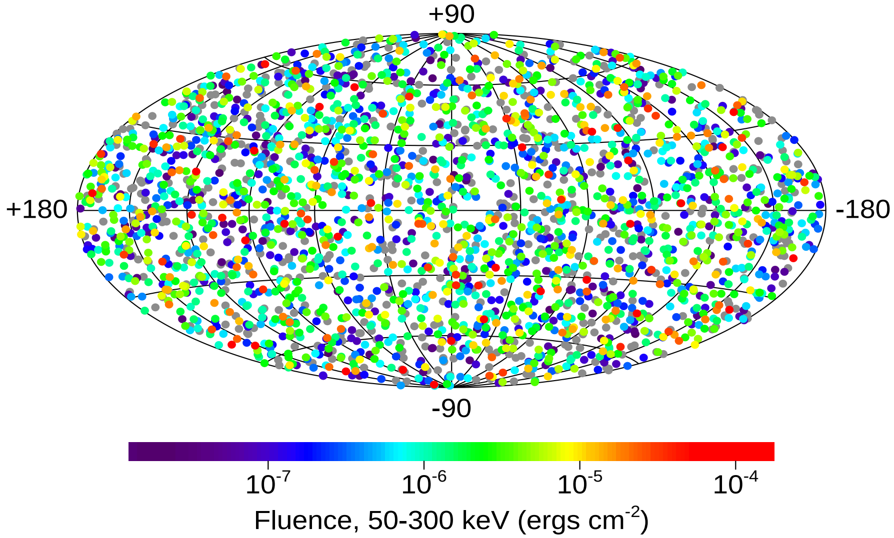
MagnetoHydroDynamics of Gamma-Ray Burst Jets



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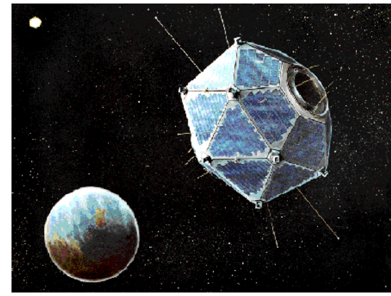
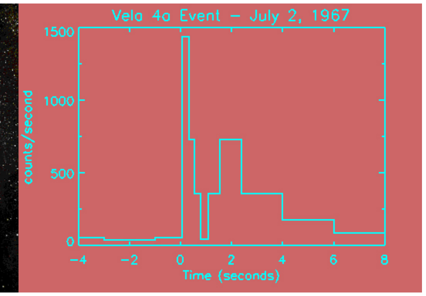
Outline

- GRBs and their afterglows
  - observations
  - our understanding
- the MHD description
  - general theory
  - the model
  - results
- Crab-like pulsar winds
  - a solution to the σ-problem

Observations

- 1967: the first GRB
  Vela satellites
  (first publication on 1973)





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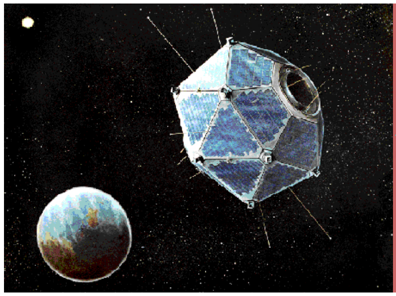
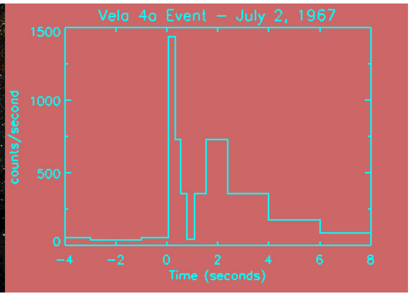

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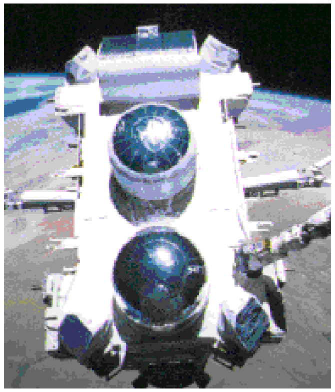

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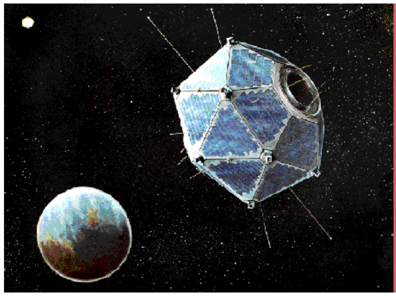

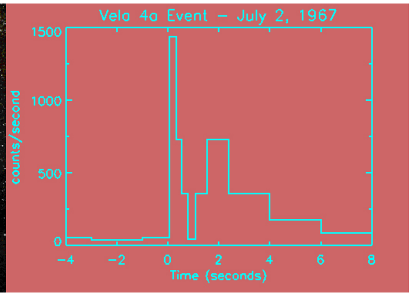

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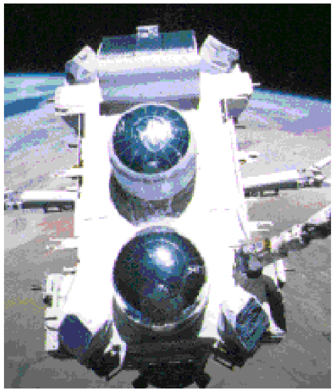

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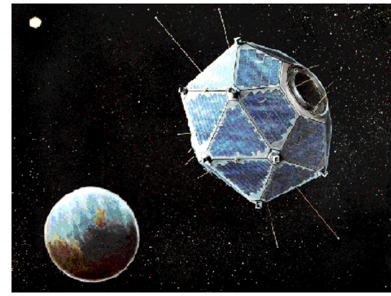

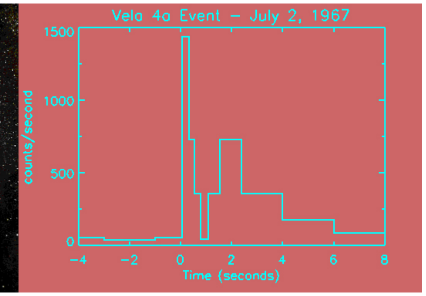

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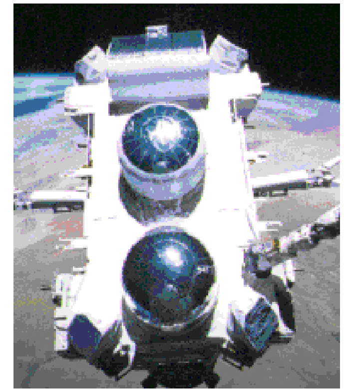

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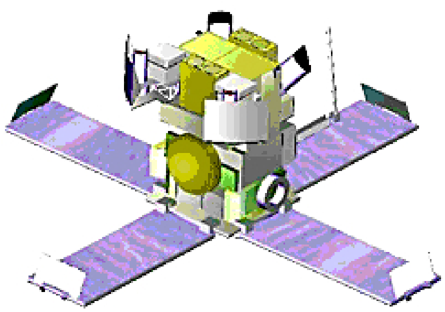

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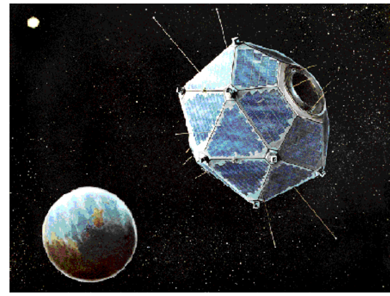

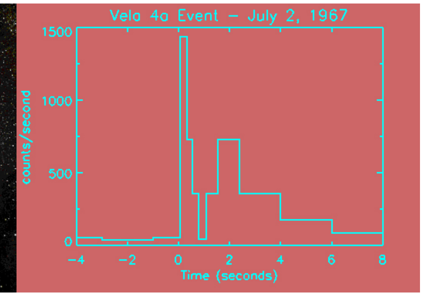

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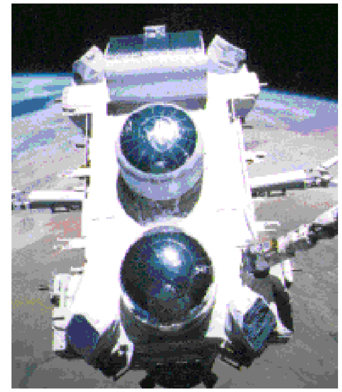

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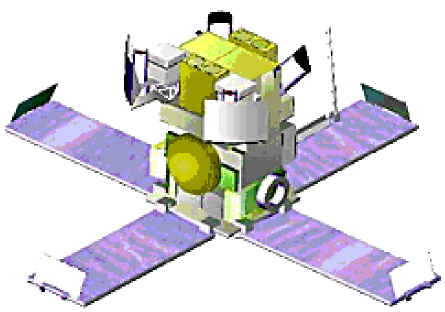
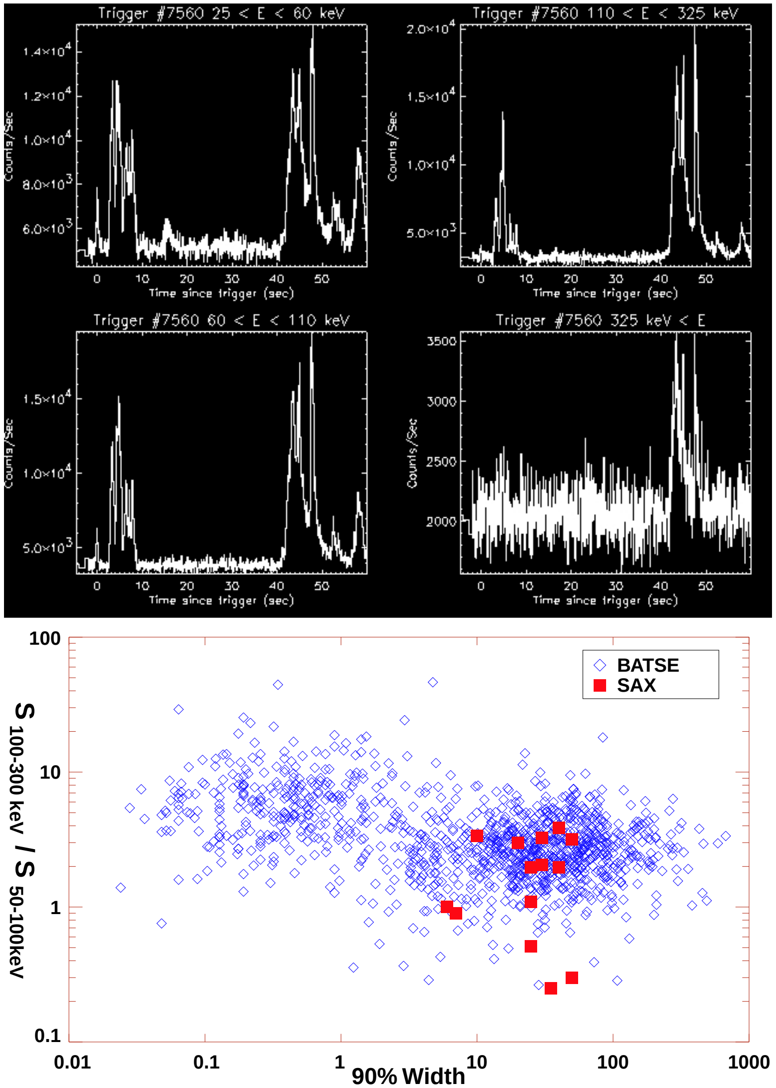


- INTERnationalGamma-RayAstrophysicsLaboratory

GRB prompt emission



(from Djorgovski et al. 2001)

- Fluence $F_\gamma = 10^{-8} - 10^{-3}$ergs/cm^2

  energy

  $$E_\gamma = 10^{53} \left(\frac{D}{3\ \mathrm{Gpc}}\right)^2 \left(\frac{F_\gamma}{10^{-4}\ \frac{\mathrm{ergs}}{\mathrm{cm}^2}}\right) \left(\frac{\Delta\omega}{4\pi}\right) \text{ergs}$$

  collimation { reduces E_γ ; increases the rate of events }

- non-thermal spectrum
- Duration $\Delta t = 10^{-3} - 10^3$s

  long bursts > 2 s, short bursts < 2 s

- Variability $\delta t = \Delta t / N\,, N = 1 - 1000$

  compact source $R < c\,\delta t \sim 1000$ km

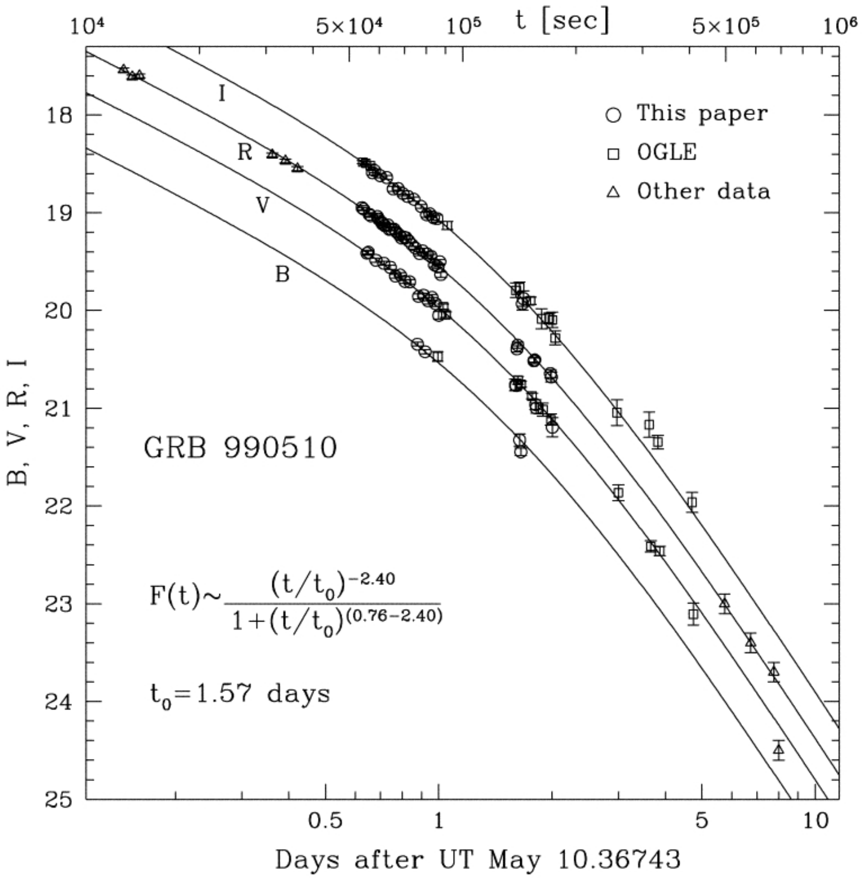
  not a single explosion

  huge optical depth for $\gamma\gamma \rightarrow e^+ e^-$

  compactness problem: how the photons escape?

  relativistic motion $\gamma \gtrsim 100$ { $R < \gamma^2 c\,\delta t$; blueshifted photon energy ; beaming ; optically thin }

Afterglow



(from Stanek et al. 1999)

- from X-rays to radio
- fading – broken power law

  panchromatic break $F_\nu \propto \begin{cases} t^{-a_1}, t < t_o \\ t^{-a_2}, t > t_o \end{cases}$
- non-thermal spectrum

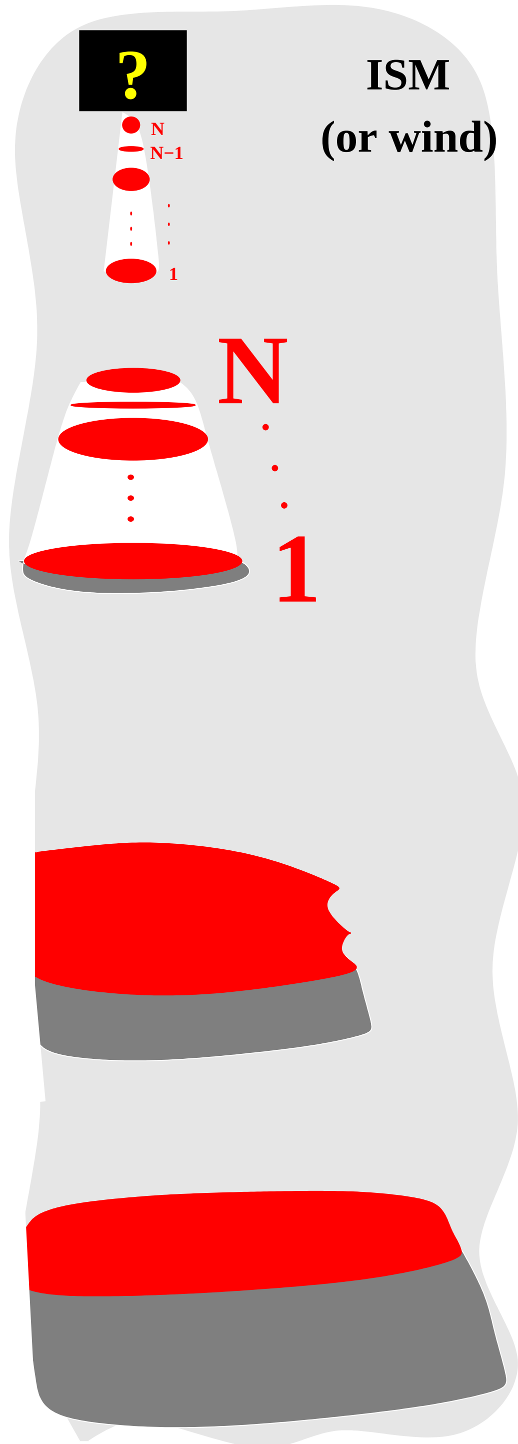
  (synchrotron + inverse Compton
  with power law electron energy distribution)

The internal–external shocks model

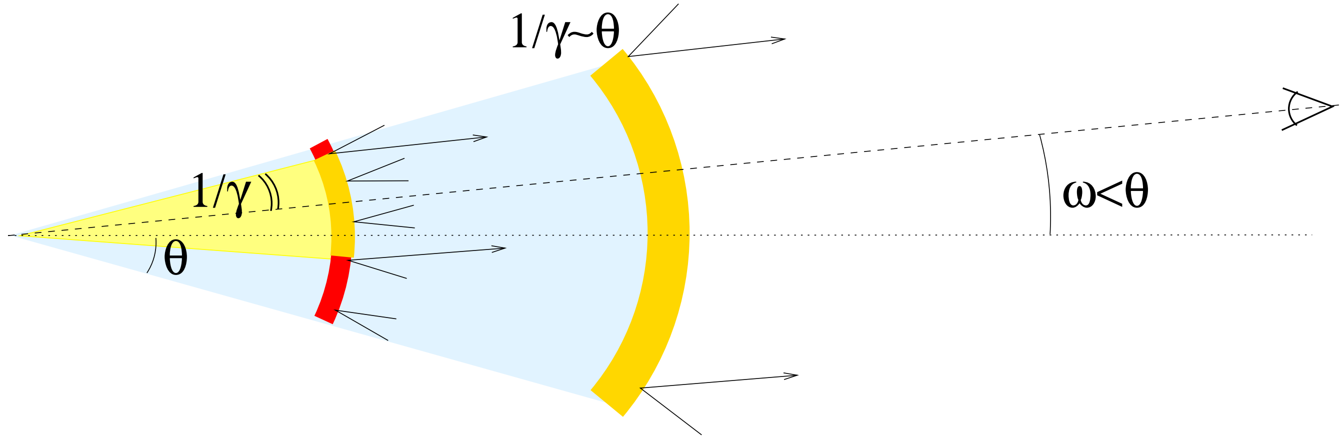
mass outflow (pancake)
N shells (moving with different $\gamma \gg 1$)
Frozen pulse
(if ℓ the path's arclength,
$s \equiv ct - \ell = const$ for each shell,
$\delta s = const$ for two shells)

internal shocks
($\sim 10\%$ of kinetic energy $\rightarrow$ **GRB**)

external shock
interaction with ISM (or wind)
(when the flow accumulates $M_{ISM} = M/\gamma$)
As γ decreases with time, kinetic energy $\rightarrow$ X-rays ... radio
$\rightarrow$ **Afterglow**



Beaming – Collimation



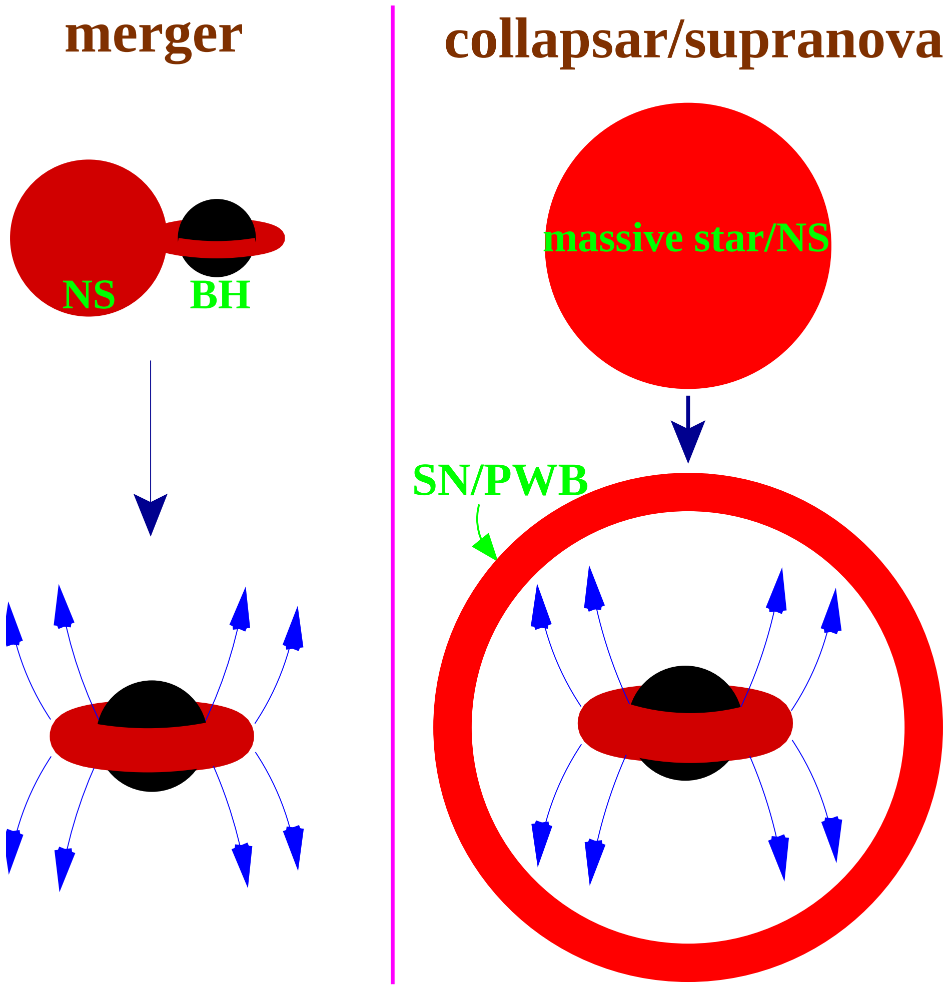
- During the afterglow γ decreases
  When $1/\gamma > \vartheta$ the $F(t)$ decreases faster
  The broken power-law justifies collimation

- orphan afterglows ?
  (for $\omega > \vartheta$)

- afterglow fits $\rightarrow$
  - opening half-angle $\vartheta = 1^{\circ} - 10^{\circ}$
  - energy $E_\gamma = 10^{50} - 10^{51}$ergs (Frail et al. 2001)
  - $E_{\text{afterglow}} = 10^{50} - 10^{51}$ergs (Panaitescu & Kumar 2002)

Imagine a Progenitor . . .

- **acceleration** and **collimation** of matter ejecta
- $E \sim$ 1% of the binding energy of a solar-mass compact object
- small $\delta t \rightarrow$ compact object
- highly relativistic $\rightarrow$ compact object
- two time scales $(\delta t\,, \Delta t)$ + energetics suggest accretion



The BH – debris-disk system

- **Energy reservoirs:**

① binding energy of the orbiting debris
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☞ viscous dissipation $\Rightarrow$ thermal energy $\Rightarrow \nu\bar{\nu} \rightarrow e^+e^- \Rightarrow e^\pm$/photon/baryon **fireball**

– unlikely that the disk is optically thin to neutrinos (Di Matteo, Perna, & Narayan 2002)
– hot, luminous photosphere $\Rightarrow$ detectable thermal emission (Daigne & Mochkovitch 2002)
– collimation ?
– highly super-Eddington $L \Rightarrow M_{\text{baryon}} \uparrow \Rightarrow \gamma \approx \dfrac{\mathcal{E}}{M_{\text{baryon}}c^2} \downarrow$

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☞ dissipation of magnetic fields

generated by the differential rotation in the torus $\Rightarrow e^\pm$/photon/baryon “magnetic” **fireball**

– collimation ?
– hot, luminous photosphere $\Rightarrow$ detectable thermal emission

☞ MHD extraction (Poynting jet)

- $\mathcal{E} = \frac{c}{4\pi} \underbrace{\frac{\varpi\Omega}{c} B_p}_{E} B_\phi \times$ area $\times$ duration $\Rightarrow$

$$\frac{B_p B_\phi}{(2\times 10^{14}\text{G})^2} = \left[\frac{\mathcal{E}}{5\times 10^{51}\text{ergs}}\right] \left[\frac{\text{area}}{4\pi\times 10^{12}\text{cm}^2}\right]^{-1} \left[\frac{\varpi\Omega}{10^{10}\text{cm s}^{-1}}\right]^{-1} \left[\frac{\text{duration}}{10\text{s}}\right]^{-1}$$

  - from the BH: $B_p \gtrsim 10^{15}$G (small B_ϕ, small area)
  - from the disk: smaller magnetic field required $\sim 10^{14}$G

- Is it possible to "use" this energy and accelerate the matter ejecta?
  Important to solve the transfield force-balance equation
  (ignoring the transfield and assuming radial flow $\longrightarrow$ tiny efficiency; Michel 1969)

(⋆ force-free electrodynamics (massless limit of the ideal MHD) – outgoing wave (Lyutikov & Blandford):
the energy remains Poynting – they ignore the transfield – no outflowing matter is needed for the GRB
⋆ Does the dissipation stops the acceleration?
dissipation $\longrightarrow$ acceleration! (Drenkhahn & Spruit 2002))

Ideal MHD

- ❒ Only one exact solution known: the steady-state, cold, r self-similar model found by Li, Chiueh, & Begelman (1992) and Contopoulos (1994).
- ❒ Generalization for non-steady GRB outflows, including radiation and thermal effects.

Ideal Magneto-Hydro-Dynamics

in collaboration with Arieh Königl (U of Chicago)

- Outflowing matter:
  - baryons (rest density ρ_0)
  - ambient electrons (neutralize the protons)
  - $e^{\pm}$ pairs (Maxwellian distribution)

- photons (blackbody distribution)

- large scale electromagnetic field $\mathbf{E}\,,\mathbf{B}$

$\tau \gg 1$ ensure local thermodynamic equilibrium

$$\left.\begin{array}{l}\text{charge density } \frac{J^0}{c} \ll \frac{\rho_0}{m_p}e \\ \text{current density } J \ll \frac{\rho_0}{m_p}ec\end{array}\right\} \text{one fluid approximation}$$

$\mathbf{V}$ bulk velocity

$P =$ total pressure (matter + radiation)

$\xi c^2 =$ specific enthalpy (matter + radiation)

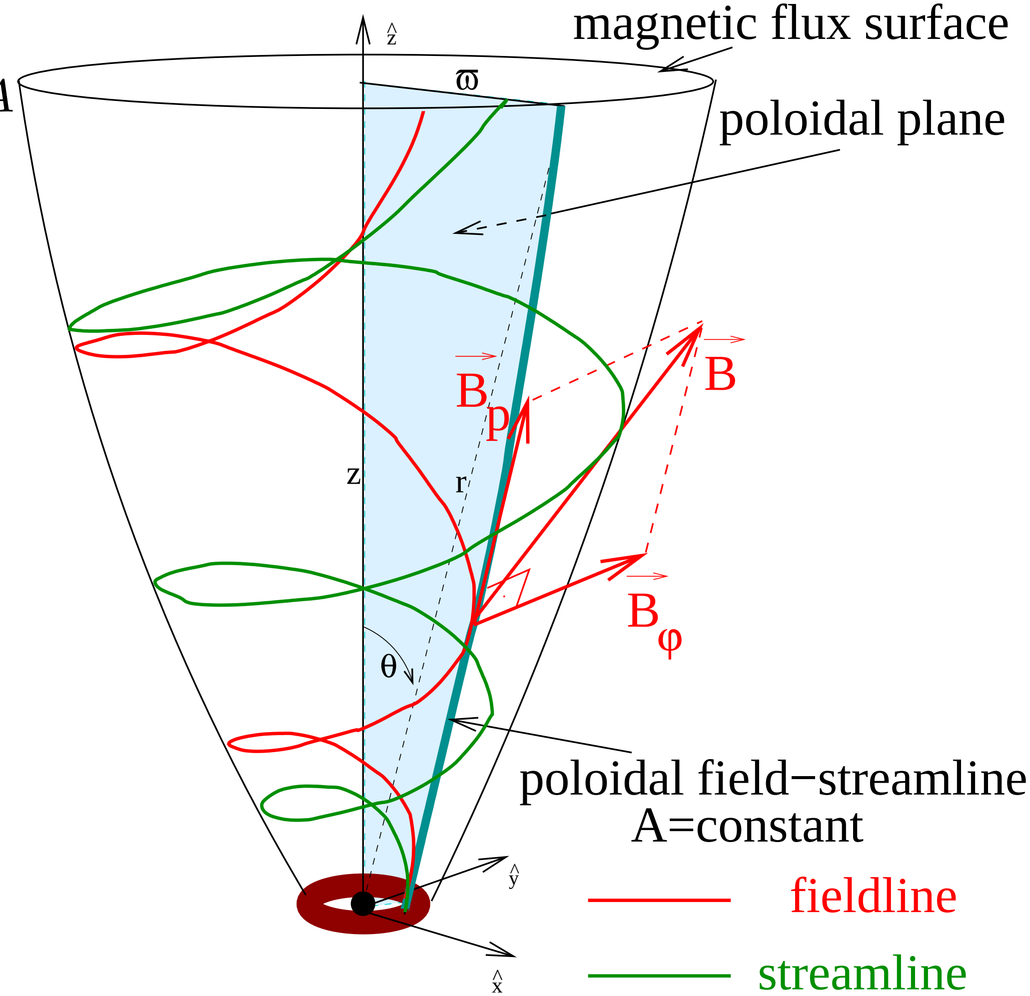
Assumptions

❶ axisymmetry
❷ highly relativistic poloidal motion
❸ quasi-steady poloidal magnetic field $\Leftrightarrow E_\phi = 0 \Leftrightarrow \mathbf{B}_p \parallel \mathbf{V}_p$

Introduce the magnetic flux function A

$\mathbf{B} = \mathbf{B}_p + \mathbf{B}_\phi\,, \quad \mathbf{B}_p = \nabla \times \left(A \frac{\hat{\phi}}{\varpi} \right)$

Faraday + Ohm $\longrightarrow \mathbf{V}_p \parallel \mathbf{B}_p$



The frozen-pulse approximation

- The arclength along a poloidal fieldline
$$\ell = \int_{\frac{s}{c}}^{t} V_p dt \approx ct - s \Rightarrow s = ct - \ell$$

- s is constant for each ejected shell. Moreover, the distance between two different shells $\ell_2 - \ell_1 = s_1 - s_2$ remains the same (even if they move with $\gamma_1 \neq \gamma_2$).

- Eliminating t in terms of s, we show that all terms with $\partial/\partial s$ are $\mathcal{O}(1/\gamma) \times$ remaining terms (generalizing the HD case examined by Piran, Shemi, & Narayan 1993). Thus we may examine the motion of each shell using steady-state equations.
$$\left(\text{e.g., } \frac{d}{dt} = (c - V_p)\frac{\partial}{\partial s} + \mathbf{V} \cdot \nabla_s \approx \mathbf{V} \cdot \nabla_s\right)$$

Integration

The full set of ideal MHD equations can be partially integrated to yield five fieldline constants (functions of A and $s = ct - \ell$):

① the mass-to-magnetic flux ratio

② the field angular velocity

③ the specific angular momentum

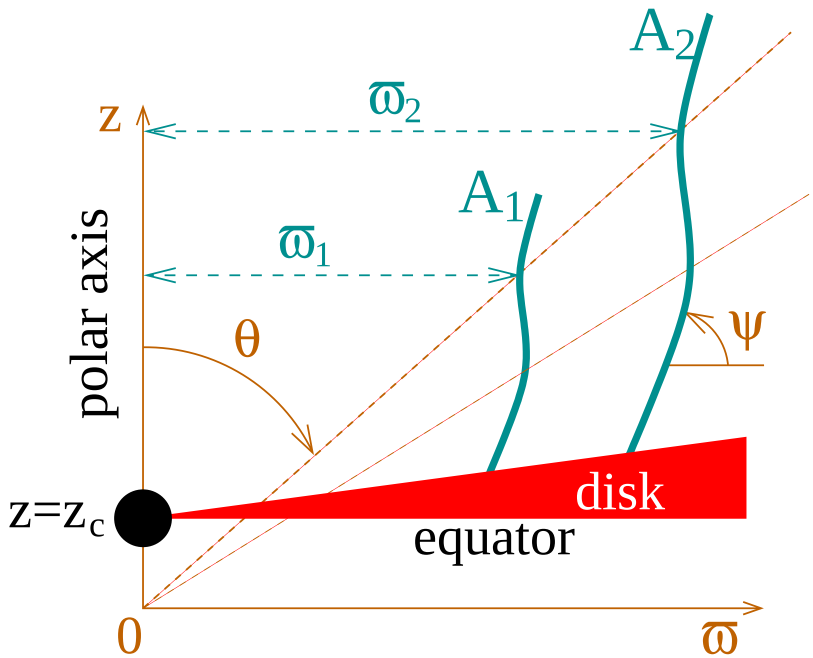
④ the total energy-to-mass flux ratio μc^2

⑤ the adiabat $P/\rho_0^{4/3}$

Two integrals remain to be performed, involving the

Bernoulli and transfield force-balance equations.

r self-similarity



self-similar ansatz $r = \mathcal{F}_1(A)\,\mathcal{F}_2(\theta)$

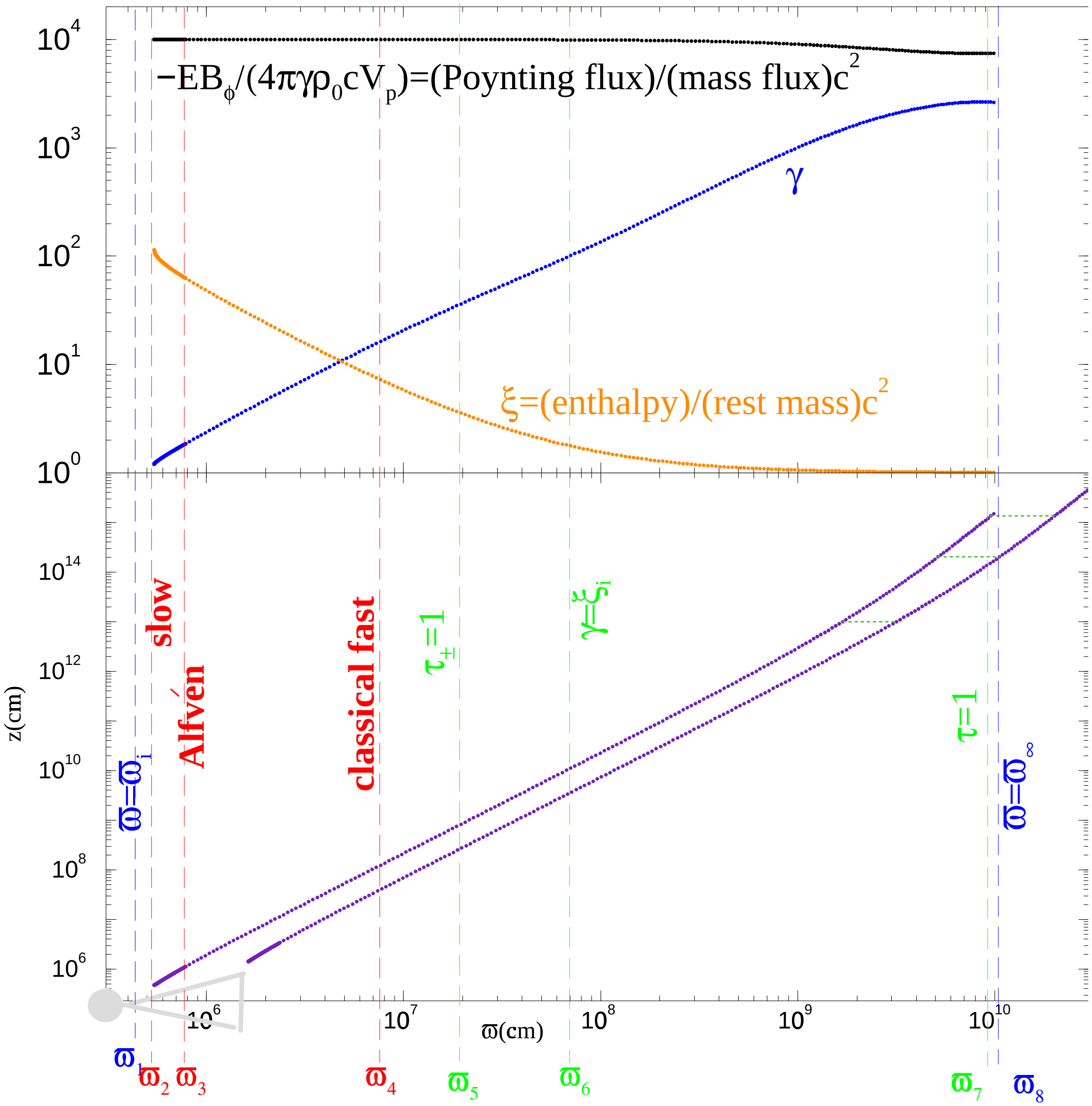
For points on the same cone $\theta = const$,
$$\frac{\varpi_1}{\varpi_2} = \frac{r_1}{r_2} = \frac{\mathcal{F}_1(A_1)}{\mathcal{F}_1(A_2)}.$$

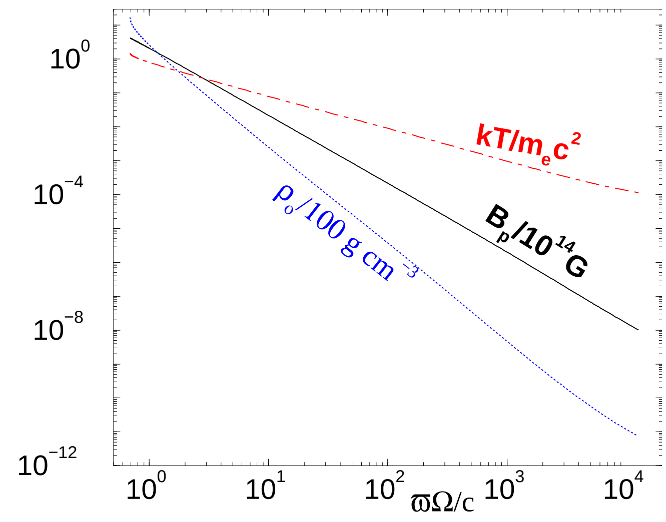
ODEs
$$\left\{\begin{array}{l} \psi = \psi(x\,, M\,, \theta)\,, \text{(Bernoulli)} \\ \dfrac{dx}{d\theta} = \mathcal{N}_0(x\,, M\,, \psi\,, \theta)\,, \text{(definition of } \psi) \\ \dfrac{dM}{d\theta} = \dfrac{\mathcal{N}(x\,, M\,, \psi\,, \theta)}{\mathcal{D}(x\,, M\,, \psi\,, \theta)}\,, \text{(transfield)} \end{array}\right\}$$

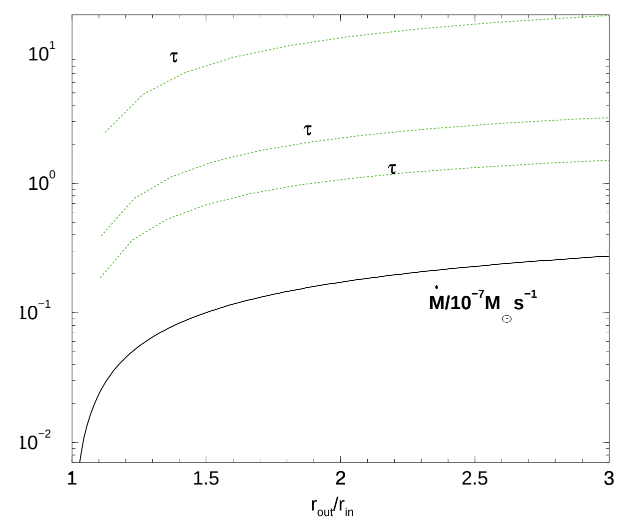
$\mathcal{D} = 0:$ singular points (Alfvén, modified slow - fast)

(start the integration from a cone $\theta = \theta_i$ and give the boundary conditions $B_\theta = -\mathcal{C}_1 r^{F-2}$, $B_\phi = -\mathcal{C}_2 r^{F-2}$, $V_r/c = \mathcal{C}_3$, $V_\theta/c = -\mathcal{C}_4$, $V_\phi/c = \mathcal{C}_5$, $\rho_0 = \mathcal{C}_6 r^{2(F-2)}$, and $P = \mathcal{C}_7 r^{2(F-2)}$, where F = parameter).

Trans-Alfvénic Jets (NV & Königl 2001, 2003a)

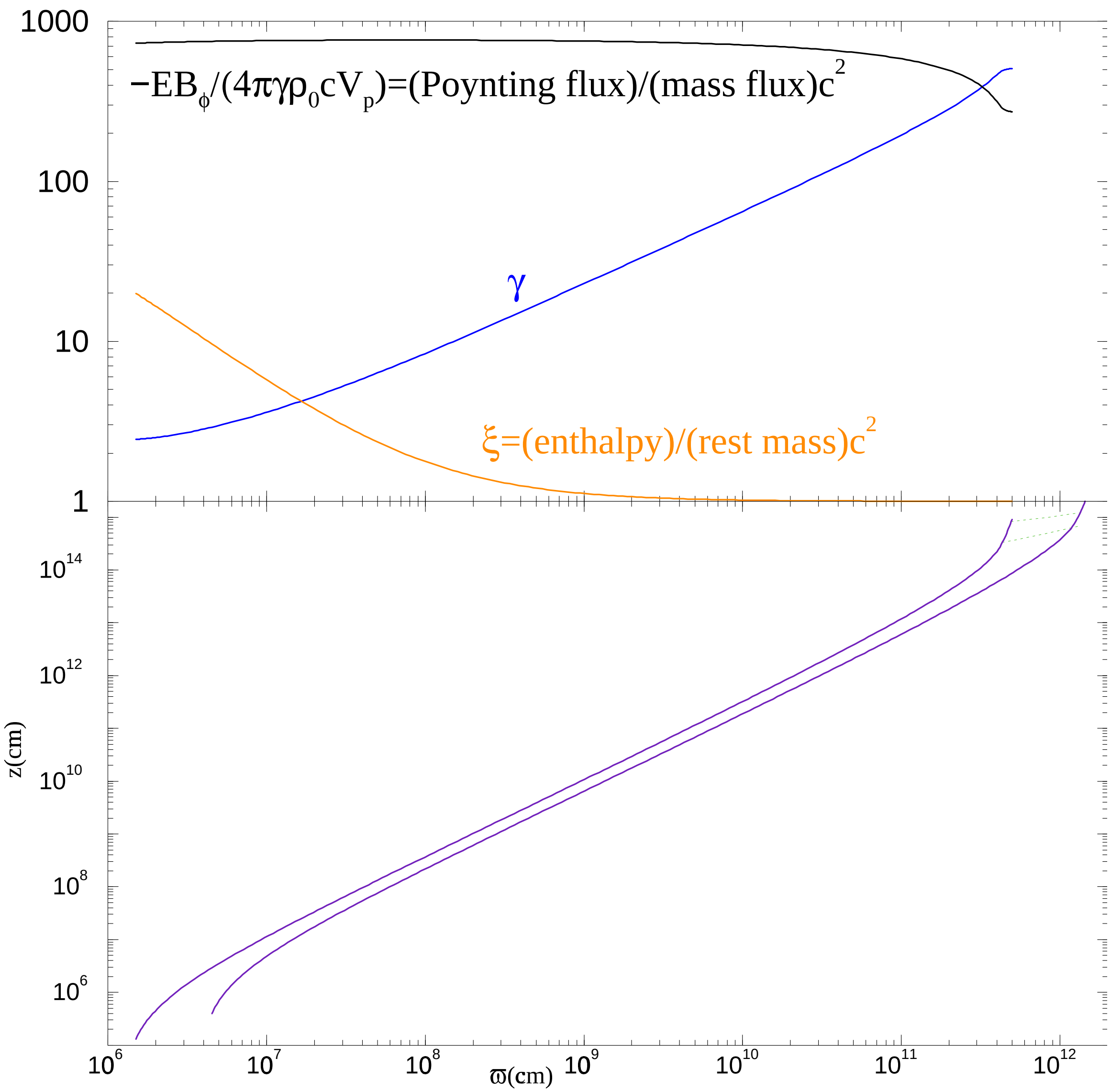


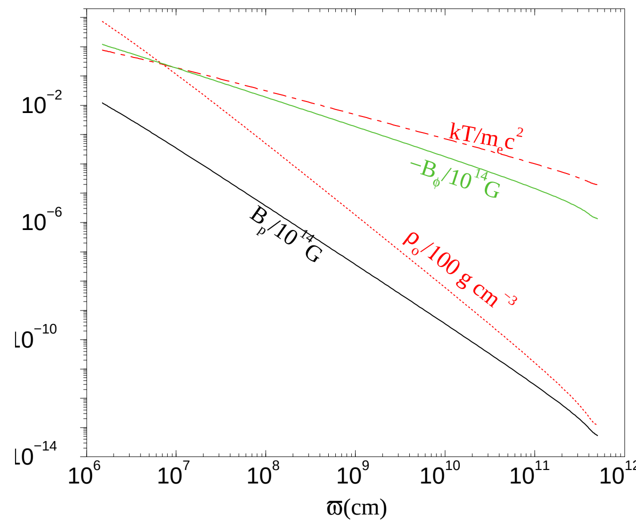


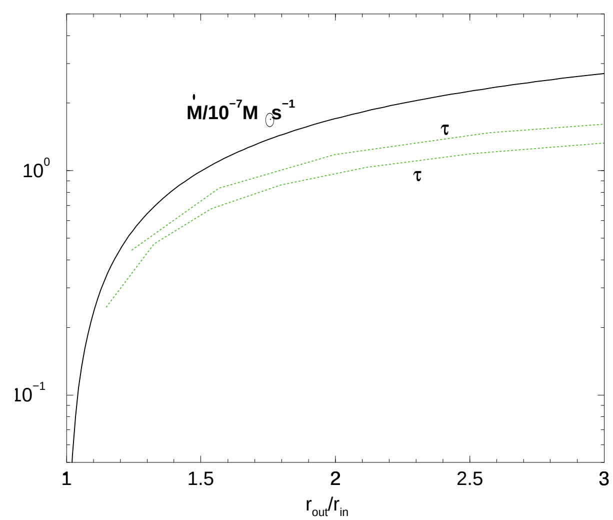


- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

Super-Alfvénic Jets (NV & Königl 2003b)







- Thermal acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$, $T \propto \varpi^{-0.8}$, $B_\phi \propto \varpi^{-1}$, $z \propto \varpi^{1.5}$)
- Magnetic acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$)
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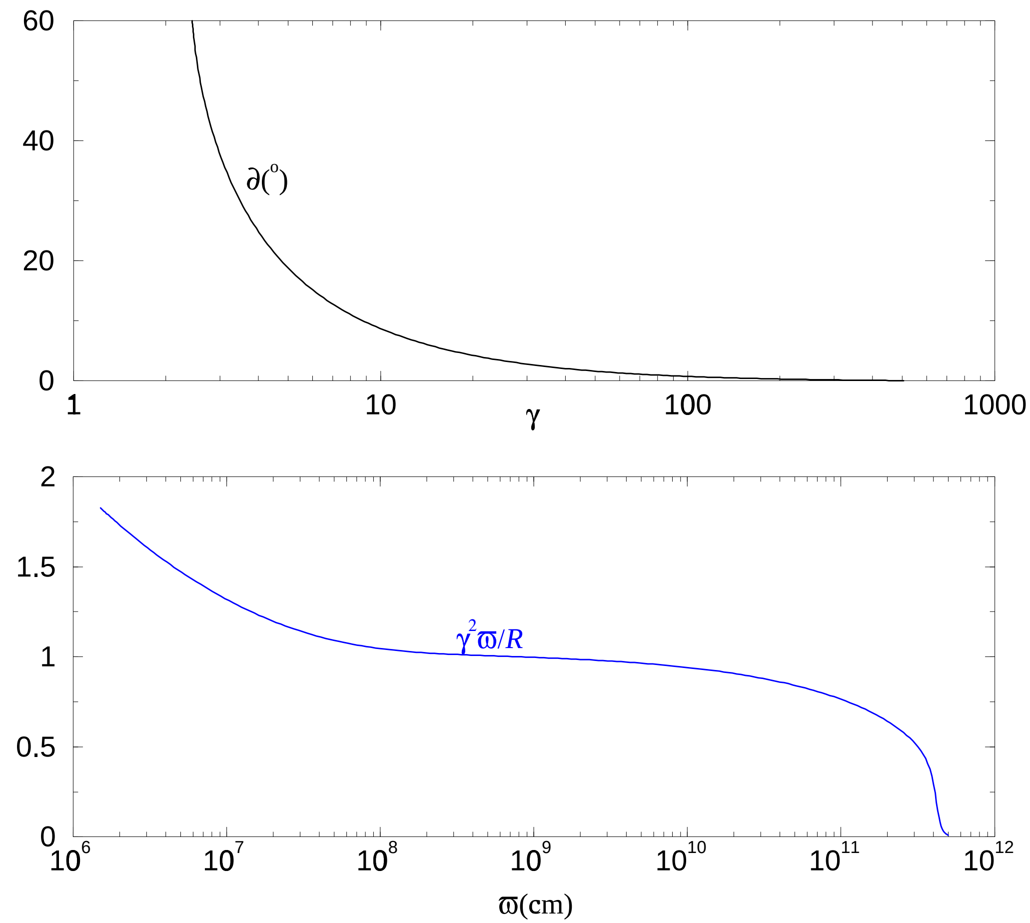
Collimation

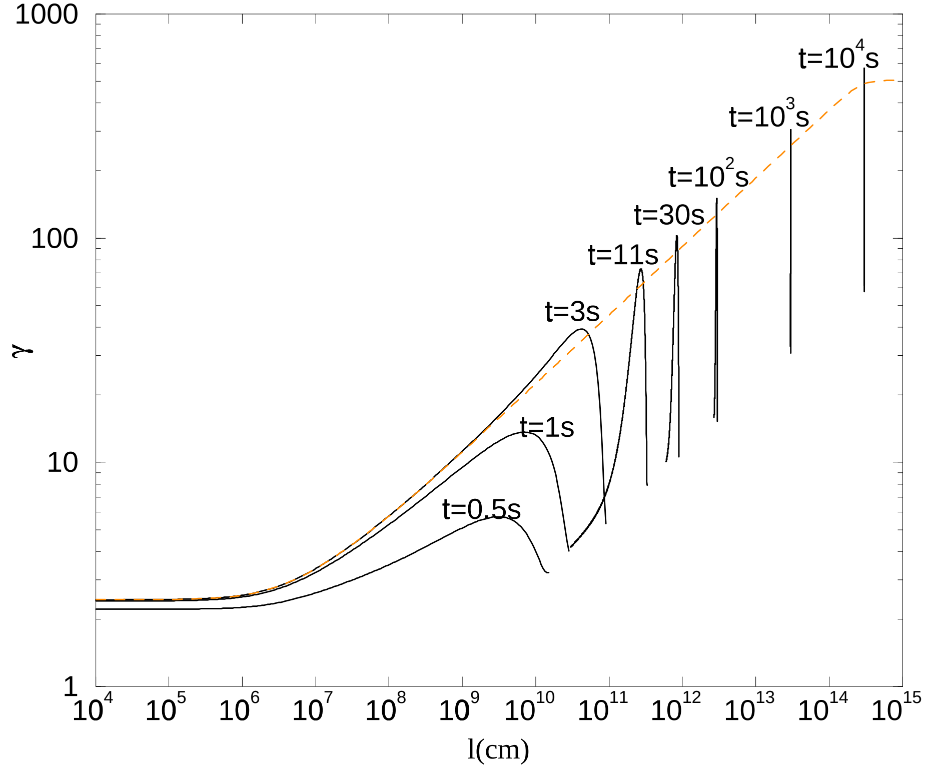


⋆ At $\varpi = 10^8$cm – where $\gamma = 10$ – the opening half-angle is already $\vartheta = 10^o$

⋆ For $\varpi > 10^8$cm, collimation continues slowly ($\mathcal{R} \sim \gamma^2 \varpi$)

Time-Dependent Effects

⋆ recovering the time-dependence:



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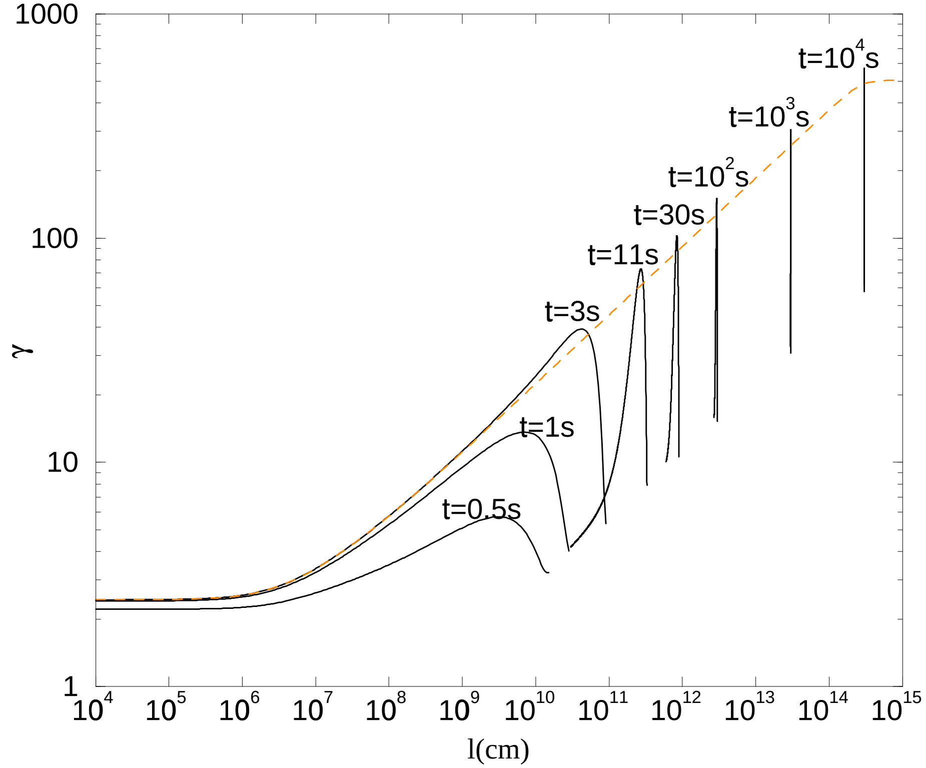

⋆ internal shocks:

The distance between two neighboring shells s_1 , $s_2 = s_1 + \delta s$

$$\delta \ell = \delta \left(\int_{\frac{s}{c}}^{t} V_p dt \right) = -\delta s - \delta \left(\int_{\frac{s}{c}}^{t} (c - V_p) dt \right) \approx -\delta s - \int_0^t \delta \left(\frac{c}{2\gamma^2} \right) dt$$

Different $V_p \Rightarrow$ collision (at $ct \approx \gamma^2 \delta s$ – inside the cylindrical regime)

Conclusions

- Solution incorporates:
  - rotation and magnetic effects (important near BH)
  - thermal–radiation effects
- The flow is initially thermally and subsequently magnetically accelerated
- The outflow is largely Poynting flux-dominated:
  - the implied lower radiative luminosity near the origin could alleviate the baryon contamination problem
  - negligible photospheric emission
- The magnetic field:
  - provide the most plausible means of extracting the rotational energy on the burst timescale
  - self-collimation
  - Lorentz acceleration ($\sim 50\%$ efficiency)
  - guiding property (internal shock mechanism)
  - could account for the observed synchrotron emission

Magnetic acceleration in general

- Non-radial flow $\rightarrow \gamma_\infty \gg \mu^{1/3}$ (cf. Michel's solution). Also, the classical fast magnetosonic point is located at a finite distance from the origin. Most of the acceleration occurs downstream of the classical fast point.

- Other applications:
  - AGN outflows: Using the same radially self-similar model, we show that the acceleration in relativistic AGN outflows (e.g., in the recently observed sub-parsec-scale jet in NGC 6251) can be attributed to magnetic driving
  - Crab-like pulsar winds

A solution to the pulsar σ-problem

- $\sigma = \frac{\text{Poynting flux}}{\text{matter energy flux}} \approx 10^4$ at the fast surface $\rightarrow \sigma \approx 10^{-3}$ at $r \approx 3 \times 10^{17}$cm (Kennel & Coroniti, Arons)

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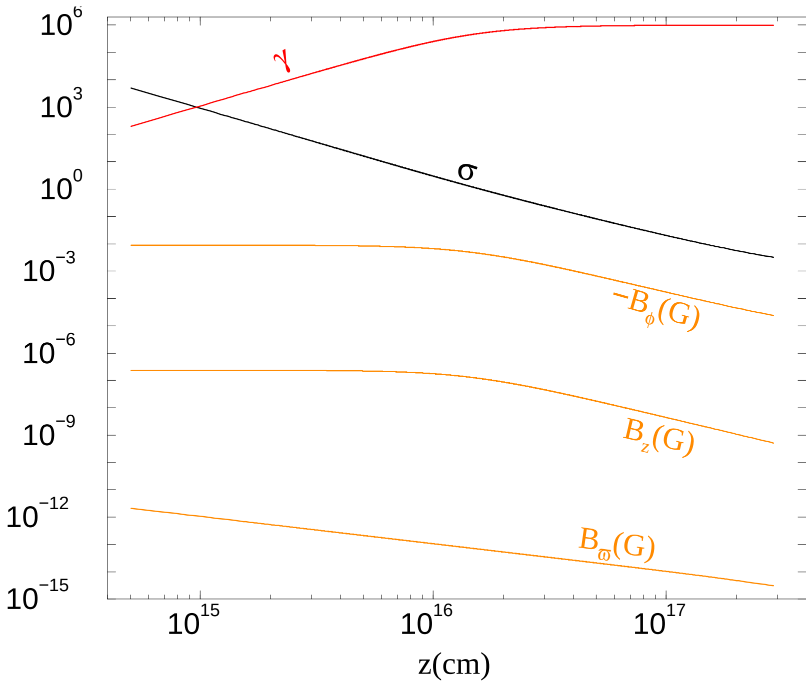
- logarithmic acceleration
  - $z = f(A)\varpi^{n(A)}$, Chiueh, Li, & Begelman (1991)
  - perturbed monopole field $A = A_0(1 - \cos\theta + \delta)$, Lyubarsky & Eichler (2001)

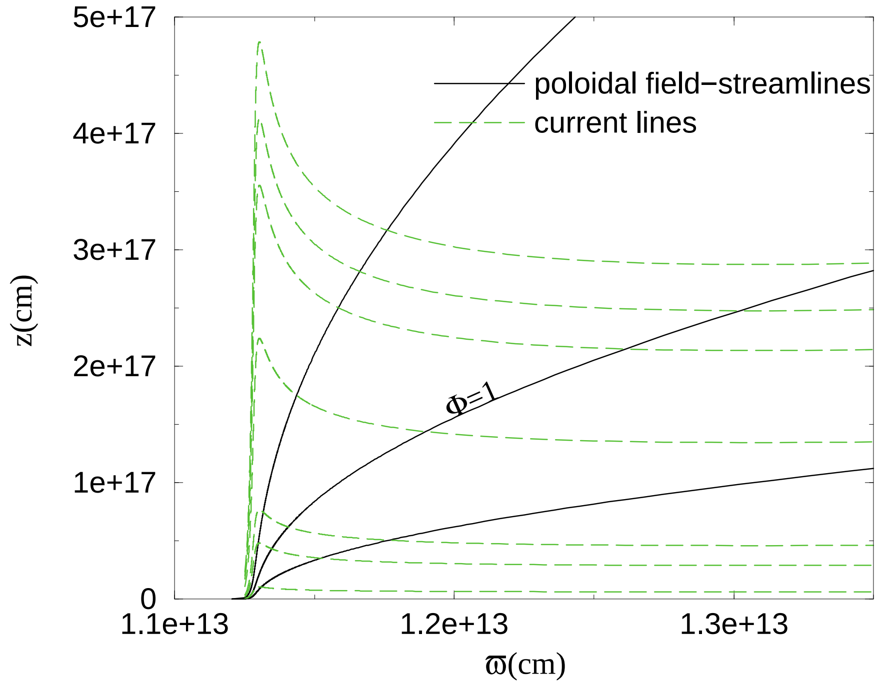
A solution to the pulsar σ-problem

- $\sigma = \frac{\text{Poynting flux}}{\text{matter energy flux}} \approx 10^4$ at the fast surface $\to \sigma \approx 10^{-3}$ at $r \approx 3 \times 10^{17}$cm (Kennel & Coroniti, Arons)

- logarithmic acceleration
  - $z = f(A)\varpi^{n(A)}$, Chiueh, Li, & Begelman (1991)
  - perturbed monopole field $A = A_0(1 - \cos\theta + \delta)$, Lyubarsky & Eichler (2001)

- The z self-similar model: $z = \Phi(A) f(\varpi)$
  $B_z \gg B_\varpi$, superAlfvénic regime





Transition from $\sigma \approx 10^4$ to $\sigma \approx 10^{-3}$ (i.e., from Poynting- to matter-dominated flow)

The ideal MHD equations

Maxwell:

$$\nabla \cdot \mathbf{B} = 0 = \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{c \partial t}\,, \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{c \partial t} + \frac{4\pi}{c}\mathbf{J}\,, \nabla \cdot \mathbf{E} = \frac{4\pi}{c} J^0$$

Ohm: $\mathbf{E} = \mathbf{B} \times \mathbf{V}/c$

baryon mass conservation (continuity):

$$\frac{d(\gamma \rho_0)}{dt} + \gamma \rho_0 \nabla \cdot \mathbf{V} = 0\,, \quad \text{where} \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla$$

energy $U_\mu T^{\mu\nu}_{,\nu} = 0$ (or specific entropy conservation, or first law for thermodynamics):

$$\frac{d\left(P/\rho_0^{4/3}\right)}{dt} = 0$$

momentum $T^{\nu i}_{,\nu} = 0$: $\gamma \rho_o \frac{d\left(\xi \gamma \mathbf{V}\right)}{dt} = -\nabla P + \frac{J^0 \mathbf{E} + \mathbf{J} \times \mathbf{B}}{c}$

Eliminating t in terms of s: $(\mathbf{V} \cdot \nabla_s)\left(\xi \gamma \mathbf{V}\right) - \frac{(\nabla_s \cdot \mathbf{E})\,\mathbf{E} + (\nabla_s \times \mathbf{B}) \times \mathbf{B}}{4\pi \gamma \rho_0} + \frac{\nabla P}{\gamma \rho_0} =$

$$(V_p - c)\frac{\partial\left(\xi \gamma \mathbf{V}\right)}{\partial s} + \frac{\partial (E + B_\phi)}{4\pi \gamma \rho_0 \partial s}\frac{\nabla_s A}{|\nabla_s A|} \times \mathbf{B} - \nabla_s A \frac{\nabla_s \ell \cdot \nabla_s A}{|\nabla_s A|^2}\frac{\partial (E^2 - B_\phi^2)}{8\pi \gamma \rho_0 \partial s}$$