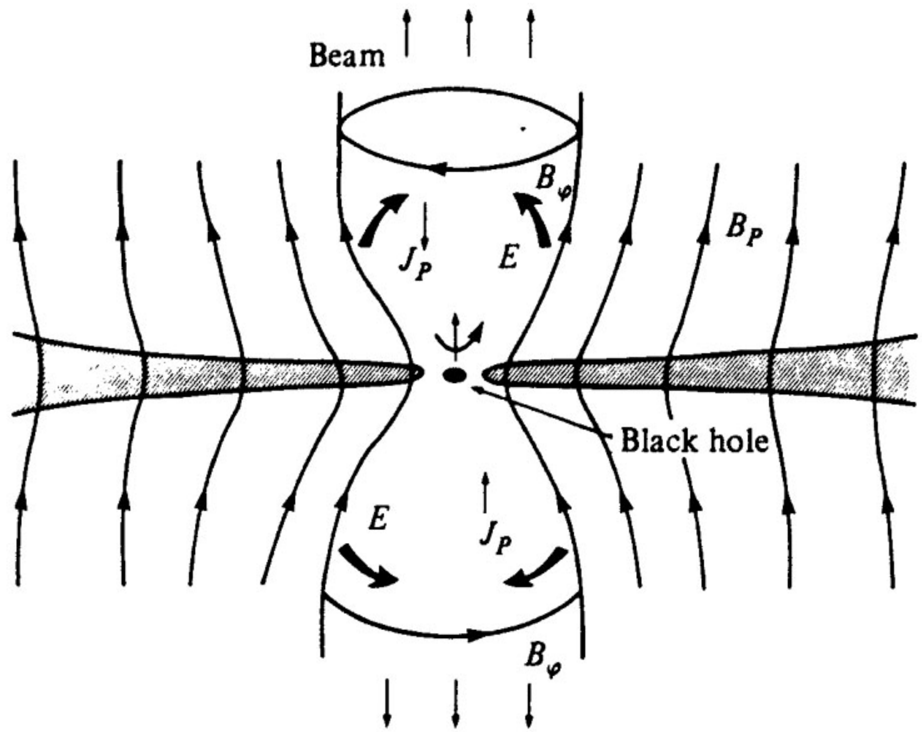
Role of magnetic field

- ⋆ extract energy (Poynting flux)
- ⋆ extract angular momentum
- ⋆ transfer energy and angular momentum to matter
- ⋆ explain relatively large-scale acceleration
- ⋆ self-collimation
- ⋆ synchrotron emission
- ⋆ polarization and Faraday RM maps

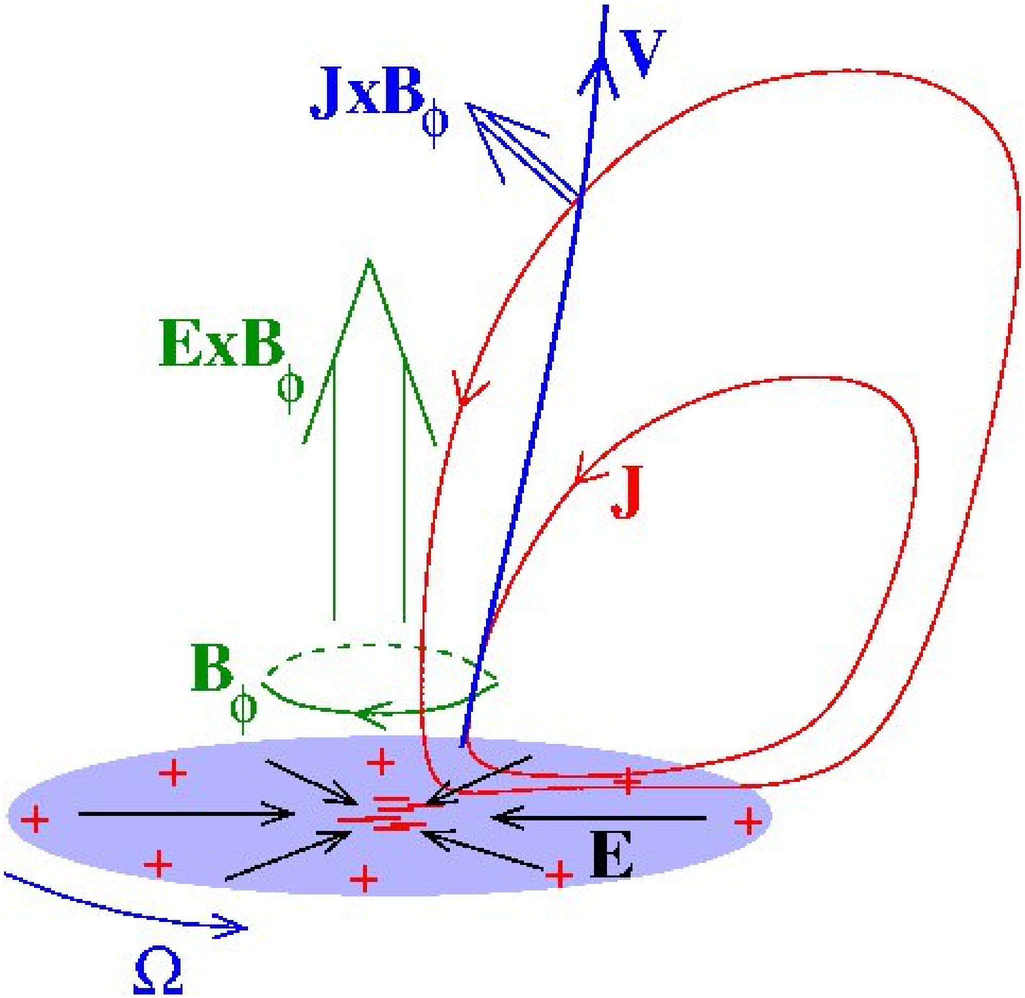
How MHD acceleration works

A unipolar inductor (Faraday disk)





magnetic field + rotation



current $\leftrightarrow B_\phi$
Poynting flux $\frac{c}{4\pi}EB_\phi$ is extracted (angular momentum as well)

The Faraday disk could be the rotating accretion disk, or the frame dragging if energy is extracted from the ergosphere of a rotating black hole (Blandford & Znajek mechanism)

The ideal MHD equations

Maxwell:

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0=\boldsymbol{\nabla}\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{c\partial t}\,,\boldsymbol{\nabla}\times\boldsymbol{B}=\frac{\partial\boldsymbol{E}}{c\partial t}+\frac{4\pi}{c}\boldsymbol{J}\,,\boldsymbol{\nabla}\cdot\boldsymbol{E}=\frac{4\pi}{c}J^0$$

Ohm: $\boldsymbol{E}=-\frac{\boldsymbol{V}}{c}\times\boldsymbol{B}$

mass conservation (continuity):

$$\frac{d(\gamma\rho_0)}{dt}+\gamma\rho_0\boldsymbol{\nabla}\cdot\boldsymbol{V}=0\,,\quad \text{where}\quad \frac{d}{dt}=\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\boldsymbol{\nabla}$$

energy $U_\mu T^{\mu\nu}_{,\nu}=0$ (or specific entropy conservation, or first law for thermodynamics):

$$\frac{d\left(P/\rho_0^\Gamma\right)}{dt}=0$$

momentum $T^{\nu i}_{,\nu}=0$: $\gamma\rho_o\frac{d\left(\xi\gamma\boldsymbol{V}\right)}{dt}=-\boldsymbol{\nabla}P+\frac{J^0\boldsymbol{E}+\boldsymbol{J}\times\boldsymbol{B}}{c}$

magnetic acceleration

- simplified momentum equation along the flow

$$\gamma\rho_0 \frac{d(\gamma V)}{dt} = -\frac{B_\phi}{4\pi\varpi}\frac{\partial(\varpi B_\phi)}{\partial \ell} \qquad = \boldsymbol{J}\times\boldsymbol{B} \text{ force}$$

(ϖ= cylindrical distance, ℓ= arclength along flow)

- simplified Ferraro's law (ignore V_ϕ – small compared to $\varpi\Omega$)

$$V_\phi = \varpi\Omega + V B_\phi/B_p \quad \Leftrightarrow \quad B_\phi \approx -\frac{\varpi\Omega B_p}{V} \quad \text{“Parker spiral”}$$

- combine the two, use the mass-to-magnetic flux $\Psi_A = \dfrac{4\pi\gamma\rho_0 V}{B_p}$

(constant due to flux-freezing)

$$m\frac{d(\gamma V)}{dt} = -\frac{\partial}{\partial \ell}\left(\frac{S}{V}\right), \quad m = \frac{\Psi_A}{A\Omega^2}, \quad S = \frac{\varpi^2 B_p}{A}$$

(A is the magnetic flux – integral)

toy model

$$m\frac{d(\gamma V)}{dt} = -\frac{\partial}{\partial \ell}\left(\frac{S}{V}\right)$$

motion of a mass $m = \dfrac{\Psi_A}{A\Omega^2}$ in a velocity-dependent potential $\dfrac{S}{V}$

corresponding energy integral = Bernoulli $\quad \gamma mc^2 + \dfrac{S}{V} = E$

The equation of particle motion can be written as a de-Laval nozzle equation

$$\frac{dV}{d\ell} = \frac{\dfrac{dS}{d\ell}}{E - \gamma^3 mc^2}$$

bunching function $S = \varpi^2 B_p / A$

using the definition of A, $$S = \frac{2\pi\varpi^2 B_p}{\int \boldsymbol{B}_p \cdot d\boldsymbol{a}}$$

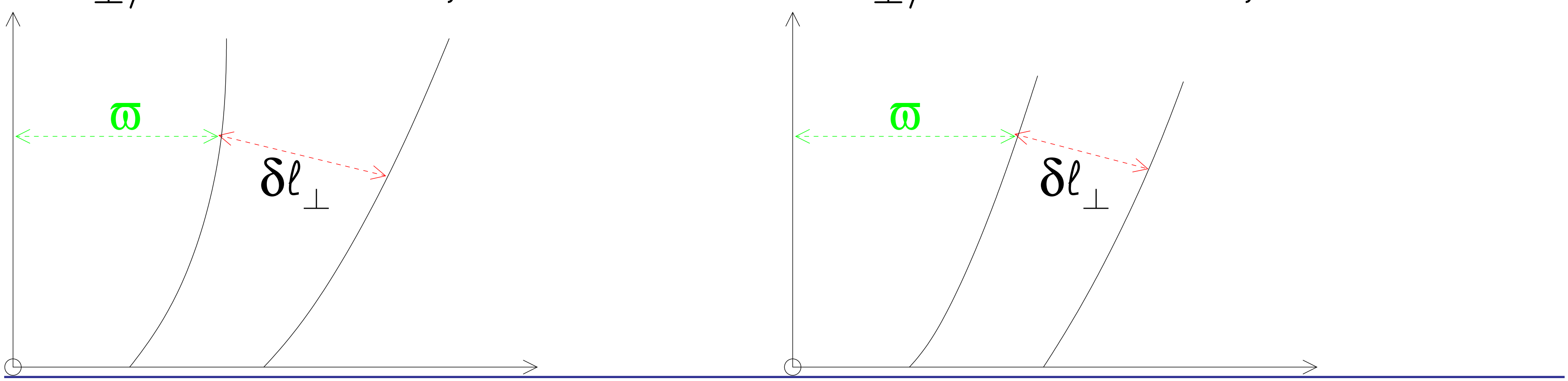
thus S measures the ratio of the local over the mean poloidal magnetic field

it measures how bunched are the fieldlines at a given point

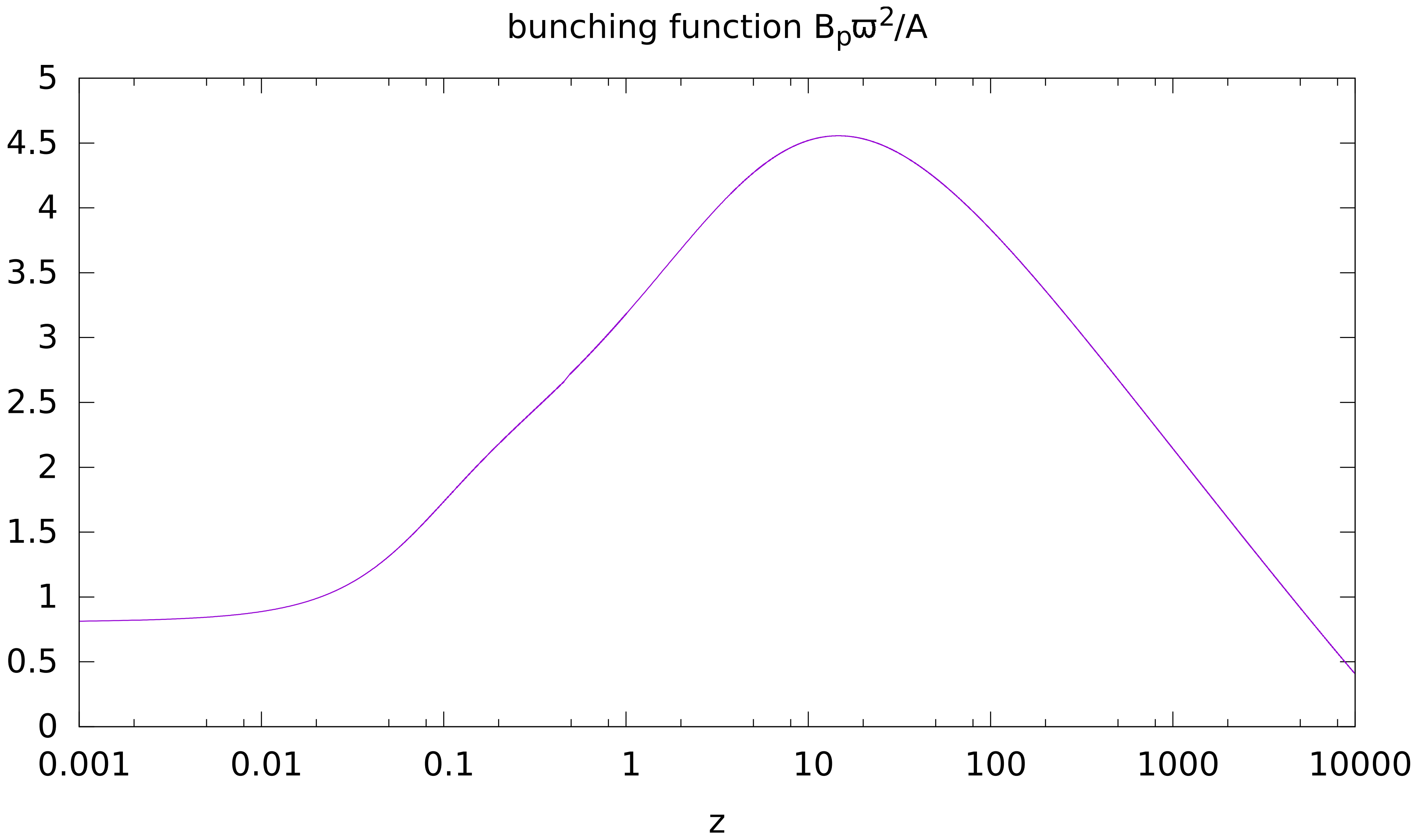
its variation along the flow measures the expansion of the flow,

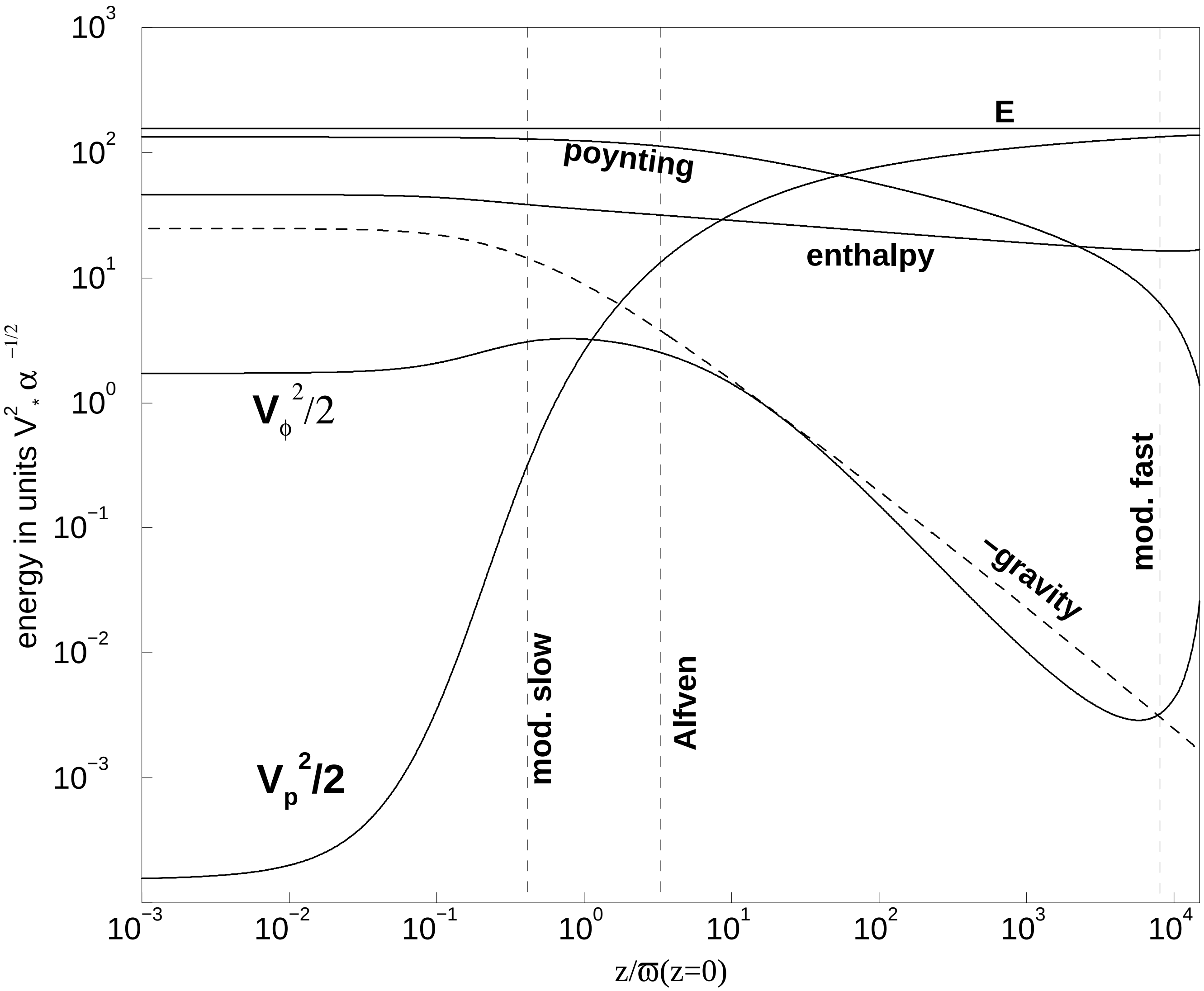
$$S \propto \underbrace{B_p\, 2\pi\varpi\delta\ell_\perp}_{\delta A} \frac{\varpi}{\delta\ell_\perp} \propto \frac{\varpi}{\delta\ell_\perp}$$

if $\delta\ell_\perp/\varpi$ increases, S decreases if $\delta\ell_\perp/\varpi$ decreases, S increases

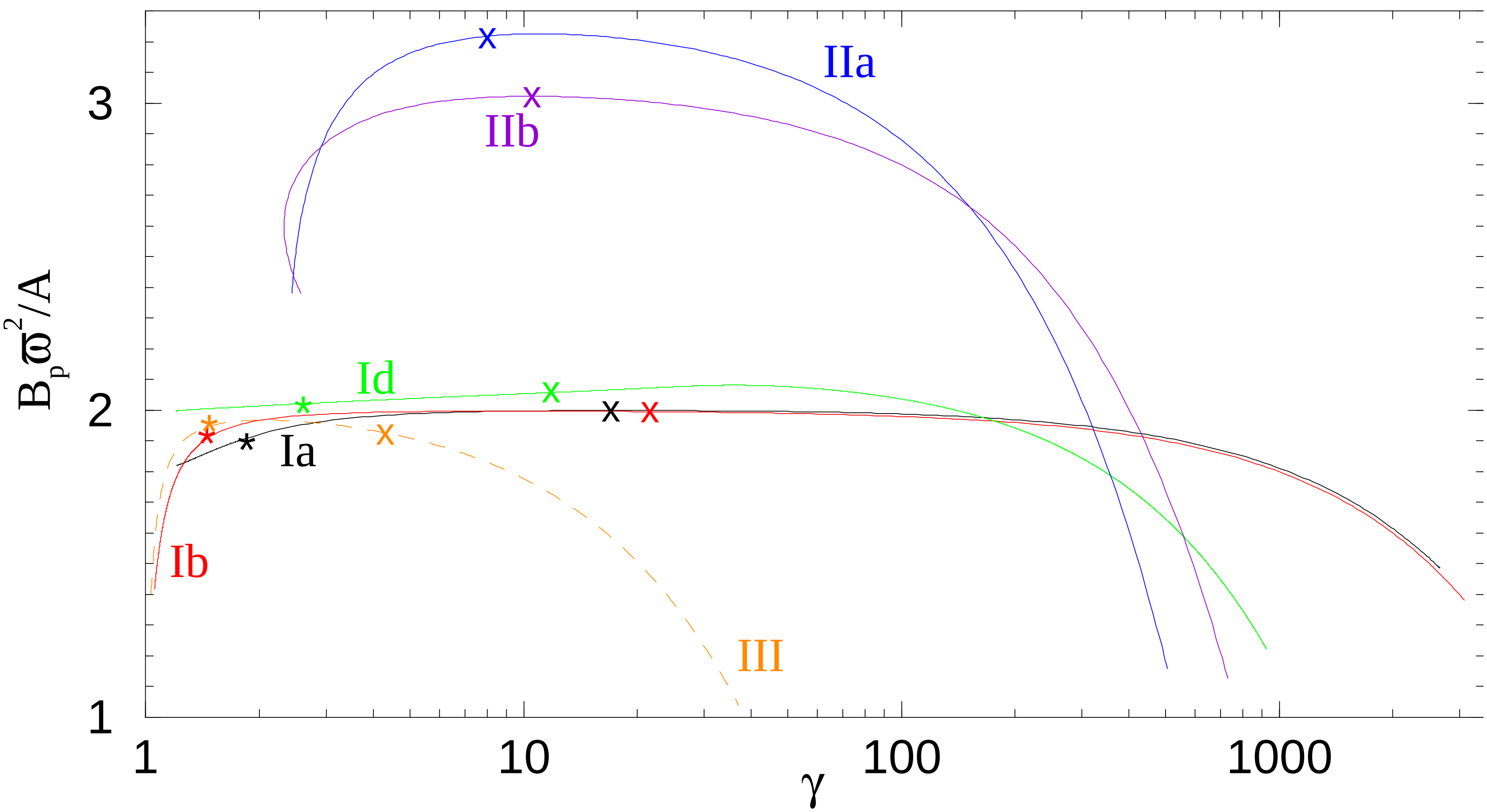


Vlahakis+2000 nonrelativistic solution

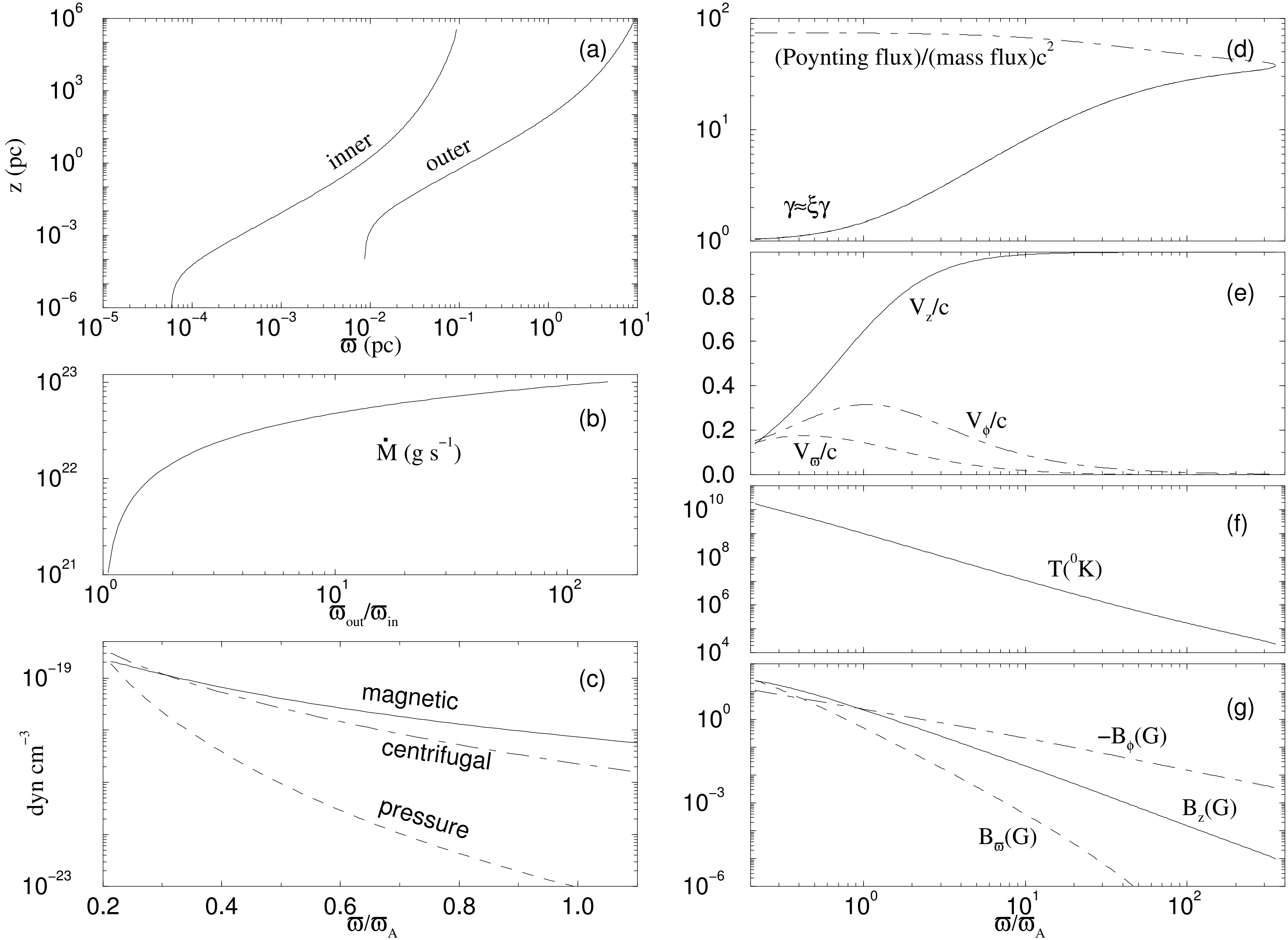



E
poynting
enthalpy
$V_\phi{}^2/2$
$V_p{}^2/2$
−gravity
mod. slow
Alfven
mod. fast
energy in units $V_*^2\,\alpha^{-1/2}$
$z/\varpi(z=0)$
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4}

Vlahakis & Königl 2003, 2004 relativistic solutions

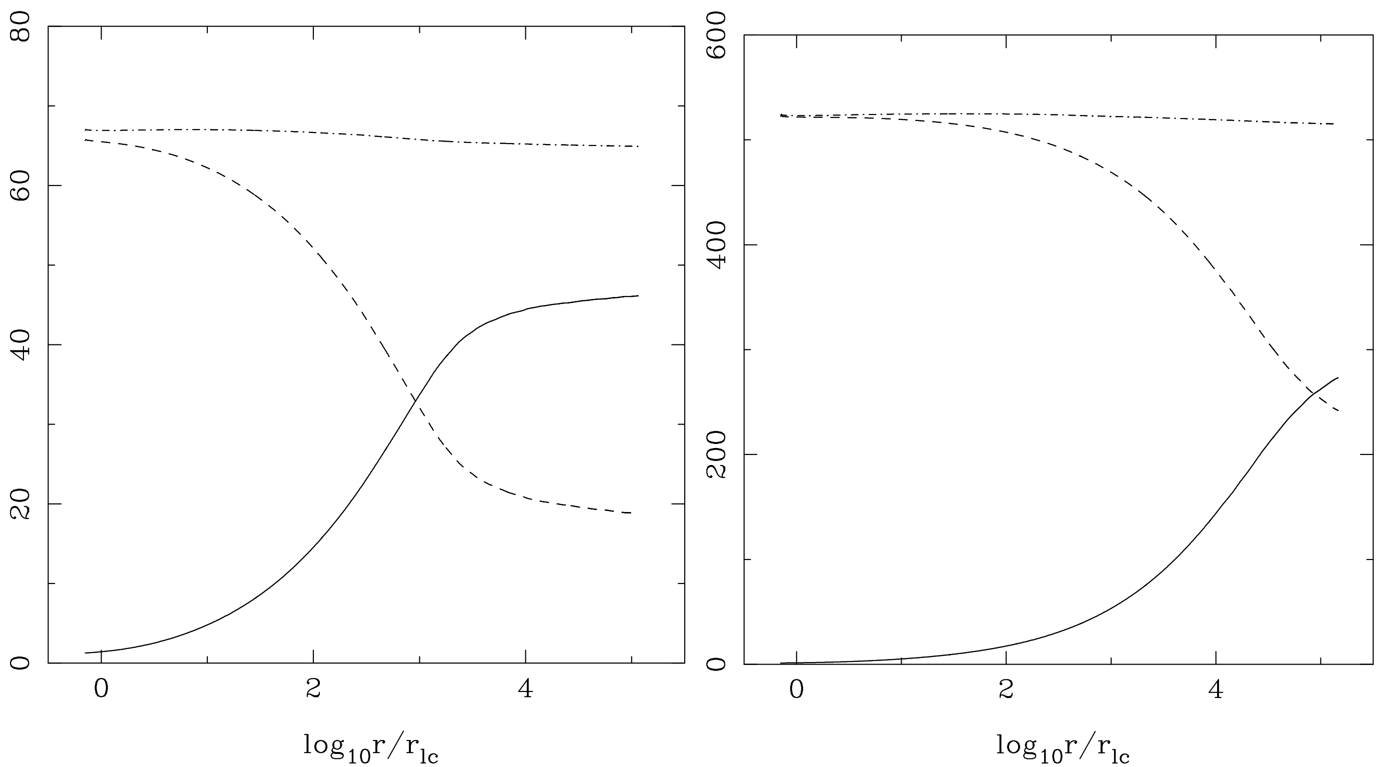


acceleration efficiency $\gtrsim 50\%$



(a)
z (pc)
ϖ (pc)
inner
outer
(b)
$\dot{M}$ (g s^{-1})
ϖ_{out}/ϖ_{in}
(c)
dyn cm^{-3}
magnetic
centrifugal
pressure
ϖ/ϖ_A
(d)
(Poynting flux)/(mass flux)c^2
$\gamma \approx \xi\gamma$
(e)
V_z/c
V_ϕ/c
V_ϖ/c
(f)
T(^{0}K)
(g)
$-B_\phi$(G)
B_z(G)
B_ϖ(G)
ϖ/ϖ_A

Simulations of special relativistic jets (e.g. Komissarov+2009)



energy flux ratios:

$\gamma = \frac{\text{kinetic}}{\text{rest mass}}$

$\gamma\sigma = \frac{\text{Poynting}}{\text{rest mass}}$

$(\sigma = \frac{\text{Poynting}}{\text{kinetic}})$

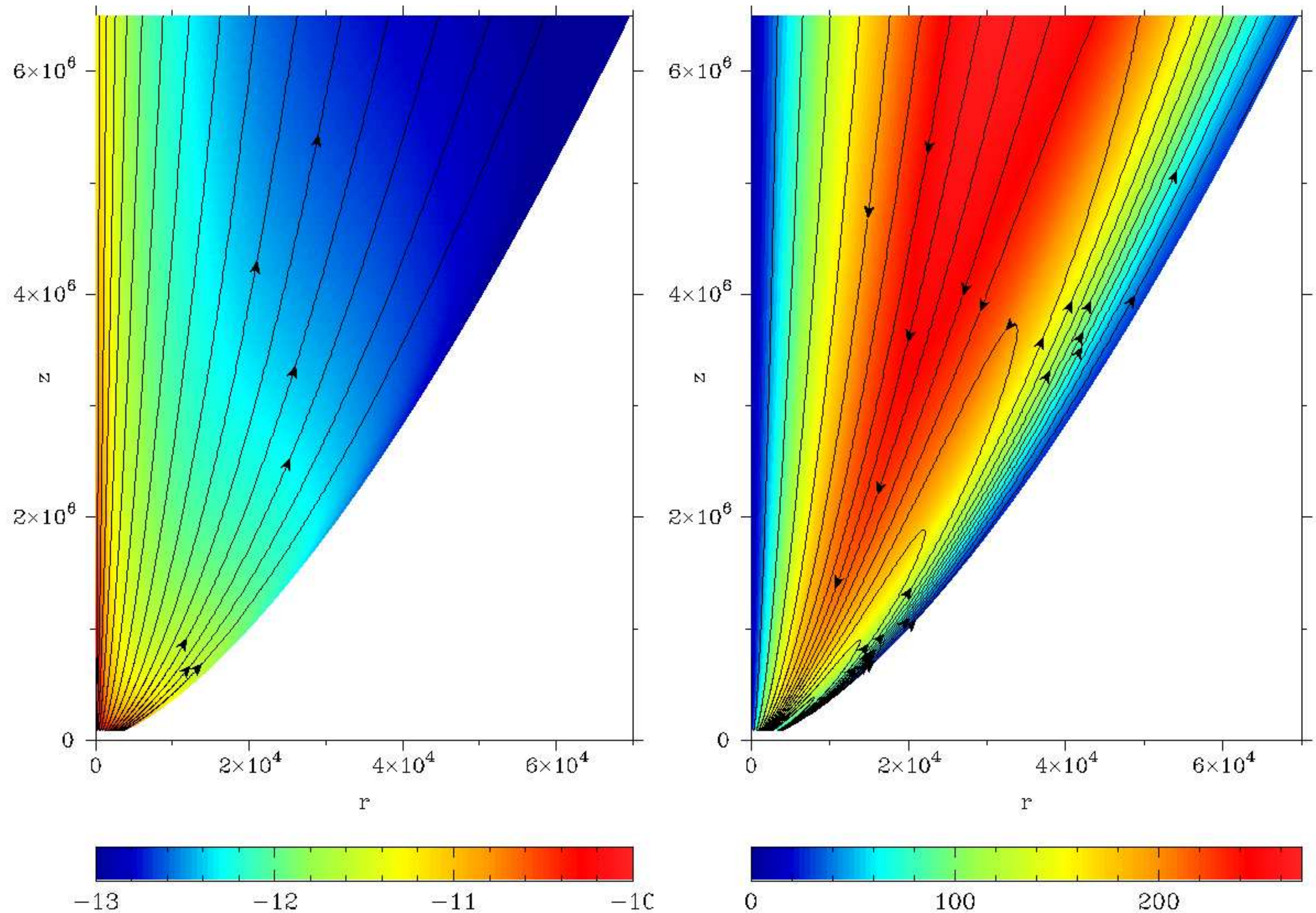
$\mu = \gamma + \gamma\sigma$

γ (increasing),

$\gamma\sigma$ (decreasing),

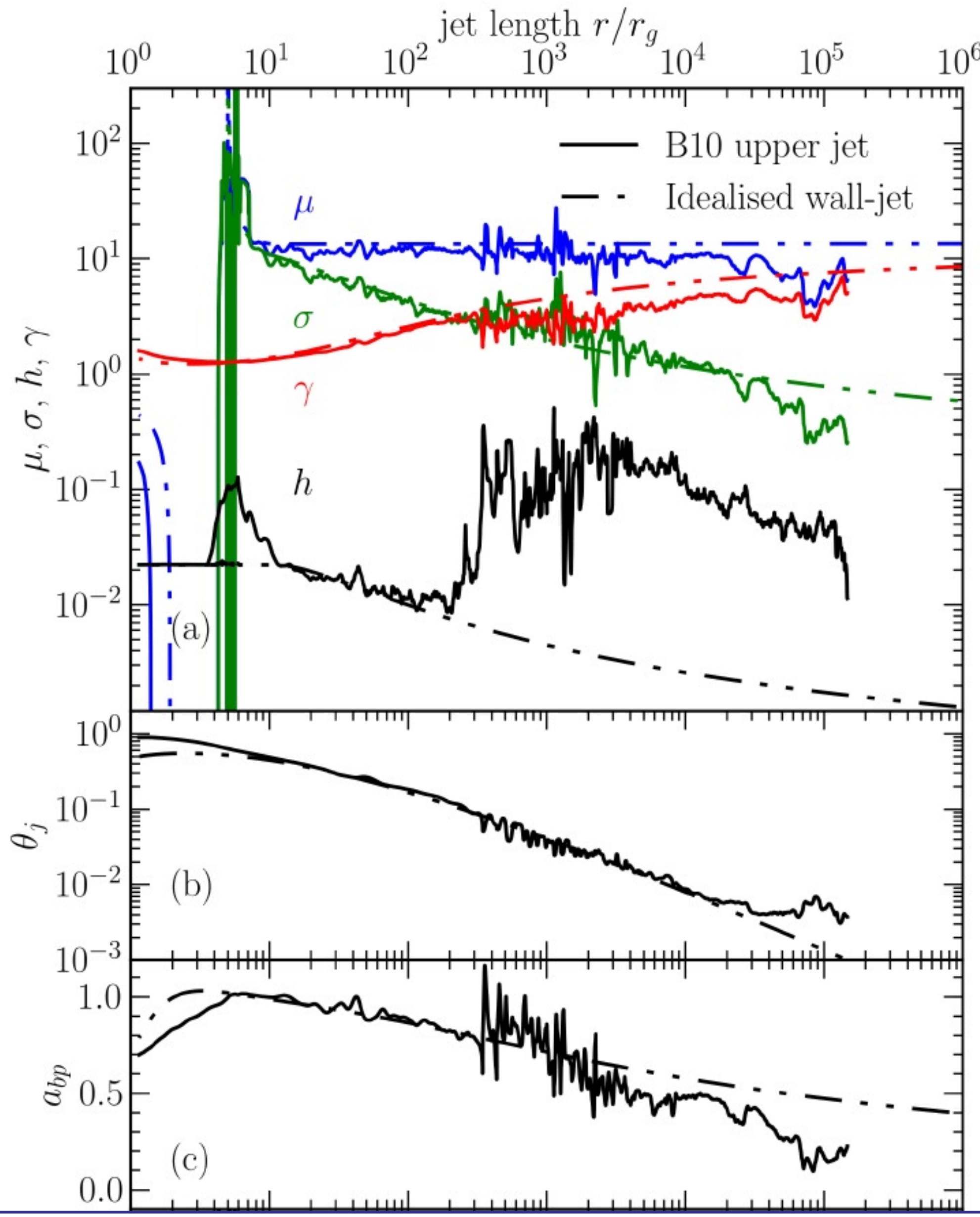
and μ (constant)

efficiency $> 50\%$



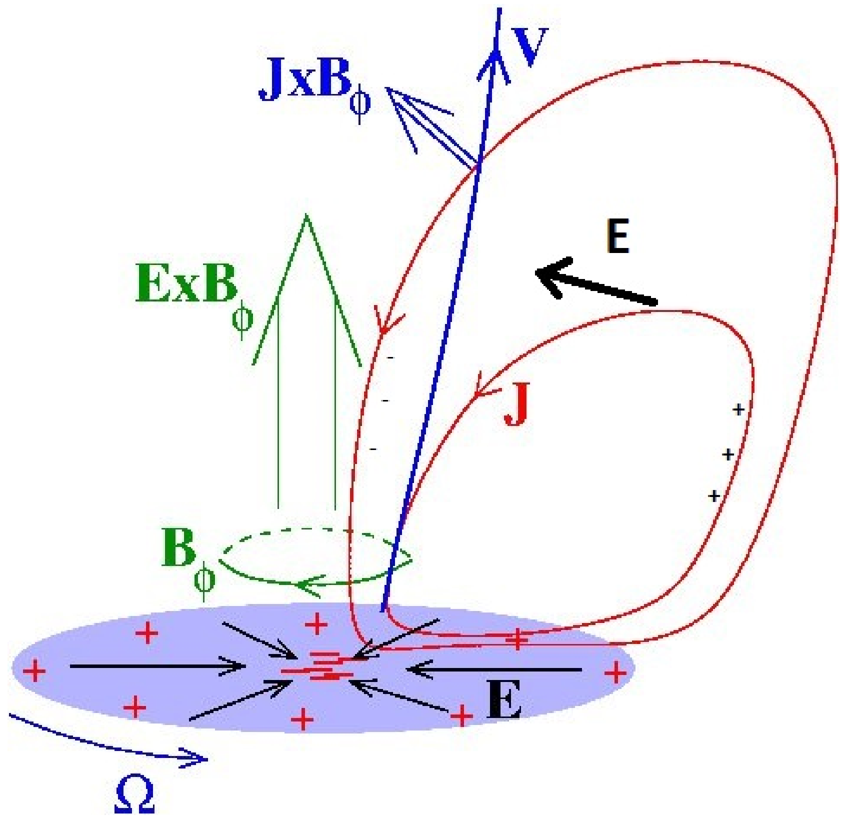
left: density/field lines, right: Lorentz factor/current lines (jet shape $z \propto r^{1.5}$)

Even in general relativistic MHD jet simulations (Chatterjee+2019 - review Mizuno 2022)



Basic questions: collimation

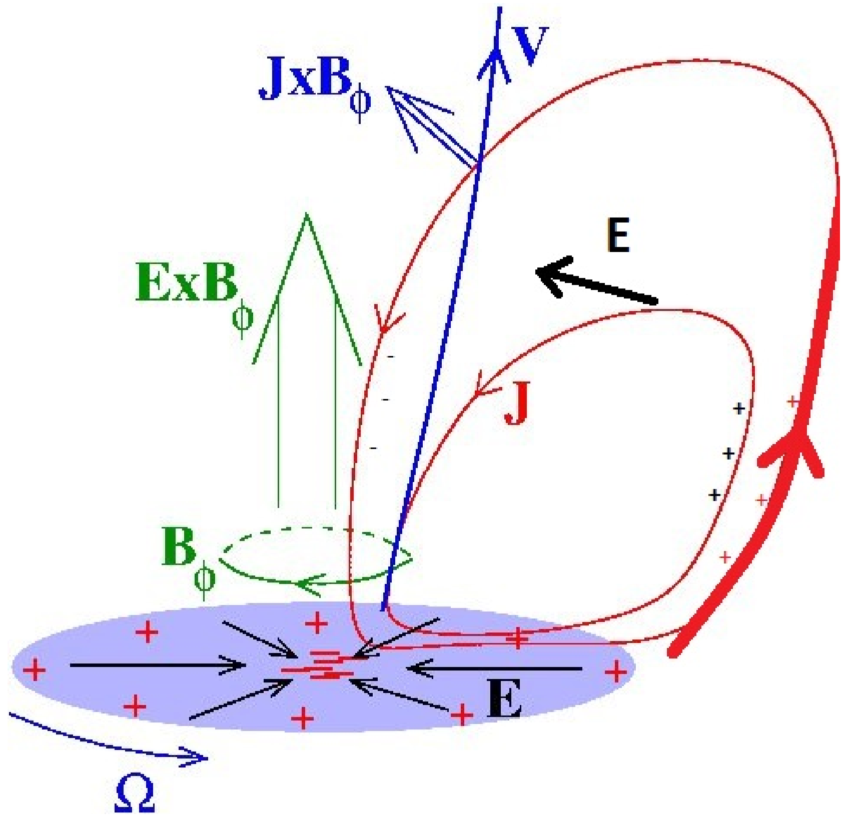
hoop-stress:



+ electric force (acts in the opposite way in the core of the jet)

degree of collimation ? Role of environment?

Spatial scale of γ



pressure equilibrium at the boundary $\frac{B^2 - E^2}{8\pi} = P_{\rm ext}$

ideal conductor $\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B}/c \Rightarrow E \approx V B_\phi / c$
$B \approx B_\phi \propto 1/\varpi$ (from Ampére with approximately constant I)
knowing $P_{\rm ext}(z)$ we find $\gamma = \sqrt{B^2/8\pi P_{\rm ext}}$

- ☞ transfield component of the momentum equation for relativistic jets simplifies to $\mathcal{R} \approx \gamma^2 \varpi$

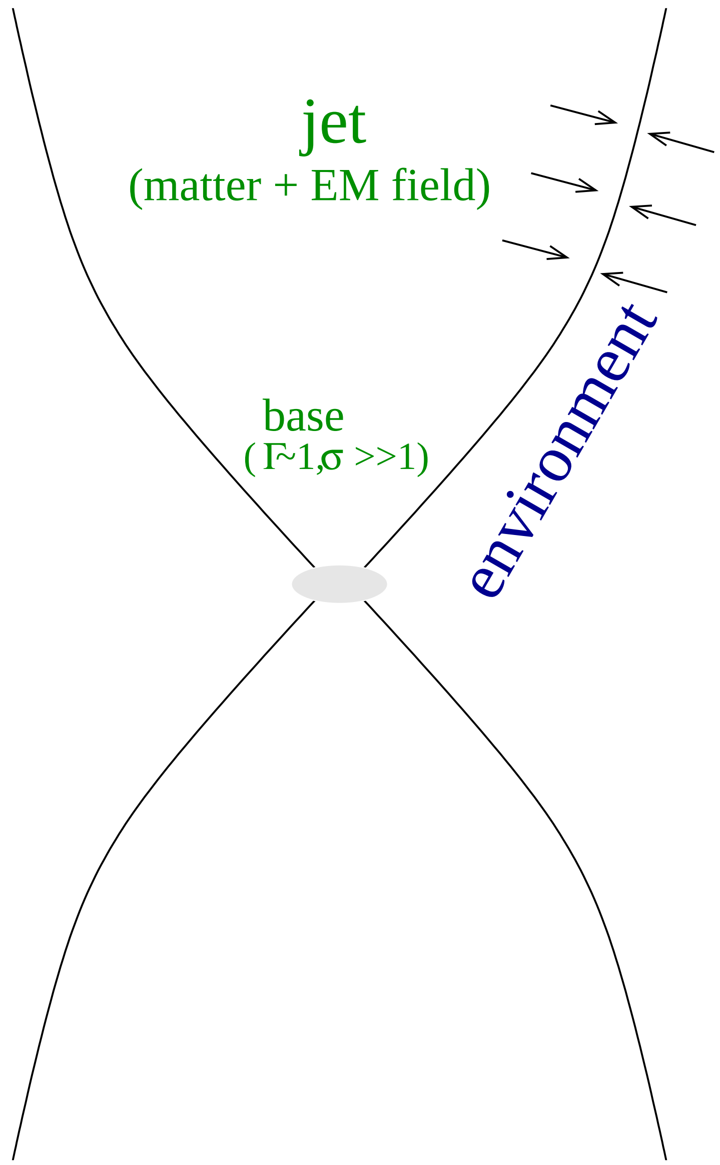
  since $\mathcal{R}^{-1} \approx -\frac{d^2\varpi}{dz^2} \approx \frac{\varpi}{z^2}$ it gives power-law $\gamma \approx z/\varpi$
  (for parabolic shapes $z \propto \varpi^a$, γ is a power of ϖ)

- ☞ role of external pressure

  combining $\mathcal{R} \approx \gamma^2 \varpi$ with $\gamma = \sqrt{B^2/8\pi P_{\rm ext}}$:

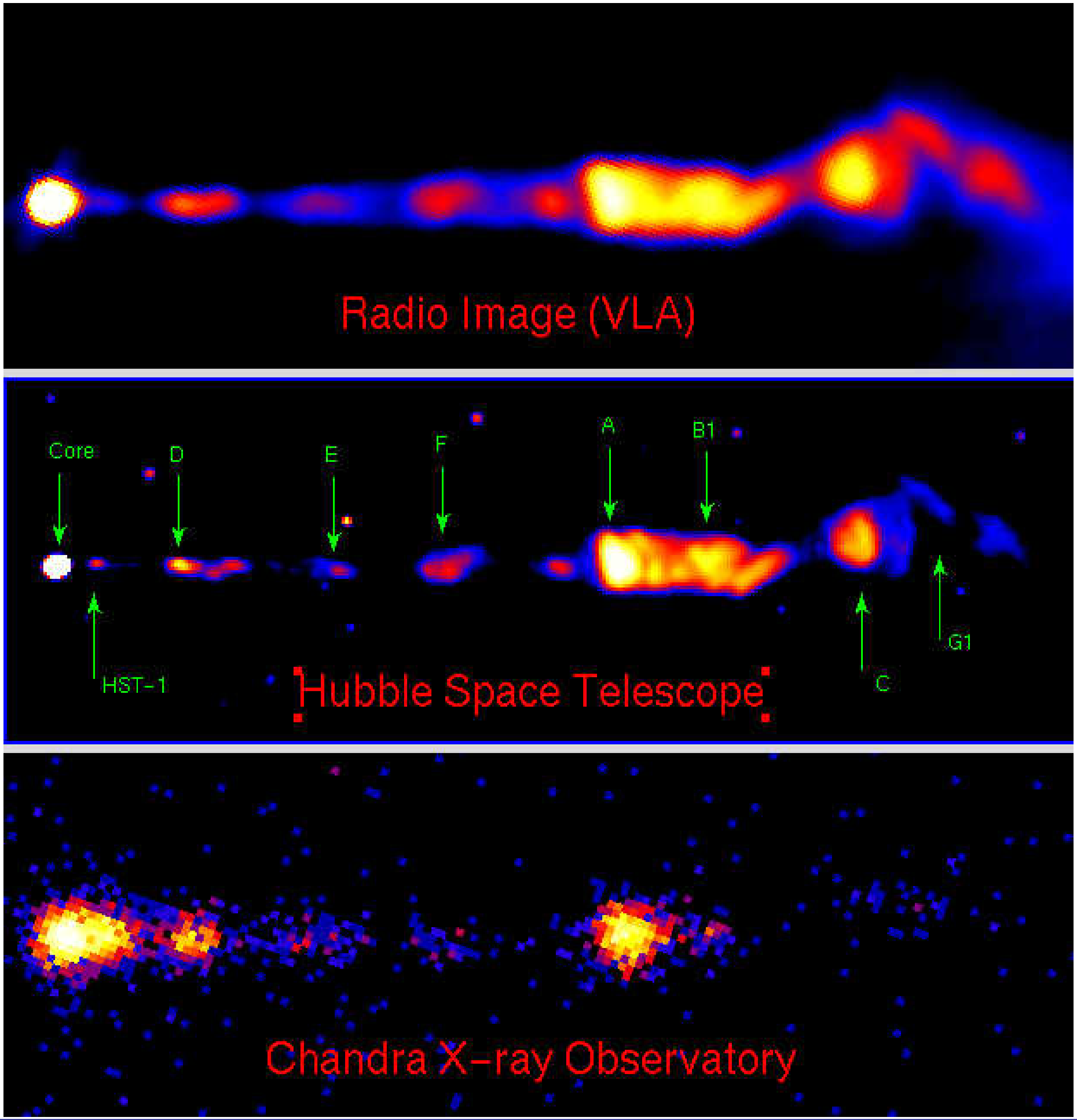
  - if the pressure drops slower than z^{-2} then
    - ⋆ shape more collimated than $z \propto \varpi^2$
    - ⋆ linear acceleration $\gamma \propto \varpi$
  - if the pressure drops as z^{-2} then
    - ⋆ parabolic shape $z \propto \varpi^a$ with $1 < a \leq 2$
    - ⋆ first $\gamma \propto \varpi$ and then power-law acceleration $\gamma \sim z/\varpi \propto \varpi^{a-1}$
  - if pressure drops faster than z^{-2} then
    - ⋆ conical shape
    - ⋆ linear acceleration $\gamma \propto \varpi$ (small efficiency)

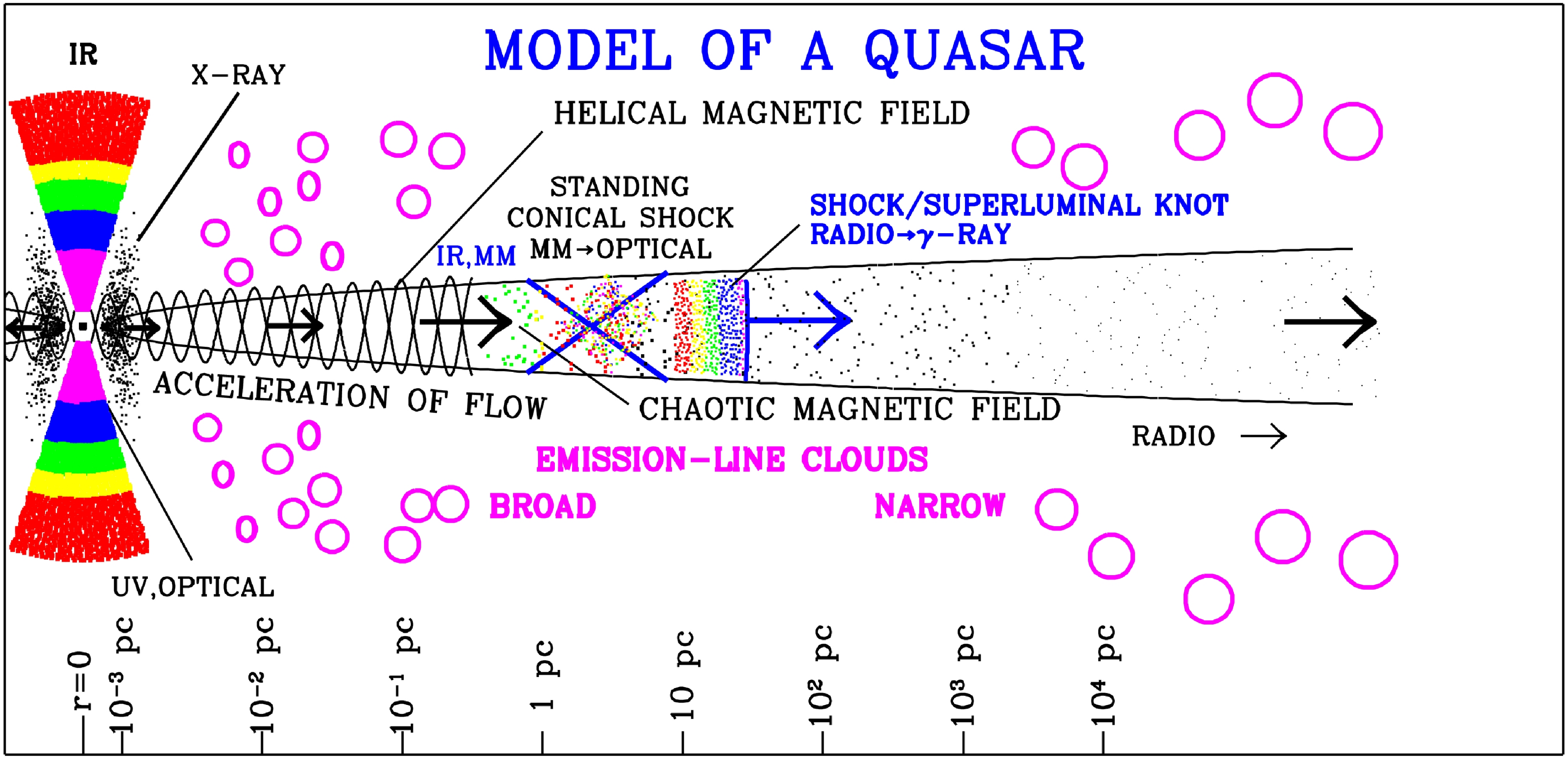
Basic questions



- source of matter/energy?
  disk or central object,
  rotation+magnetic field
- bulk acceleration ✓
- collimation ✓
- role of environment? ✓

2nd level of understanding



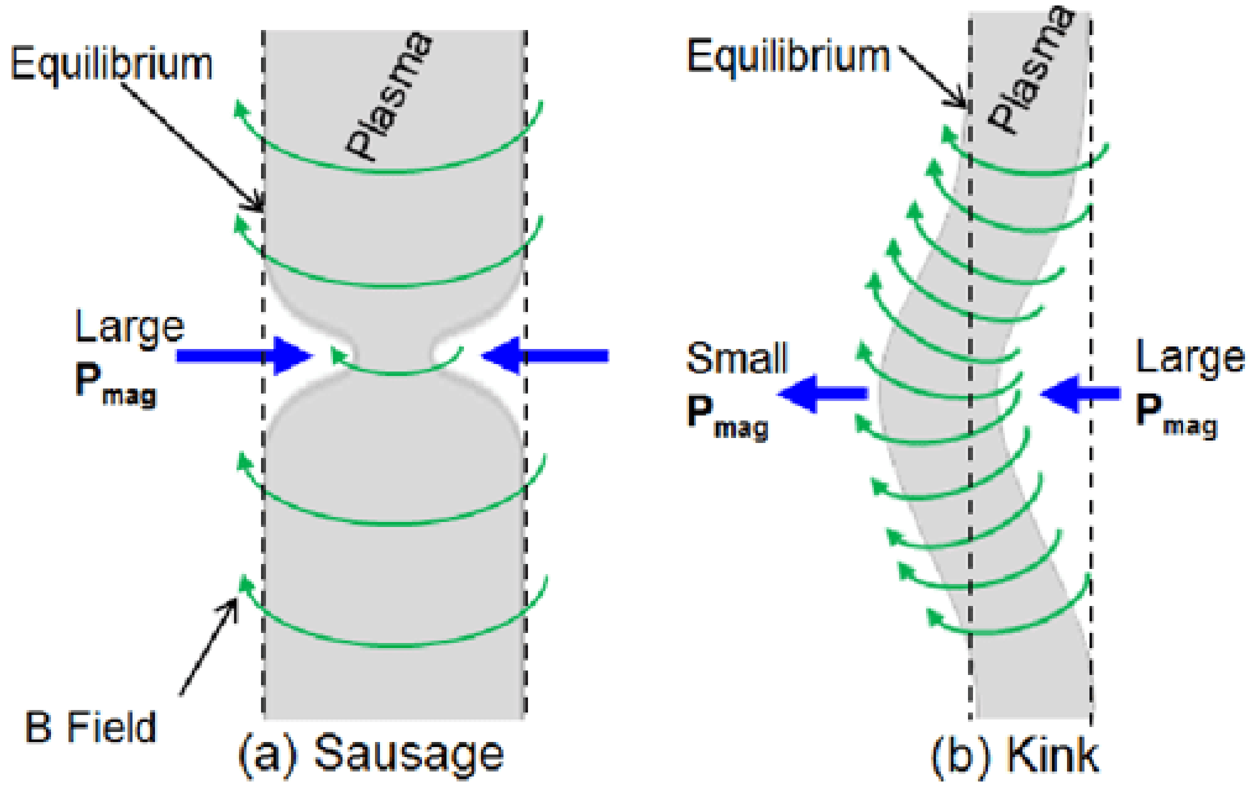


credit: Boston University Blazar Group

- ☞ jet stability (Kelvin-Helmholtz? current driven? centrifugal?)
- ☞ nonthermal radiation – particle acceleration

  shocks or reconnection ? connection with instabilities ?
- ☞ polarization maps and comparison with observations
- ☞ role of resistivity?
- ☞ kinetic description ?
  (combination with magnetohydrodynamics)

Current-driven instabilities

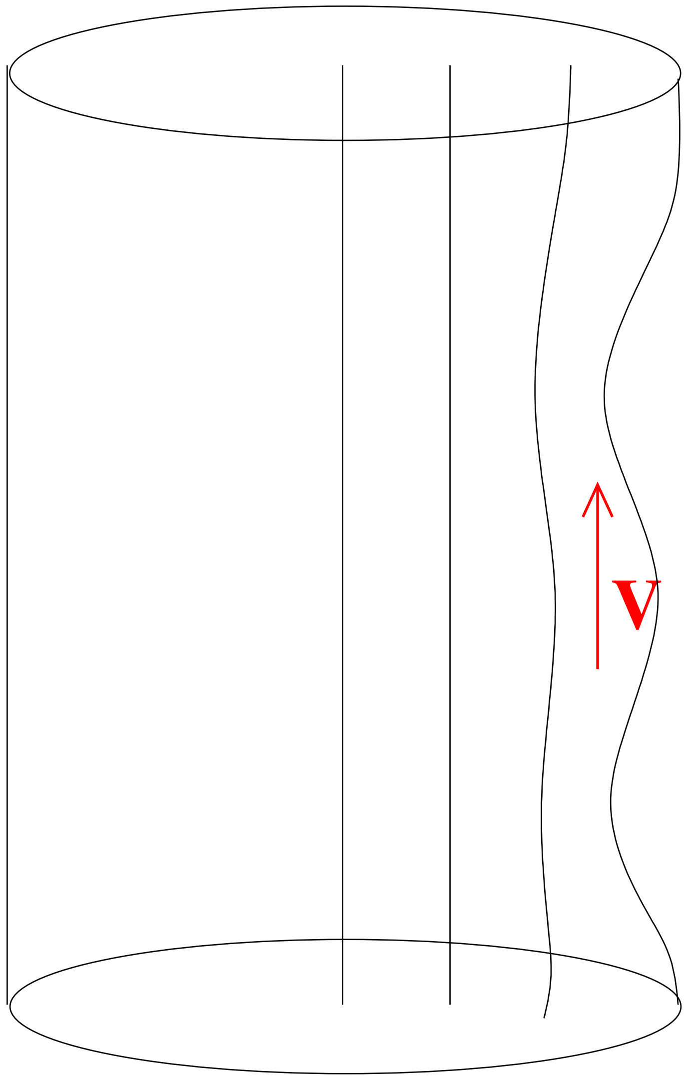


(sketch from Yager-Elorriaga 2017)

Role of B_z? of inertia?

At large distances distances the field is mainly toroidal (since $B_p \propto 1/\varpi^2$, $B_\phi \propto 1/\varpi$)

Kinetic instabilities



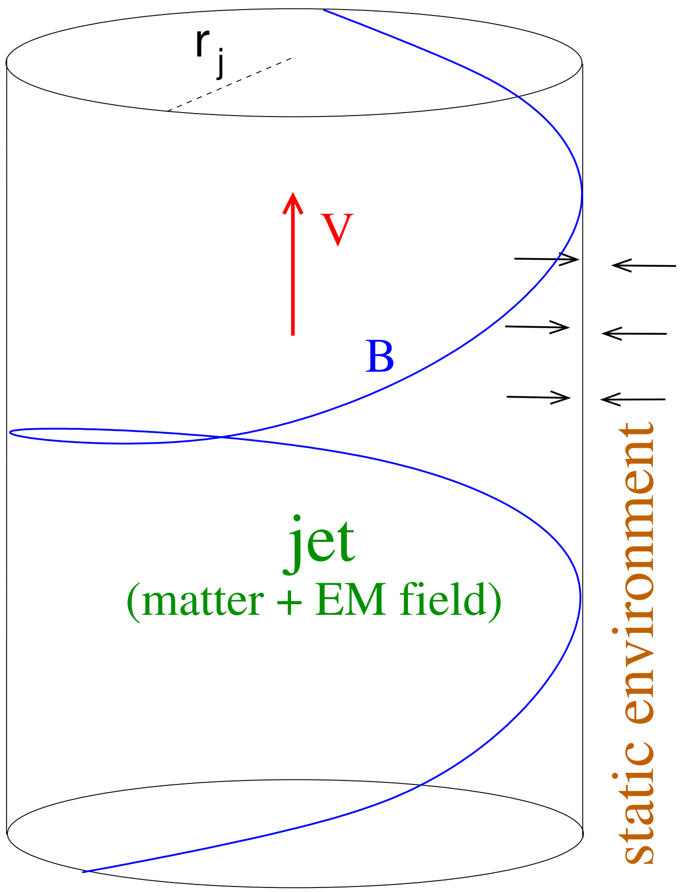
Relative motion drives
Kelvin-Helmholtz instability

For astrophysical jets we need to combine

- magnetic field
- compressibility
- relativity (in bulk speed, sound speed, Alfvén speed)
- cylindrical geometry

Linear stability analysis

Charis Sinnis' PhD work



Unperturbed state:

- Cylindrical jet, cold, with constant speed $V_0\hat{z}$, constant density ρ_0, and helical magnetic field

$$B_{0z} = \frac{B_0}{1+(\varpi/\varpi_0)^2}, \quad B_{0\phi} = B_{0z}\gamma\frac{\varpi}{\varpi_0}$$

(satisfying the force balance equation).
B_0 controls the magnetization $\sigma = \dfrac{B_{\rm co}^2}{4\pi\rho_0 c^2}$, ϖ_0 controls the $\dfrac{B_\phi}{B_z}$

- Environment: uniform, static, with density $\eta\rho_{0\,jet}$, either hydrodynamic or cold with uniform B_{0z}

• Add perturbations in all quantities
$Q(\varpi\,,z\,,\phi\,,t) = Q_0(\varpi) + Q_1(\varpi)e^{i(kz+m\phi-\omega t)}$
with integer m, real k, and complex ω (temporal approach),
i.e. $Q = Q_0(\varpi) + Q_1(\varpi)e^{\Im\omega t}e^{i(kz+m\phi-\Re\omega t)}$
(instability corresponds to $\Im\omega > 0$)

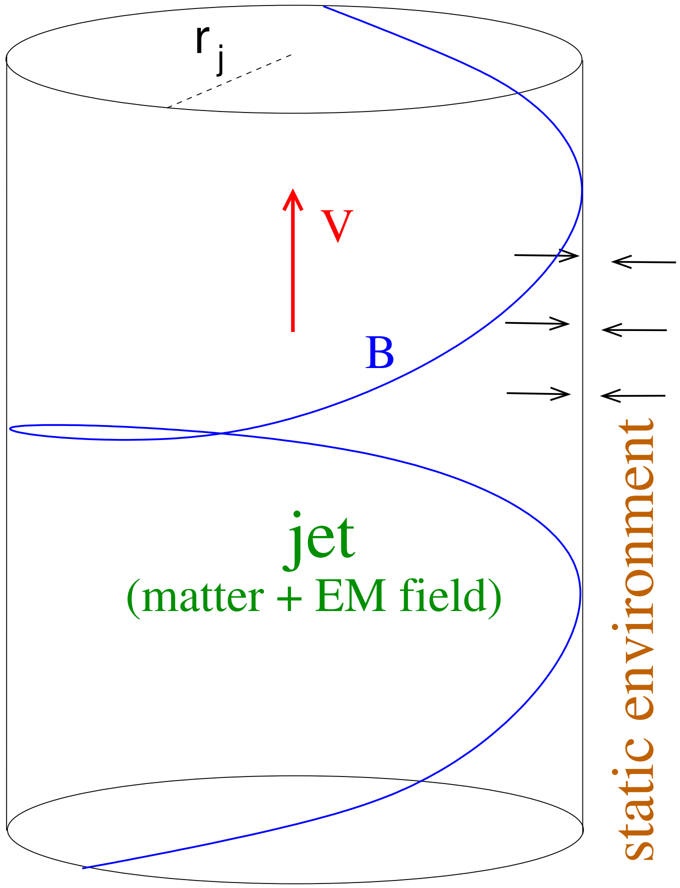
Linearization of the ideal relativistic magnetohydrodynamic equations reduces to two equations with (complex) unknowns:

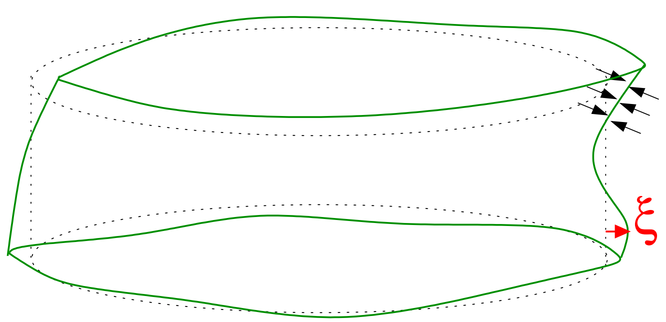
• the Lagrangian displacement of each fluid element in the radial direction y_1

• the perturbation of the total pressure at the displaced position y_2

These should be continuous everywhere (at the interface as well)

Eigenvalue problem





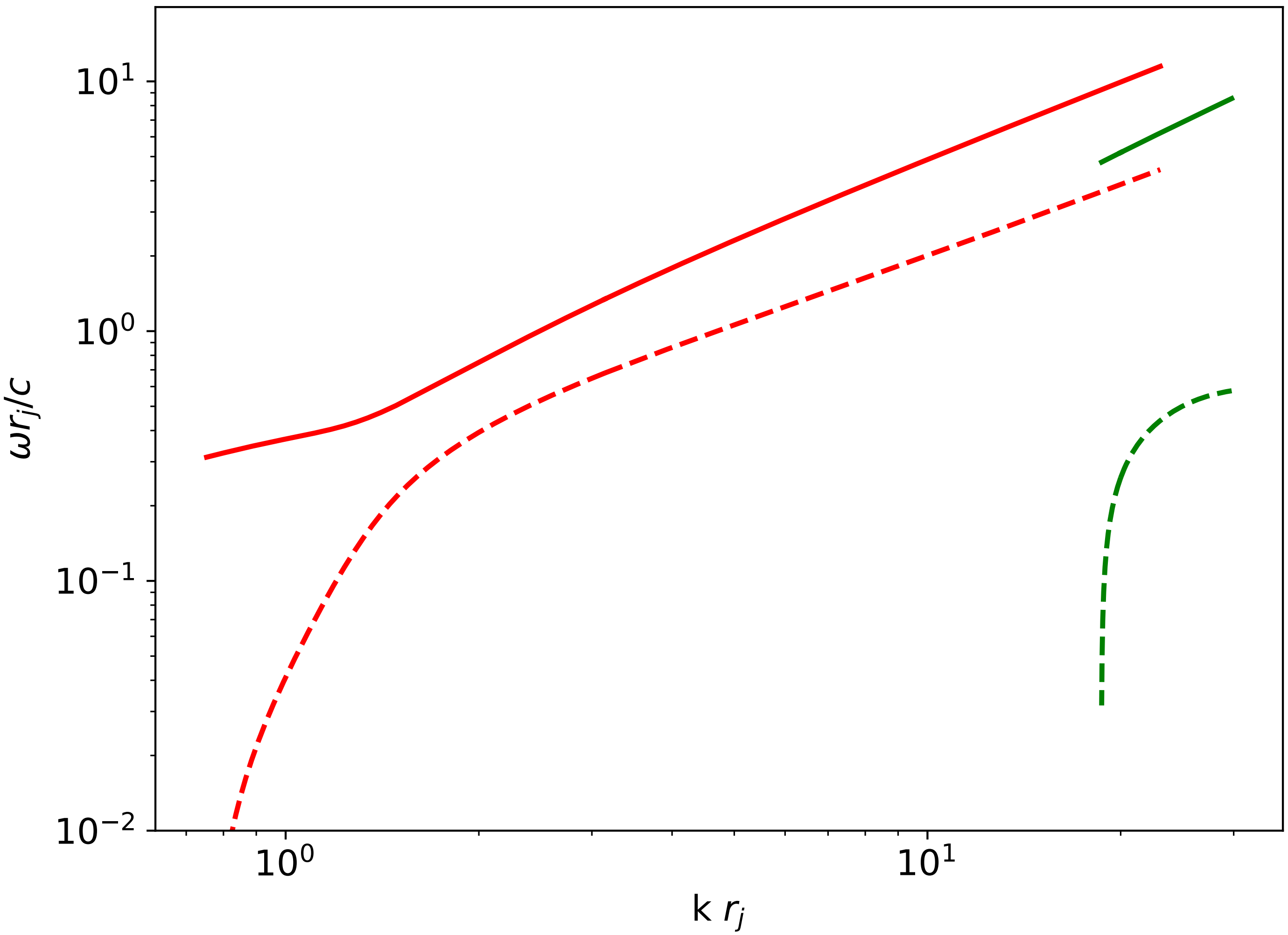
- integrate the equations inside the jet (attention to regularity condition on the axis)

- integrate the equations in the environment (solution vanishes at $\varpi \gg \varpi_j$)

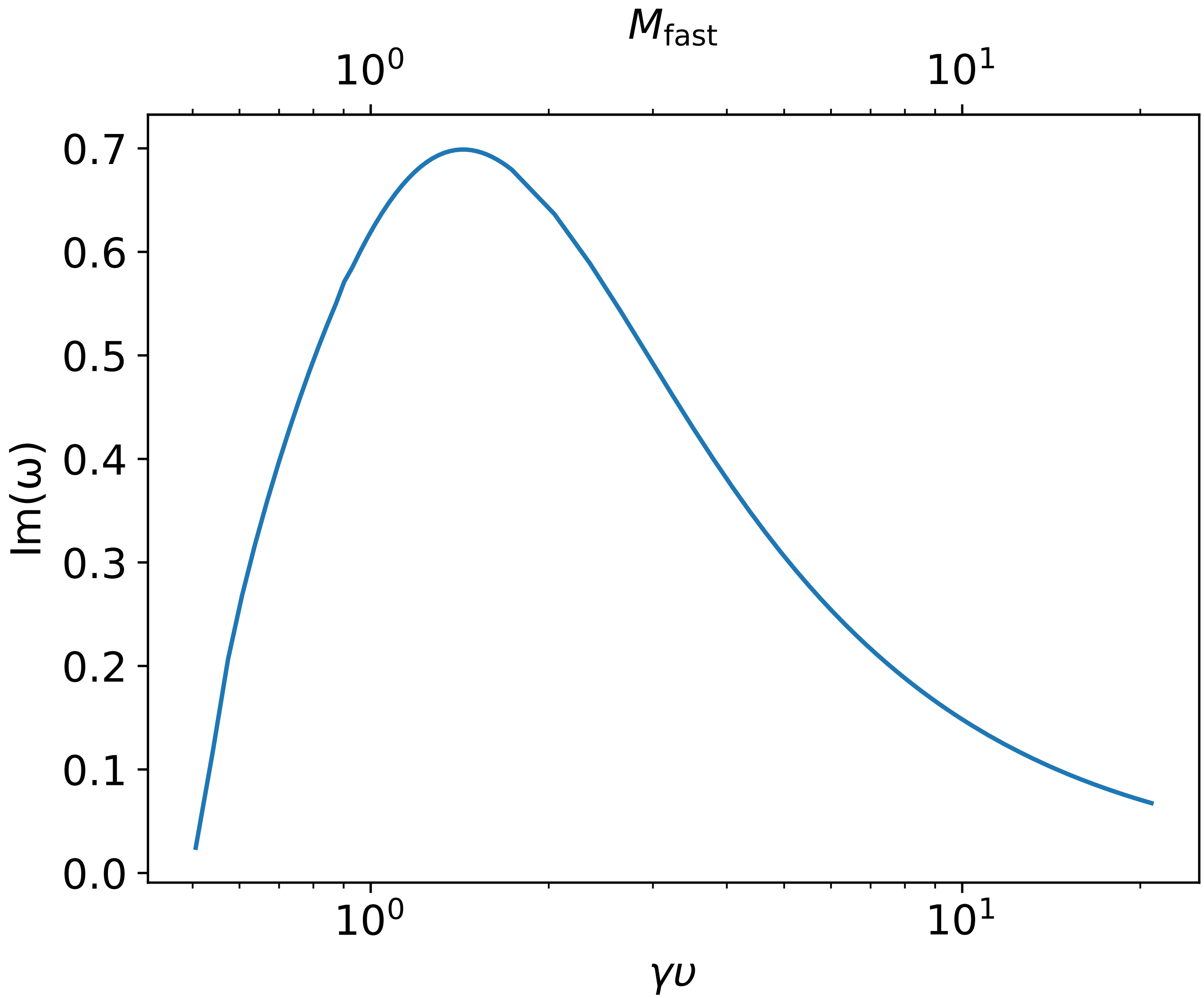
- Match the solutions at ϖ_j: find ω for which y_1 and y_2 are continuous $\longrightarrow$ dispersion relation

- The solution depends on γ, σ, ϖ_0, η, and the wavenumbers k, m

Result for the dispersion relation (Re=solid, Im=dashed), for $\gamma = 2$, $\sigma = 1$ (at ϖ_j), $\varpi_0 = 0.1$, $\eta = 10$, and $m = 0$.
K-H is the most unstable mode.

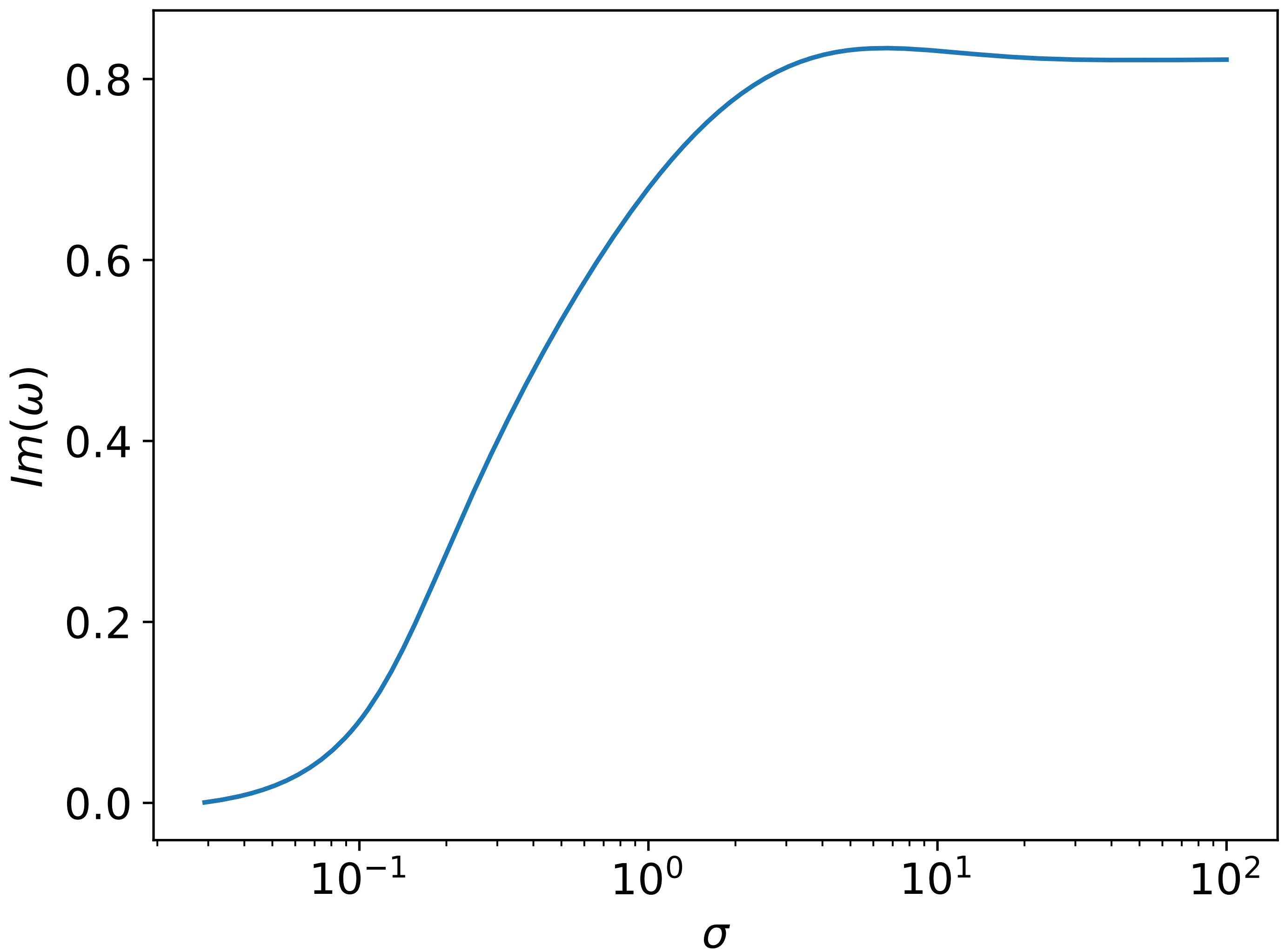


We explore in the following a fiducial case with $k = \pi$

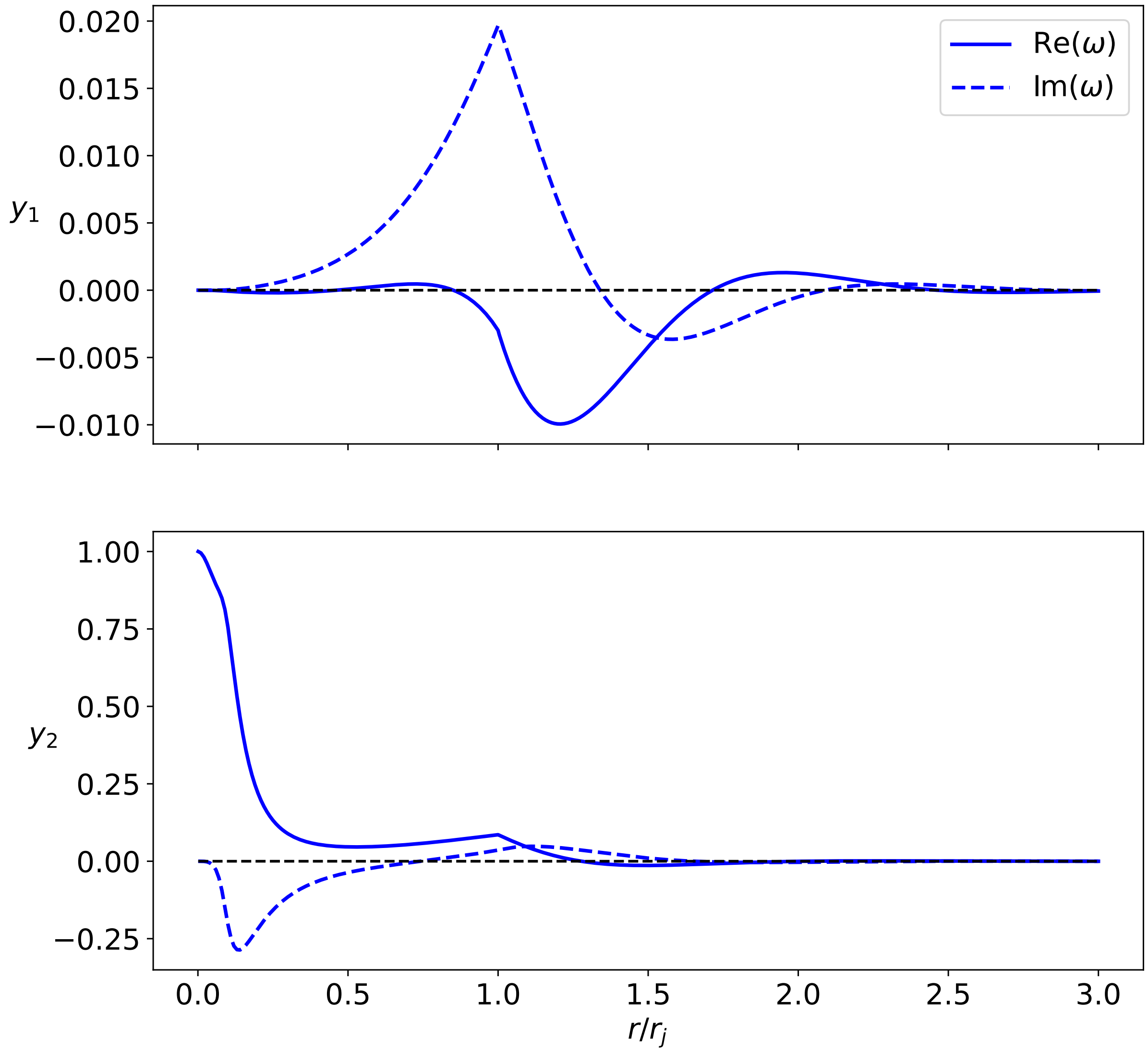


For small speeds $\Im\omega \propto V$ while sufficiently large M_{fast} stabilizes

Dependence on the jet magnetization (at ϖ_j)



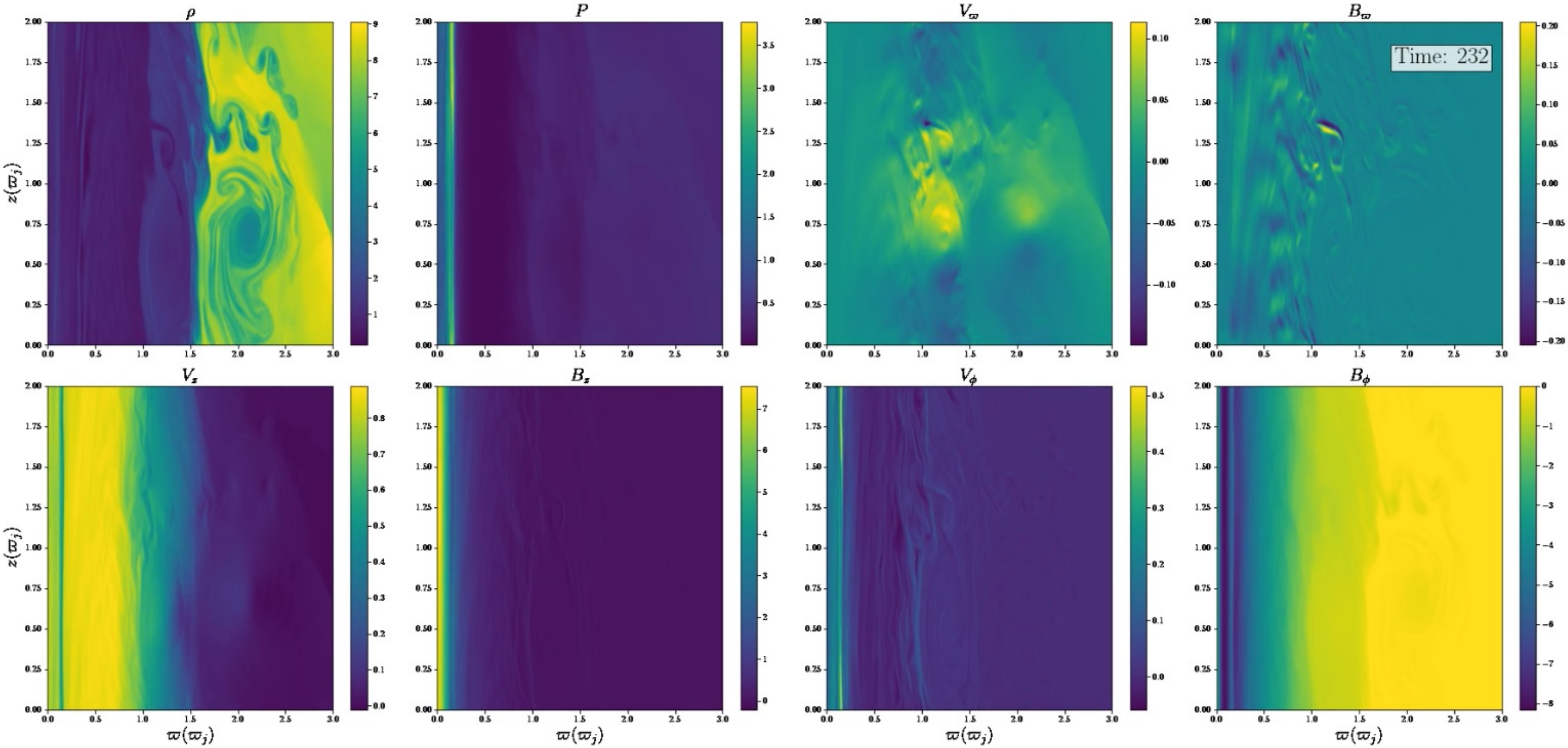
Locality of the eigenfunction y_1 (Lagrangian displacement)



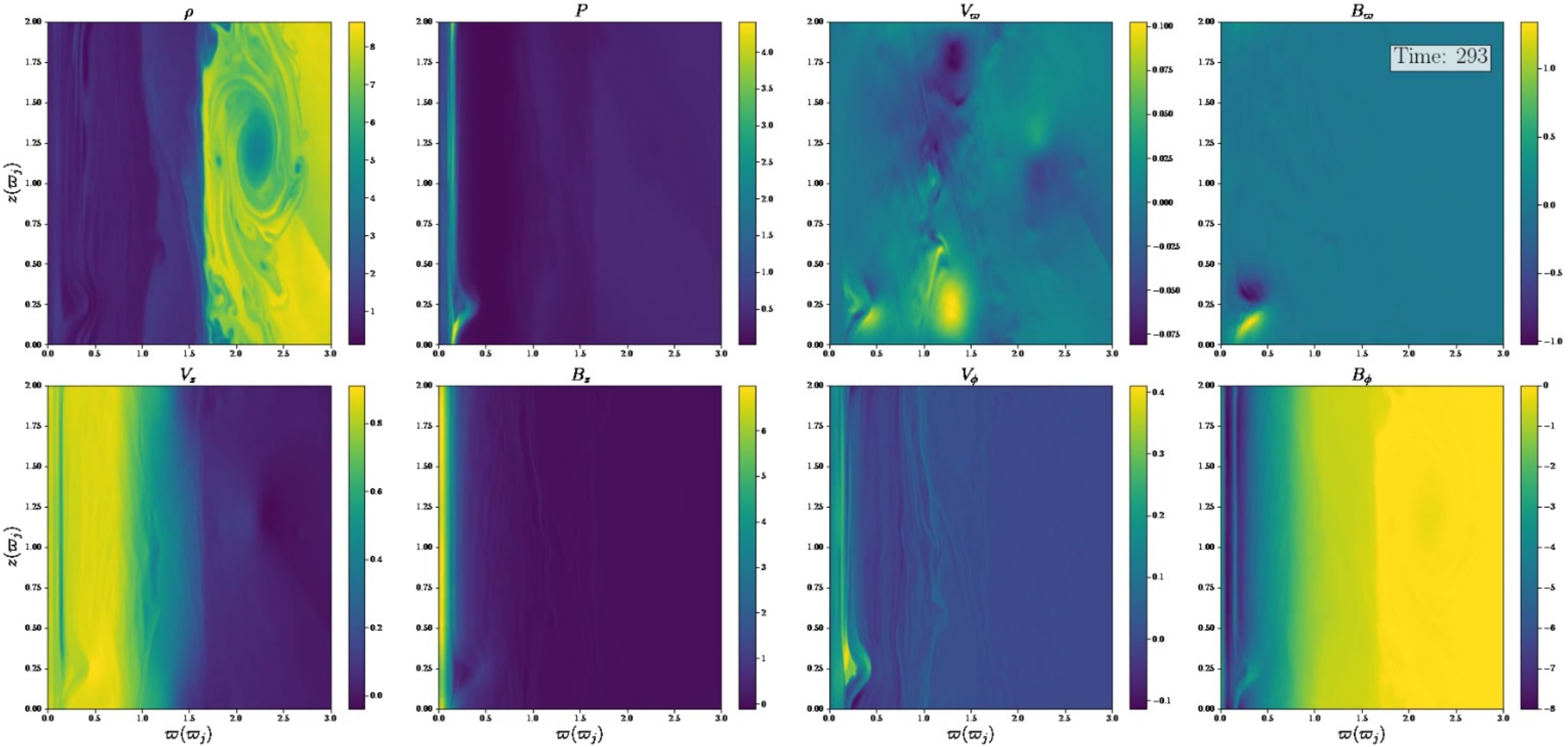
Nonlinear evolution

Thodoris Nousias' master thesis

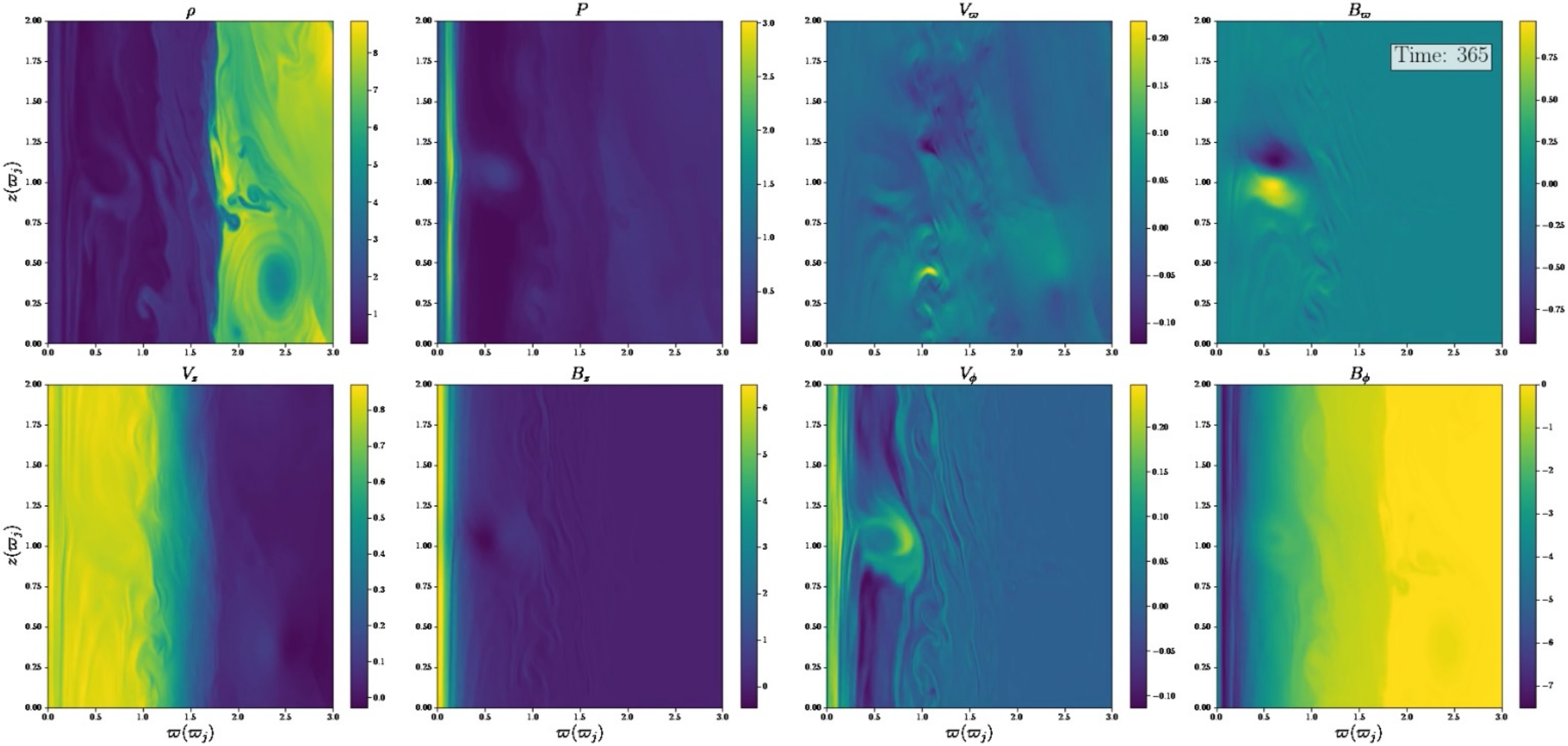
Simulation using the PLUTO code, with initial condition eigenfunction of the linear analysis (fiducial case).



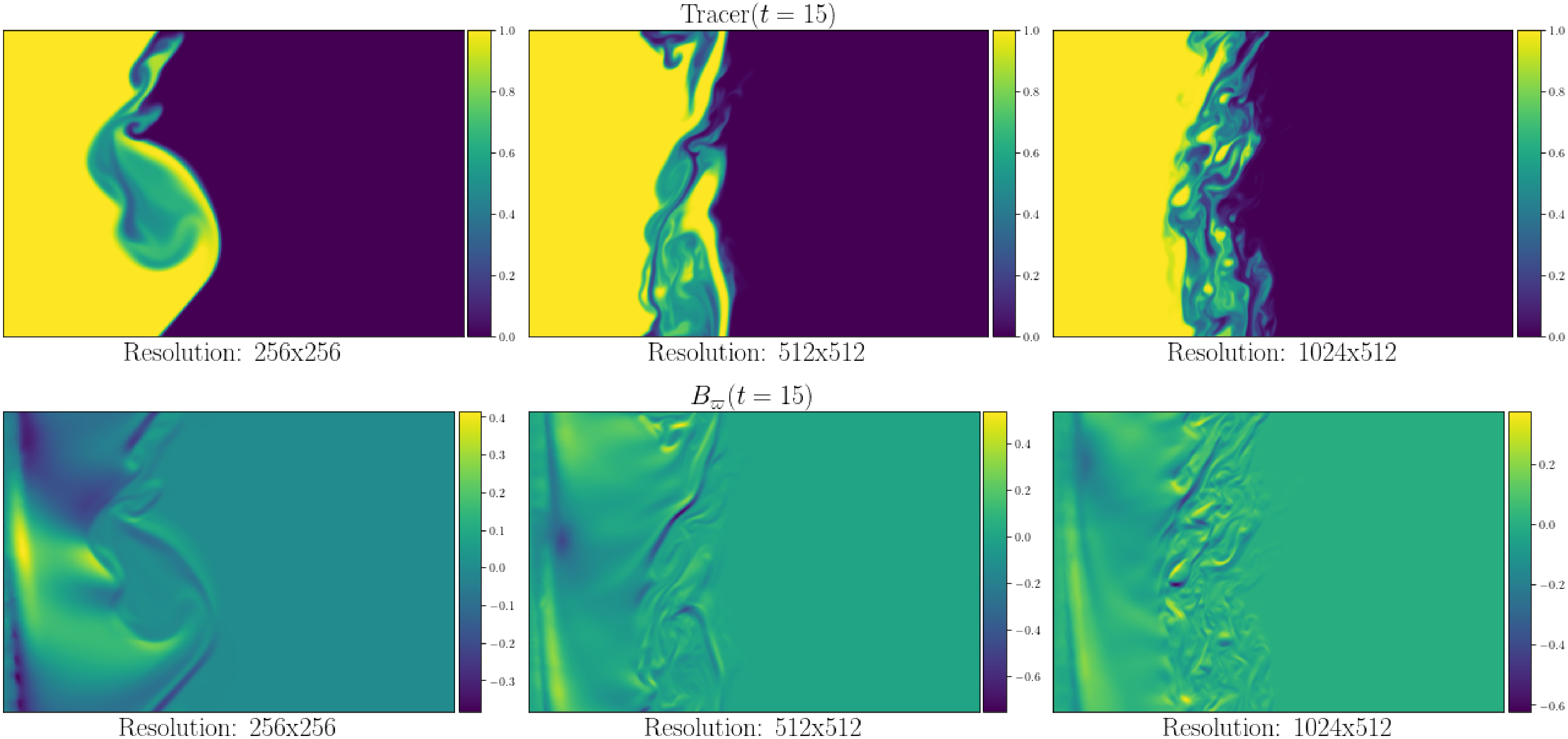
ρ
P
Vϖ
Bϖ
Time: 232
Vz
Bz
Vφ
Bφ
z(ϖj)
ϖ(ϖj)



ρ
P
V_ϖ
B_ϖ
Time: 293
V_z
B_z
V_ϕ
B_ϕ
$z(\varpi_j)$
$\varpi(\varpi_j)$


ρ
P
V_ϖ
B_ϖ
Time: 365
V_z
B_z
V_ϕ
B_ϕ
$z(\varpi_j)$
$\varpi(\varpi_j)$

Resolution effects



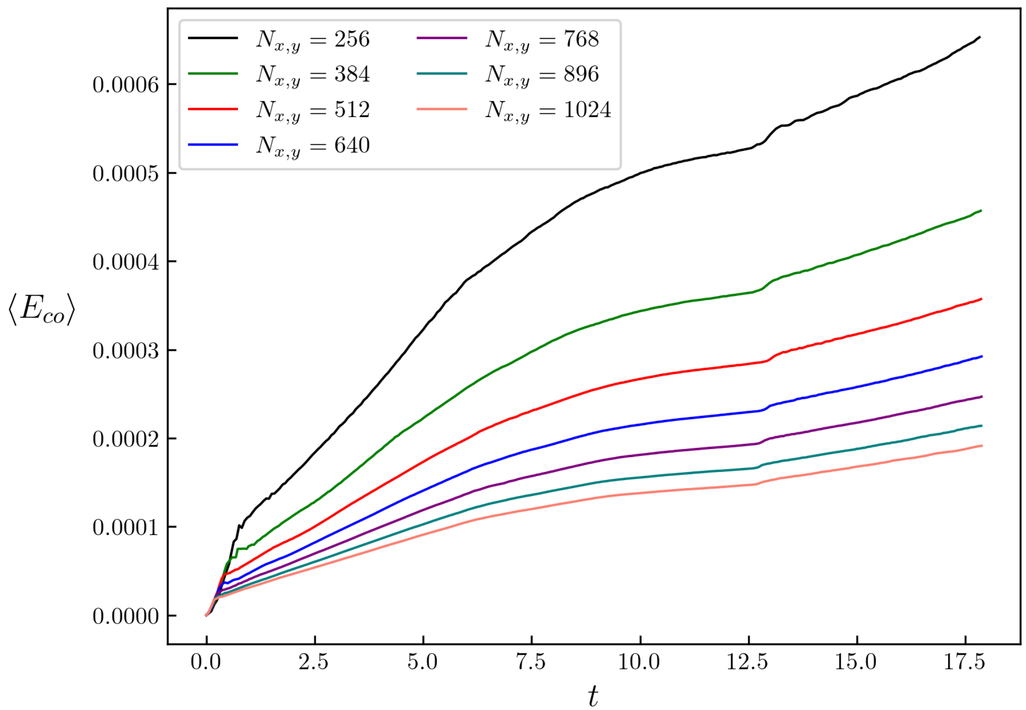
- for kinetic instabilities growth rate increases with k
- cannot be fully resolved
- numerical errors mimic physical diffusion effects

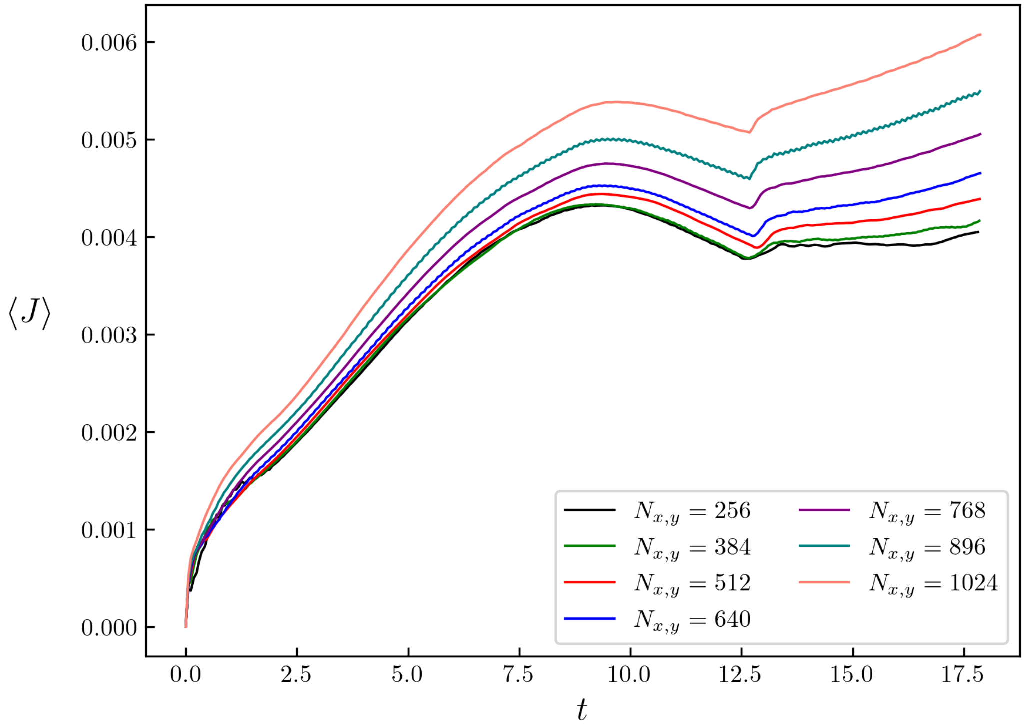
Numerical magnetic diffusivity

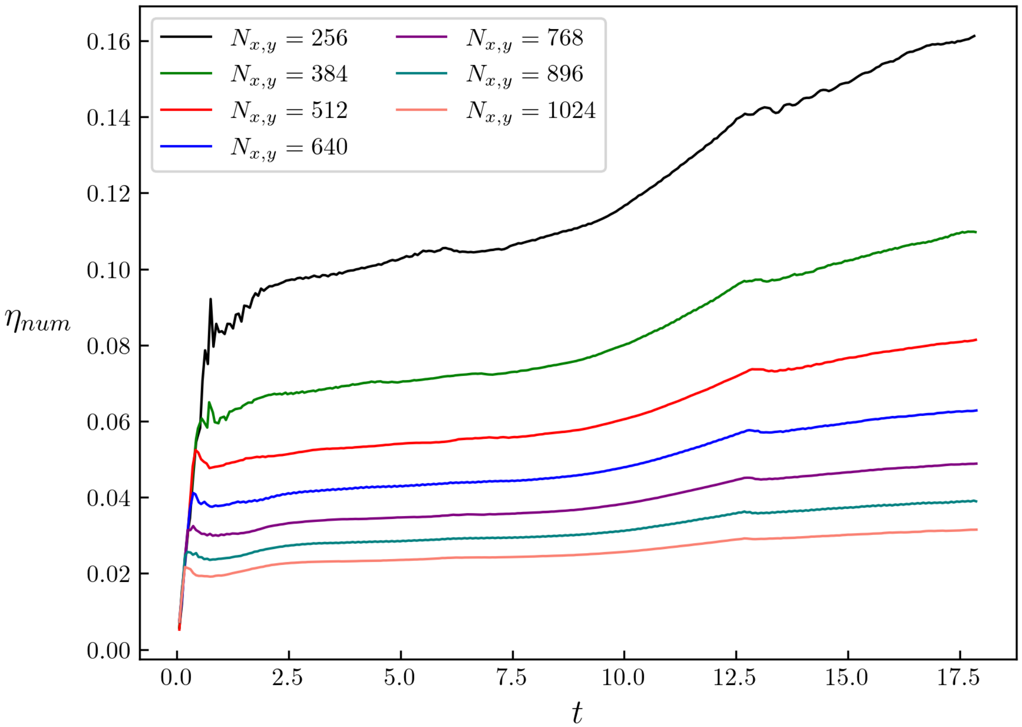
Argyris Loules' PhD thesis

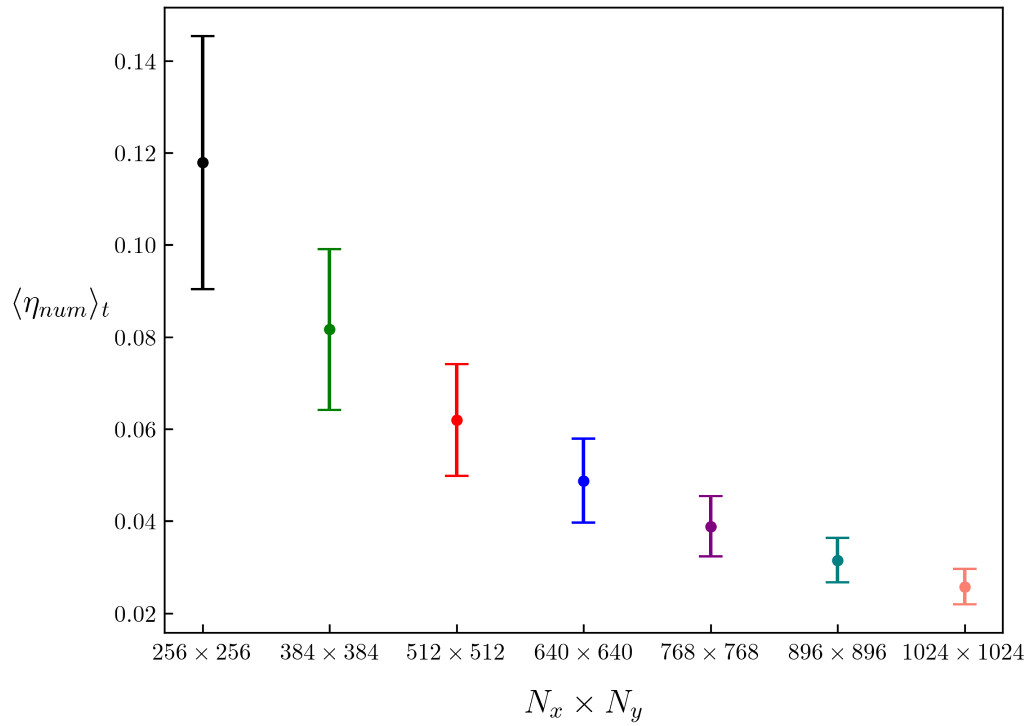
Estimation from "Ohm's law" $\langle J \rangle = \frac{c^2}{4\pi\eta}\langle E \rangle$

(using a numerical experiment of a blast wave in a homogeneous magnetic field)









The cell size defines the magnetic diffusivity ($\eta_{\text{num}} \propto 1/N$).

Effects of physical resistivity cannot be seen if $\eta < \eta_{\text{num}}$

Physical magnetic diffusivity
Argyris Loules' PhD thesis

Magnetic diffusivity affects magnetic field through

the diffusion equation $\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B})$

corresponding Reynolds number $\mathcal{R}_m = \frac{UL}{\eta}$

but also through the Joule heating in the energy equation

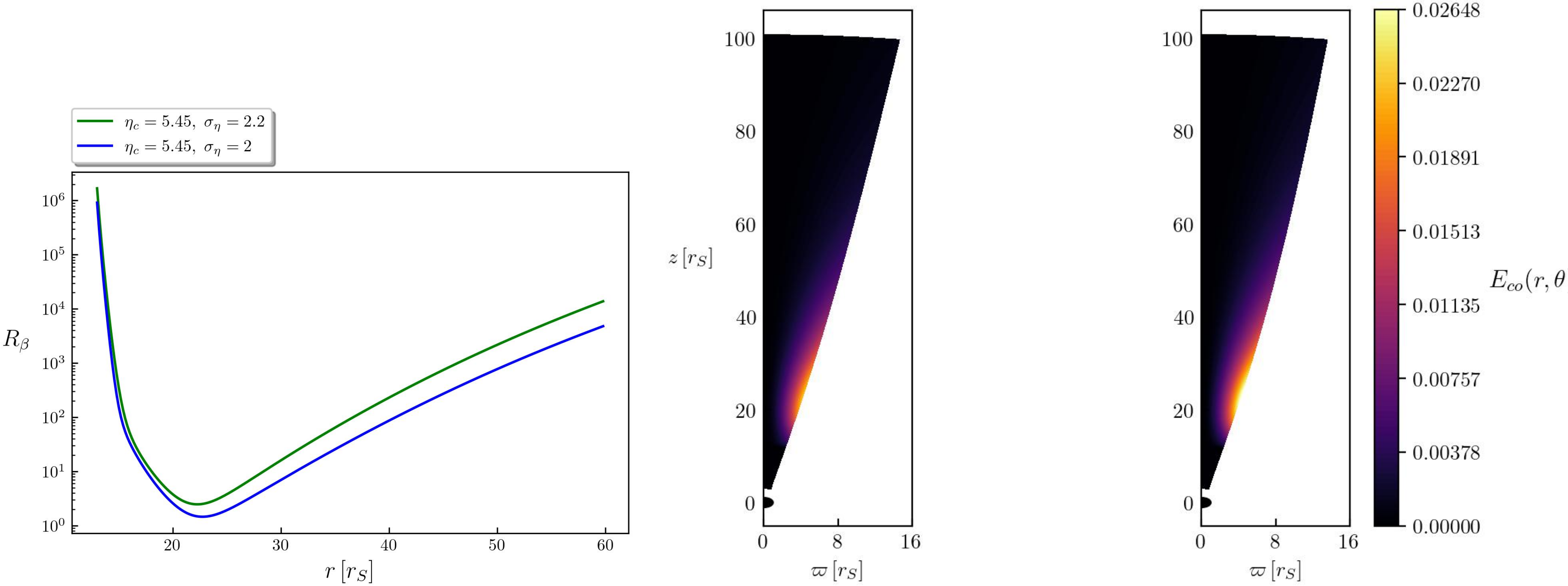
$$\frac{de}{dt} + P\frac{d(1/\rho)}{dt} = \frac{\eta}{4\pi\rho}(\nabla \times \boldsymbol{B})^2$$

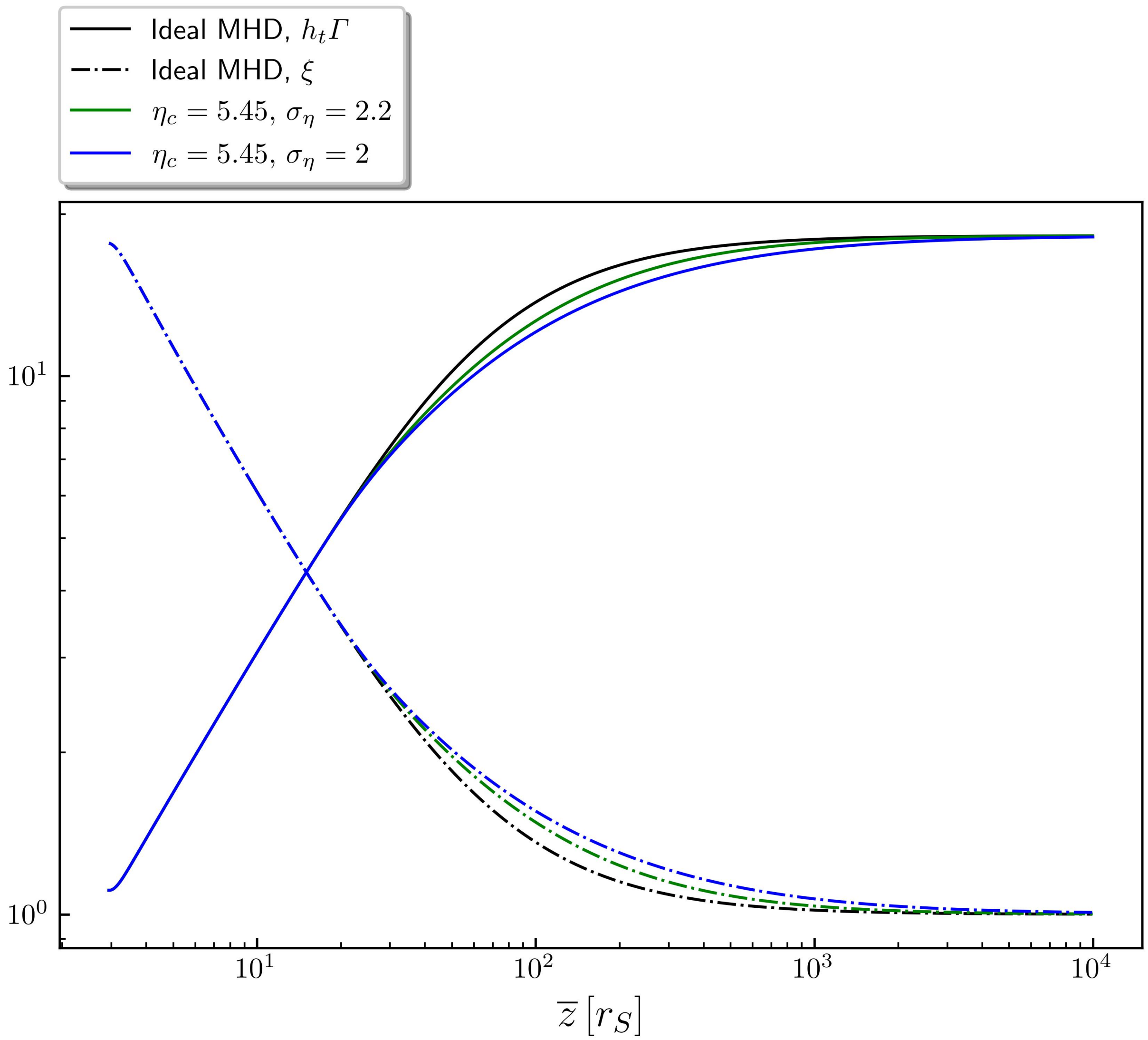
corresponding Reynolds number $\mathcal{R}_\beta = \frac{\beta}{2}\mathcal{R}_m$ (Čemeljić+2008)

Similarly in RMHD.

Analytical results

(based on expansion wrt polar angle θ near the symmetry axis of the jet)





the Joule heating temporarily compensates adiabatic cooling

Summary

- ⋆ magnetic field + rotation $\rightarrow$ Poynting flux extraction
- ⋆ the collimation-acceleration mechanism is very efficient – provides a viable explanation for the bulk jet acceleration
- ⋆ environment significantly affects jet dynamics in the acceleration-collimation zone (jet-shape, spatial scale of γ)
- ⋆ Kelvin-Helmholtz is the most unstable mode for heavy jets with mildly relativistic speeds, high magnetizations
- ⋆ interesting to analyze the nonlinear evolution via simulations but also analytically isolating features

Thank you for your attention