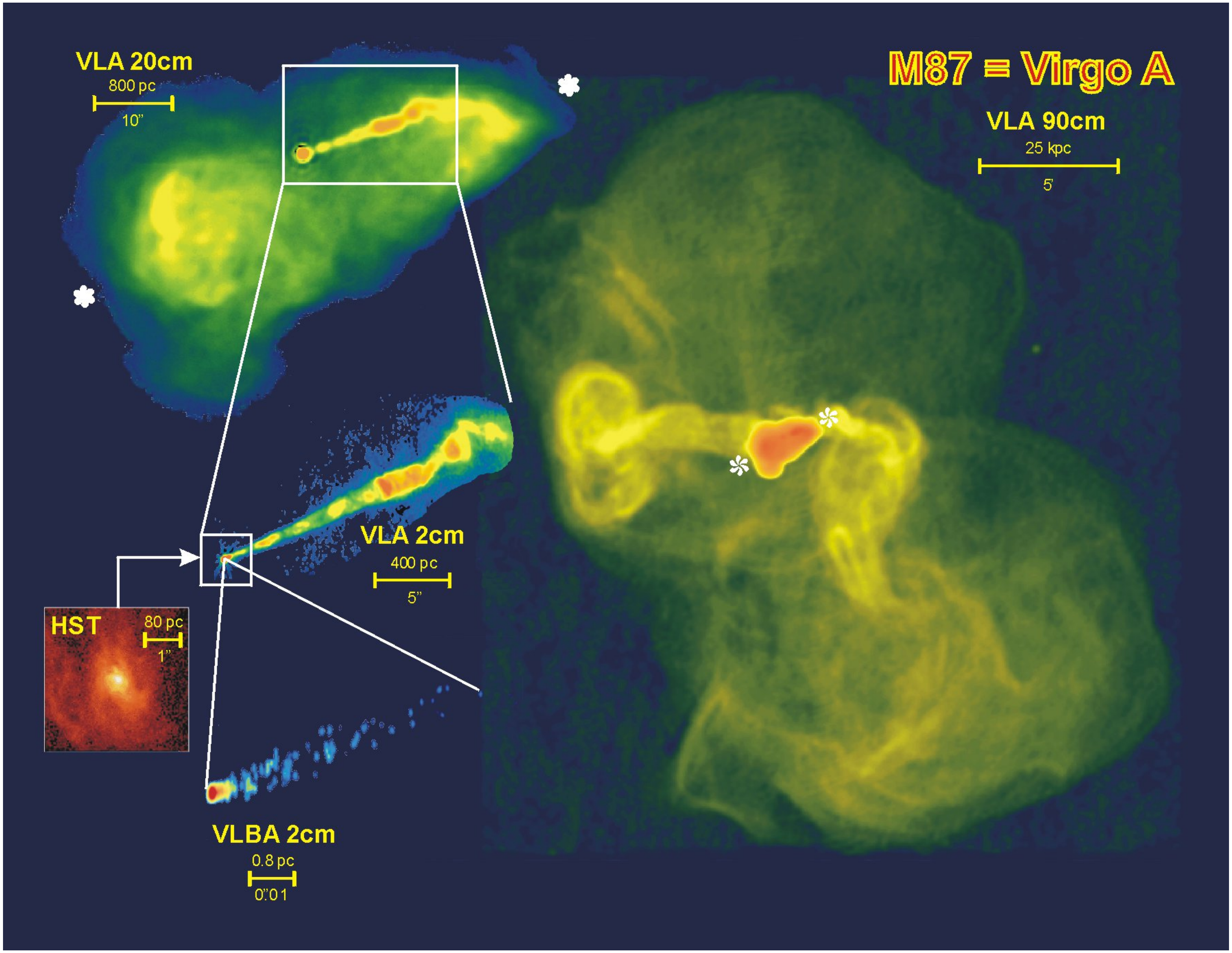
Dynamics of relativistic magnetized jets

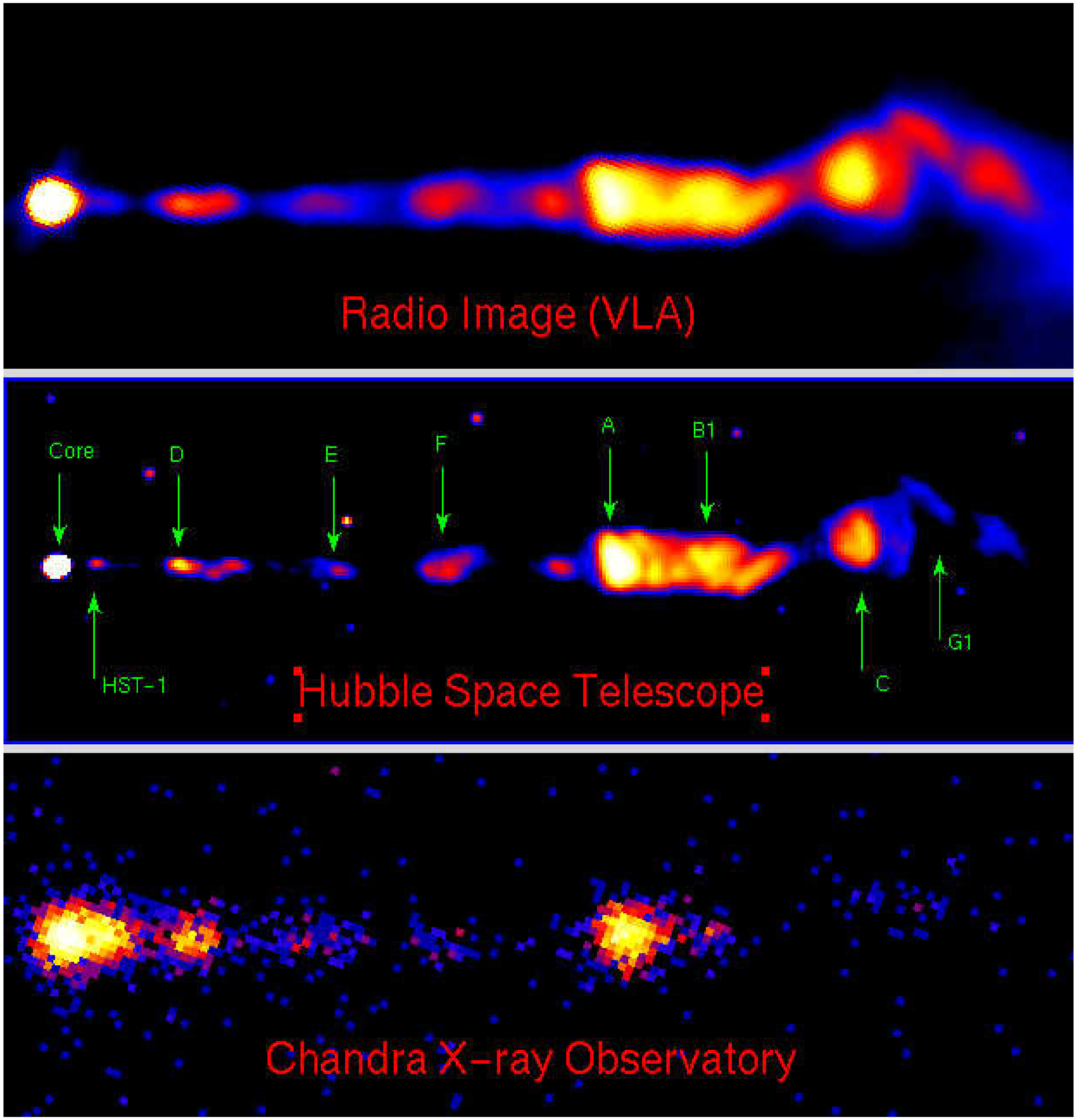
Nektarios Vlahakis
University of Athens

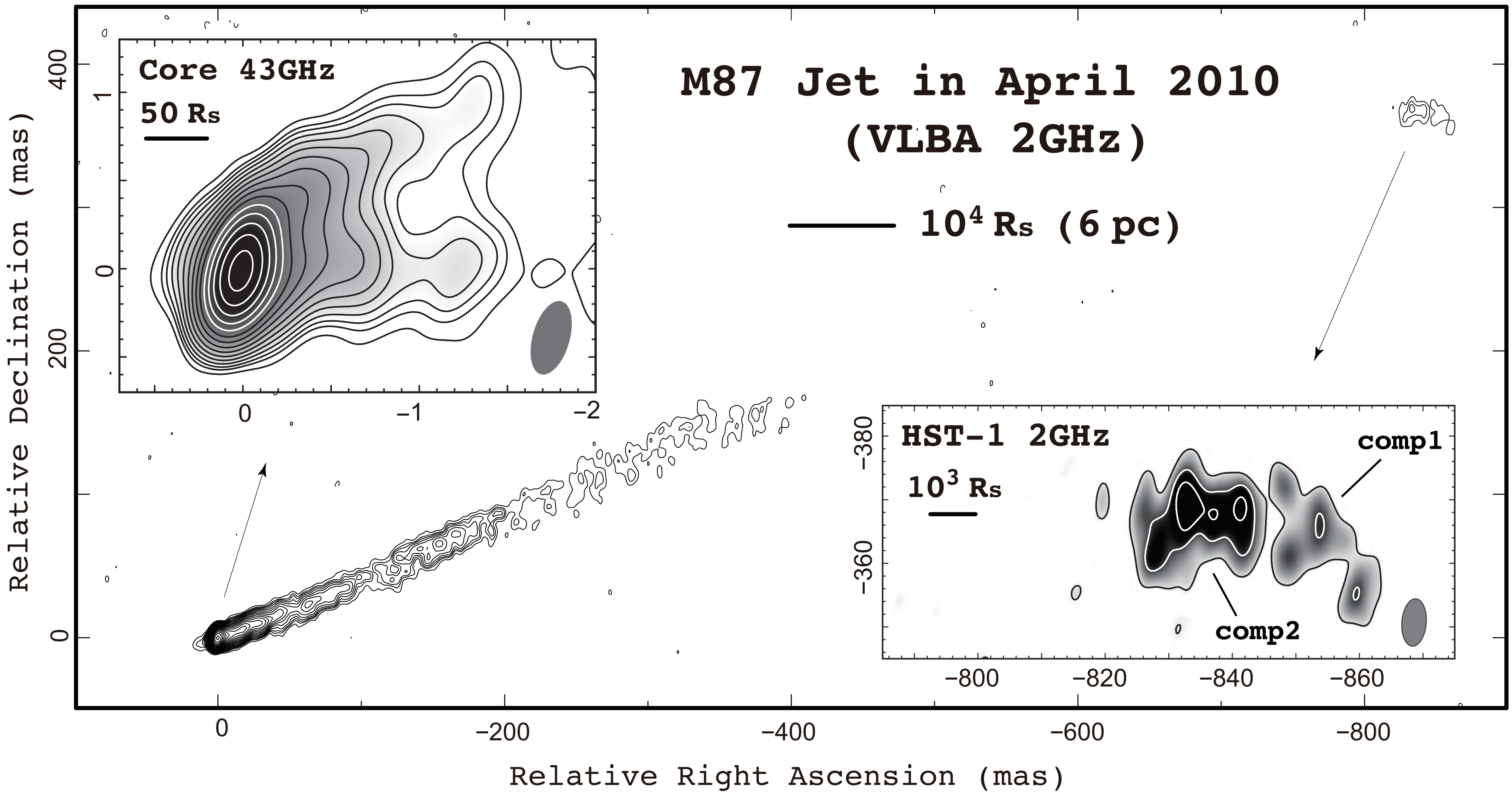
Outline

- introduction
- collimation-acceleration paradigm
- rarefaction acceleration in GRB outflows



M87 = Virgo A
VLA 20cm
800 pc
10"
VLA 90cm
25 kpc
5'
VLA 2cm
400 pc
5"
HST
80 pc
1"
VLBA 2cm
0.8 pc
0".01

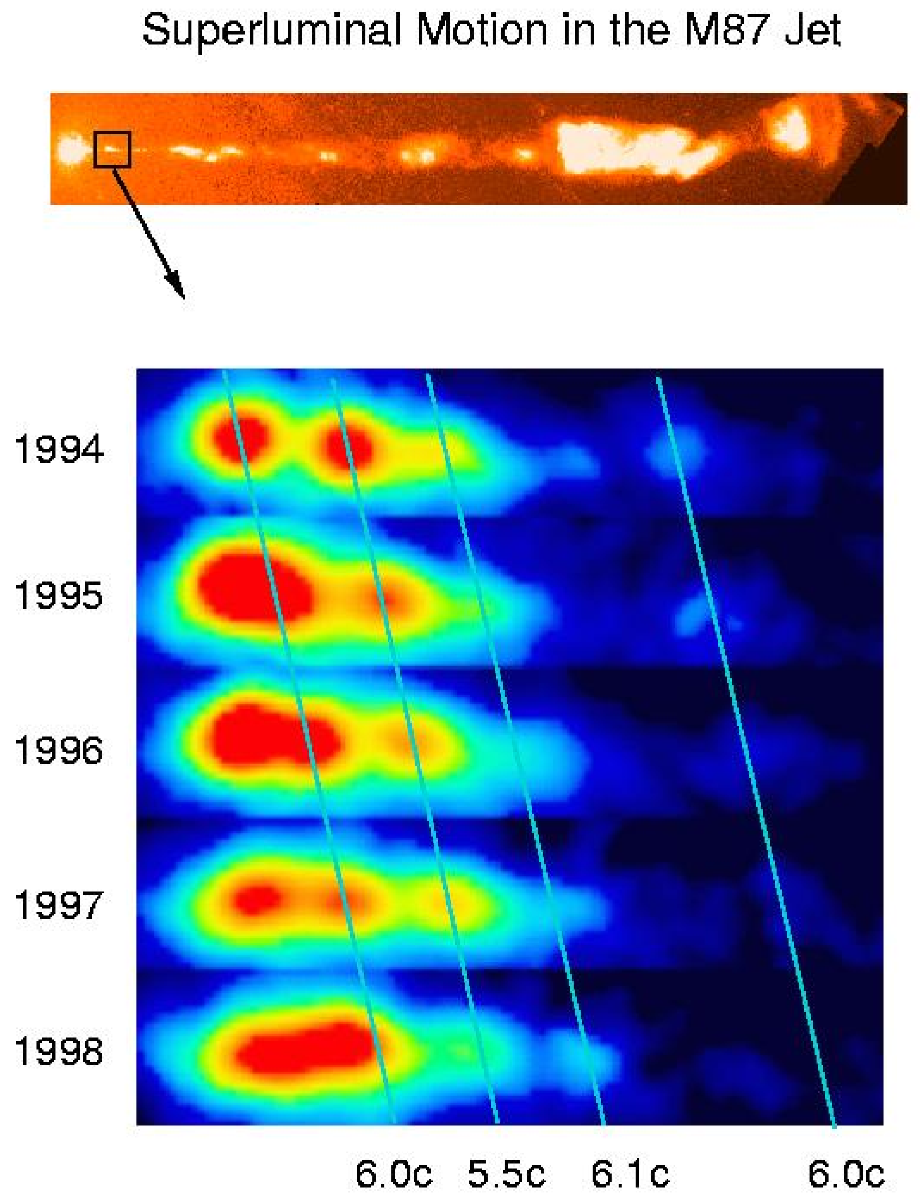

Radio Image (VLA)
Core
D
E
F
A
B1
HST-1
Hubble Space Telescope
C
G1
Chandra X-ray Observatory



M87 Jet in April 2010
(VLBA 2GHz)
10^4 Rs (6 pc)
Core 43GHz
50 Rs
HST-1 2GHz
10^3 Rs
comp1
comp2
Relative Declination (mas)
Relative Right Ascension (mas)

(Hada et al)

Jet speed


Superluminal Motion in the M87 Jet
1994
1995
1996
1997
1998
6.0c
5.5c
6.1c
6.0c

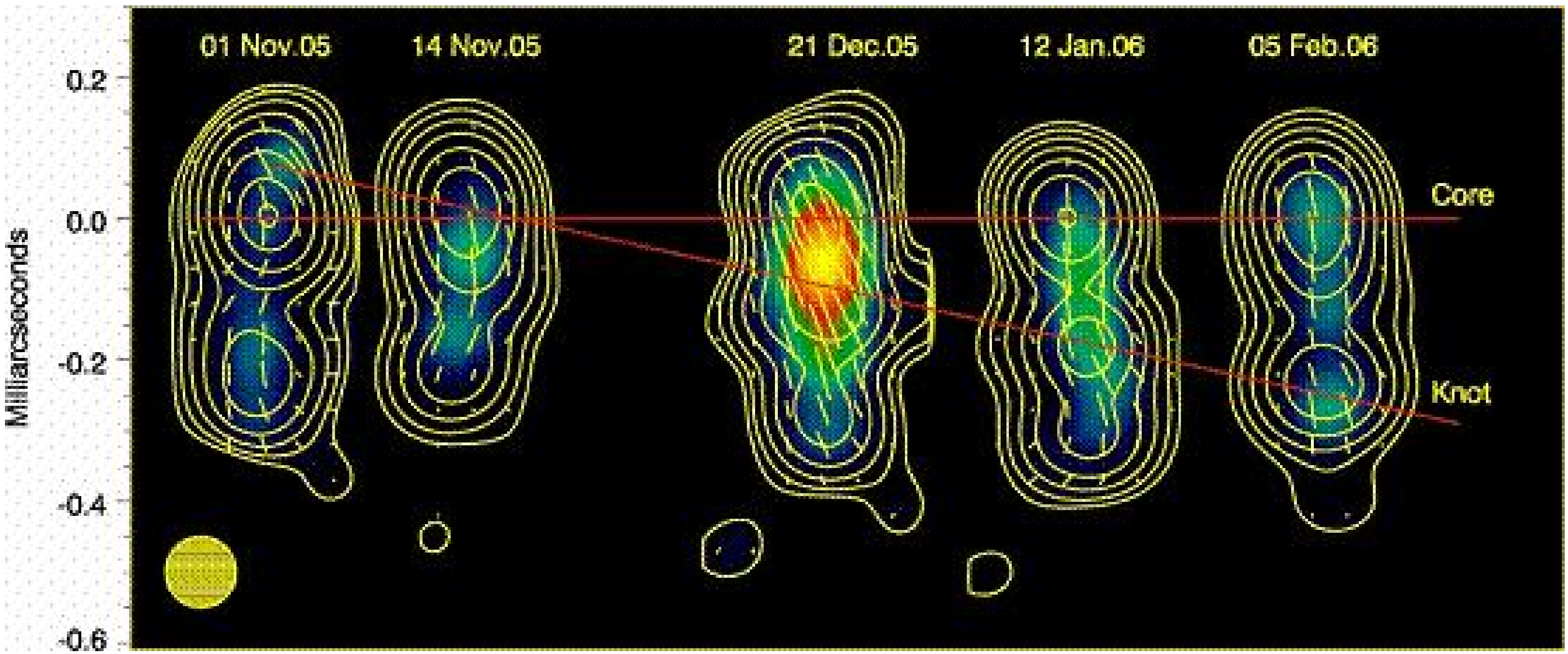
On the bulk acceleration

- More distant components have higher apparent speeds
- Brightness temperature increases with distance (Lee, Lobanov, et al)
- A more general argument on the acceleration (Sikora et al):
  - ⋆ lack of bulk-Compton features → small ($\gamma < 5$) bulk Lorentz factor at $\lesssim 10^3 r_g$
  - ⋆ the γ saturates at values $\sim$ a few 10 around the blazar zone $(10^3 - 10^4 r_g)$

  So, relativistic AGN jets undergo the bulk of their acceleration on parsec scales ($\gg$ size of the central black hole)

- Sikora et al also argue that the protons are the dynamically important component in the outflow.

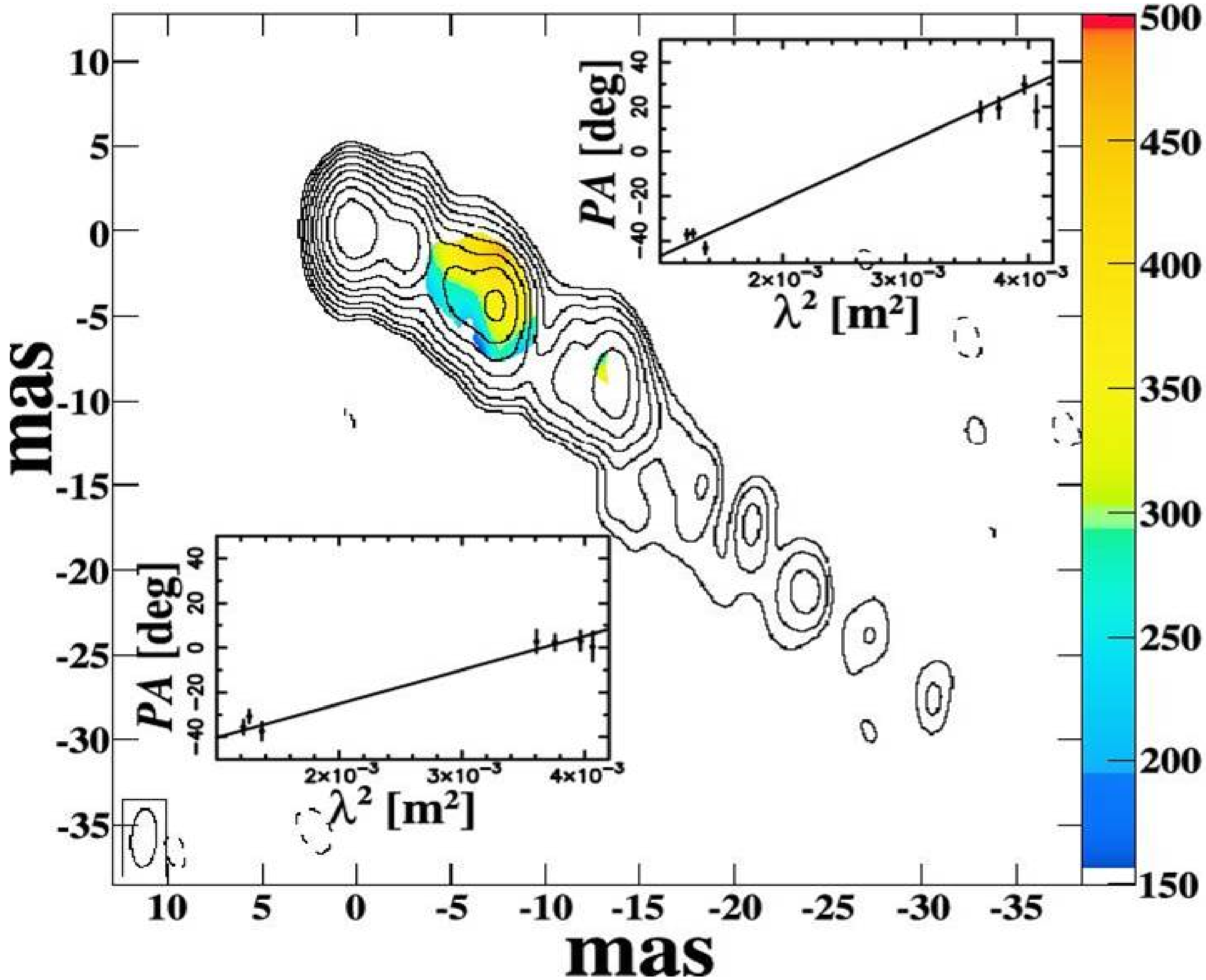
Polarization

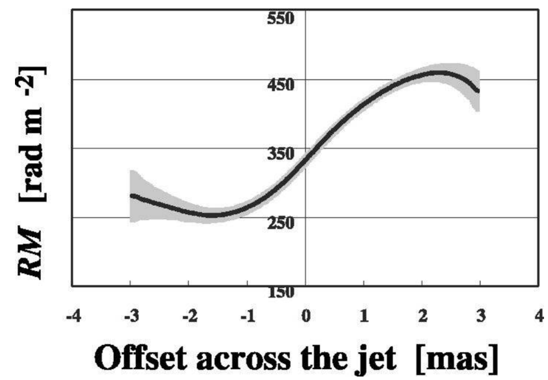


(Marscher et al 2008, Nature)

observed $\boldsymbol{E}_{\rm rad} \perp \boldsymbol{B}_{\perp\rm los}$
(modified by Faraday rotation and relativistic effects)

Faraday RM gradients across the jet



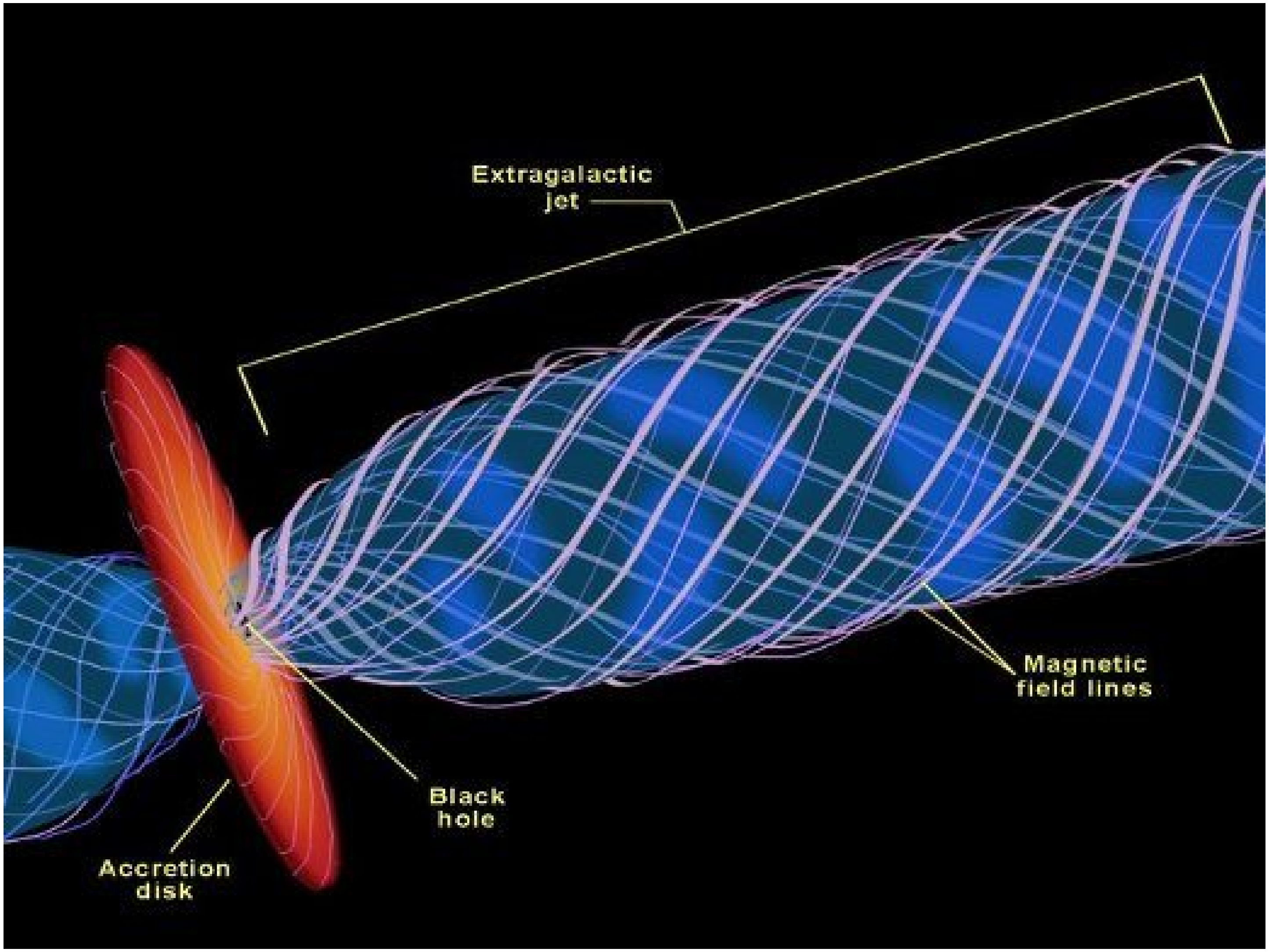


(Asada et al)

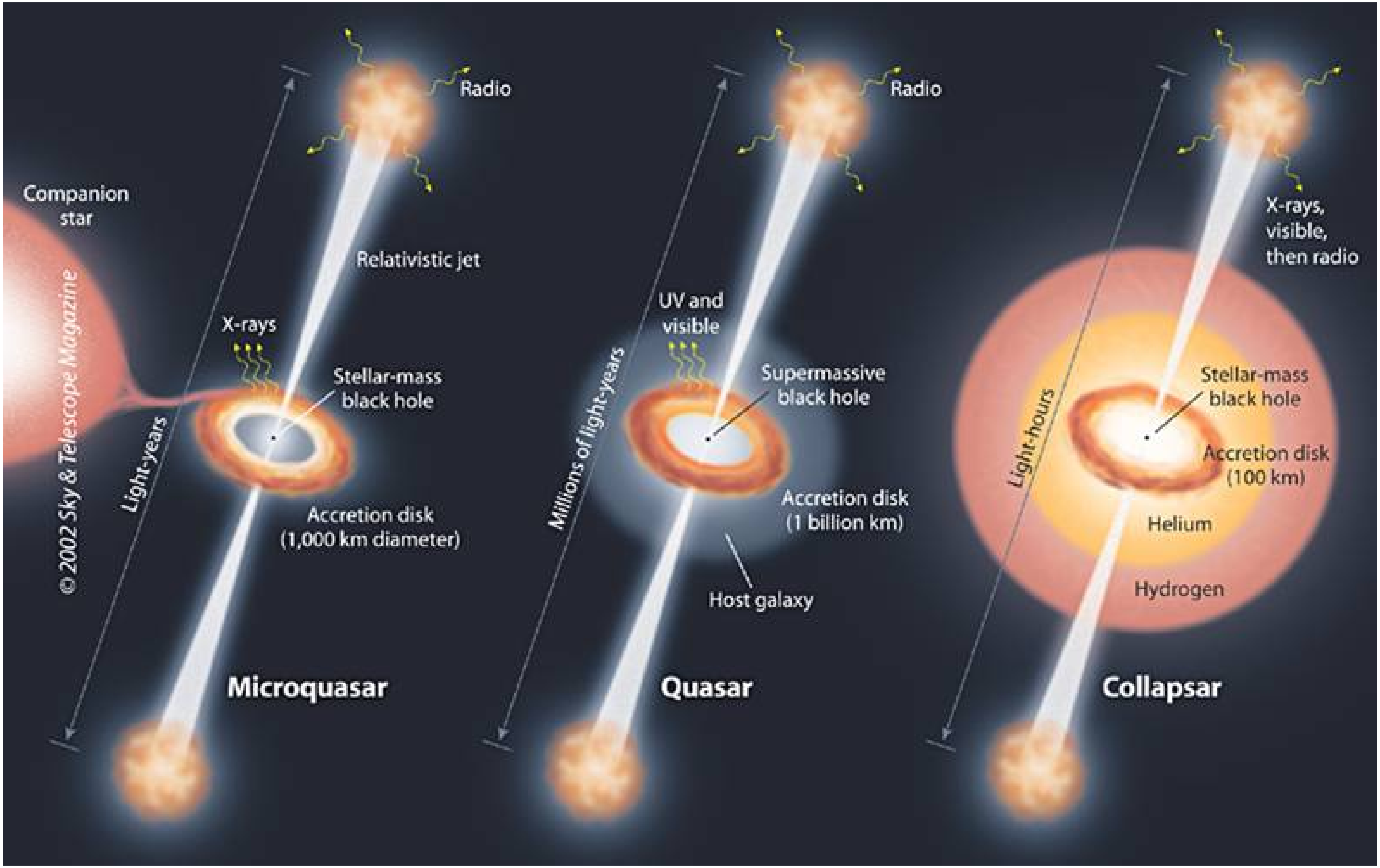
helical field surrounding the emitting region (Gabuzda)

What magnetic fields can do

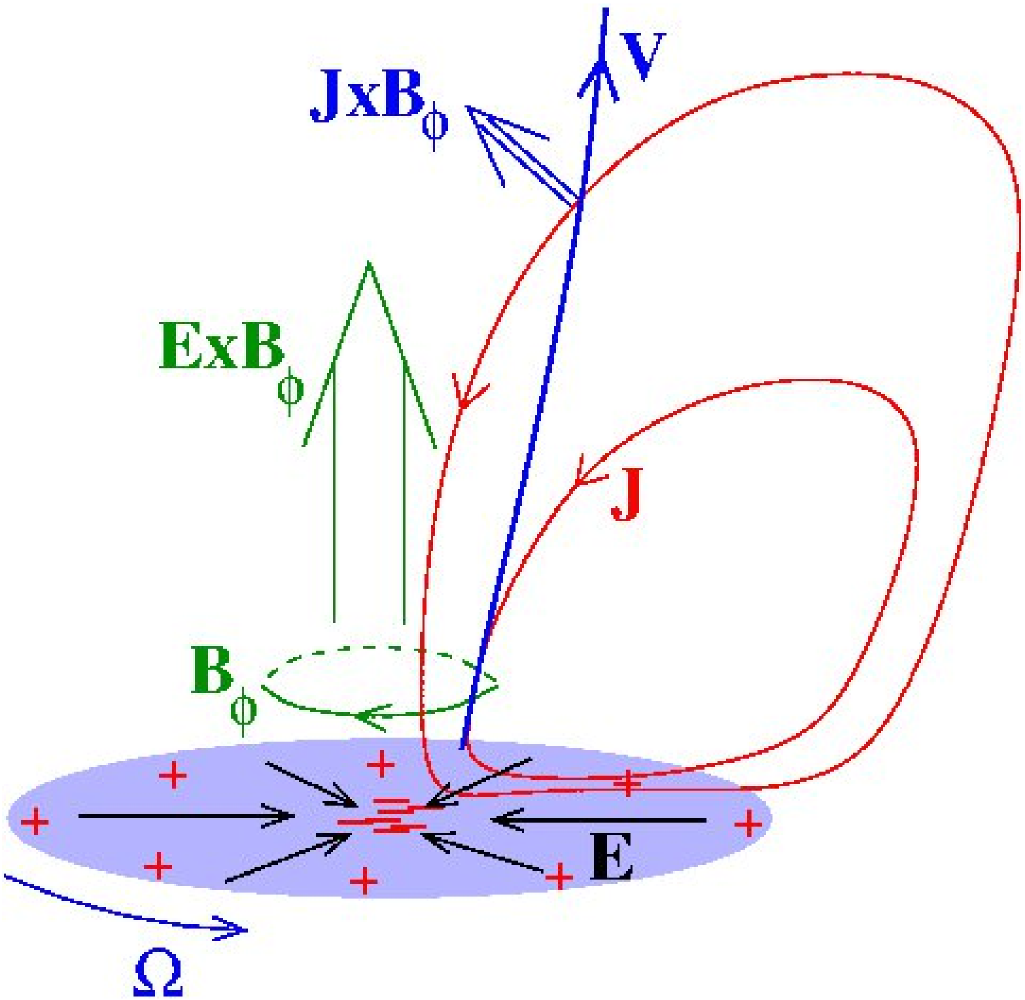
- extract energy (Poynting flux)
- extract angular momentum
- transfer energy and angular momentum to matter
- explain relatively large-scale acceleration
- self-collimation
- synchrotron emission
- polarization and RM maps



B field from advection, or dynamo, or cosmic battery (Poynting-Robertson drag).


Radio
Radio
Companion star
Relativistic jet
X-rays, visible, then radio
UV and visible
X-rays
Stellar-mass black hole
Supermassive black hole
Stellar-mass black hole
Light-years
Millions of light-years
Light-hours
Accretion disk (100 km)
Accretion disk (1 billion km)
Accretion disk (1,000 km diameter)
Helium
Host galaxy
Hydrogen
© 2002 Sky & Telescope Magazine
Microquasar
Quasar
Collapsar

A unipolar inductor



current $\leftrightarrow B_\phi$
Poynting flux $\frac{c}{4\pi} E B_\phi$ is extracted (angular momentum as well)

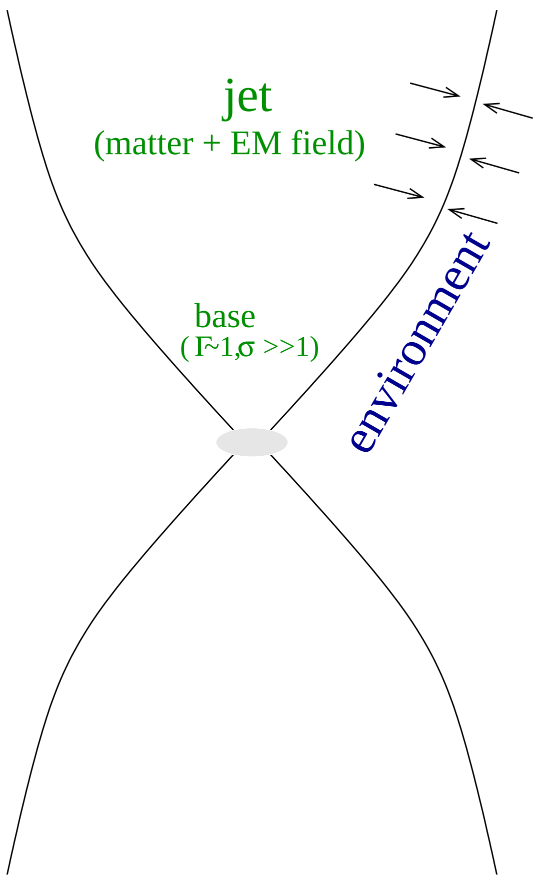
The Faraday disk could be the rotating accretion disk, or the frame dragging if energy is extracted from the ergosphere of a rotating black hole (Blandford & Znajek mechanism)

How to model magnetized outflows?

⋆ as pure electromagnetic energy (force-free, magnetodynamics, electromagnetic outflows):
– ignore matter inertia (reasonable near the origin)
– this by assumption does not allow to study the transfer of energy form Poynting to kinetic
– wave speed $= c \rightarrow$ no shocks
– there may be some dissipation (e.g. reconnection) $\rightarrow$ radiation

⋆ as magneto-hydro-dynamic flow
– the force-free case is included as the low inertia limit
– the back reaction from the matter to the field is included

Magnetized outflows



- Extracted energy per time $\dot{\mathcal{E}}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)

$$\dot{\mathcal{E}} = \frac{c}{4\pi} \underbrace{\frac{r}{r_{\rm lc}} B_p}_{E} \; B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{\mathcal{E}}/\dot{M}c^2$ gives the maximum possible bulk Lorentz factor of the flow

- Magnetohydrodynamics: matter (velocity, density, pressure) + large scale electromagnetic field

Basic questions

☞ bulk acceleration

- thermal (due to ∇P) $\longrightarrow$ velocities up to C_s
- magnetocentrifugal $\longrightarrow$ velocities up to $V_{\phi i}$
- relativistic thermal (thermal fireball) gives $\gamma \sim \left(\frac{\text{enthalpy}}{\text{mass} \times c^2}\right)_{\text{i}}$.
- magnetic ($\boldsymbol{J} \times \boldsymbol{B}$ force)
  acceleration efficiency $\gamma_\infty / \mu =$?
  terminal γ_∞ ?

☞ collimation

hoop-stress + electric force
degree of collimation ?
jet opening angle ?

some key steps on MHD modeling

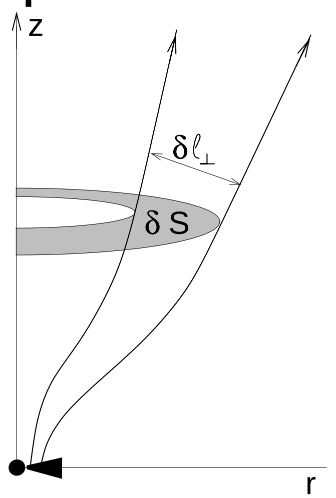
- Michel 1969: assuming monopole flow (crucial) $\rightarrow$ inefficient acceleration with $\gamma_\infty \approx \mu^{1/3} \ll \mu$
- Li, Chiueh & Begelman 1992; Contopoulos 1994: cold self-similar model $\rightarrow \gamma_\infty \approx \mu/2$ (50% efficiency)
- Vlahakis & Königl 2003: generalization of the self-similar model (including thermal and radiation effects) $\rightarrow \gamma_\infty \approx \mu/2$ (50% efficiency)
- Vlahakis 2004: complete asymptotic transfield force-balance connect the flow-shape (collimation) with acceleration explain why Michel's model is inefficient
- Beskin & Nokhrina 2006: parabolic jet with $\gamma_\infty \approx \mu/2$

some key steps (cont'd)

- Komissarov, Barkov, Vlahakis & Königl 2007 and Komissarov, Vlahakis, Königl & Barkov 2009:
  possible for the first time to simulate high γ MHD flows and follow the acceleration up to the end
  + analytical scalings
  + role of causality, role of external pressure
- Tchekhovskoy, McKinney & Narayan, 2009: simulations of nearly monopolar flow (more detailed than in Komissarov et al 2009)
  Even for nearly monopolar flow the acceleration is efficient near the rotation axis
- Lyubarsky 2009:
  generalization of the analytical results of Vlahakis 2004 and Komissarov et al 2009

“Standard” model for magnetic acceleration

☞ component of the momentum equation



$$\gamma n(\boldsymbol{V}\cdot\nabla)(\gamma w\boldsymbol{V}) = -\nabla p + J^0\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}$$

along the flow (wind equation): $\gamma \approx \mu - \mathcal{F}$

where $\mathcal{F} \propto r^2 n\gamma V_p = r^2\times$ mass flux

since mass flux $\times\delta S =$ const,

$\mathcal{F} \propto r^2/\delta S \propto r/\delta\ell_\perp$

acceleration requires the separation between streamlines to increase faster than the cylindrical radius

the collimation-acceleration paradigm:
$\mathcal{F}\downarrow$ through stronger collimation of the inner streamlines relative to the outer ones (differential collimation)

☞ external pressure plays important role

☞ transfield component of the momentum equation

$$\frac{\gamma^2 r}{\mathcal{R}} \approx \frac{\left(\frac{2I}{\Omega B_p r^2}\right)^2 r \nabla_\perp \ln \left|\frac{I}{\gamma}\right|}{1 + \frac{w}{\rho c^2} \frac{4\pi \rho u_p^2}{B_p^2} \frac{r_{\rm lc}^2}{r^2}} - \gamma^2 \frac{r_{\rm lc}^2}{r^2} \nabla_\perp r, \text{ with } \nabla_\perp \sim 1/r,$$

simplifies to $\underbrace{\frac{\gamma^2 r}{\mathcal{R}}}_{inertia} \approx \underbrace{1}_{EM} - \underbrace{\gamma^2 \frac{r_{\rm lc}^2}{r^2}}_{centrifugal}$

• if centrifugal negligible then $\gamma \approx z/r$ (since $\mathcal{R}^{-1} \approx -\frac{d^2 r}{dz^2} \approx \frac{r}{z^2}$)
power-law acceleration regime
(for parabolic shapes $z \propto r^a$, γ is a power of r)

• if inetria negligible then $\gamma \approx r/r_{\rm lc}$ linear acceleration regime

• if electromagnetic negligible then ballistic regime

☞ role of external pressure

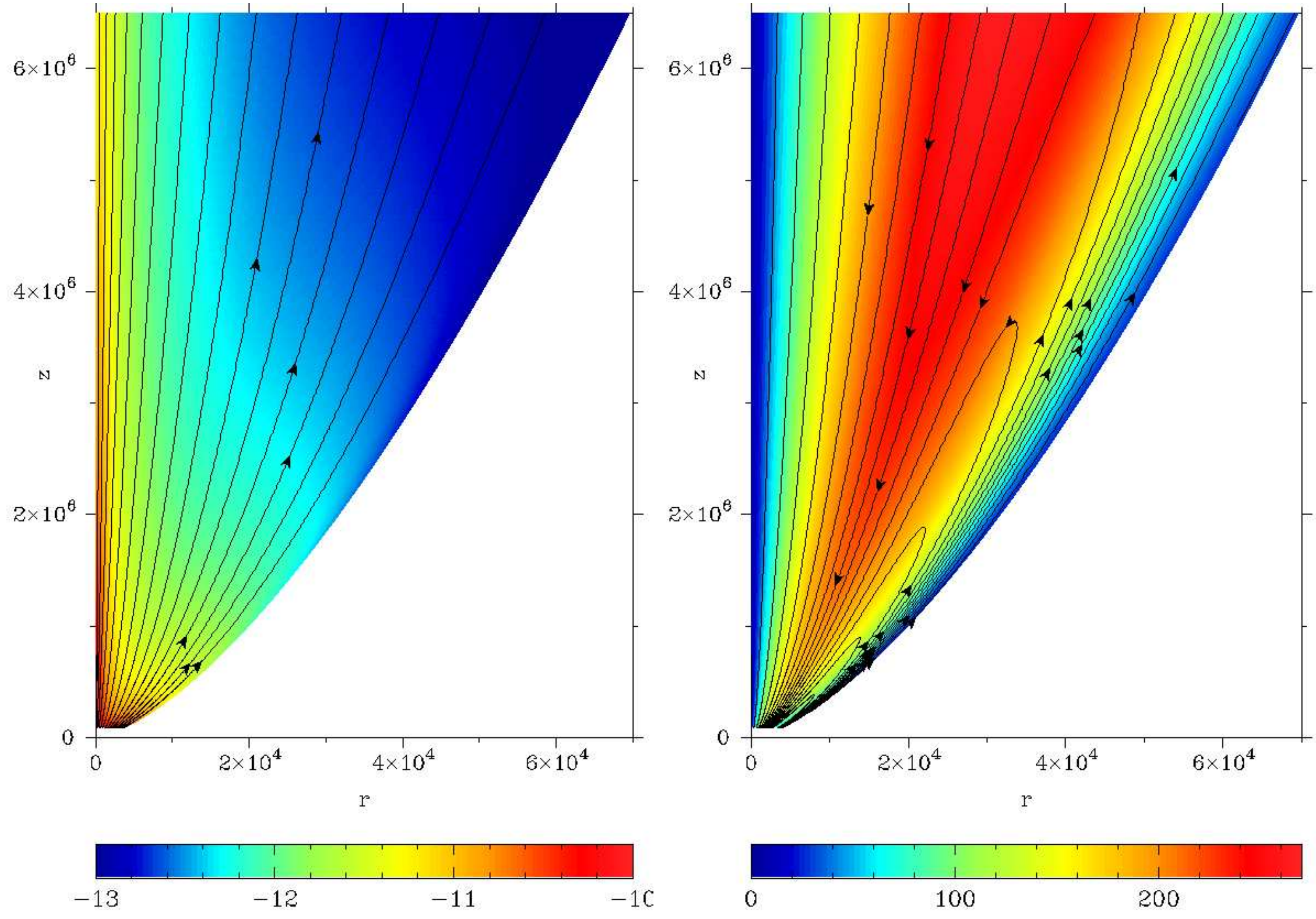
$p_{\rm ext} = B_{\rm co}^2/8\pi \simeq (B^{\hat\phi})^2/8\pi\gamma^2 \propto 1/r^2\gamma^2$

Assuming $p_{\rm ext} \propto z^{-\alpha_p}$ we find $\gamma^2 \propto z^{\alpha_p}/r^2$.

Combining with the transfield $\frac{\gamma^2 r}{\mathcal{R}} \approx 1 - \gamma^2 \frac{r_{\rm lc}^2}{r^2}$ we find the funnel shape (we find the exponent a in $z \propto r^a$).

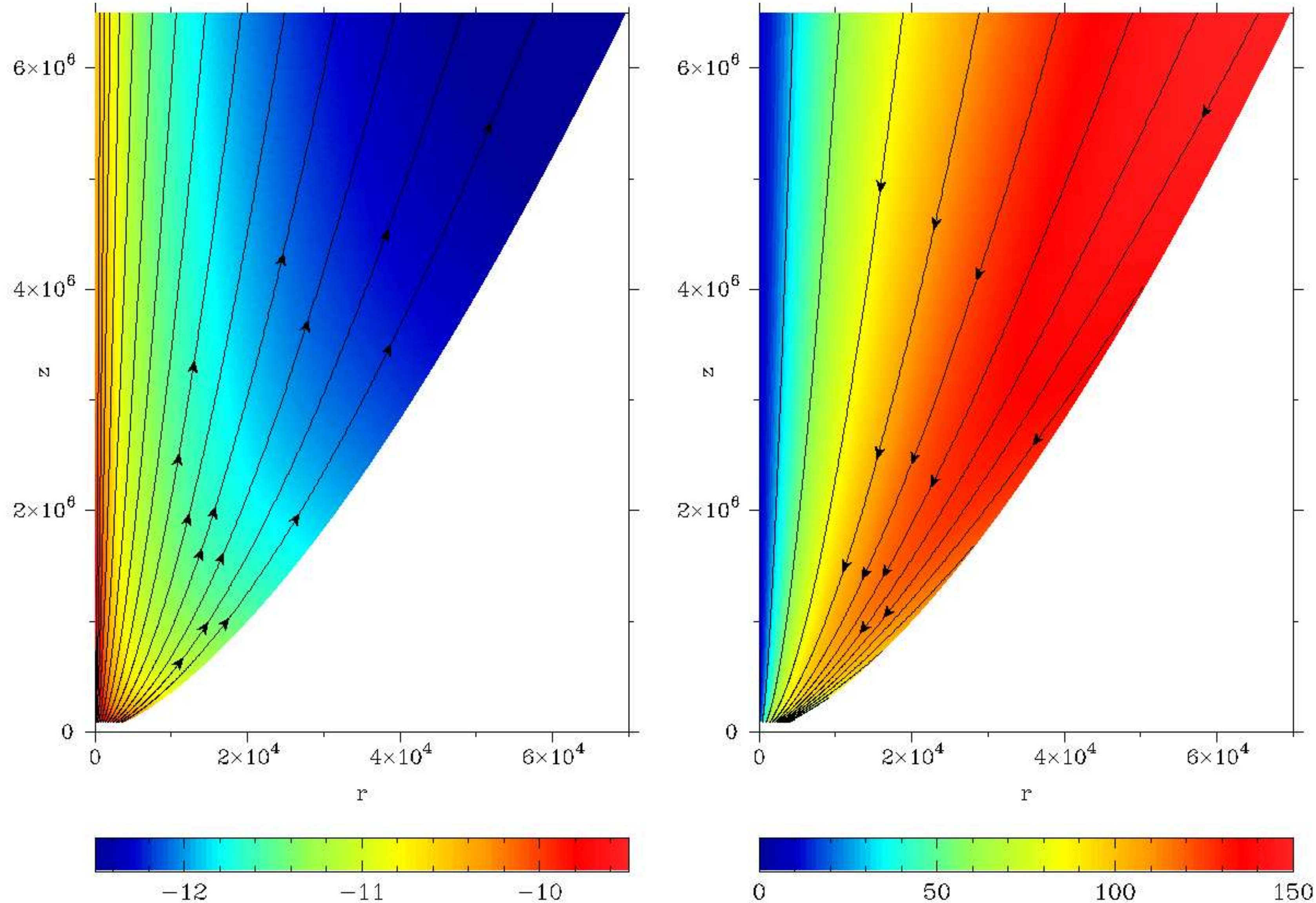
- if the pressure drops slower than z^{-2} then
  - ⋆ shape more collimated than $z \propto r^2$
  - ⋆ linear acceleration $\gamma \propto r$
- if the pressure drops as z^{-2} then
  - ⋆ parabolic shape $z \propto r^a$ with $1 < a \leq 2$
  - ⋆ first $\gamma \propto r$ and then power-law acceleration $\gamma \sim z/r \propto r^{a-1}$
- if pressure drops faster than z^{-2} then
  - ⋆ conical shape
  - ⋆ linear acceleration $\gamma \propto r$ (small efficiency)

Komissarov, Vlahakis, Königl, & Barkov 2009

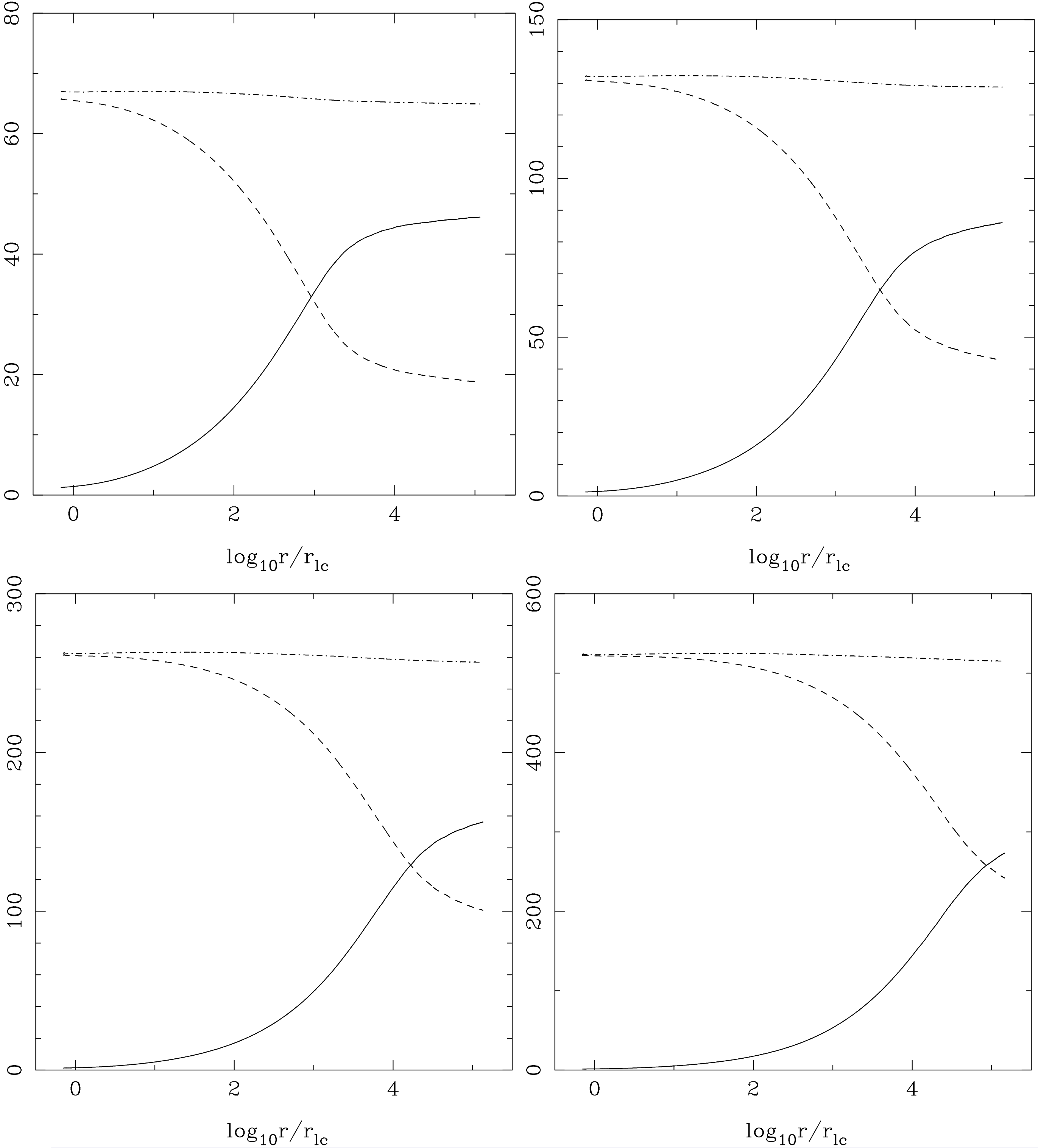


left: density/field lines, right: Lorentz factor/current lines (wall shape $z \propto r^{1.5}$)

Differential rotation $\rightarrow$ slow envelope



Uniform rotation $\rightarrow \gamma$ increases with r



energy flux ratios:

$$\gamma = \frac{\text{kinetic}}{\text{rest mass}}$$

$$\gamma\sigma = \frac{\text{Poynting}}{\text{rest mass}}$$

$$(\sigma = \frac{\text{Poynting}}{\text{kinetic}})$$

$$\mu = \gamma + \gamma\sigma$$

γ (increasing),

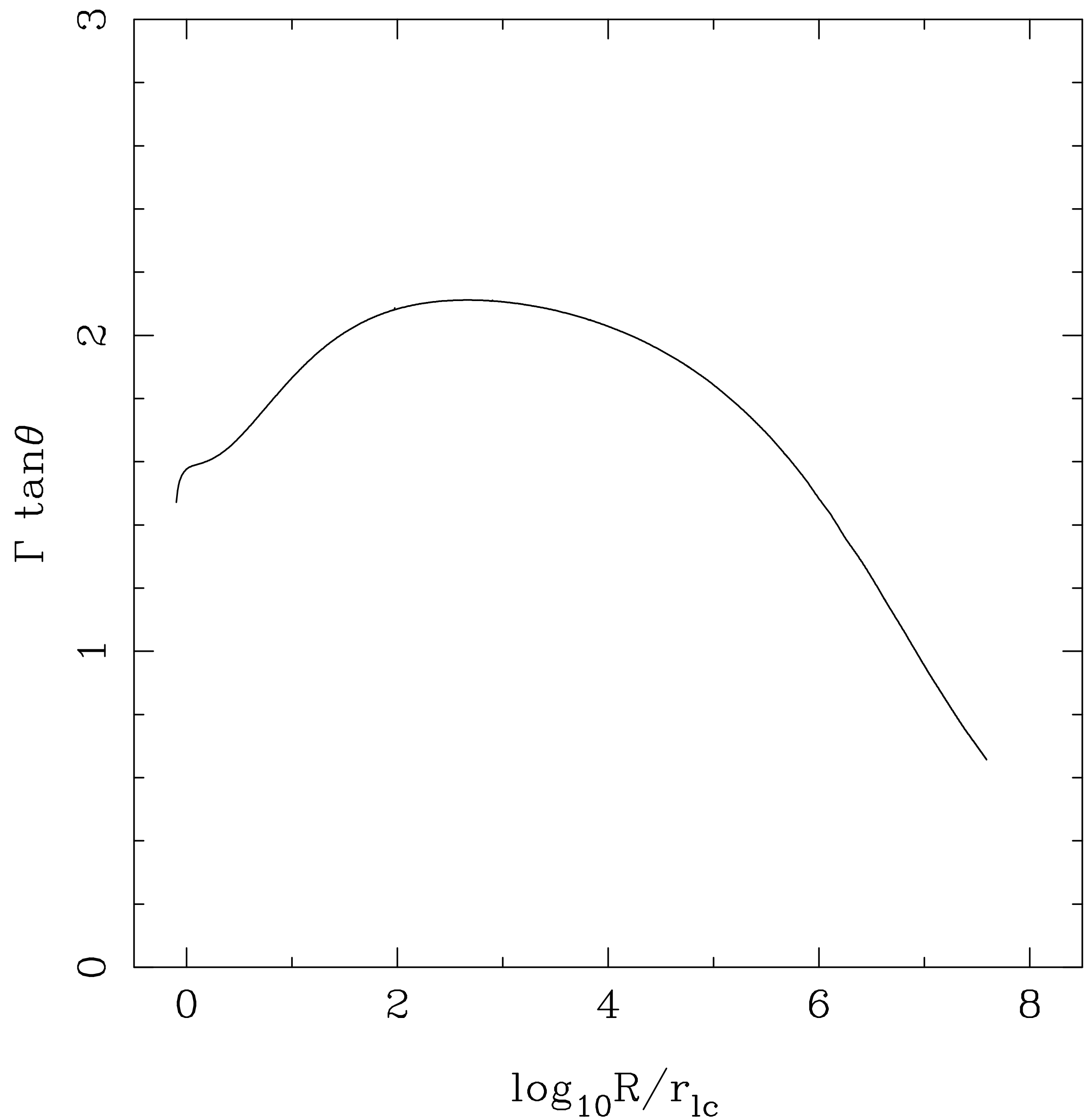
$\gamma\sigma$ (decreasing),

and μ (constant)

efficiency $> 50\%$

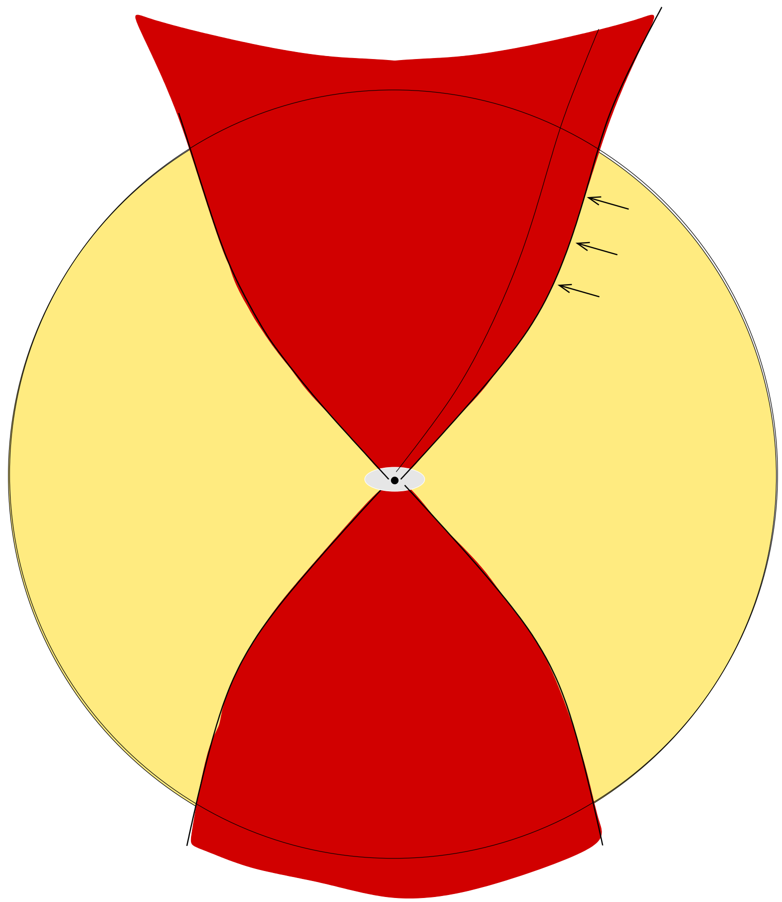
Caveat: $\gamma\vartheta \sim 1$ (for high γ)

- very narrow jets ($\vartheta < 1^\circ$ for $\gamma > 100$) $\longrightarrow$ early breaks or no breaks at all
- this is a result of causality (across jet): outer lines need to know that there is space to expand
- Mach cone half-opening θ_m should be $> \vartheta$
  With $\sin\theta_m = \dfrac{\gamma_f c_f}{\gamma V_p} \approx \dfrac{\sigma^{1/2}}{\gamma}$ the requirement for causality yields $\gamma\vartheta < \sigma^{1/2}$.
  For efficient acceleration ($\sigma \sim 1$ or smaller) we always get $\gamma\vartheta \sim 1$

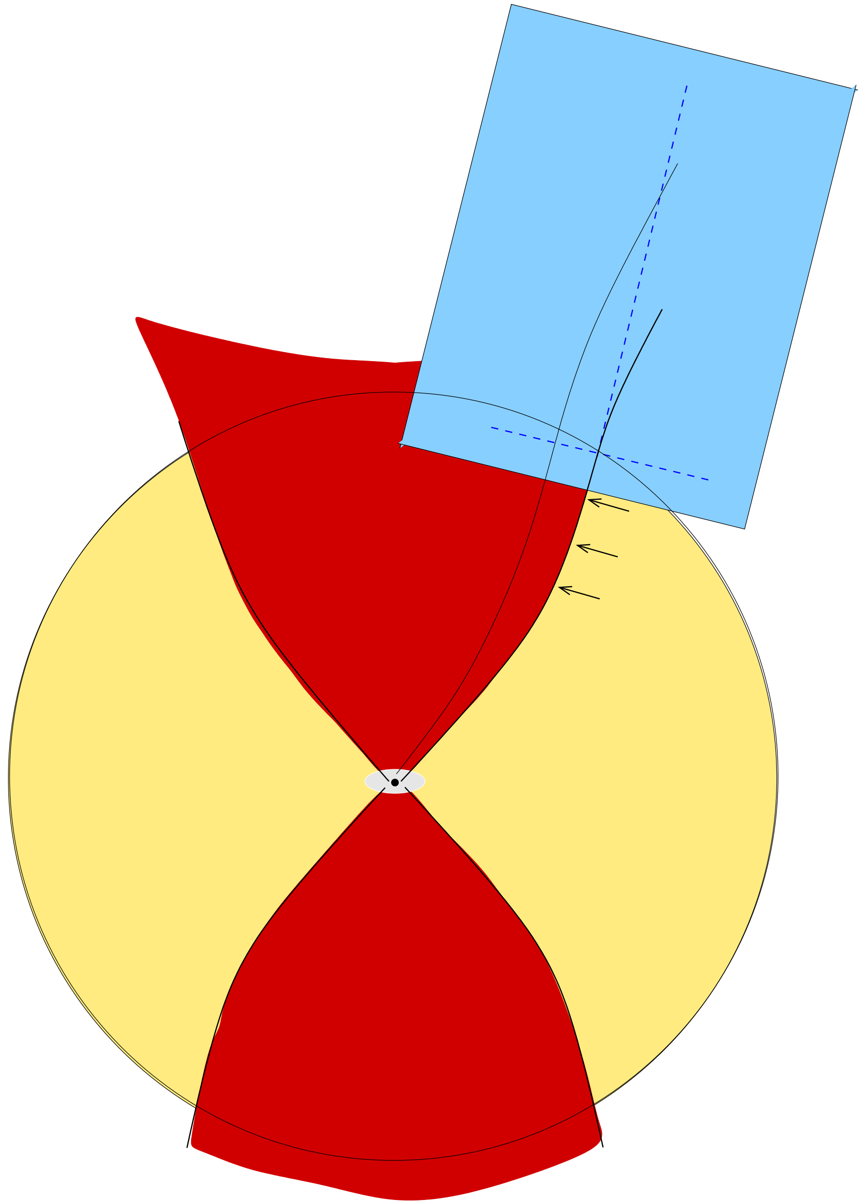


$\Gamma\ \tan\theta$
0
1
2
3
0
2
4
6
8
$\log_{10} R/r_{lc}$

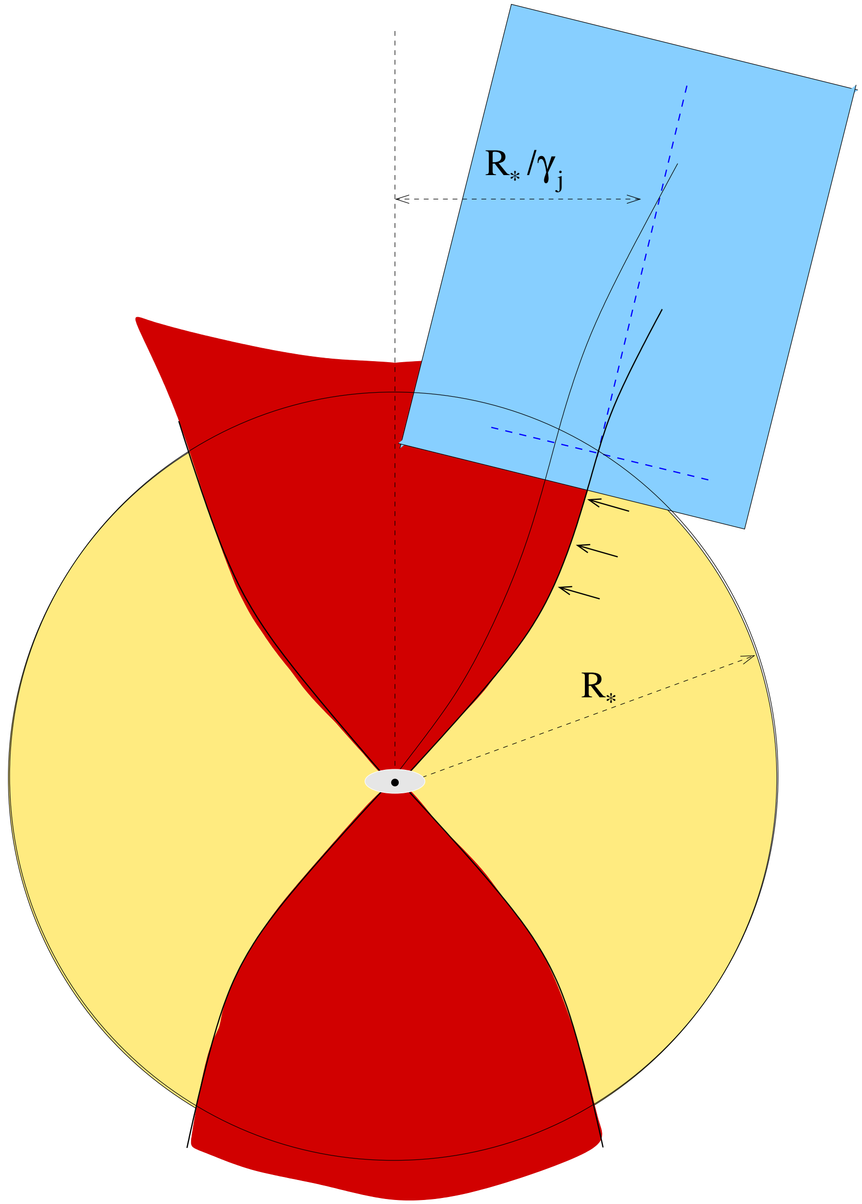
Rarefaction acceleration



Rarefaction acceleration

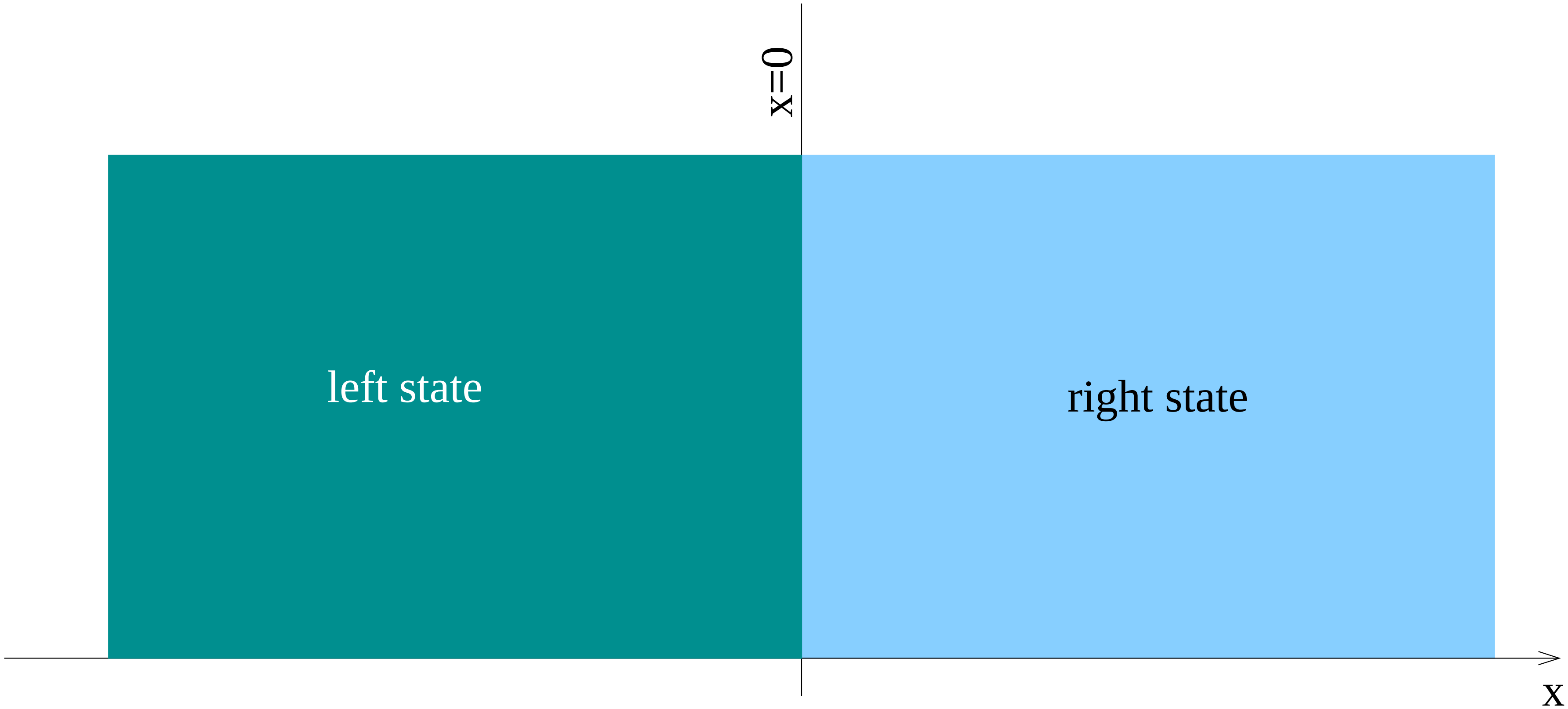


Rarefaction acceleration



Rarefaction simple waves

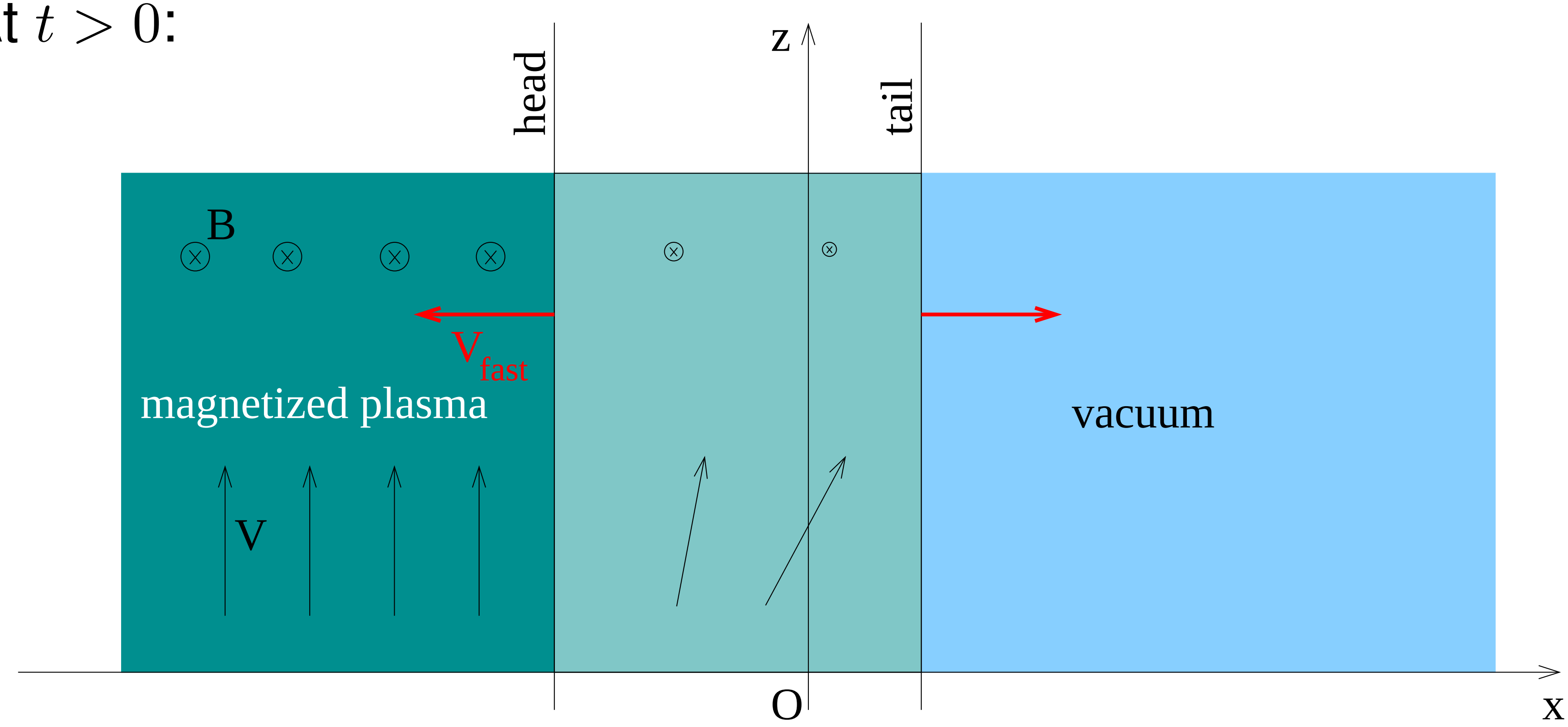
At $t = 0$ two uniform states are in contact:



This Riemann problem allows self-similar solutions that depend only on $\xi = x/t$.

- when right=vacuum, simple rarefaction wave

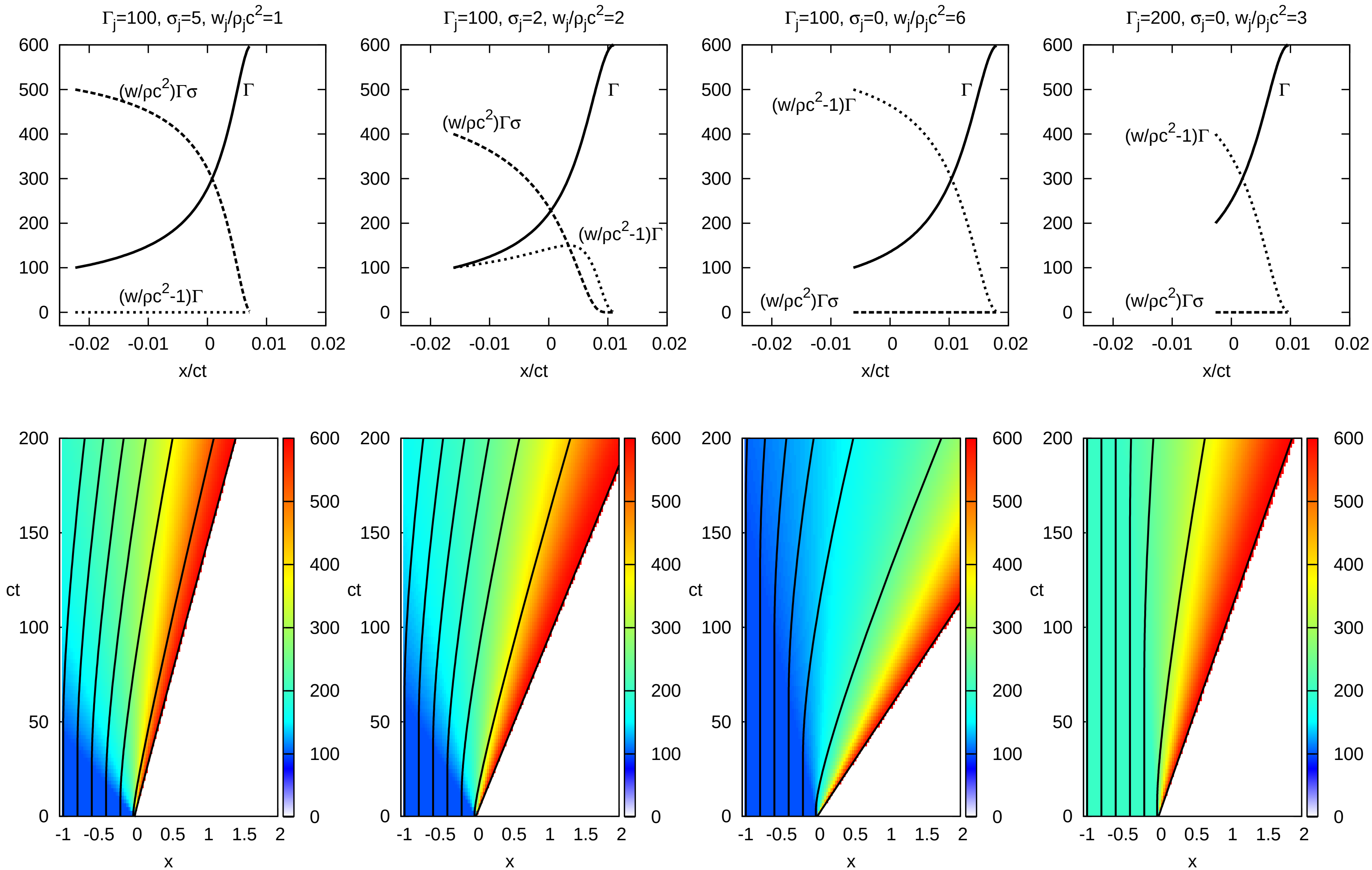
At $t > 0$:



for the cold case the Riemann invariants imply

$$v_x = \frac{1}{\gamma_j}\frac{2\sigma_j^{1/2}}{1+\sigma_j}\left[1-\left(\frac{\rho}{\rho_j}\right)^{1/2}\right],\ \gamma = \frac{\gamma_j\,(1+\sigma_j)}{1+\sigma_j\rho/\rho_j},\ \rho = \frac{4\rho_j}{\sigma_j}\sinh^2\left[\frac{1}{3}\mathrm{arcsinh}\left(\sigma_j^{1/2}-\frac{\mu_j}{2}\frac{x}{t}\right)\right]$$

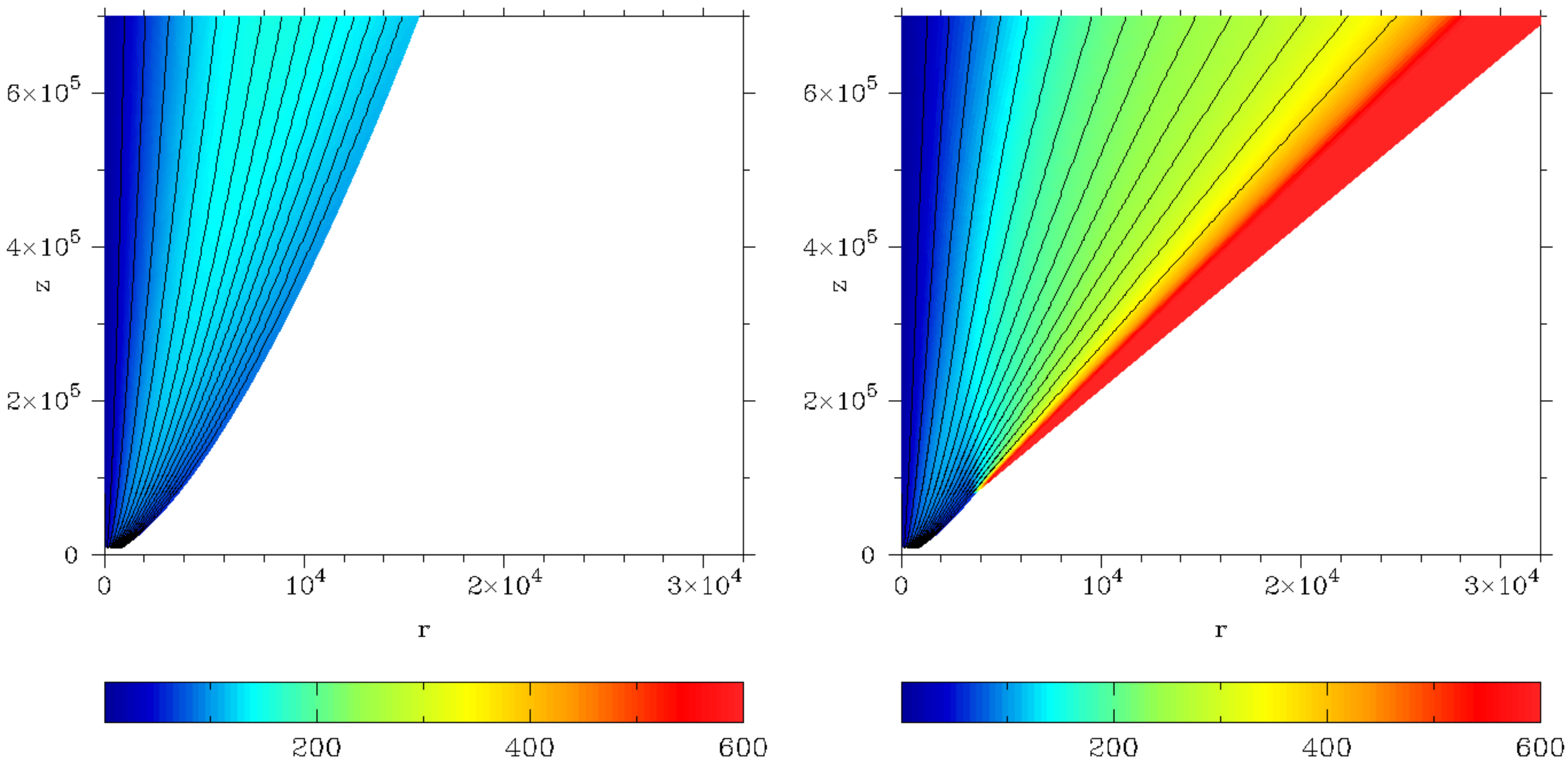
$$V_{head} = -\frac{\sigma_j^{1/2}}{\gamma_j}, \qquad V_{tail} = \frac{1}{\gamma_j}\frac{2\sigma_j^{1/2}}{1+\sigma_j}, \qquad \Delta\vartheta = V_{tail} < 1/\gamma_i$$

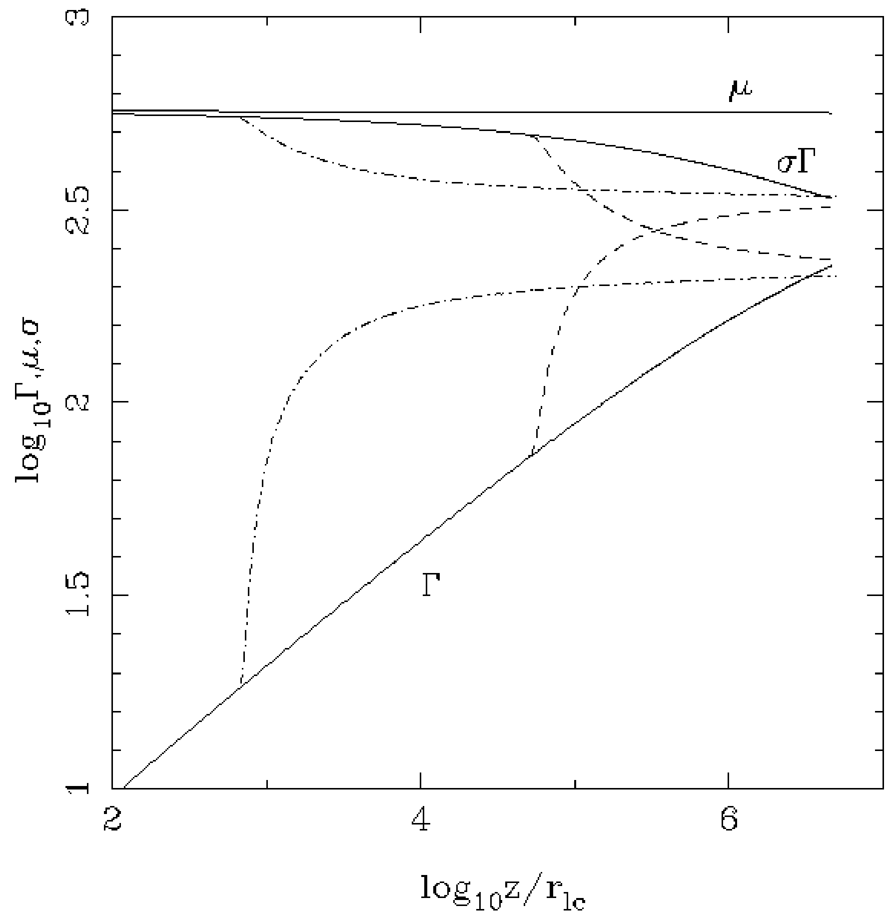


The colour image in the Minkowski diagrams represents the distribution of the Lorentz factor and the contours show the worldlines of fluid parcels initially located at $x_i = -1, -0.8, -0.6, -0.4, -0.2, -0.02, 0$.

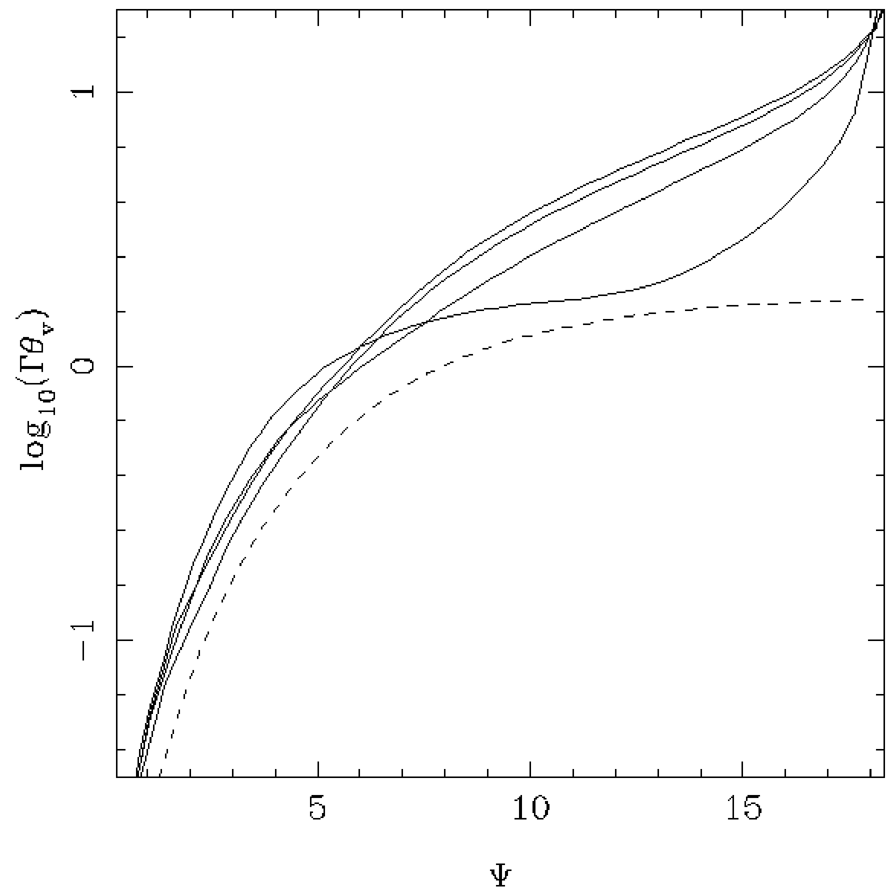
Simulation results

Komissarov, Vlahakis & Königl 2010





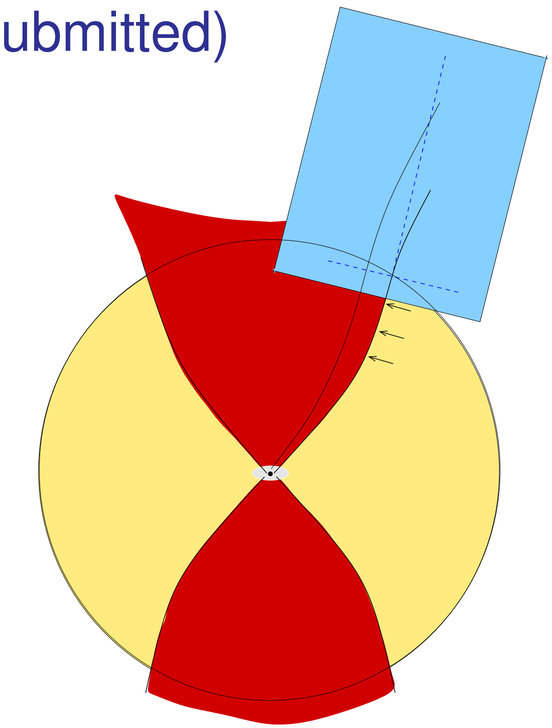
μ
σΓ
Γ
$\log_{10}\Gamma, \mu, \sigma$
$\log_{10} z/r_{lc}$
3
2.5
2
1.5
1
2
4
6



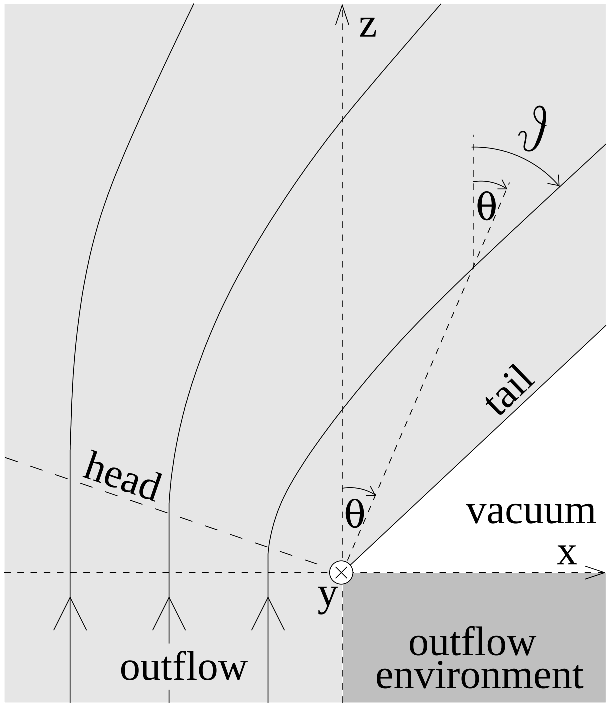
$\log_{10}(\Gamma\theta_v)$
Ψ
1
0
−1
5
10
15

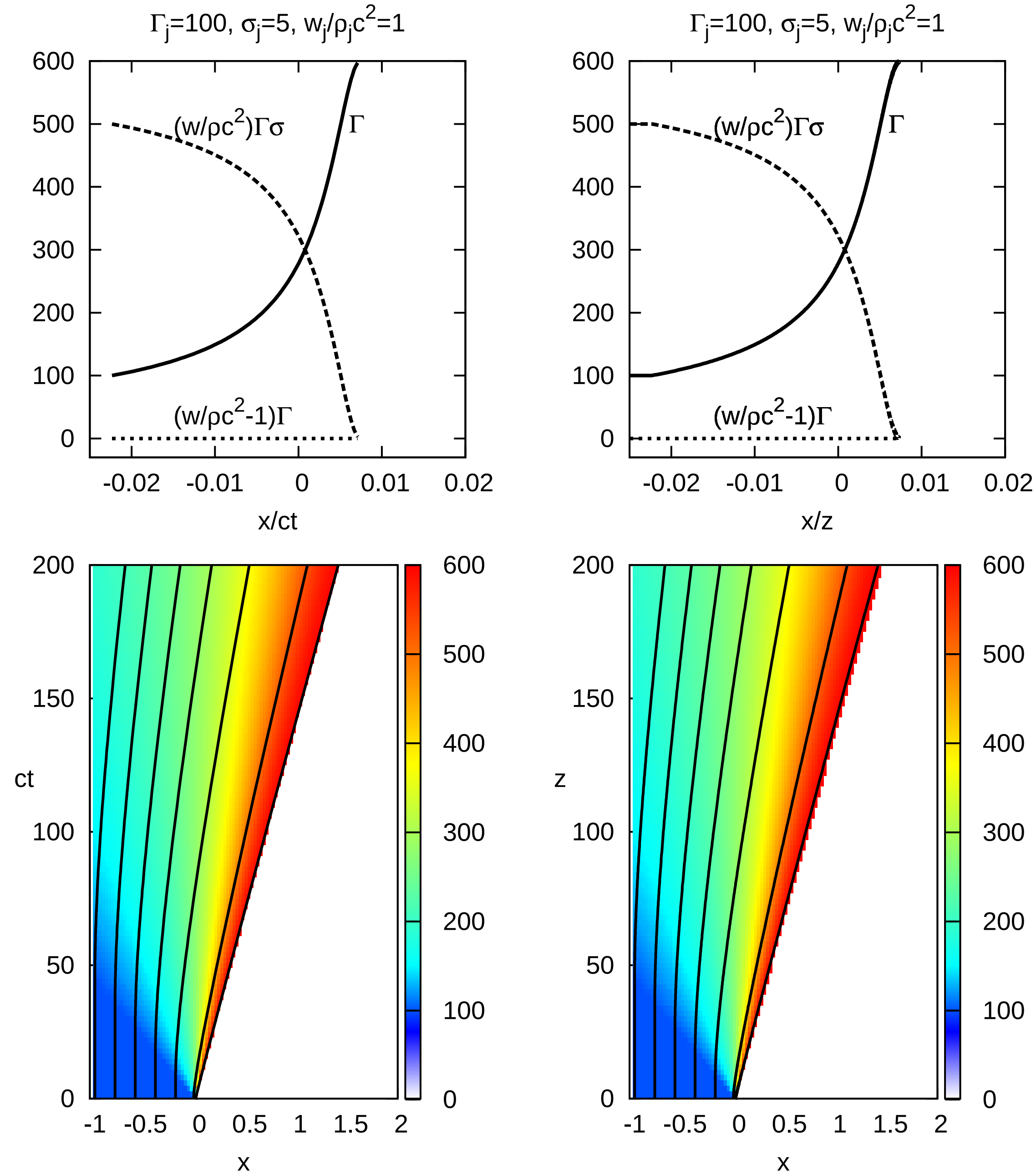
Steady-state rarefaction wave

Sapountzis & Vlahakis (MNRAS submitted)



- “flow around a corner”
- planar geometry
- ignoring B_p (nonzero B_y)
- similarity variable x/z (angle θ)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)
- in addition, allow for inhomogeneity in the “left” state





$$\theta_{\rm head} = -\frac{\sigma_j^{1/2}}{\gamma_j}$$

$$\theta_{\rm tail} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)}$$

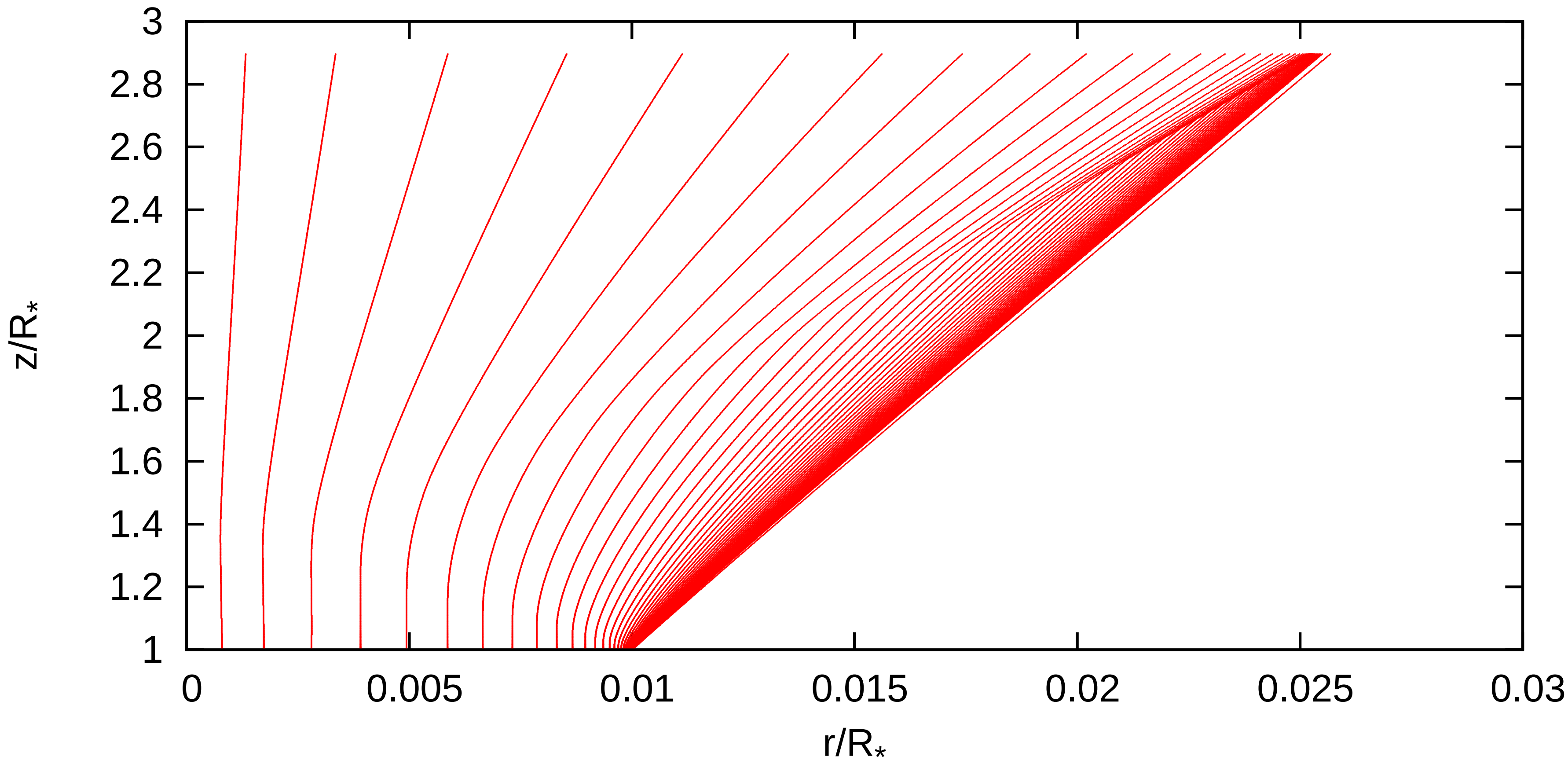
$$\sigma = (\sigma_j \gamma_j x_i / z)^{2/3}$$

$\sigma = 1$ at $r = \sigma_j \gamma_j |x_i| = 7{\times}10^{11} \sigma_j \left(\frac{|x_i|}{R_\star/\gamma_j}\right) \left(\frac{R_\star}{10 R_\odot}\right)$ cm

time-dependent (left) and steady-state (right) rarefaction (similar; $ct \to z$)
(distance unit $= R_\star/\gamma_j \sim 10^{10}$ cm)

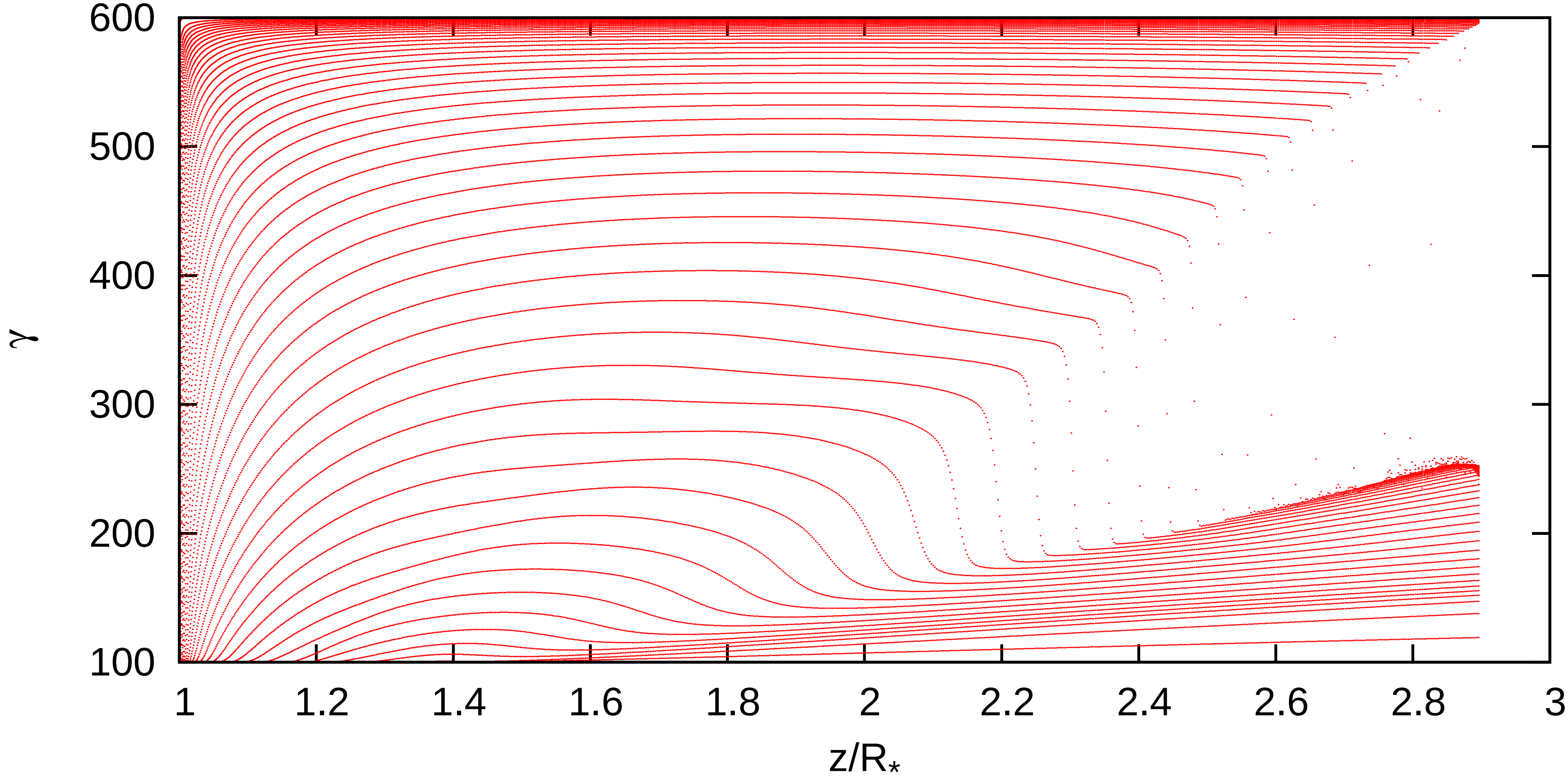
Axisymmetric model

Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)

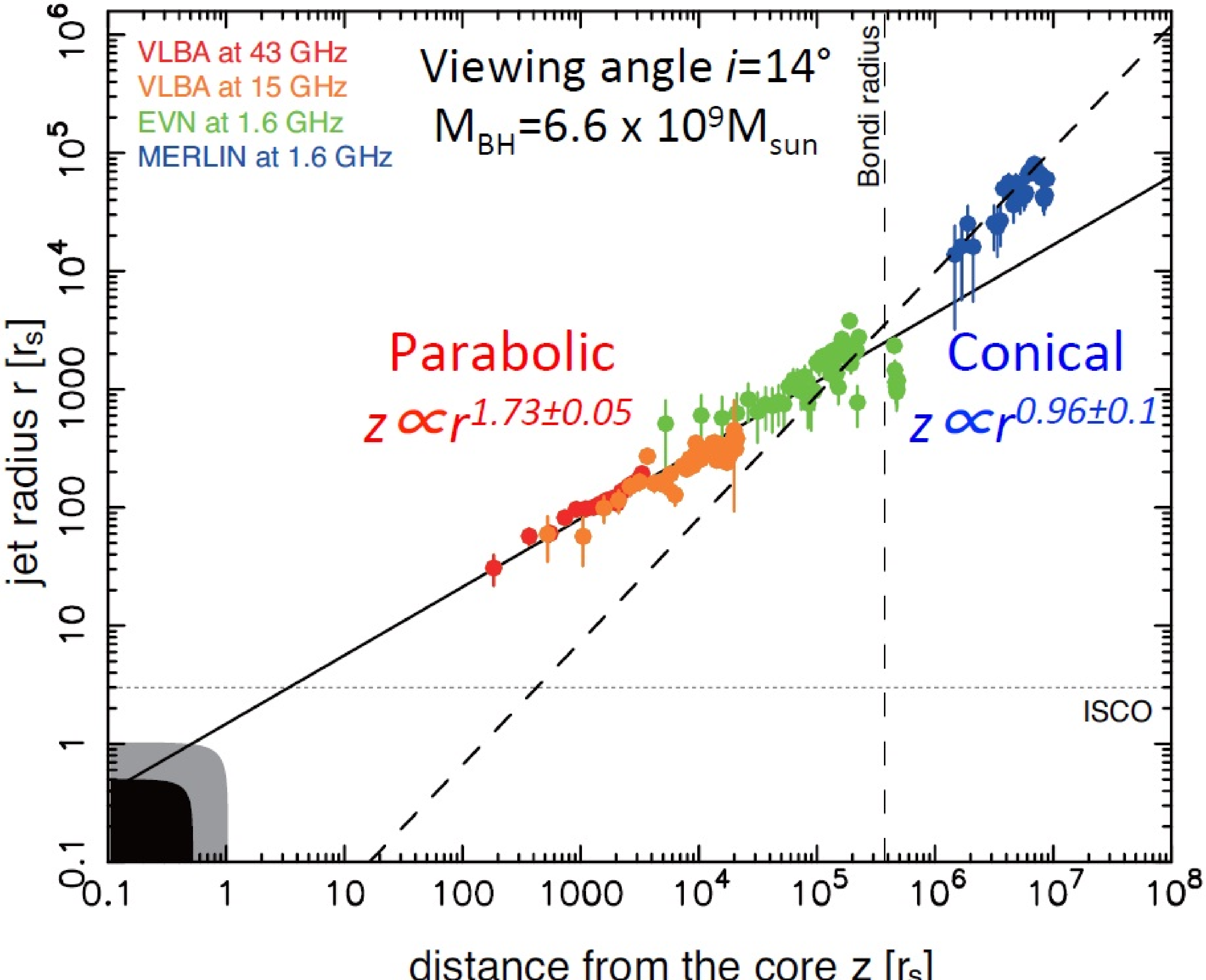


(not in scale!)

typical value of $R_\star = 10^{12}$ cm


600
500
400
300
200
100
γ
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
z/R⋆

(Asada & Nakamura 2011)



Summary

- ⋆ The collimation-acceleration paradigm provides a viable explanation of the dynamics of relativistic jets
- ⋆ bulk acceleration up to Lorentz factors $\gamma_\infty \gtrsim 0.5 \frac{\mathcal{E}}{Mc^2}$
  caveat: in ultrarelativistic GRB jets $\vartheta \sim 1/\gamma$
- ⋆ Rarefaction acceleration
  - further increases γ
  - makes GRB jets with $\gamma\vartheta \gg 1$
- ⋆ Future work
  - apply other stratified jet models
  - attention to the shock from reflection on the rotation axis
  - use realistic pressure distributions
    inside the star (from stellar-evolution models),
    and outside – shock formation