Rarefaction waves in magnetized astrophysical jets

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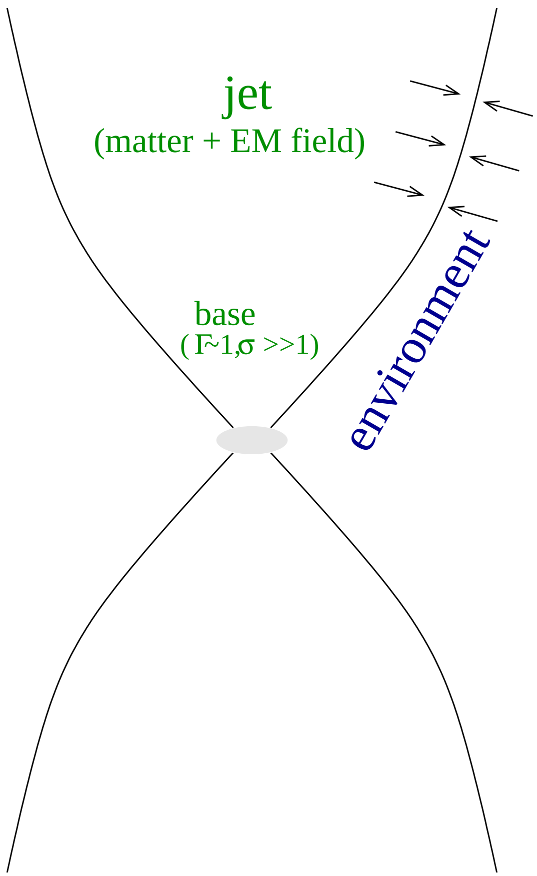
Outline

- “standard” magnetic acceleration (related to collimation)
- rarefaction acceleration
- models – application to GRBs – discussion for AGNs

Magnetized outflows



- Extracted energy per time $\dot{\mathcal{E}}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)

$$\dot{\mathcal{E}} = \frac{c}{4\pi} \underbrace{\frac{r}{r_{\rm lc}} B_p}_{E} B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

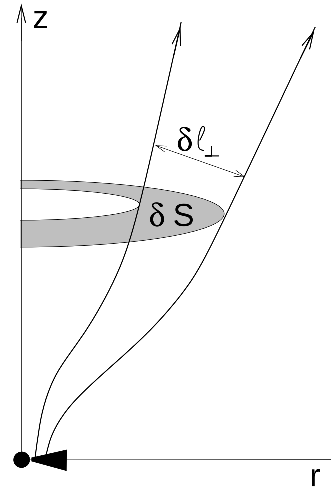
- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{\mathcal{E}} / \dot{M} c^2$ gives the maximum possible bulk Lorentz factor of the flow

- Magnetohydrodynamics: matter (velocity, density, pressure) + large scale electromagnetic field

“Standard” model for magnetic acceleration

☞ component of the momentum equation



$$\gamma\rho_0(\boldsymbol{V}\cdot\nabla)\,(\gamma w\boldsymbol{V}) = -\nabla p + J^0\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}$$

along the flow (wind equation): $\gamma \approx \mu - \mathcal{F}$

where $\mathcal{F} \propto r^2 n\gamma V_p = r^2\times$ mass flux

since mass flux $\times\delta S =$ const,

$\mathcal{F} \propto r^2/\delta S \propto r/\delta\ell_\perp$

acceleration requires the separation between streamlines to increase faster than the cylindrical radius

the collimation-acceleration paradigm:

$\mathcal{F}\downarrow$ through stronger collimation of the inner streamlines relative to the outer ones (differential collimation)

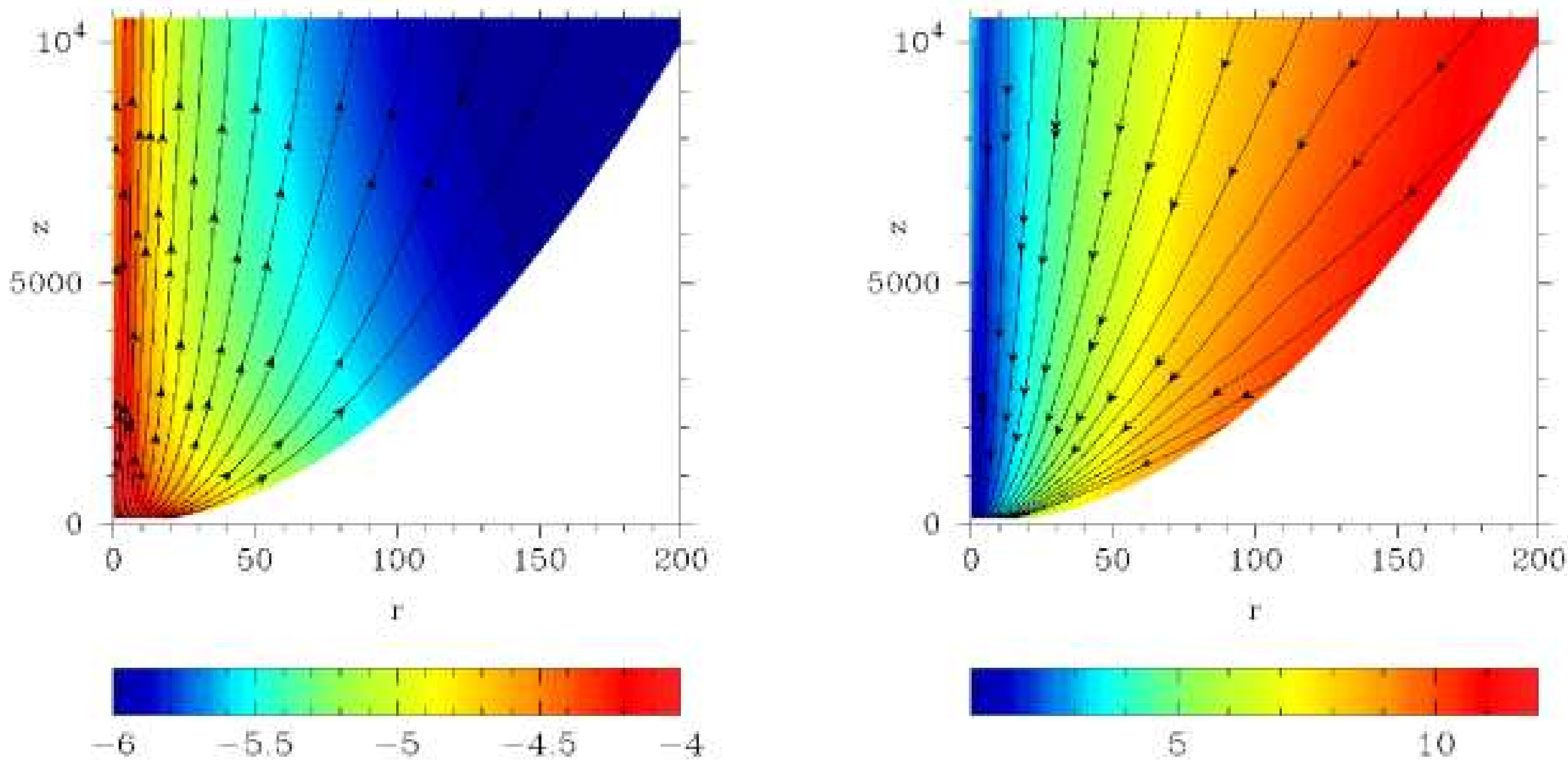
☞ transfield component of the momentum equation

$$\frac{\gamma^2 r}{\mathcal{R}} \approx \frac{\left(\frac{2I}{\Omega B_p r^2}\right)^2 r \nabla_\perp \ln\left|\frac{I}{\gamma}\right|}{1 + \frac{4\pi w \rho_0 u_p^2}{B_p^2}\frac{r_{\rm lc}^2}{r^2}} - \gamma^2 \frac{r_{\rm lc}^2}{r^2}\nabla_\perp r,$$ with $\nabla_\perp \sim \frac{1}{r}$, $r_{\rm lc} = \frac{c}{\Omega}$,

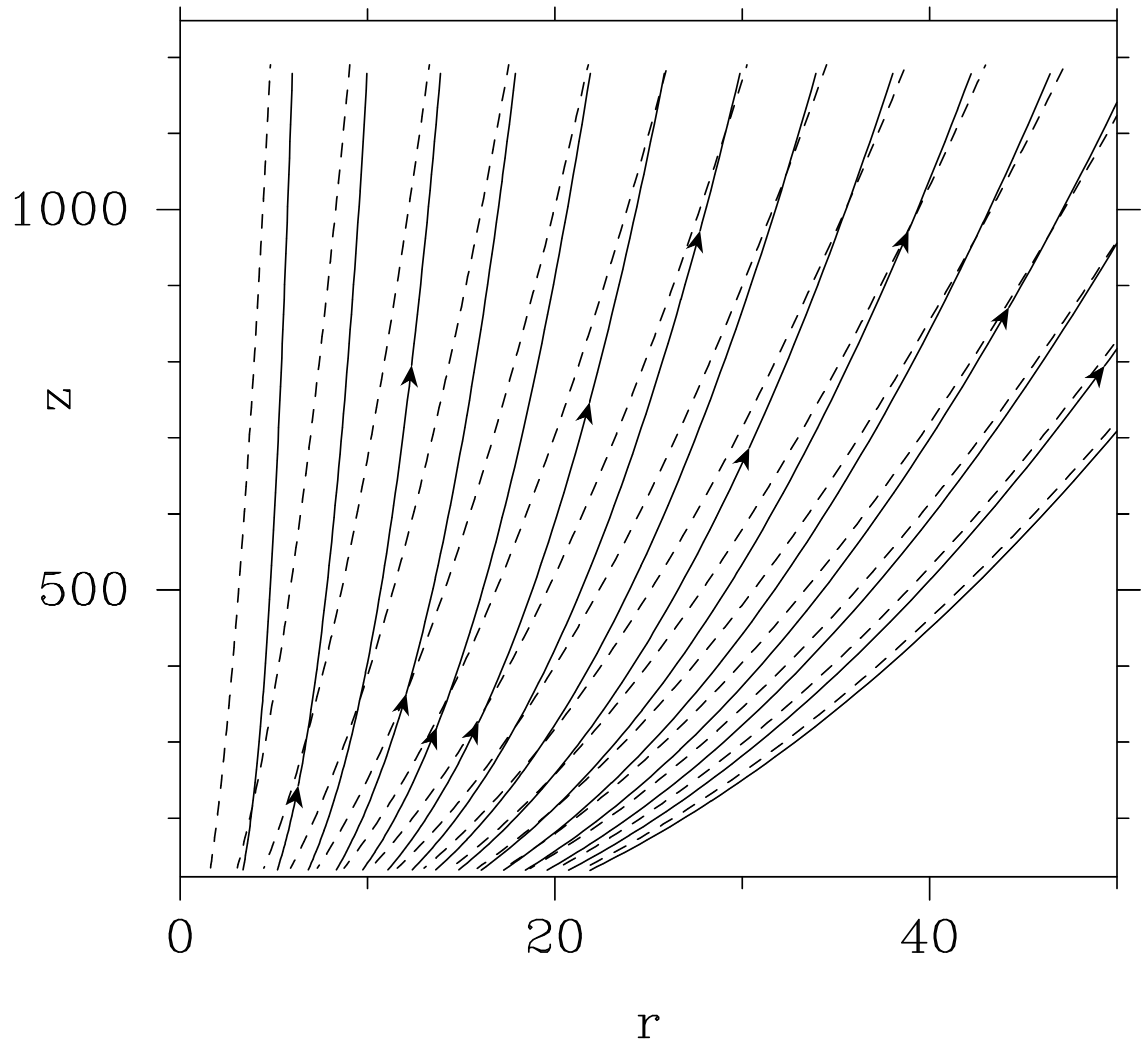
simplifies to $$\underbrace{\frac{\gamma^2 r}{\mathcal{R}}}_{inertia} \approx \underbrace{1}_{EM} - \underbrace{\gamma^2 \frac{r_{\rm lc}^2}{r^2}}_{centrifugal}$$

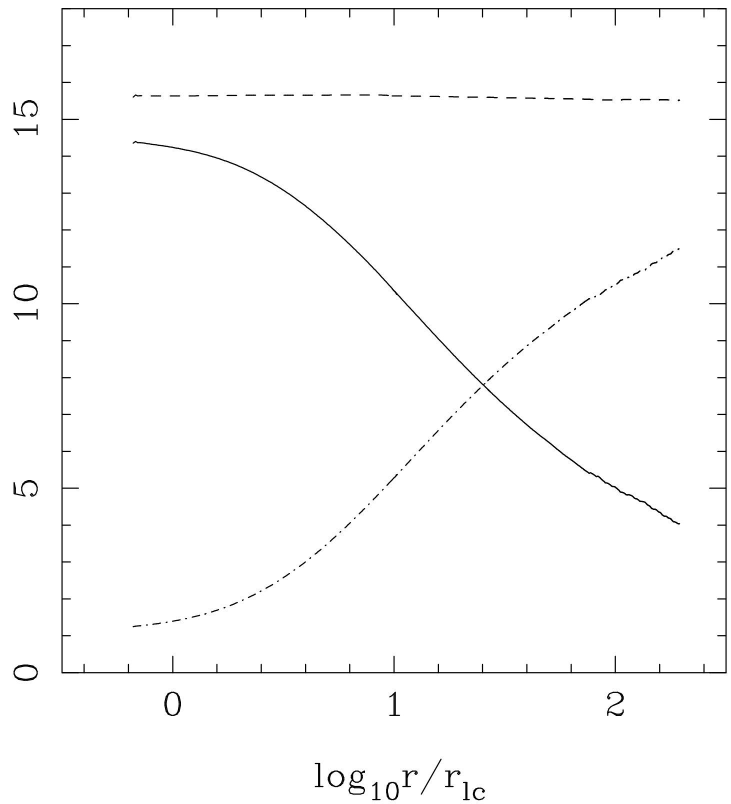
- if centrifugal negligible then $\gamma \approx z/r$ (since $\mathcal{R}^{-1} \approx -\frac{d^2 r}{dz^2} \approx \frac{r}{z^2}$)
  power-law acceleration regime
  (for parabolic shapes $z \propto r^a$, γ is a power of r)
- if inetria negligible then $\gamma \approx r/r_{\rm lc}$ linear acceleration regime
- if electromagnetic negligible then ballistic regime

Simulations of relativistic jets
Komissarov, Barkov, Vlahakis, & Königl (2007)



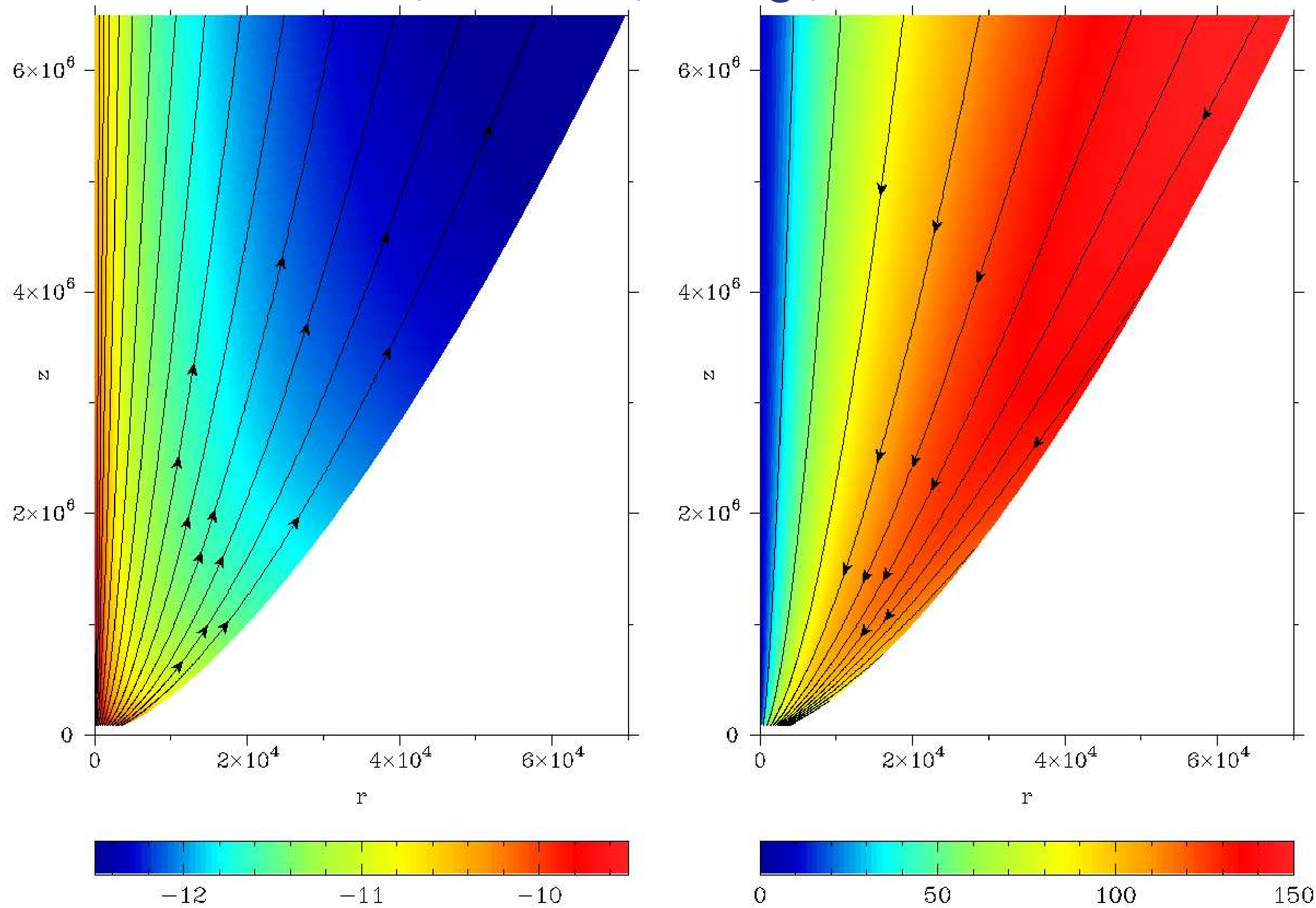
Left panel shows density (colour) and magnetic field lines. Right panel shows the Lorentz factor (colour) and the current lines.

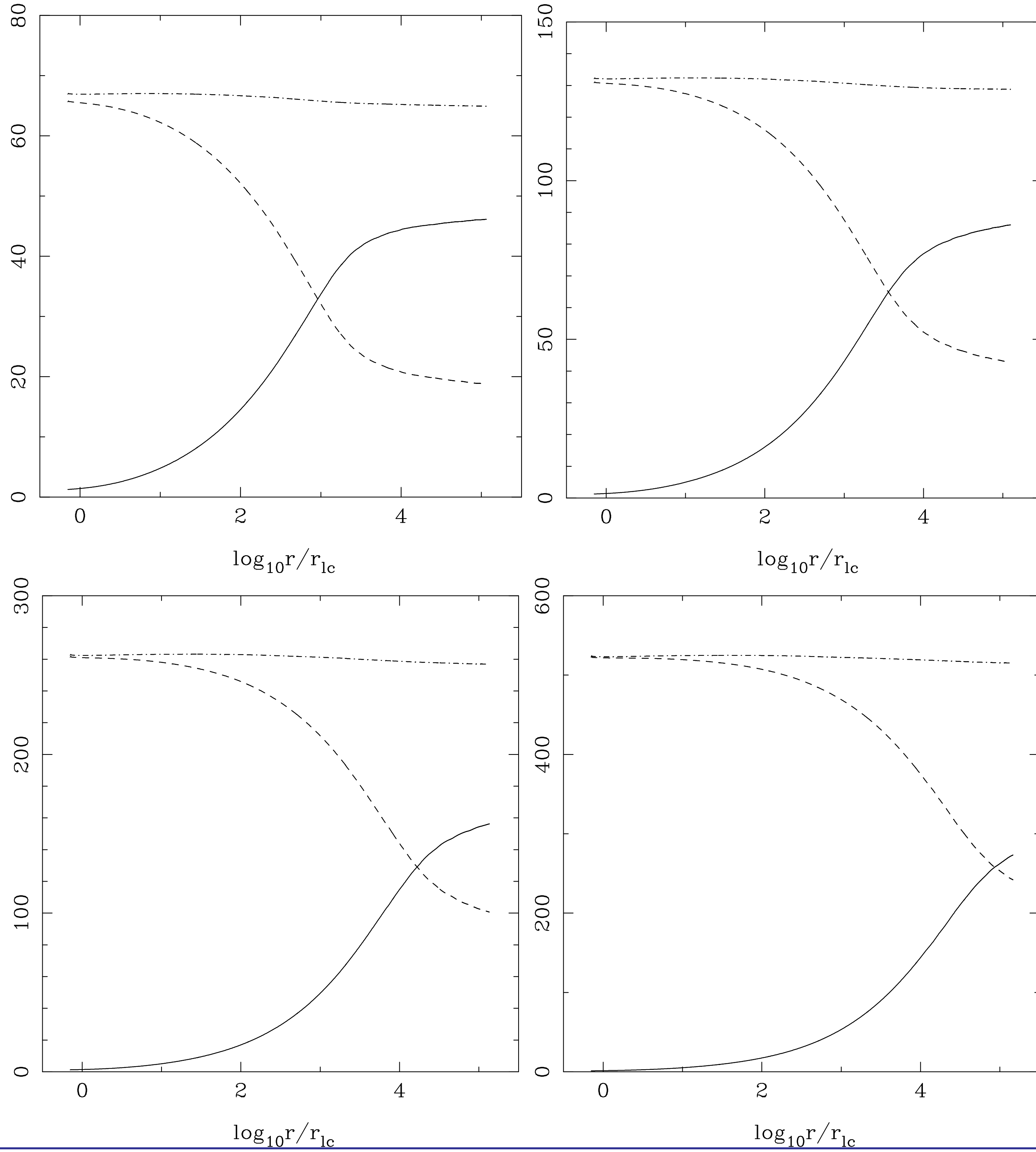

1000
500
z
0
20
40
r



$\gamma\sigma$ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius

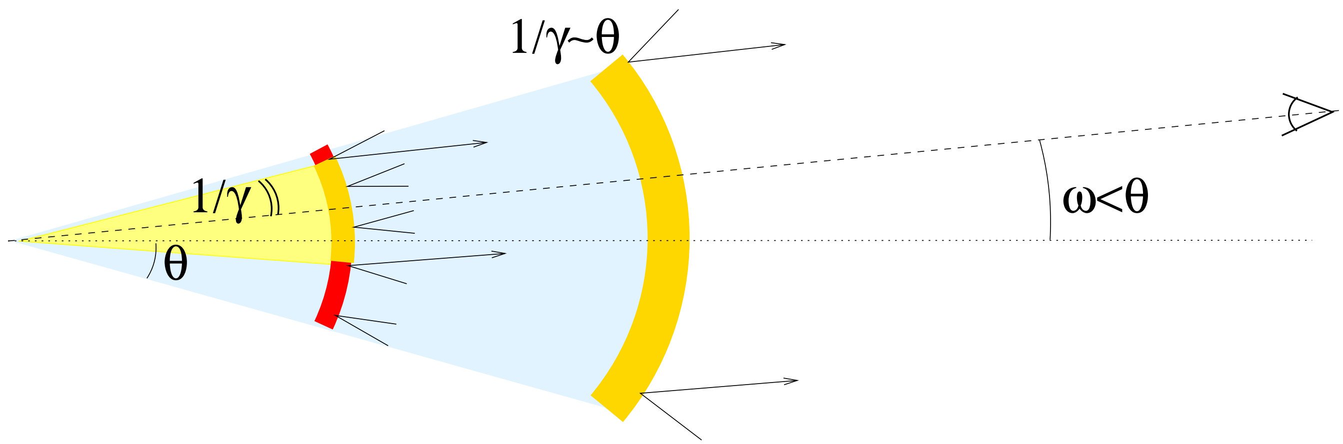
Komissarov, Vlahakis, Königl, & Barkov 2009





0
20
40
60
80
0
2
4
$\log_{10} r/r_{lc}$
0
50
100
150
0
2
4
$\log_{10} r/r_{lc}$
0
100
200
300
0
2
4
$\log_{10} r/r_{lc}$
0
200
400
600
0
2
4
$\log_{10} r/r_{lc}$

Caveat: $\gamma\vartheta \sim 1$ (for high γ)



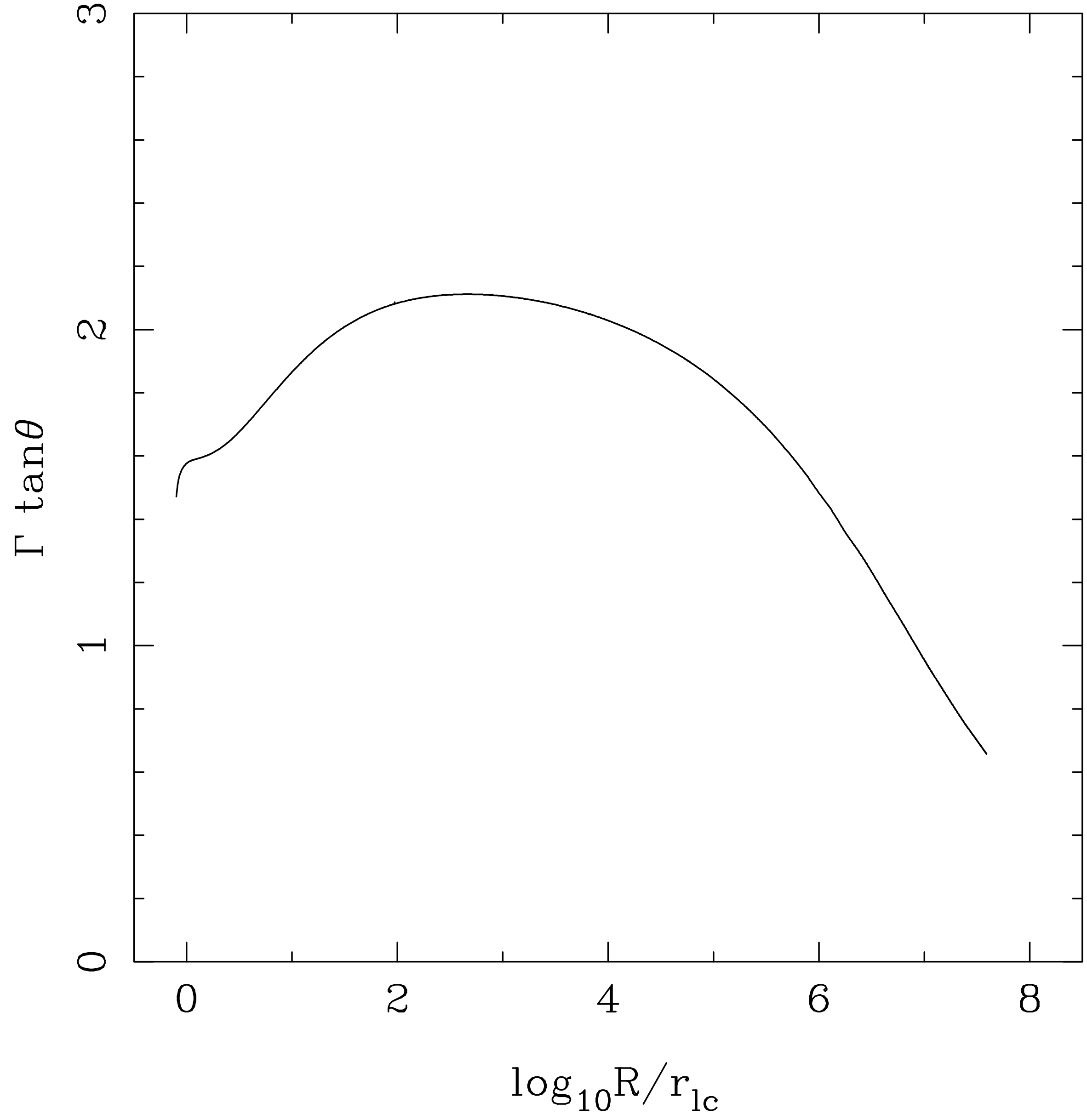
During the afterglow γ decreases
When $1/\gamma > \vartheta$ the observed flux decreases faster with time

- with $\gamma\vartheta \sim 1$ very narrow jets ($\vartheta < 1^\circ$ for $\gamma > 100$) $\longrightarrow$ early breaks or no breaks at all

- this is a result of causality (across jet): outer lines need to know that there is space to expand

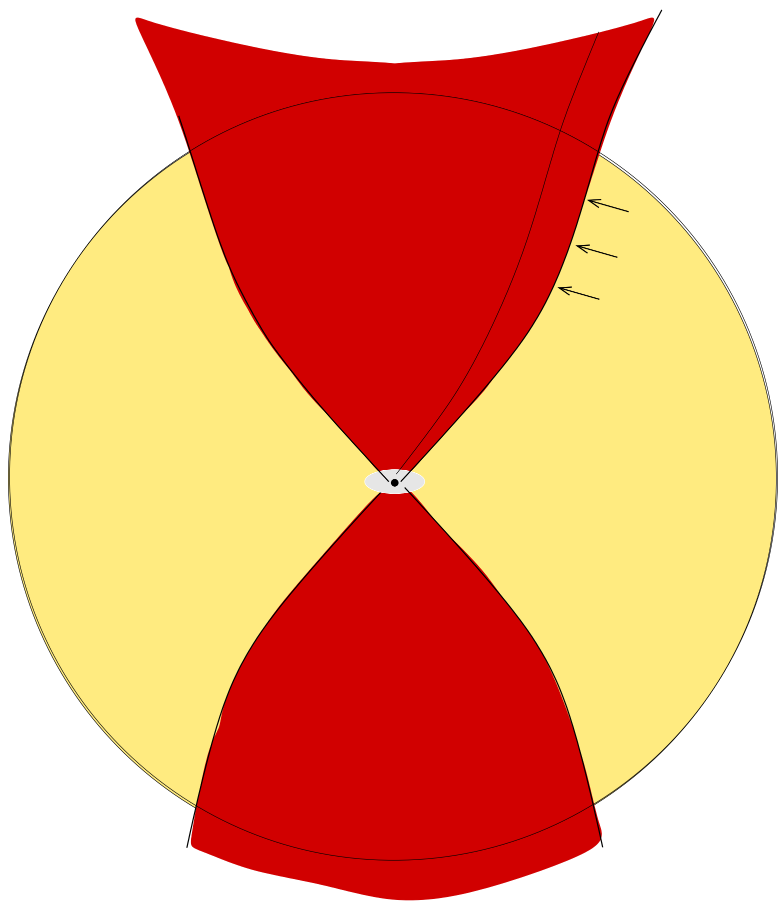
- equivalent to $\mathcal{R} \approx \gamma^2 r$ (transfield force balance)

- Mach cone half-opening θ_m should be $> \vartheta$
  With $\sin\theta_m = \dfrac{\gamma_f c_f}{\gamma V_p} \approx \dfrac{\sigma^{1/2}}{\gamma}$ the requirement for causality yields $\gamma\vartheta < \sigma^{1/2}$.
  For efficient acceleration ($\sigma \sim 1$ or smaller) we always get $\gamma\vartheta \sim 1$

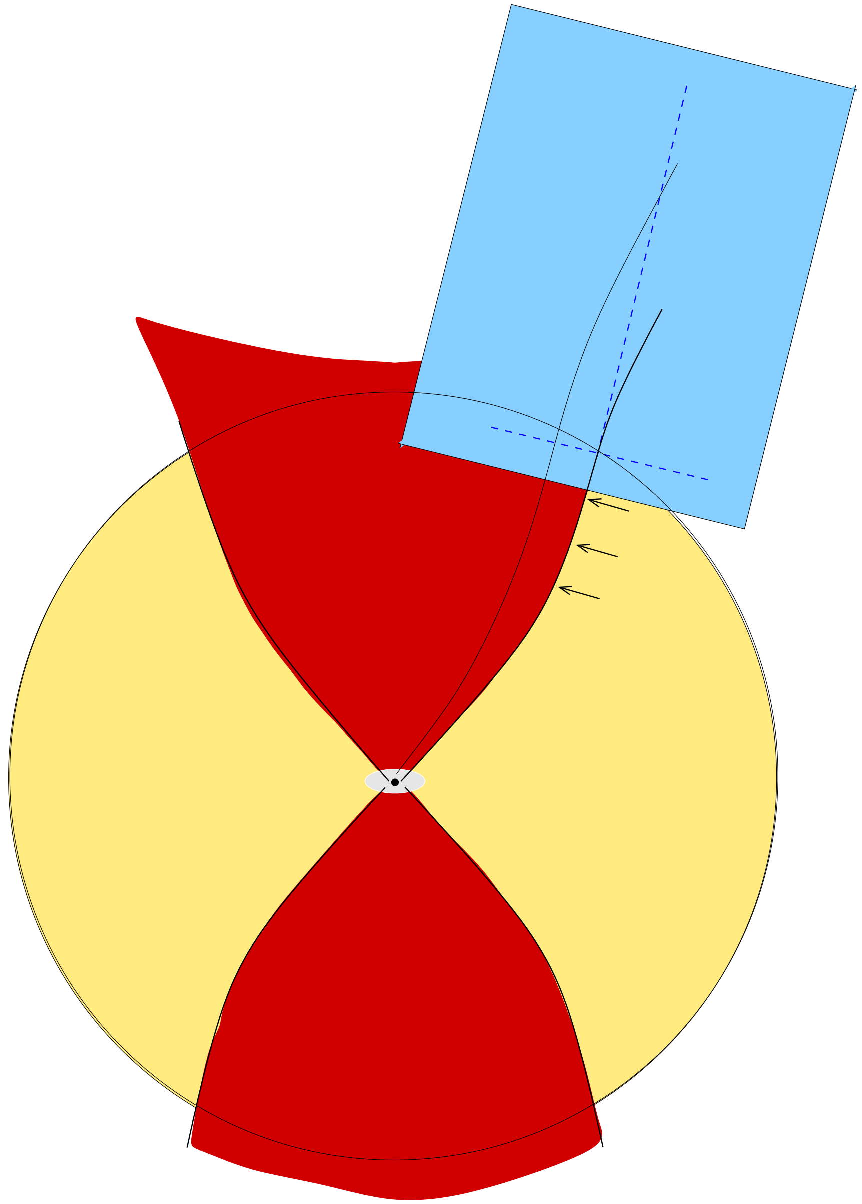


3
2
1
0
Γ tanθ
0
2
4
6
8
log10R/rlc

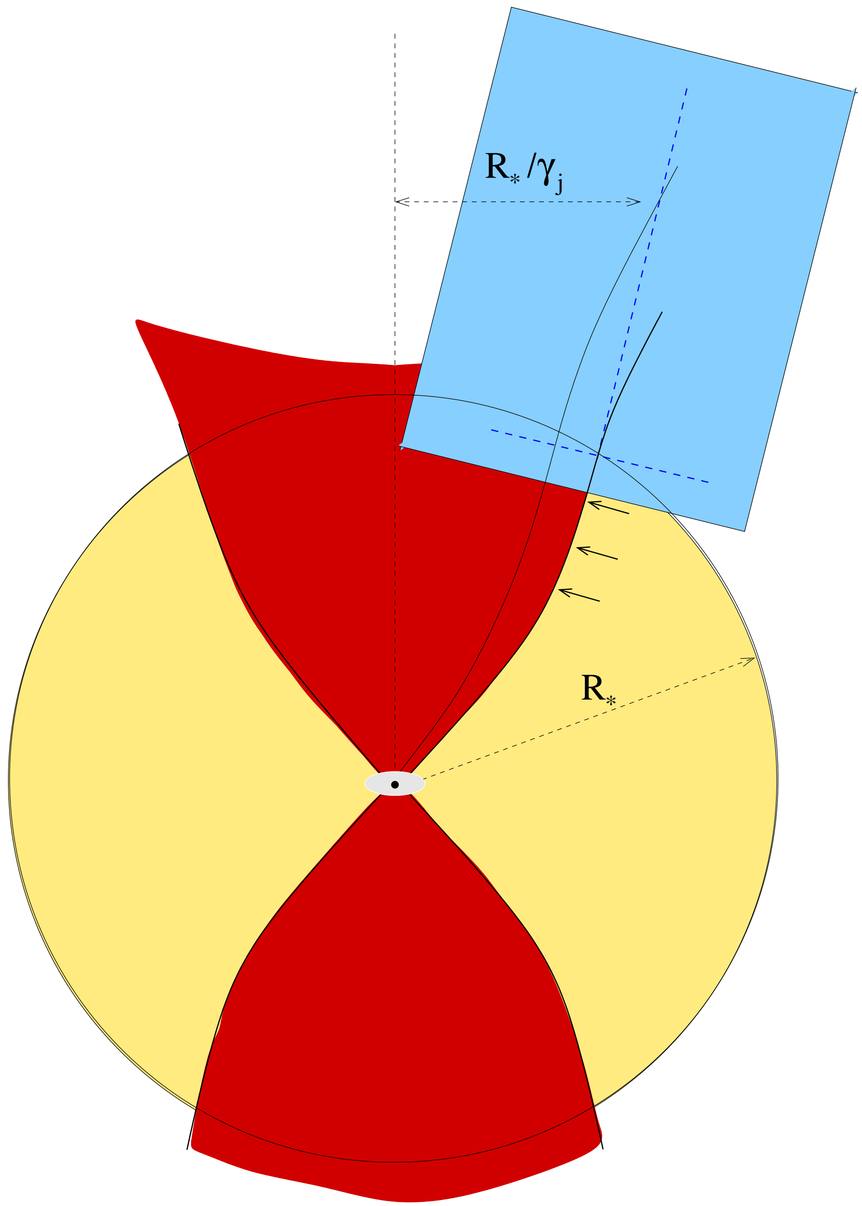
Rarefaction acceleration



Rarefaction acceleration

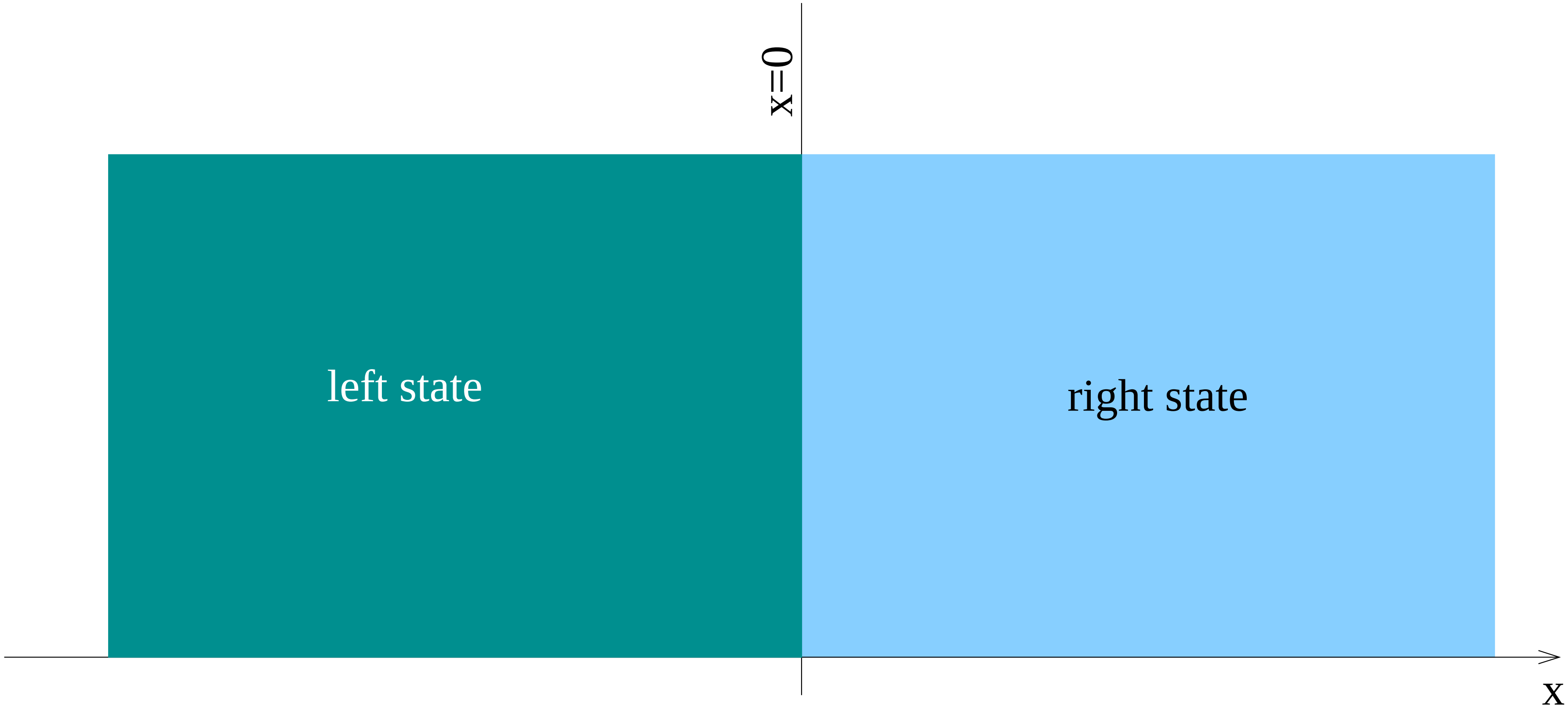


Rarefaction acceleration



Rarefaction simple waves

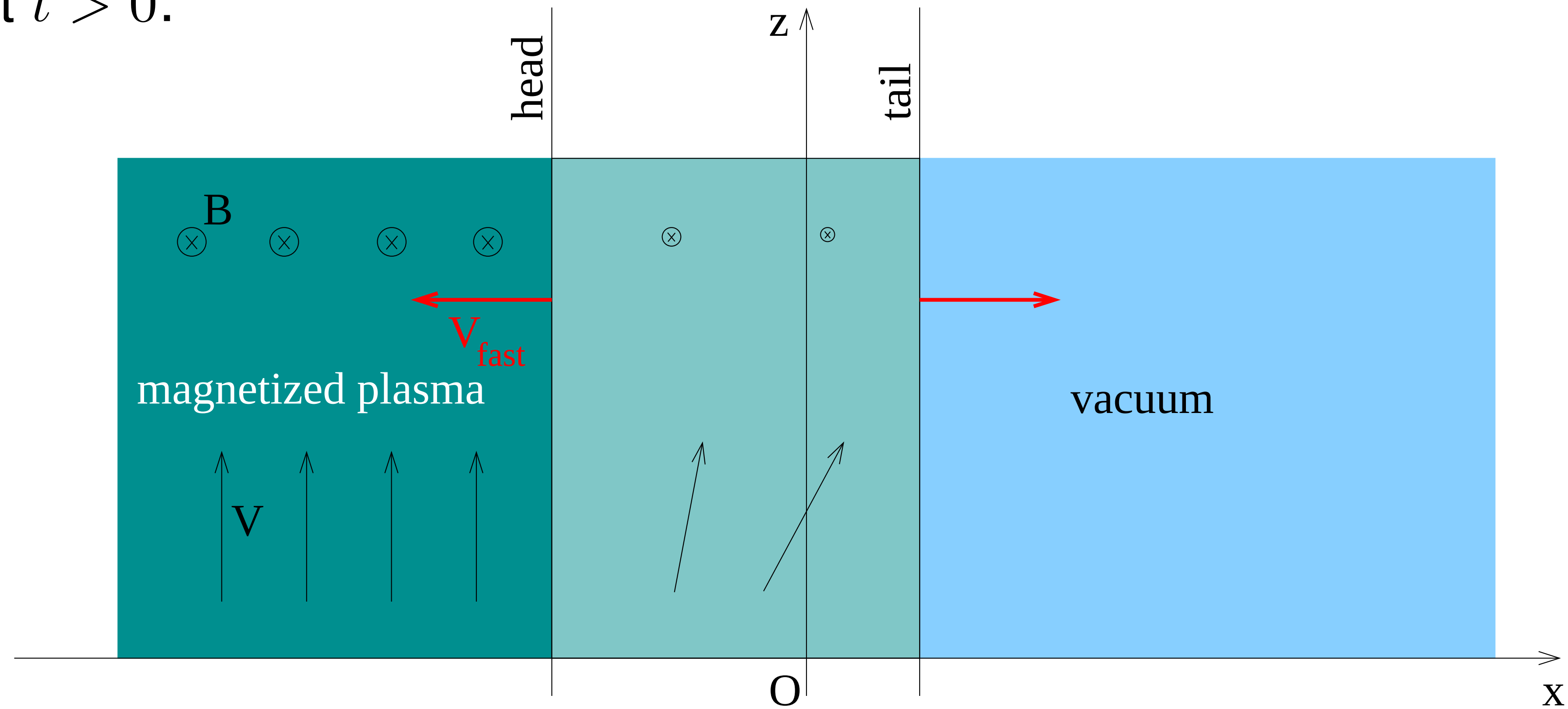
At $t = 0$ two uniform states are in contact:



This Riemann problem allows self-similar solutions that depend only on $\xi = x/t$.

- when $\rho_R/\rho_L = 0$ simple rarefaction wave

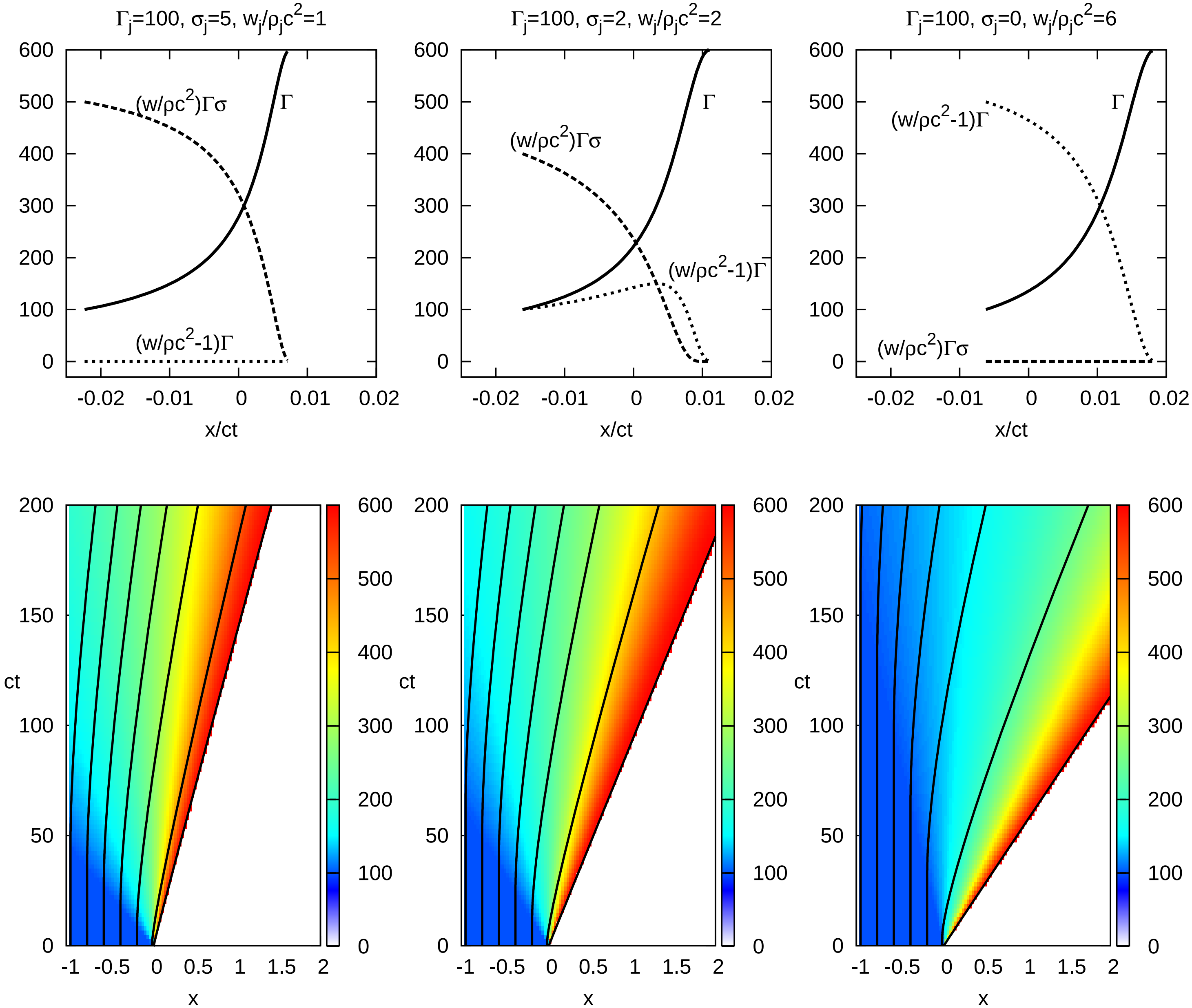
At $t > 0$:



for the cold case the Riemann invariants imply

$$v_x = \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1+\sigma_j} \left[1 - \left(\frac{\rho}{\rho_j} \right)^{1/2} \right], \; \gamma = \frac{\gamma_j \left(1+\sigma_j\right)}{1+\sigma_j \rho/\rho_j}, \; \rho = \frac{4\rho_j}{\sigma_j} \sinh^2 \left[\frac{1}{3} \mathrm{arcsinh} \left(\sigma_j^{1/2} - \frac{\mu_j}{2} \frac{x}{t} \right) \right]$$

$$V_{head} = -\frac{\sigma_j^{1/2}}{\gamma_j}, \qquad V_{tail} = \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1+\sigma_j}, \qquad \Delta\vartheta = V_{tail} < 1/\gamma_i$$

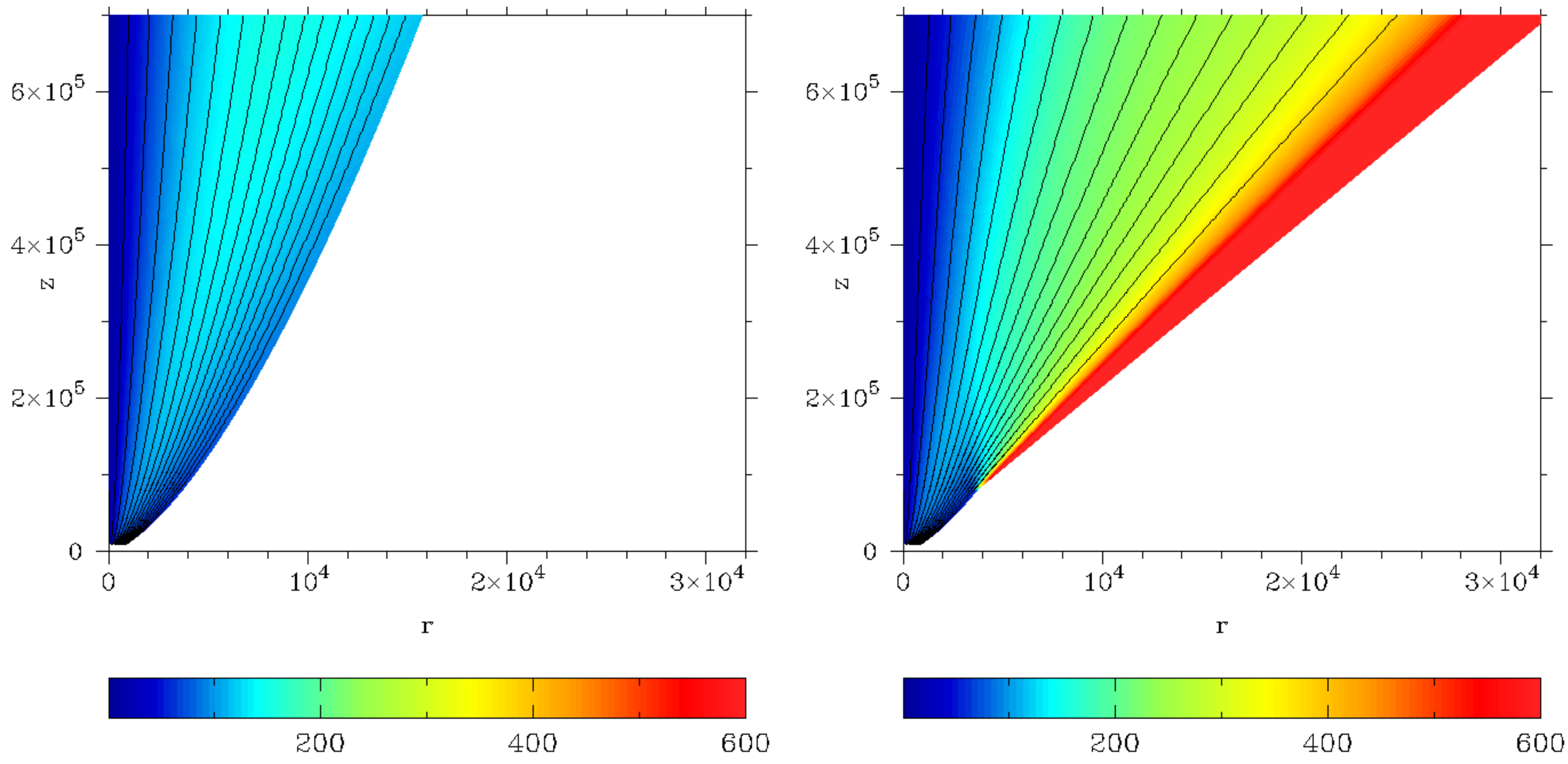


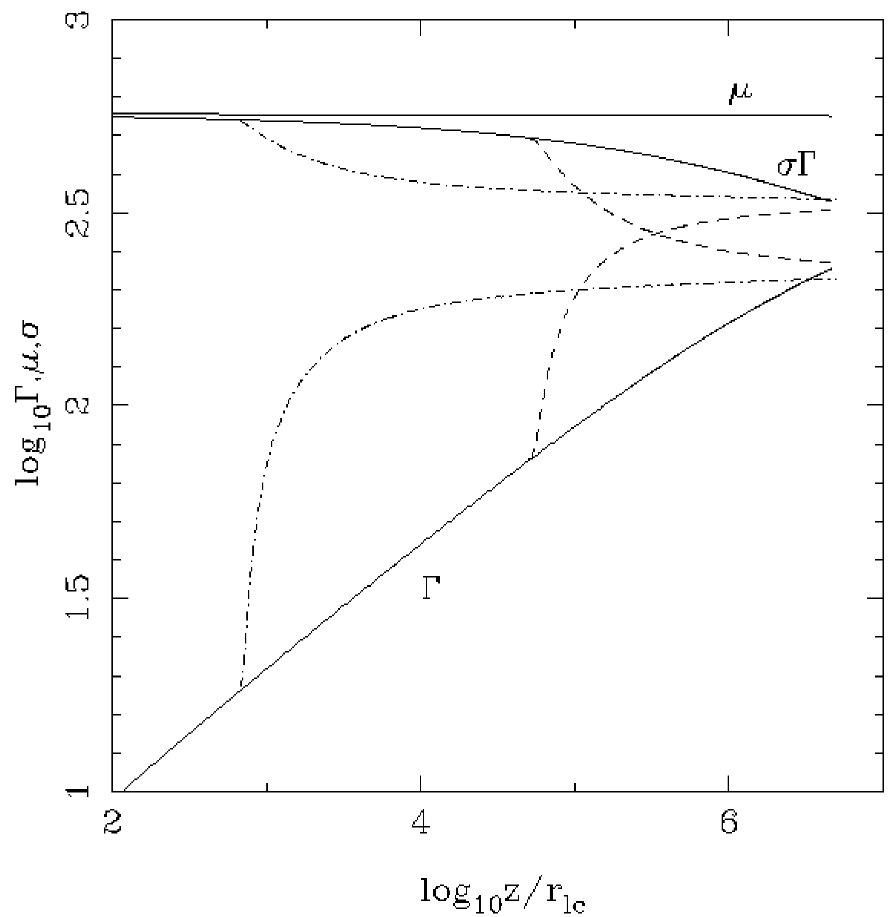
The colour image in the Minkowski diagrams represents the distribution of the Lorentz factor and the contours show the worldlines of various fluid parcels. (see also Aloy & Rezzolla 2006 for HD, Mizuno+2008 for MHD)

Simulation results

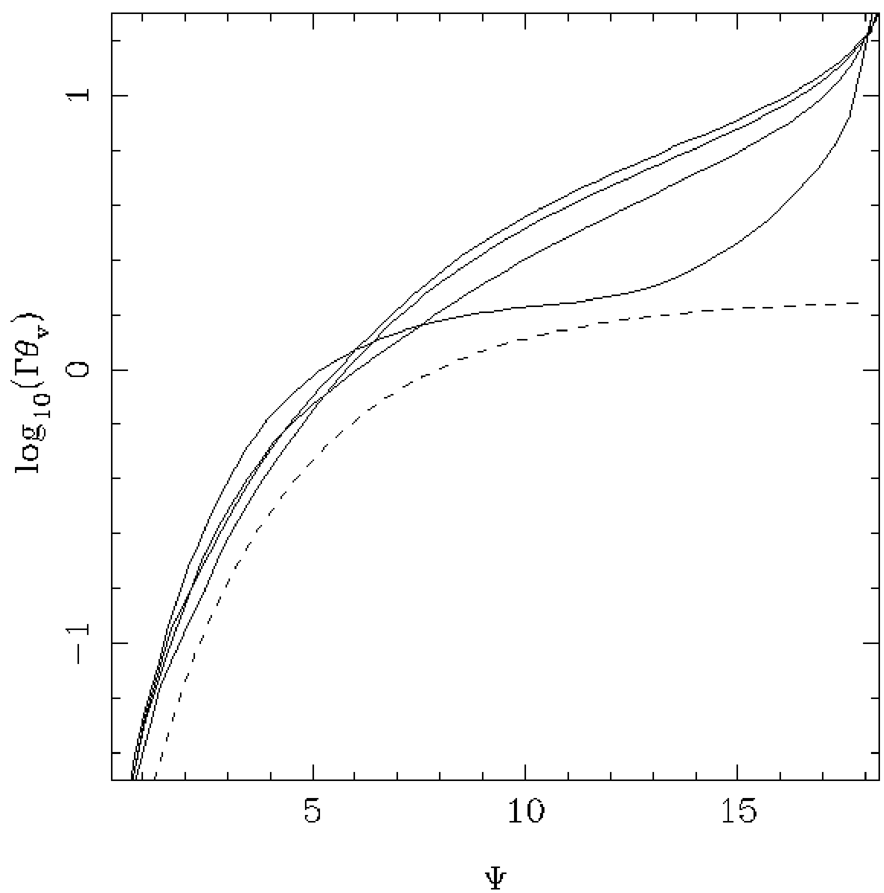
Komissarov, Vlahakis & Königl 2010

(see also Tchekhovskoy, Narayan & McKinney 2010)





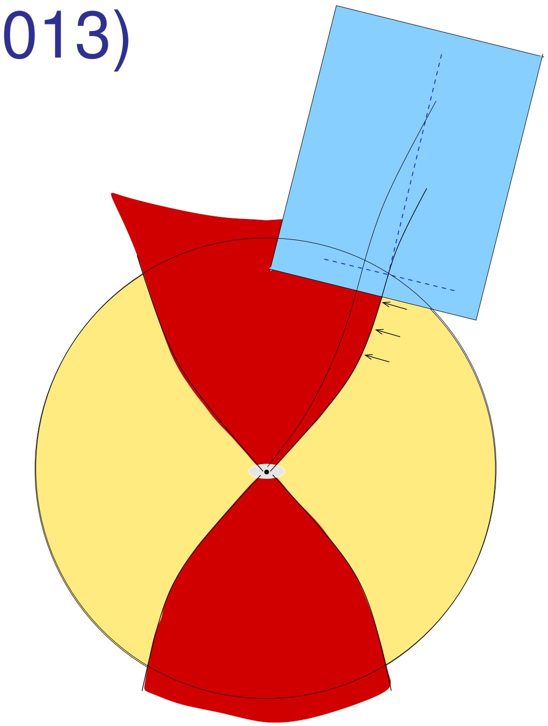
3
2.5
2
1.5
1
2
4
6
$\log_{10}\Gamma,\mu,\sigma$
$\log_{10}z/r_{lc}$
μ
$\sigma\Gamma$
Γ



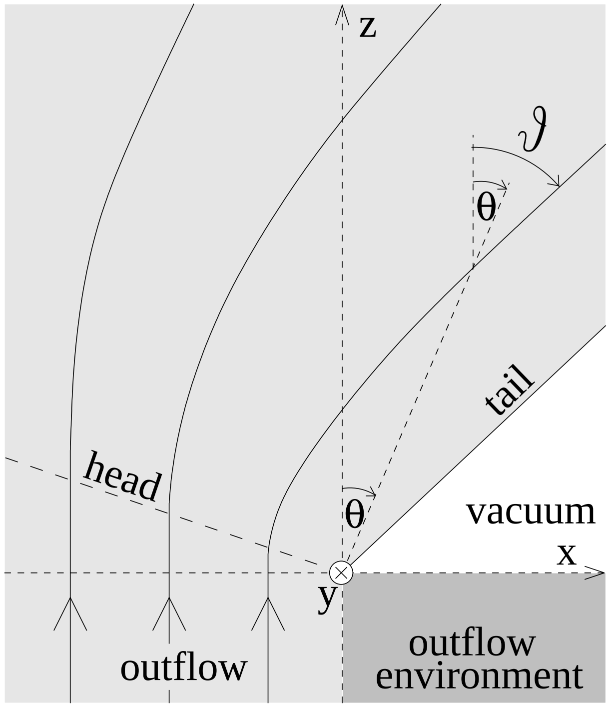
1
0
−1
5
10
15
$\log_{10}(\Gamma\theta_v)$
Ψ

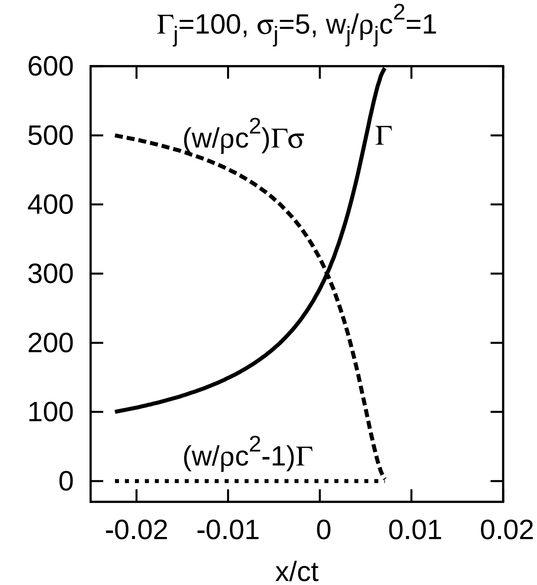
Steady-state rarefaction wave

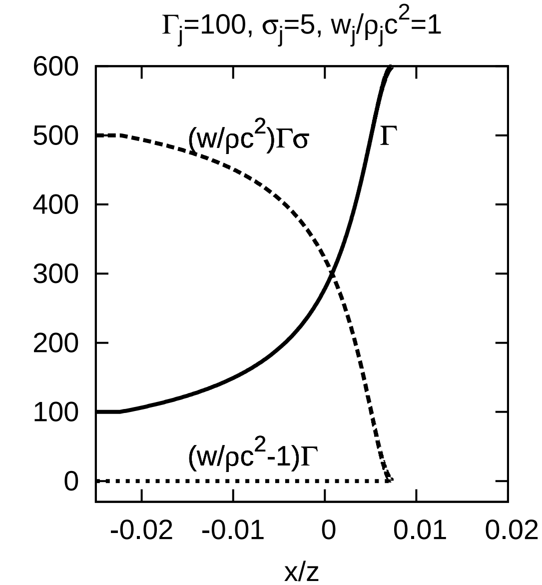
Sapountzis & Vlahakis (2013)

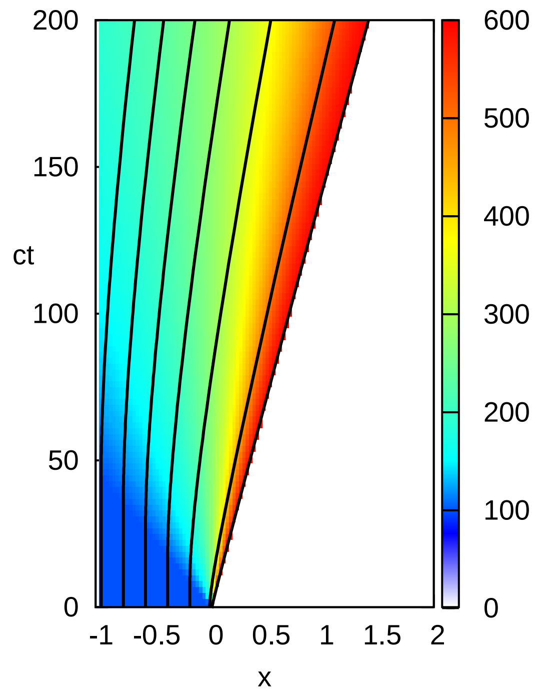


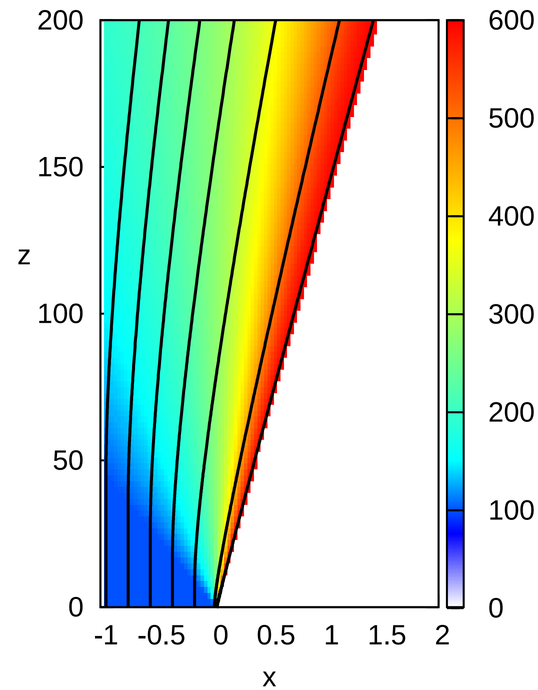
- "flow around a corner"
- planar geometry
- ignoring B_p (nonzero B_y)
- similarity variable x/z (angle θ)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)











$$\theta_{\rm head} = -\frac{\sigma_j^{1/2}}{\gamma_j}$$

$$\theta_{\rm tail} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)}$$

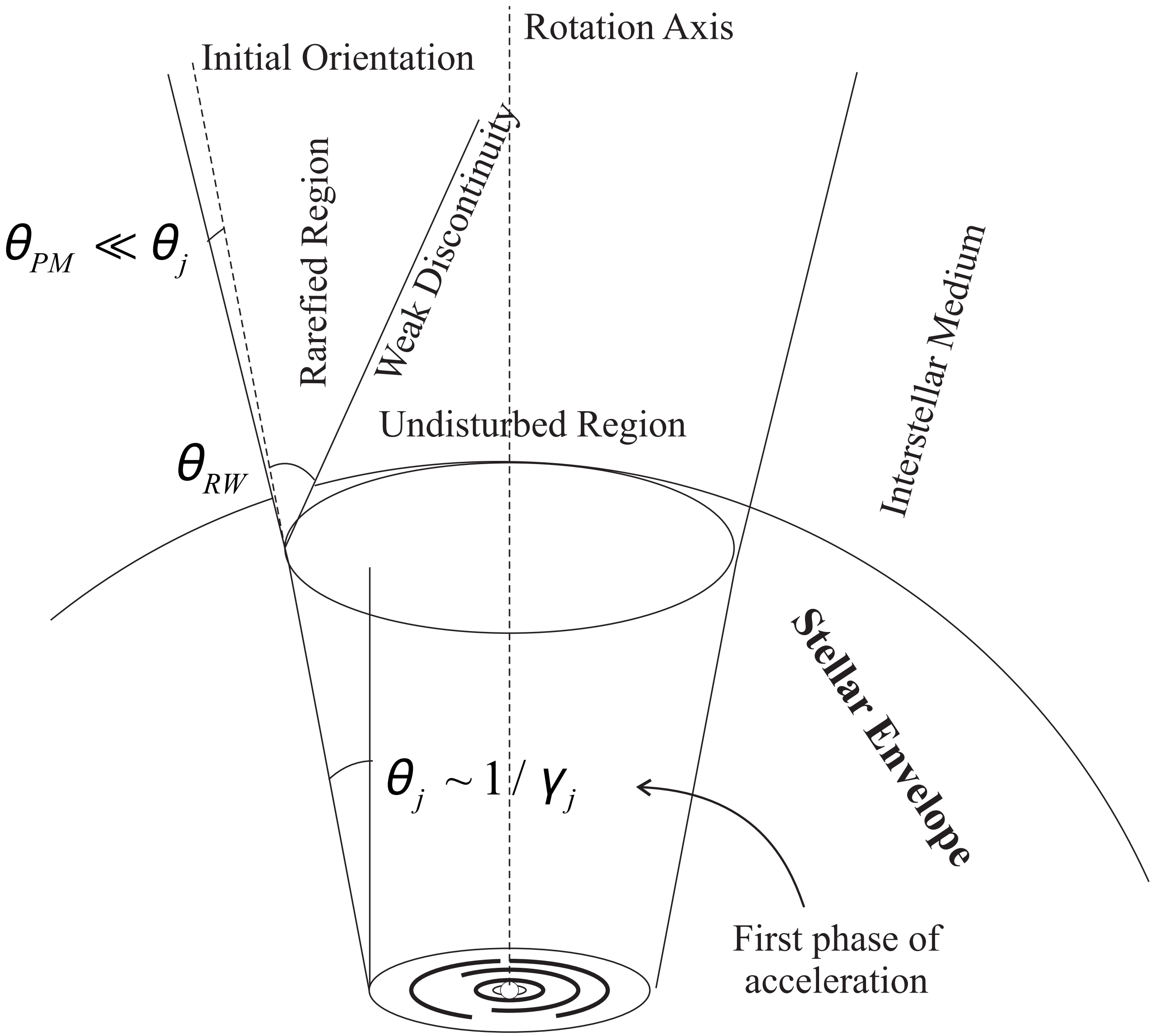
$$\sigma = (\sigma_j \gamma_j x_i / z)^{2/3}$$

$\sigma = 1$ at $r = \sigma_j \gamma_j |x_i| = 7 \times 10^{11} \sigma_j \left(\frac{|x_i|}{R_\star/\gamma_j}\right) \left(\frac{R_\star}{10 R_\odot}\right)$ cm

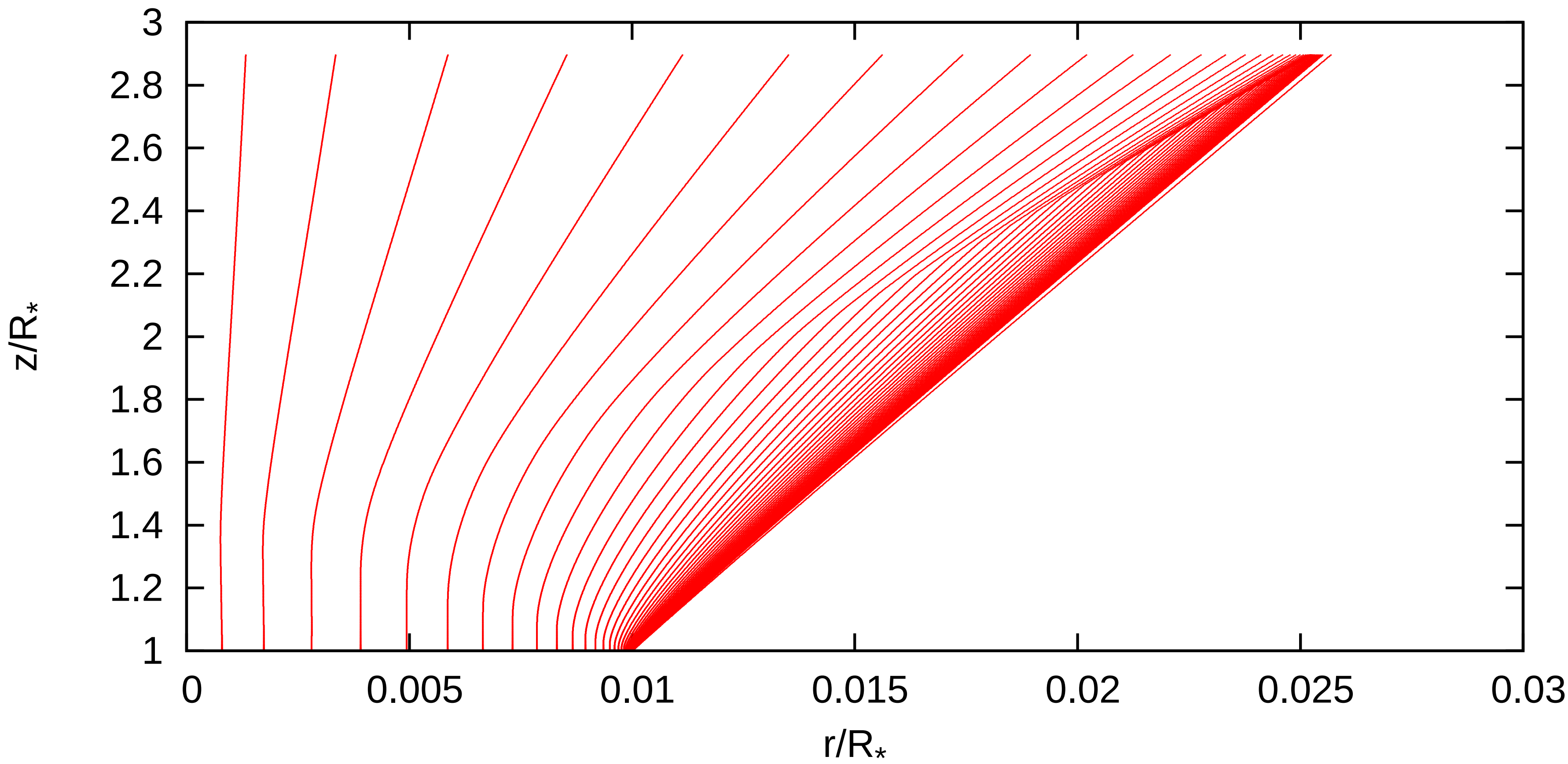
time-dependent (left) and steady-state (right) rarefaction (similar; $ct \to z$)
(distance unit $= R_\star/\gamma_j \sim 10^{10}$ cm)

Axisymmetric model

Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)

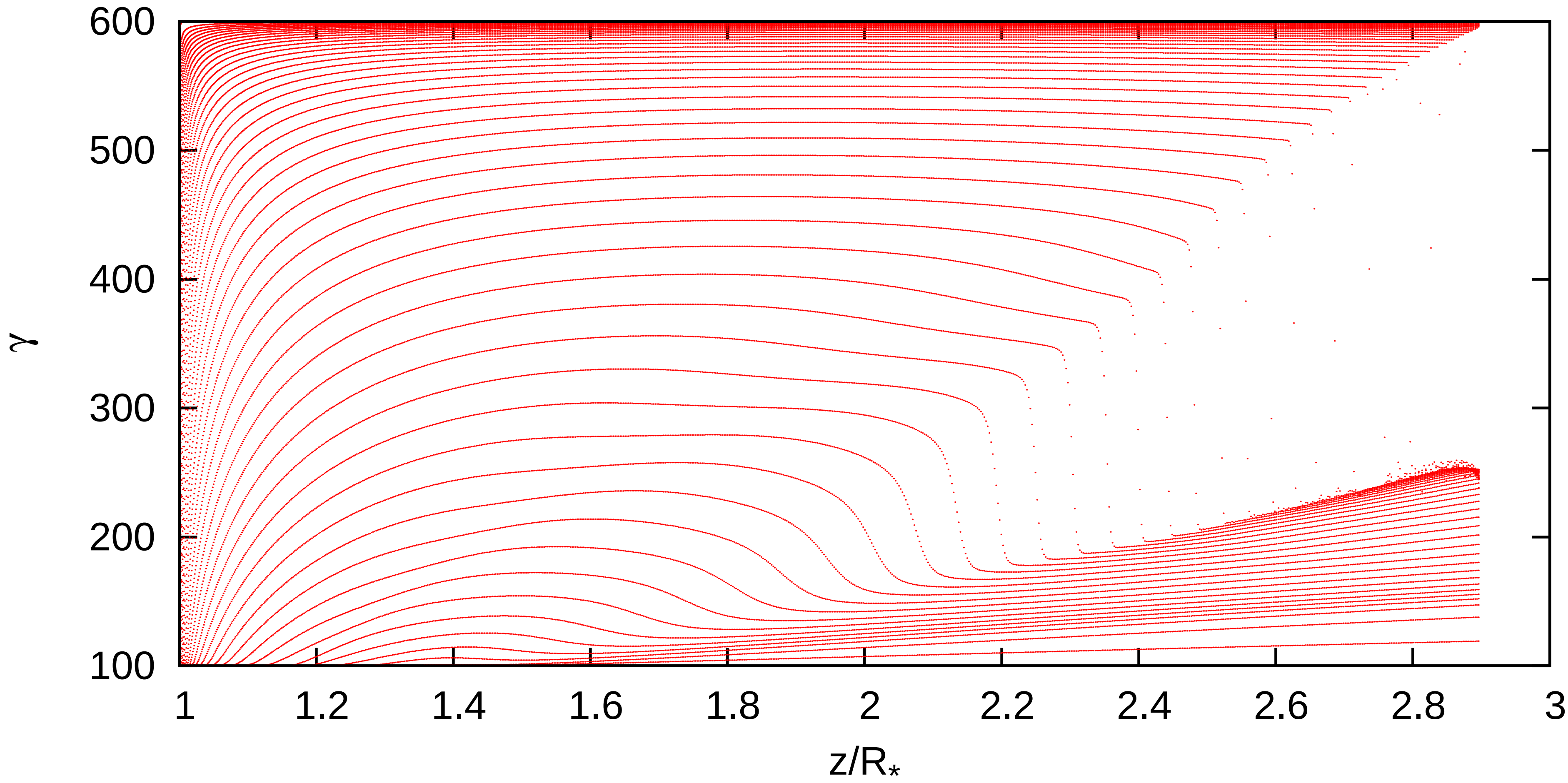


$$\gamma_j = 100, \sigma_j = 1, \rho_{ext} = 0$$

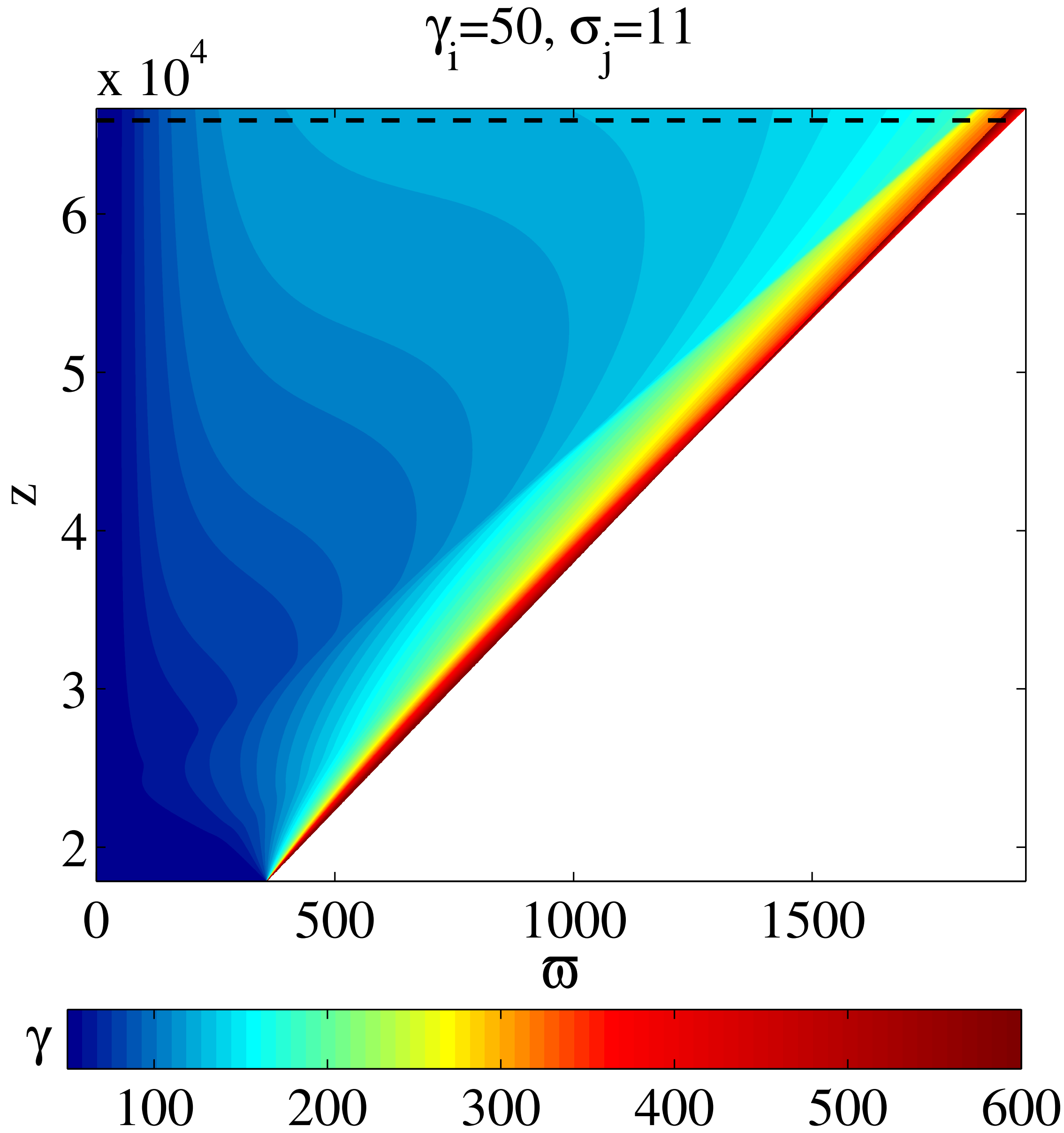


(not in scale!)

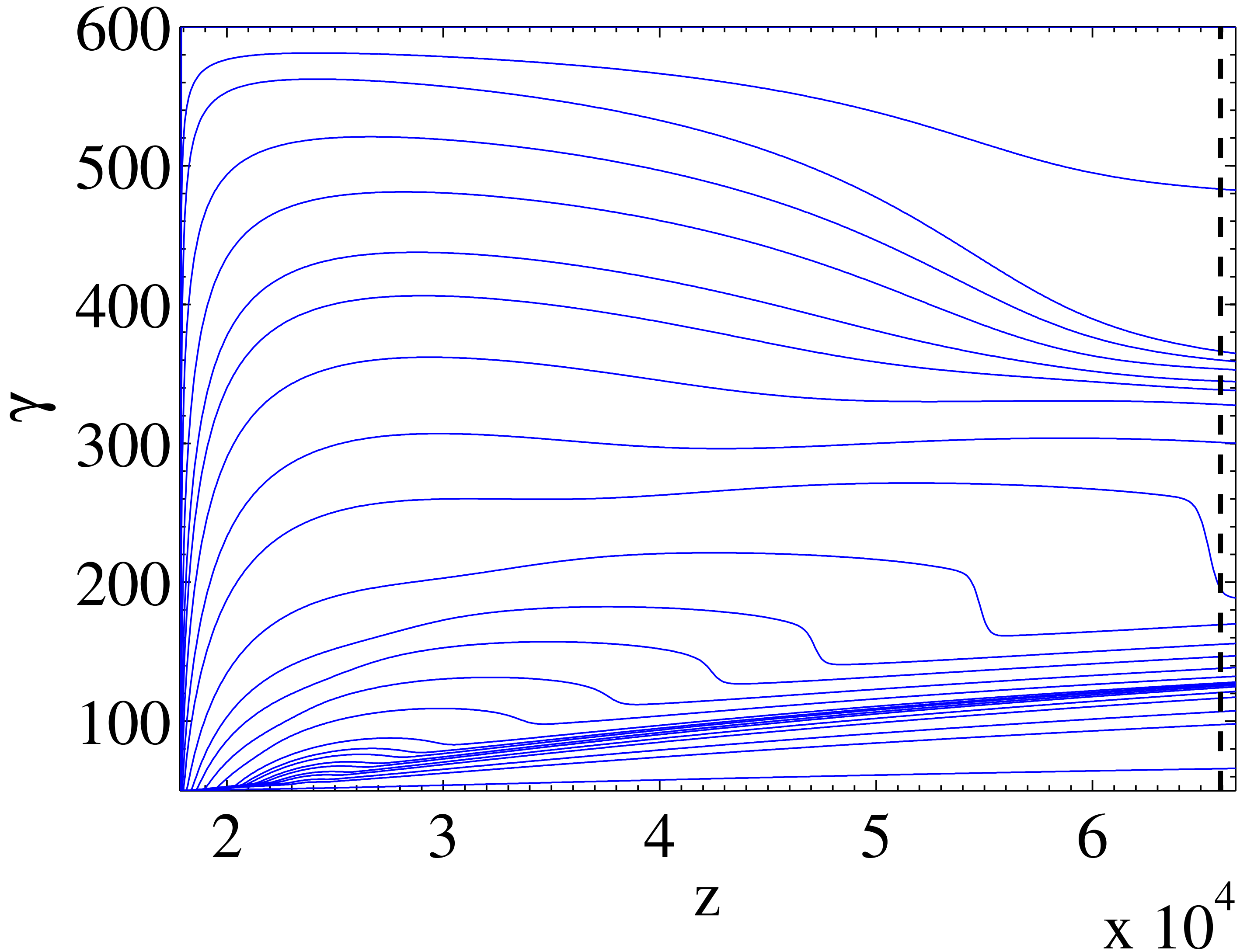
Reflection of the wave from the axis



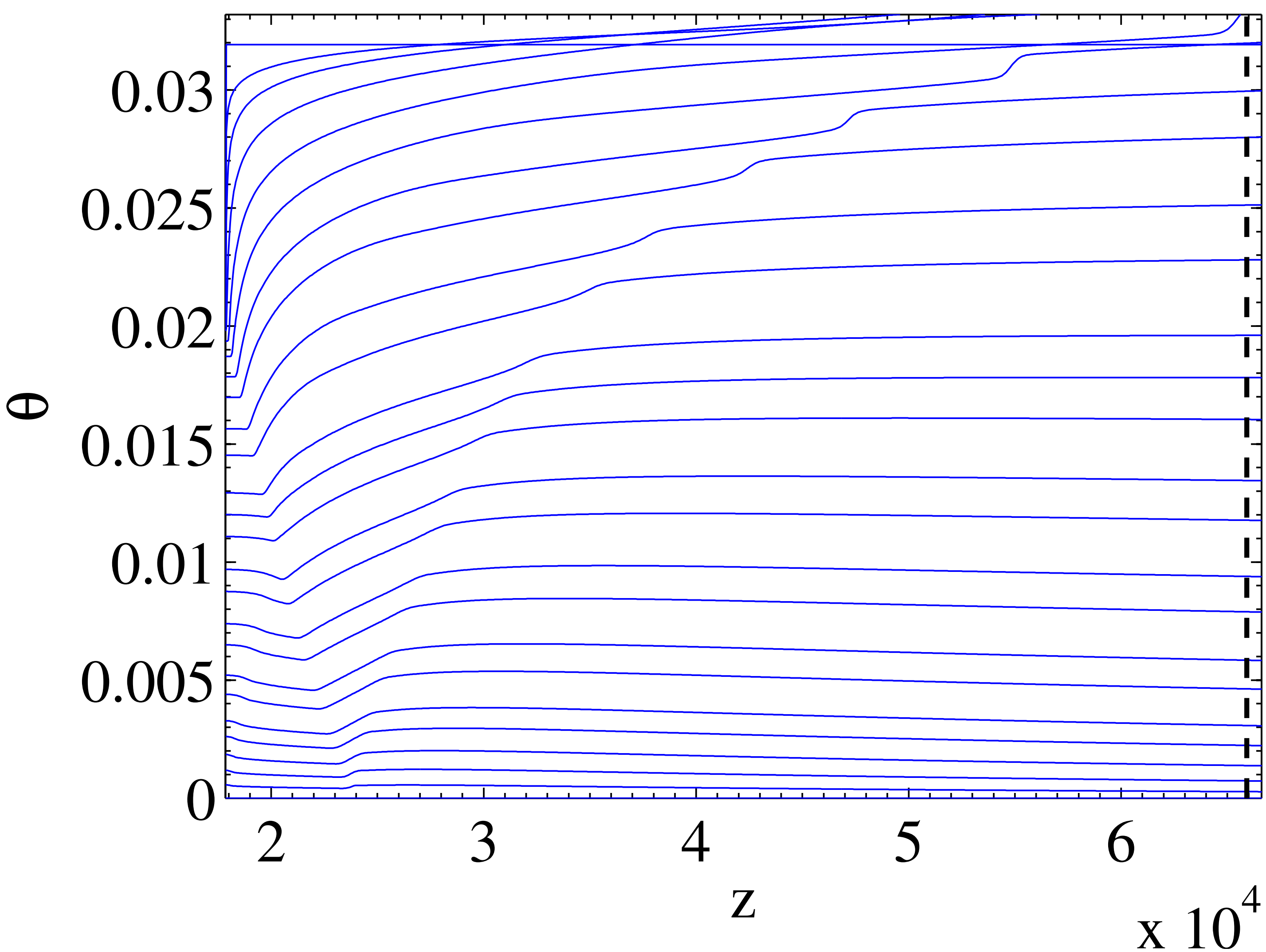
Reflection causes sudden deceleration – standing shock (?)



γ_i=50, σ_j=11
x 10^4
z
6
5
4
3
2
0
500
1000
1500
ϖ
γ
100
200
300
400
500
600

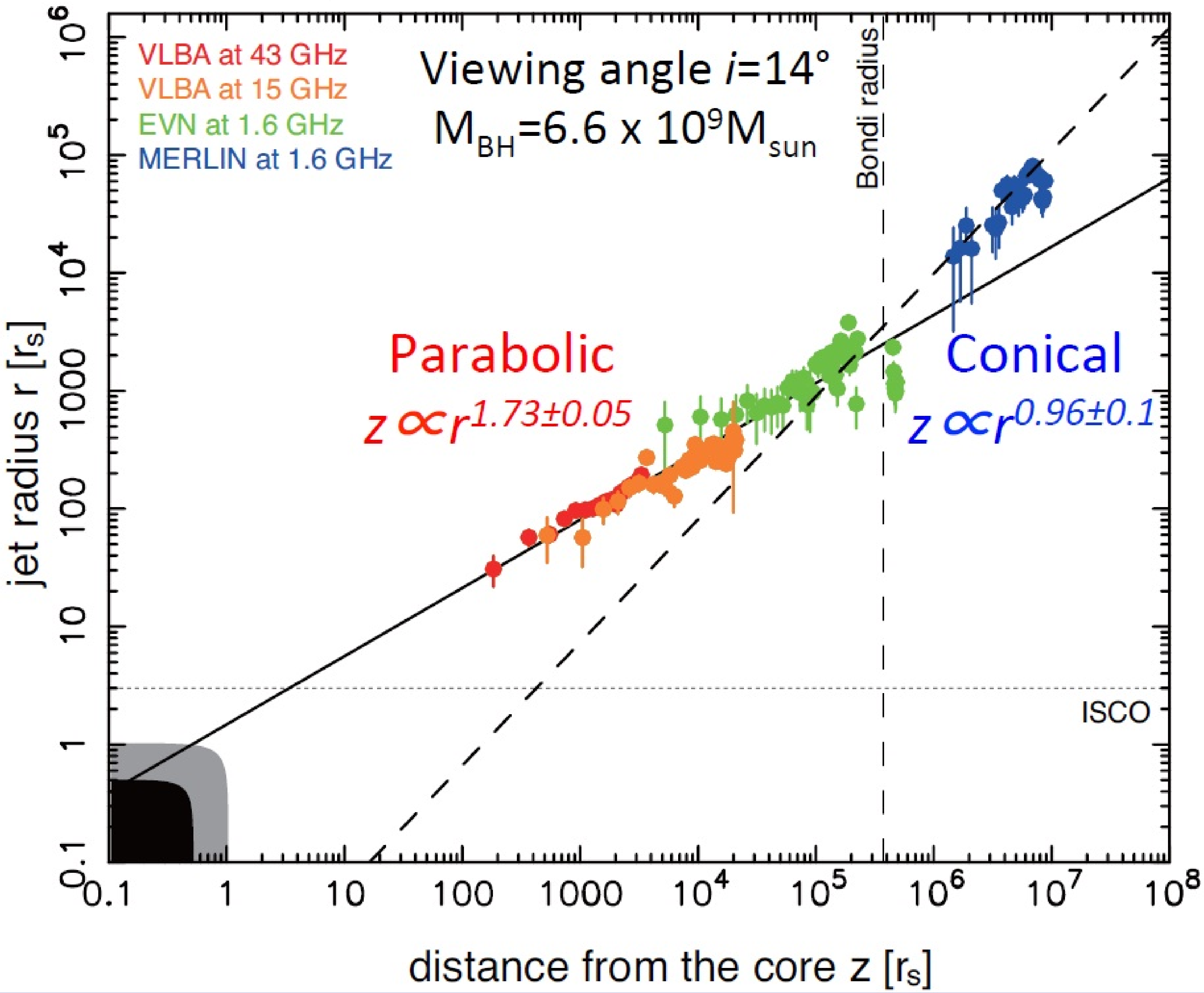


600
500
400
300
200
100
γ
2
3
4
5
6
z
x 10^4


0.03
0.025
0.02
0.015
0.01
0.005
0
θ
2
3
4
5
6
z
x 10^4

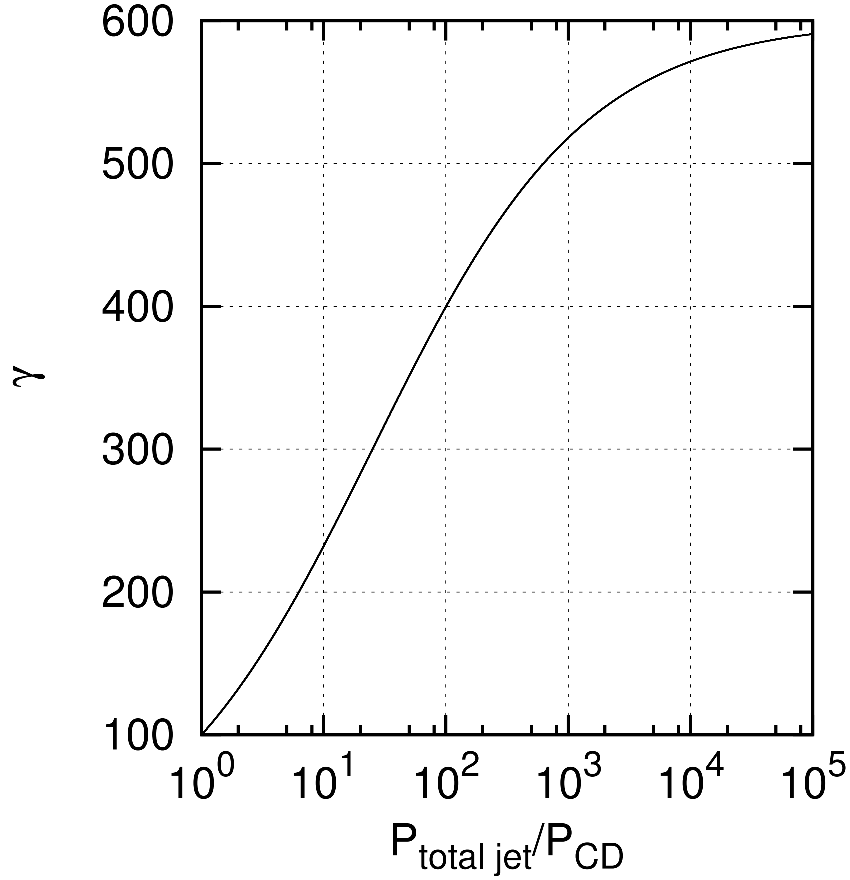
Does it work in AGNs?

(Asada & Nakamura 2011)



The role of the environment

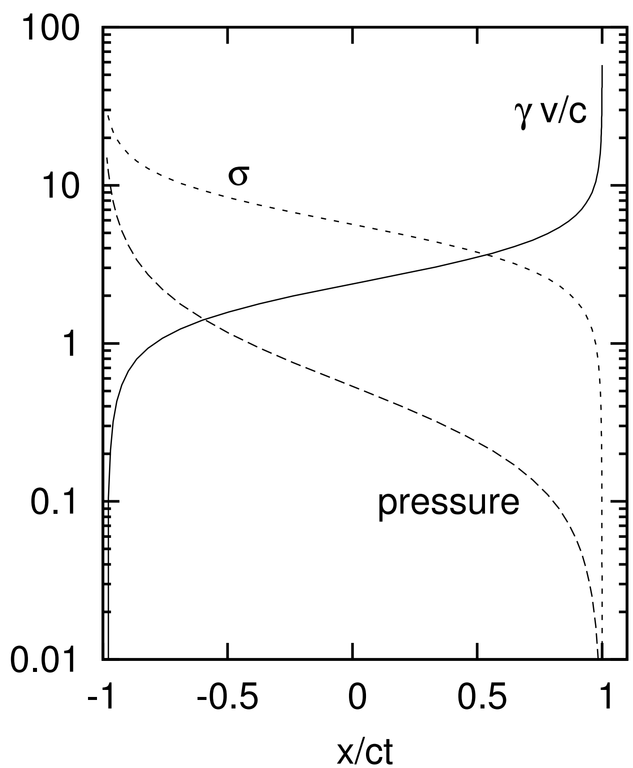
- for nonzero ρ_{ext} Riemann problem: rarefaction on the left state / contact discontinuity / shock on the right



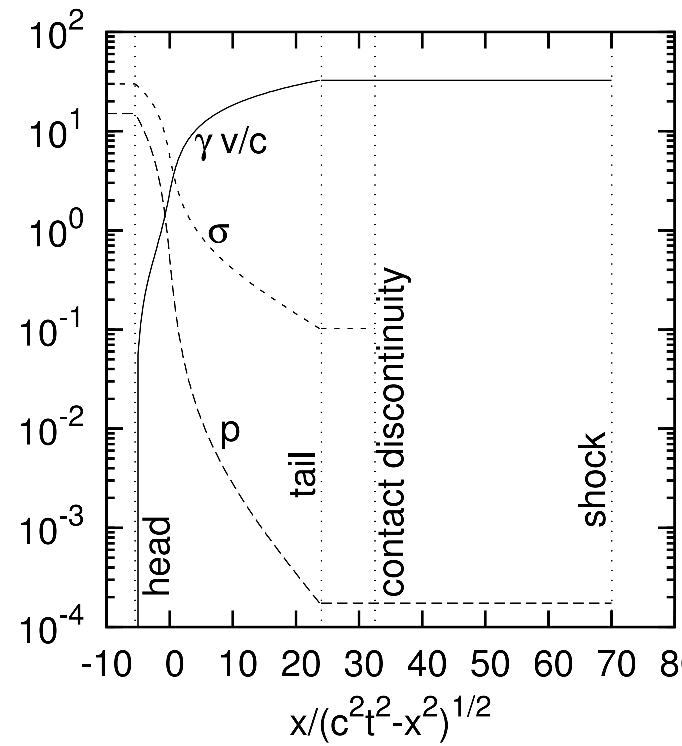
(for $\gamma_j = 100$, $\sigma_j = 1$)

- matching of speed and total pressure at the contact discontinuity gives the solution on the left and right
  (Marti+1994, Lyutikov 2010 for time-dependent problem; Katsoulakos & Vlahakis in preparation for the steady-state)

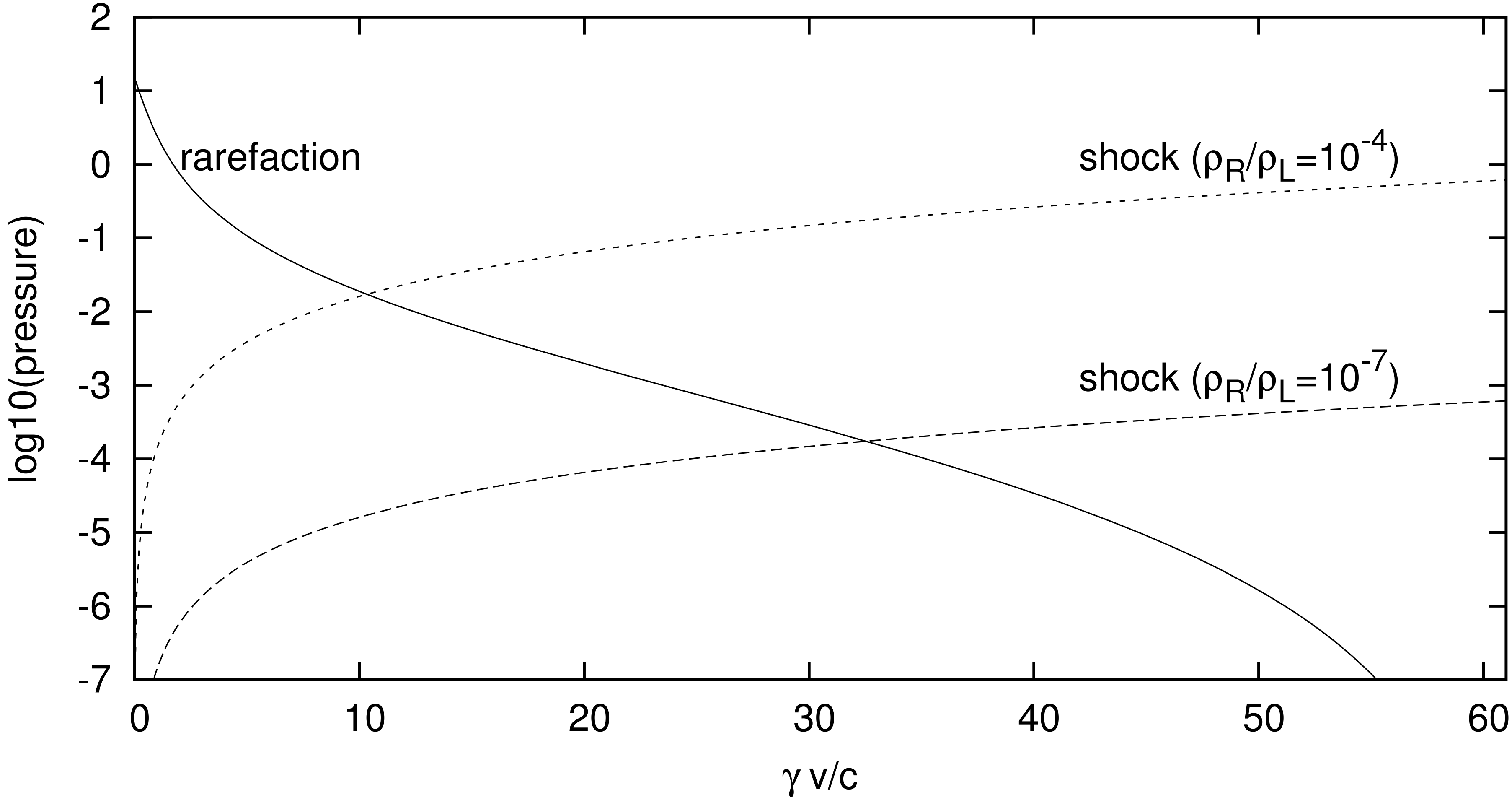
- time-dependent example: impulsive acceleration
  (Granot, Komissarov & Spitkovsky 2011)



for $\rho_R/\rho_L = 0$



for $\rho_R/\rho_L = 10^{-7}$, $P_R = 0$



- in AGNs $\rho_{ext}/\rho_j \gg 1$, so rarefaction unlikely to work
- not clear, see Millas' talk

Summary

- ⋆ The collimation-acceleration paradigm provides a viable explanation of the dynamics of relativistic jets
- ⋆ bulk acceleration up to Lorentz factors $\gamma_\infty \gtrsim 0.5 \frac{\mathcal{E}}{Mc^2}$
  caveat: in ultrarelativistic GRB jets $\vartheta \sim 1/\gamma$
- ⋆ Rarefaction acceleration
  - further increases γ
  - makes GRB jets with $\gamma\vartheta \gg 1$
  - steady shock creation (?)
  - unlikely to work in AGN jets
- ⋆ The jet-environment interaction is complicated but important to clarify

Acknowledgments

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