Disk-Jet Connection

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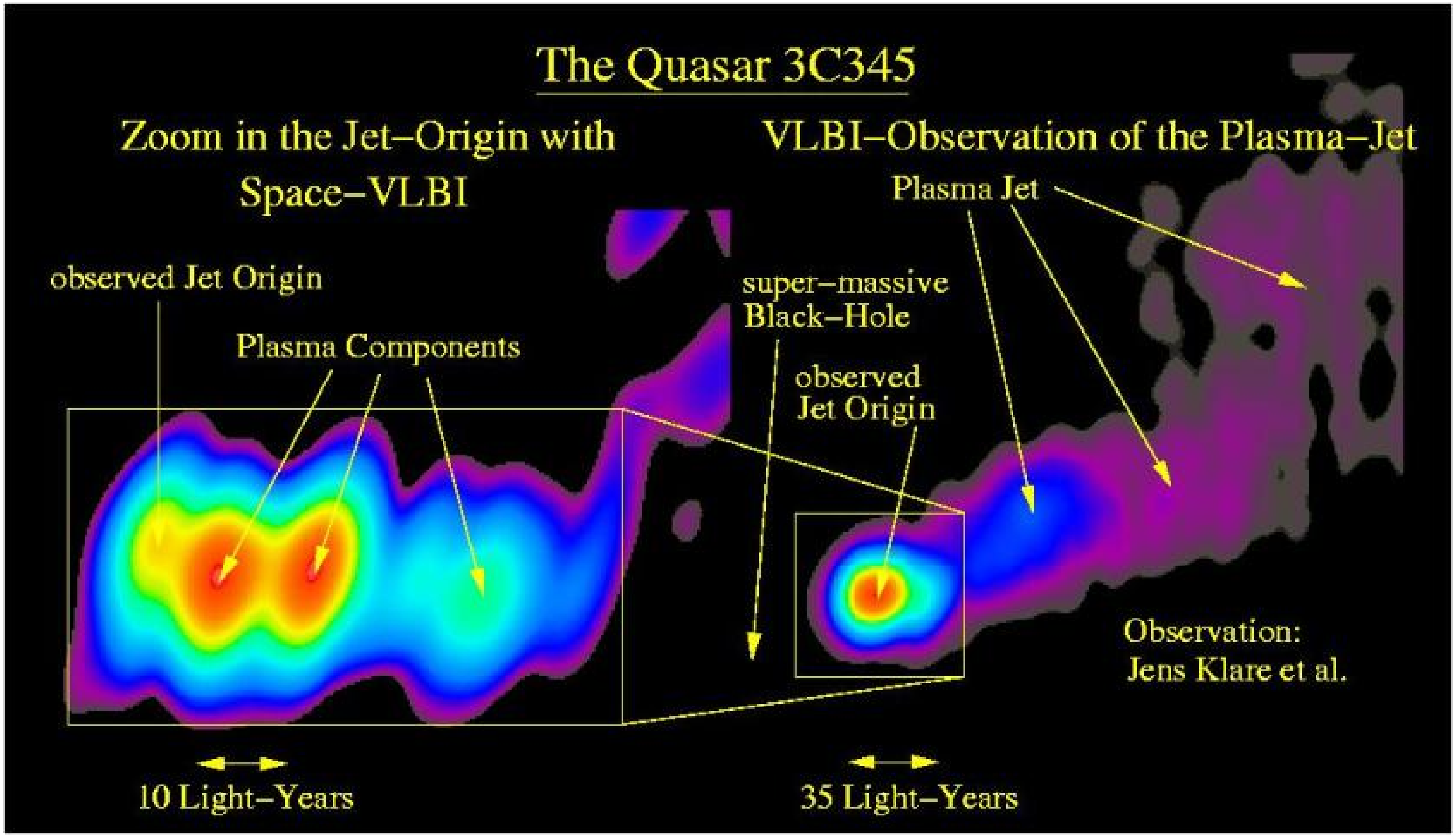
Outline

- observations
- inner jet magneto-hydro-dynamics
  - self-similar models
  - validity of s-s models using simulations
  - jet kinematics
- how the conditions at the top of the disk affect the jet velocity

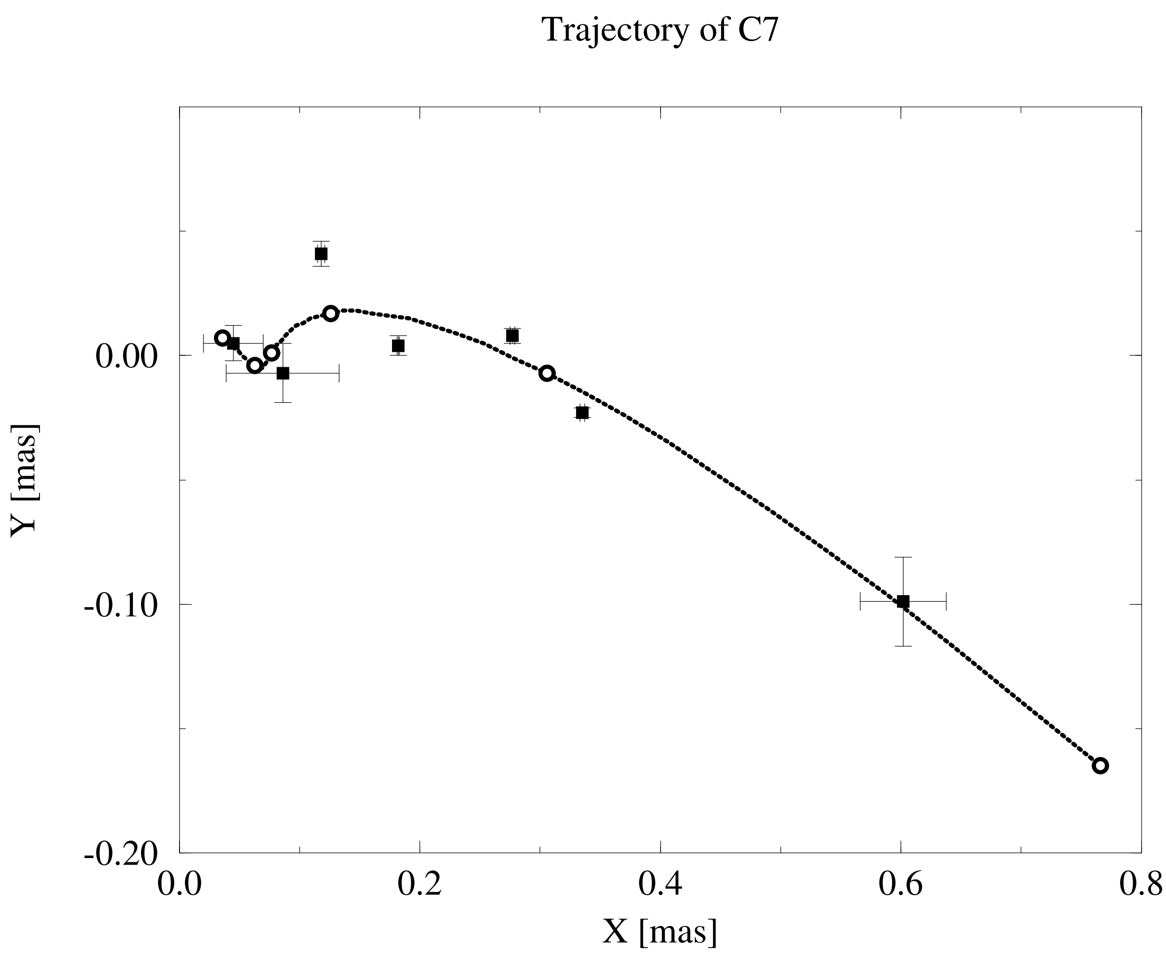


(credit: Klare at al)

The plasma components move with an apparent speed of 3-20c

These plasma components travel on curved trajectories

These trajectories differ from one component to the other



(from Lobanov's PhD thesis)

- Superluminal apparent motion $\Rightarrow \beta_{\rm app}$

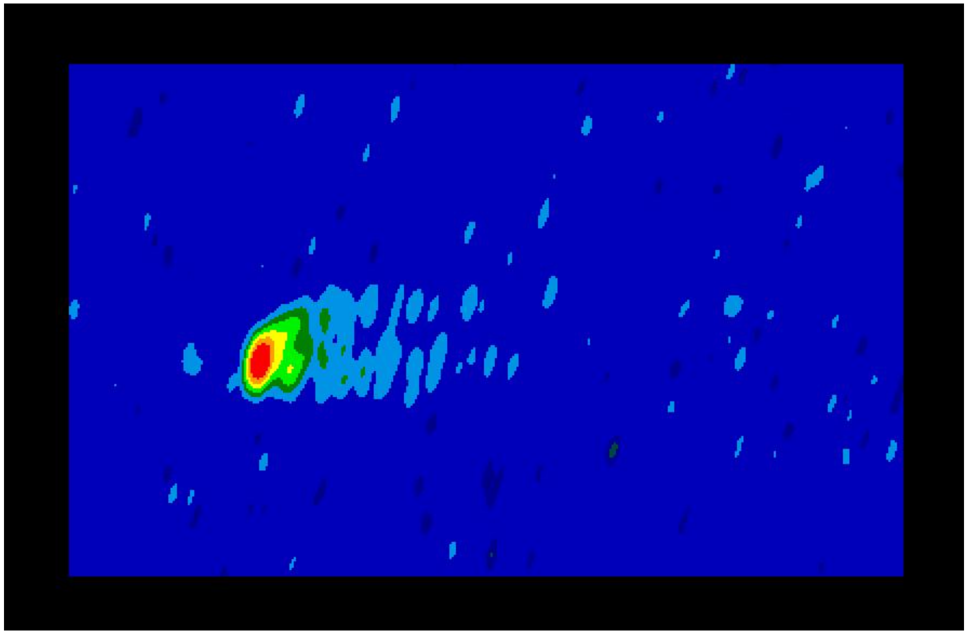
- From $\delta(t_{\rm obs}) \equiv \dfrac{1}{\gamma\,(1-\beta\cos\theta_V)}$ and $\beta_{\rm app}(t_{\rm obs}) = \dfrac{\beta\sin\theta_V}{1-\beta\cos\theta_V}$ we find $\beta(t_{\rm obs})$, $\gamma(t_{\rm obs})$ and $\theta_V(t_{\rm obs})$ **if** we know δ

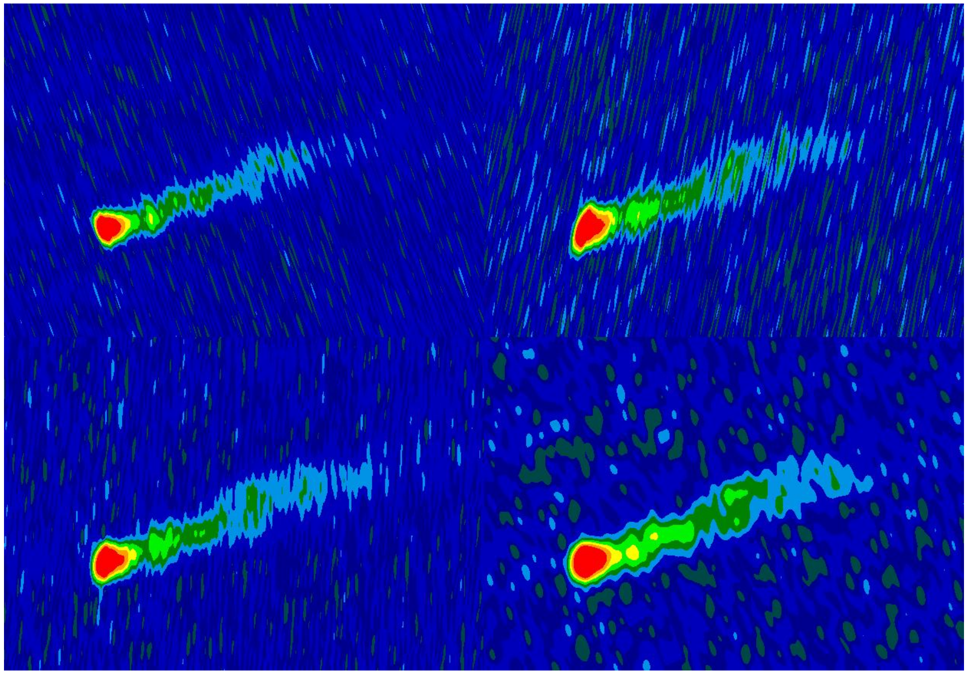
- Compare radio- and high energy emission (SSC) $\Rightarrow \delta$ (e.g., Unwin et al 1997)

For the C7 component of 3C 345 Unwin et al. (1997) inferred that it accelerates from $\gamma \sim 5$ to $\gamma \sim 10$ over the (deprojected) distance range (measured from the core) $\sim 3-20\ {\rm pc}$. Also the angle θ_V changes from ≈ 2 to $\approx 10^o$ and the Doppler factor changes from ≈ 12 to ≈ 4. ($t_{\rm obs} =$ 1992 – 1993.)

- More on jet kinematics (talk by Jorstad, Lister, poster by Piner)
- Sikora, Begelman, Madejski, & Lasota (ApJ in press):
  - kinematics and dynamics imply that jets are likely dominated by protons
  - the Poynting flux may be comparable ($\sigma \lesssim 1$) or smaller ($\sigma \ll 1$) than the kinetic flux
  - ⋆ are the claimed pc-scale accelerations of the VLBI features (e.g. Unwin et al) real?
  - ⋆ lack of bulk-Compton features $\rightarrow$ small ($\gamma < 5$) bulk Lorentz factor at $\lesssim 10^3 r_g$
  - ⋆ the γ saturates at values $\sim$ a few 10 around the blazar zone $(10^3 - 10^4 r_g)$

So, **acceleration is real**, takes at least 10^3 r$_g$, and is almost complete at 10^4 r$_g$





(left Global VLBI + VSOP, right Global VLBI)

Collimation in action (at approximately $100 r_g$) in M87. In the formation region, the jet is seen opening widely, at an angle of about 60 degrees, nearest the black hole, but is squeezed down to only 6 degrees a few light-years away.

(from Junor, Biretta, & Livio 1999)

Hydro-Dynamic Acceleration

- In case $n_e \sim n_p$, $\gamma_{\max} \sim kT_i/m_p c^2 \sim 1$ even with $T_i \sim 10^{12} K$
- If $n_e \neq n_p$, $\gamma_{\max} \sim (n_e/n_p) \times (kT_i/m_p c^2)$ could be $\gg 1$
- With some heating source, $\gamma_{\max} \gg 1$ is in principle possible

However, even in the last two cases, HD is unlikely to work because the HD acceleration saturates at distances comparable to the sonic surface where gravity is still important, i.e., very close to the disk surface (certainly at $\ll 10^3 r_g$)

Relativistic Magneto-Hydro-Dynamics

- Outflowing matter
- large scale electromagnetic field
- thermal pressure

We need to solve:

- Maxwell + Ohm equations
- mass + entropy conservation
- momentum equation

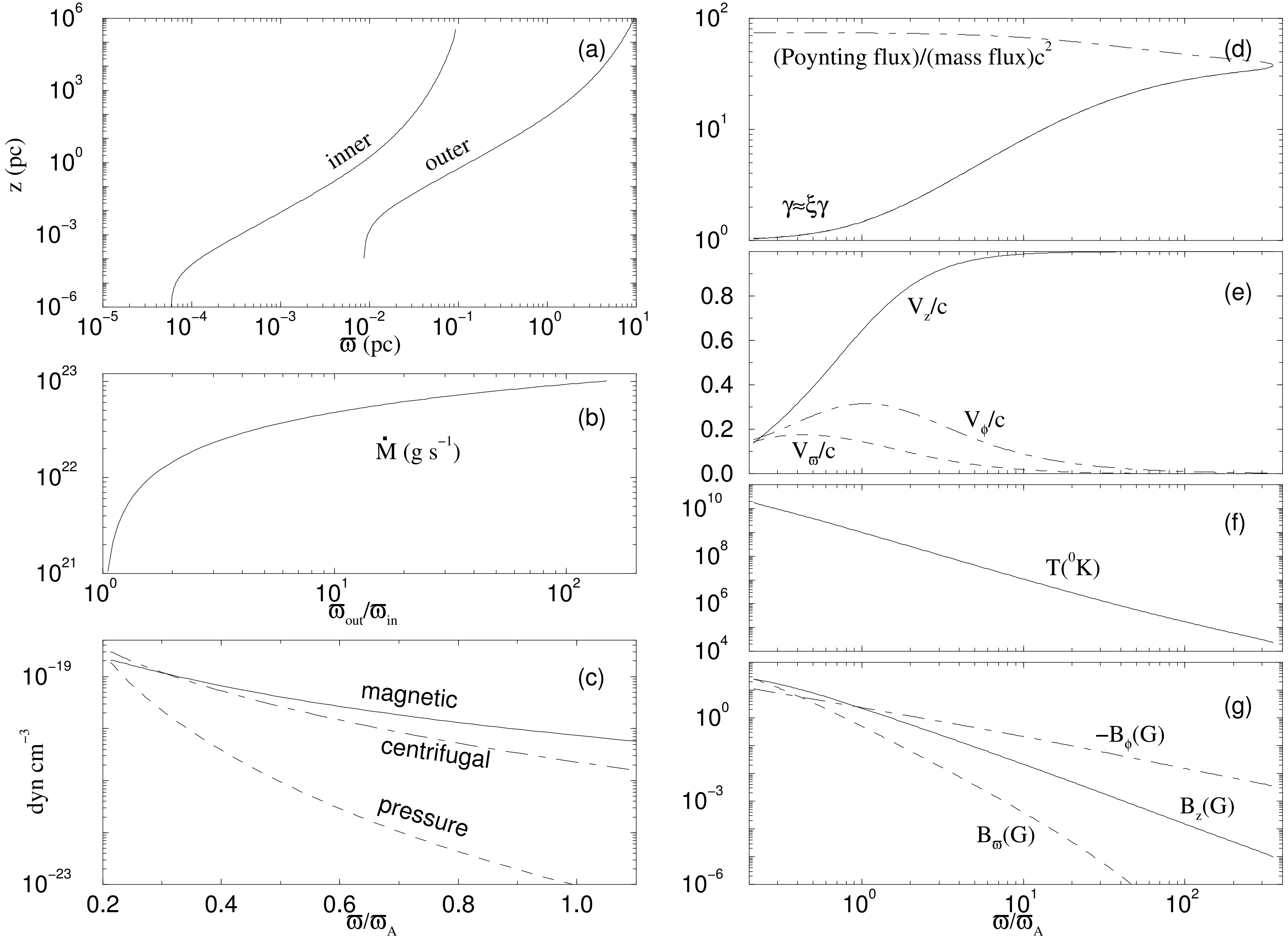
Self-similar, relativistic disk wind models

- axisymmetry
- steady-state
- ideal MHD (no resistivity)
- special relativity

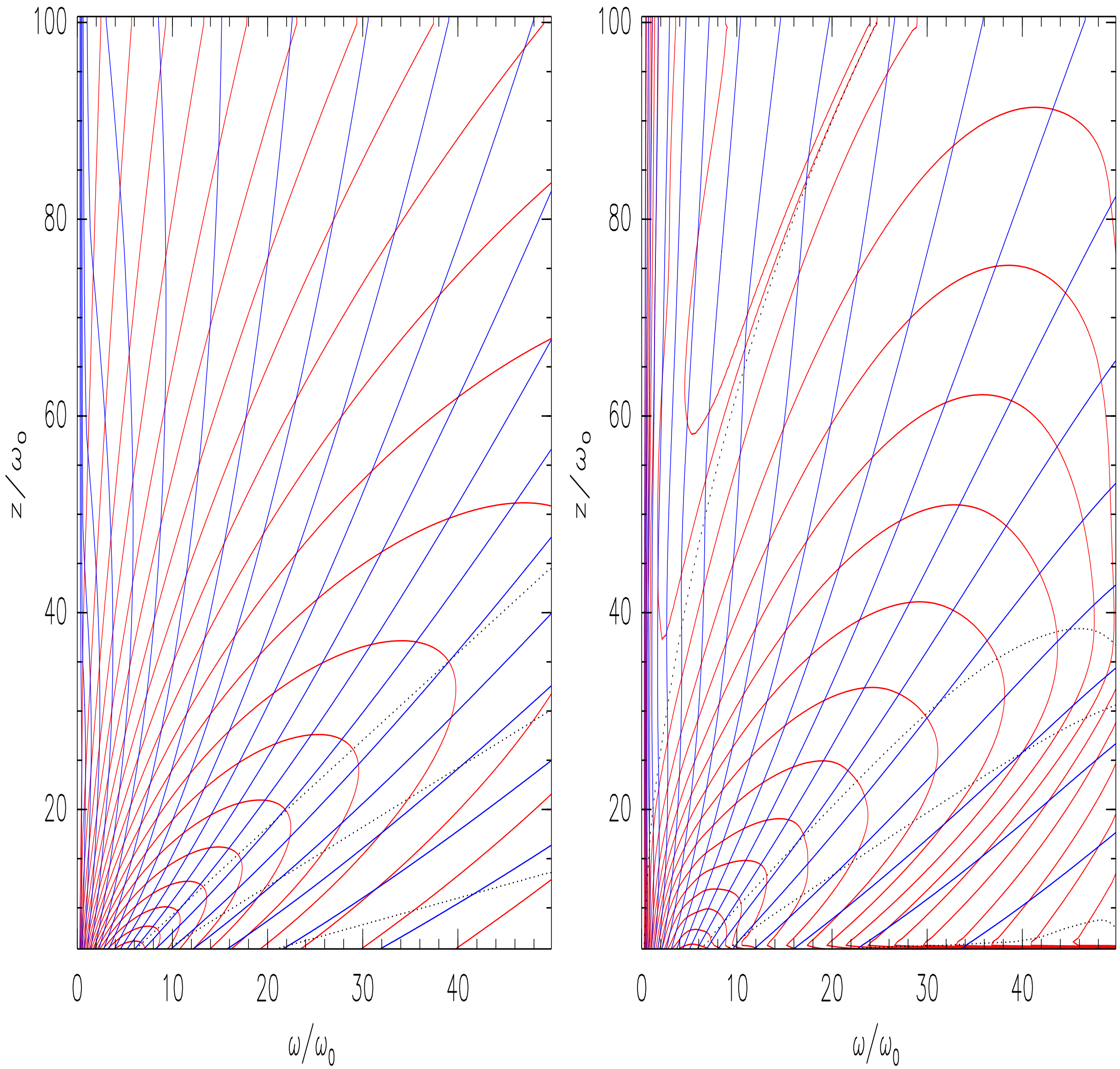
The problem reduces to the two components of the momentum equation: one along the flow and one in the transfield direction. The unknowns are γ and A (the latter is the magnetic flux function $A = \int \boldsymbol{B}_p \cdot d\boldsymbol{S}/2\pi$ that determines the field- and stream-line shape).

- boundary conditions of the form $r^x \times f(\theta)$
  - similar to the nonrelativistic model of Blandford & Payne 1982
  - cold versions of the model: Li et al 1992, Contopoulos 1994

Vlahakis & Königl, ApJ (2004) – application to 3C345



Self-similar models *vs* simulations

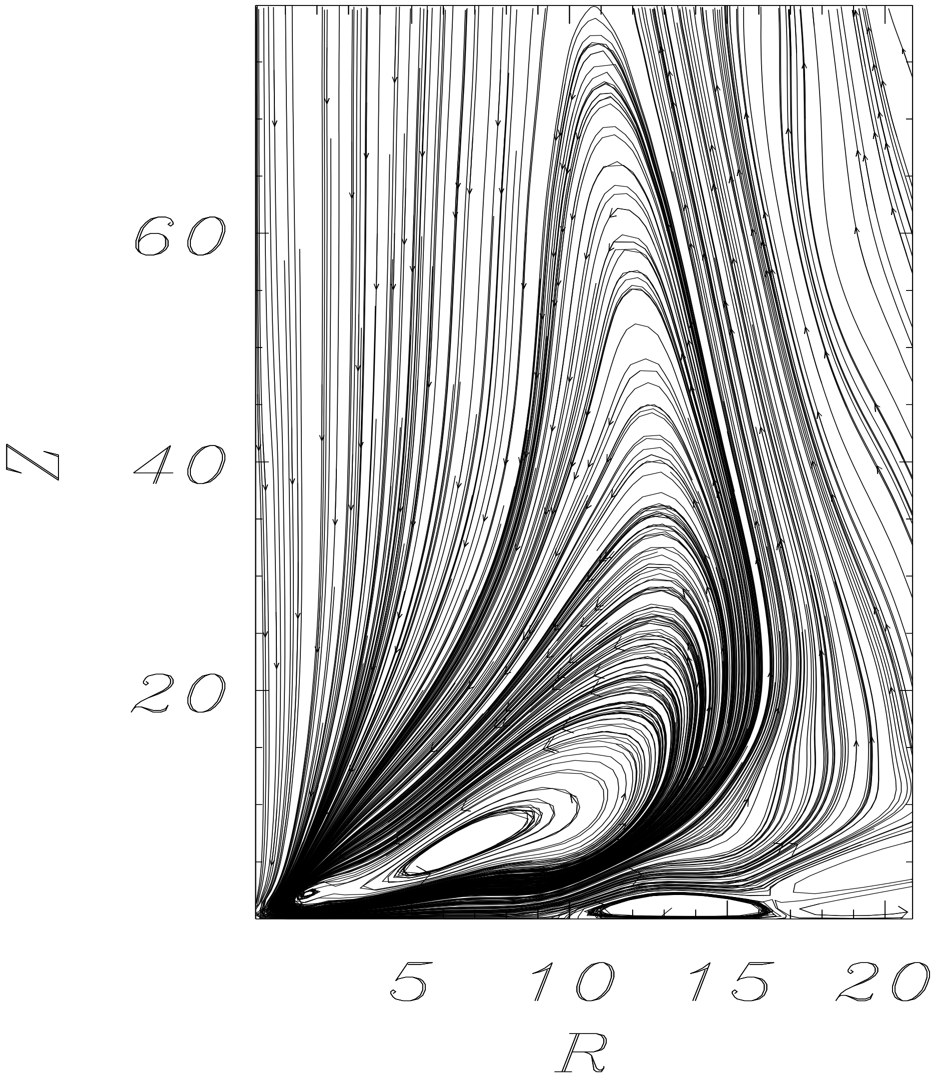


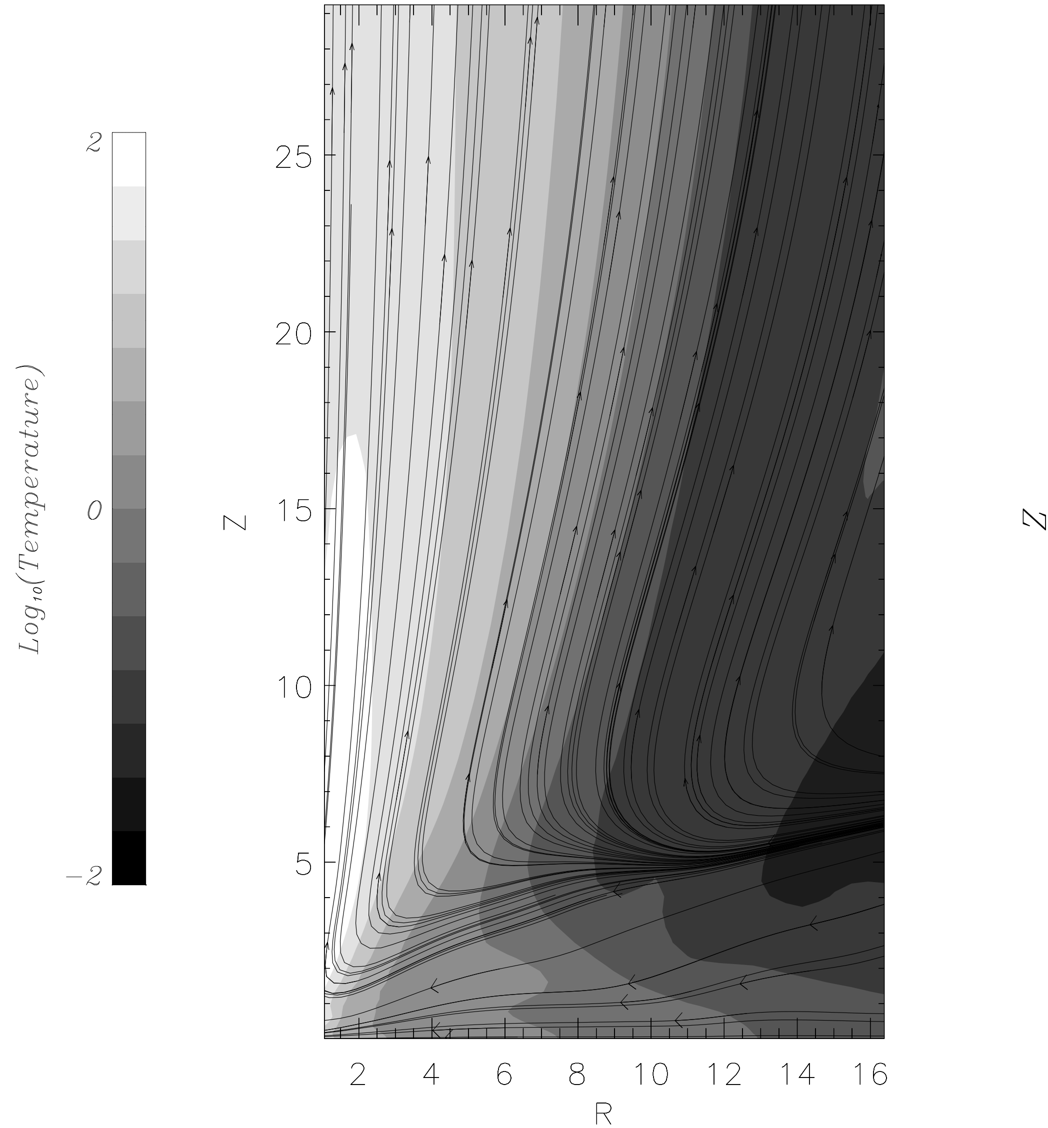
blue: flow-lines
red: current-lines
(left: self-similar,
right: simulation)

(Gracia, Vlahakis, & Tsinganos, to be submitted)

similar work by Krasnopolsky et al, ApJ (1999, 2003)

Including the disk





with an α–prescription for the anomalous resistivity

$\eta = \alpha V_{\rm A}|_{z=0} H \exp\left(-2z^2/H^2\right)$, Casse & Keppens, ApJ (2004)

Relativistic MHD simulations:

- still difficult numerically to simulate a flow with high Lorentz factor

- difficulties related to $\nabla \cdot \boldsymbol{B} = 0 \rightarrow$ simulations are usually halted after a few disk rotation periods

We may still learn using self-similar models;
they are not so bad as we originally thought

Jet kinematics

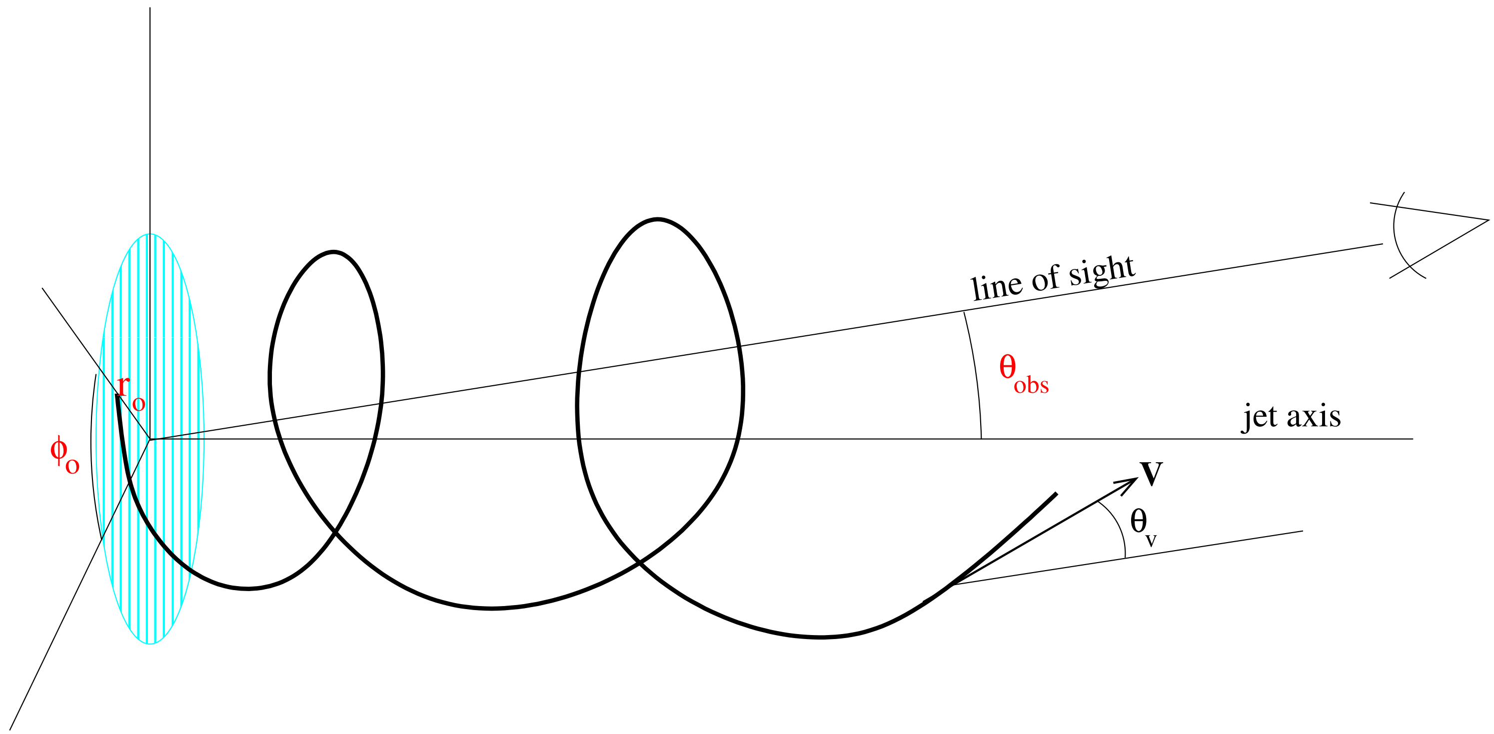
- due to precession? (e.g., Caproni & Abraham)
- instabilities? (e.g., Hardee, Meier)

bulk jet flow may play at least a partial role

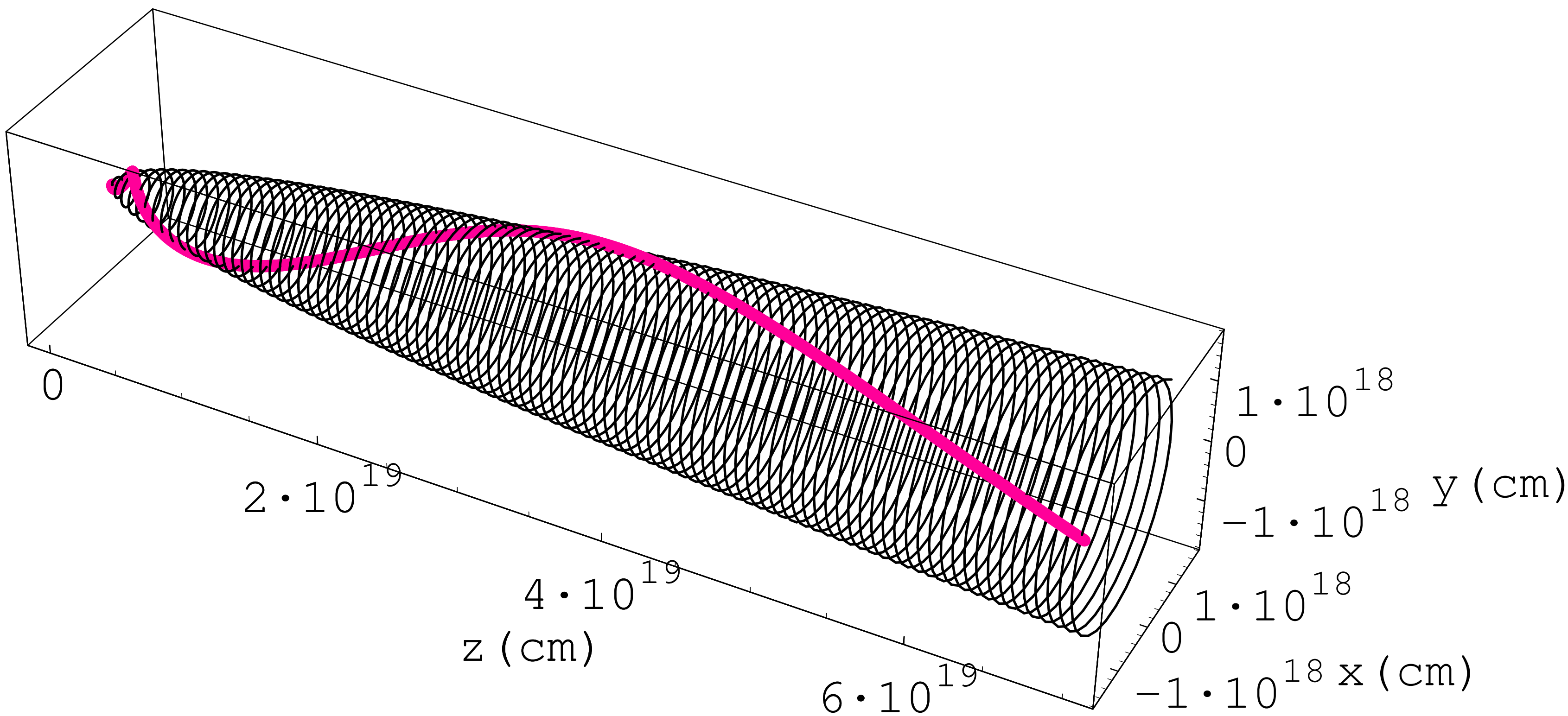
to explore this possibility, we used the relativistic self-similar model (Vlahakis & Königl 2004)

since the model gives the velocity (3D) field, we can follow the motion of a part of the flow

For given $\theta_{\rm obs}$ (angle between jet axis and line of sight) and ejection area on the disk (r_o, ϕ_o), we project the trajectory on the plane of sky and compare with observations. Find the best-fit parameters r_o, $\theta_{\rm obs}$, ϕ_o.

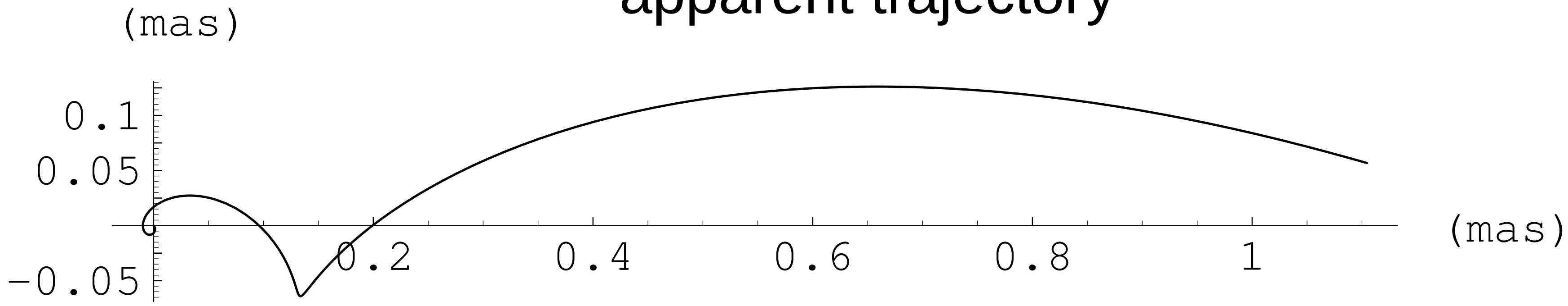


Preliminary results (Vlahakis & Königl in preparation)

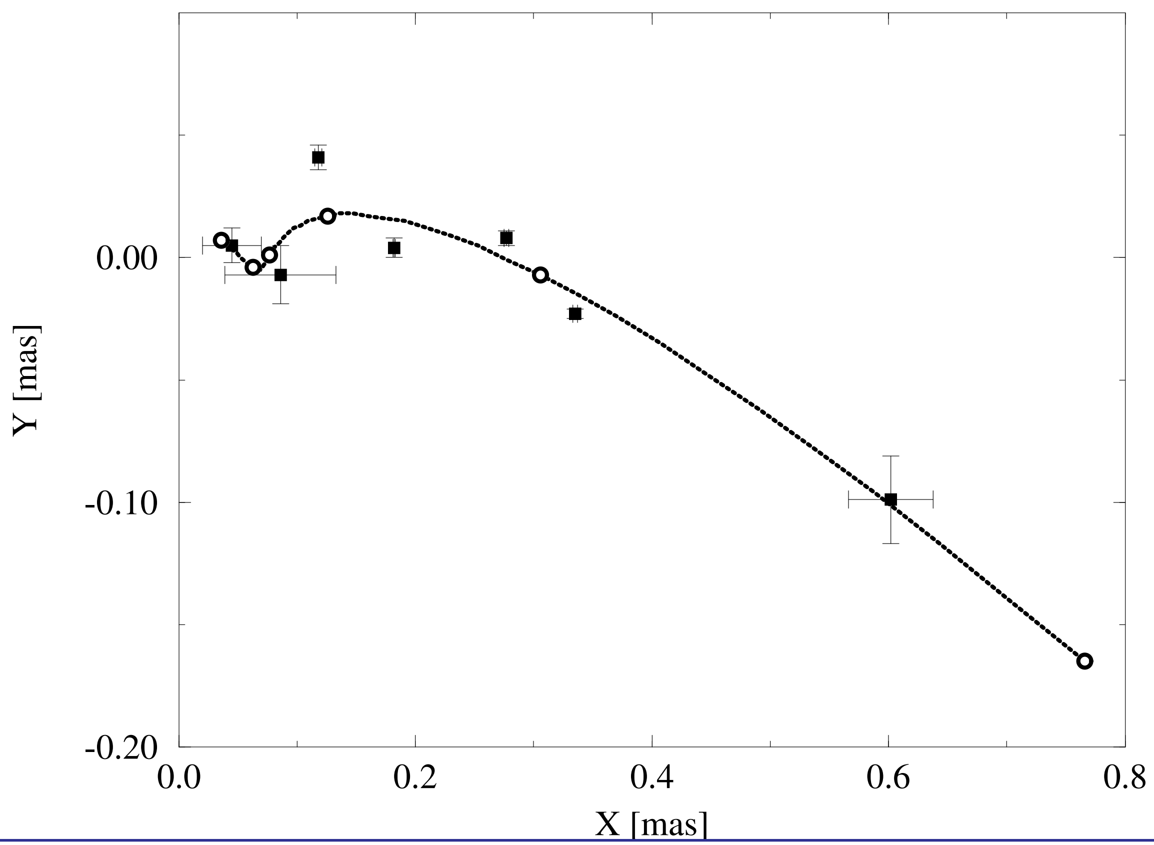


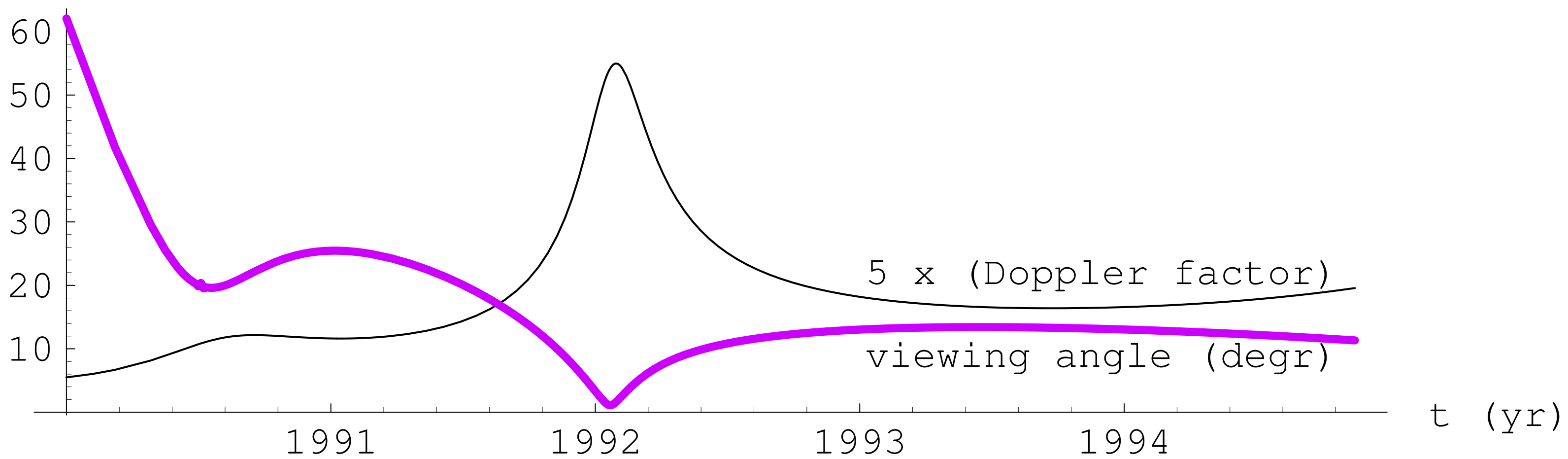
best-fit to Unwin et al results: $r_o \approx 2 \times 10^{16}$cm, ϕ_o=180^o and $\theta_{\rm obs}$=9^o

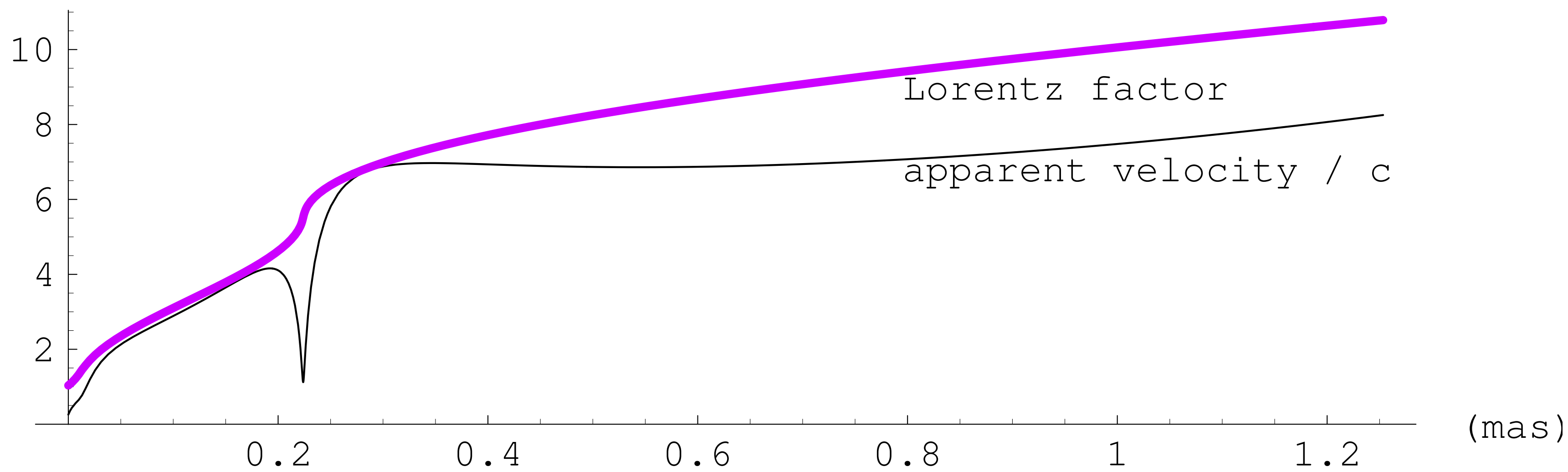
apparent trajectory



Trajectory of C7




60
50
40
30
20
10
5 x (Doppler factor)
viewing angle (degr)
t (yr)
1991
1992
1993
1994


10
8
6
4
2
Lorentz factor
apparent velocity / c
(mas)
0.2
0.4
0.6
0.8
1
1.2

Relation between boundary conditions and jet velocity

- In the self-similar solution for the jet in 3C 345 we find the efficiency of the magnetic acceleration to be 50%, or, $\gamma_\infty \approx \mu/2$ ($\mu c^2 = \frac{\text{total energy flux}}{\text{mass flux}}$; μ is the maximum possible γ), or, $\sigma_\infty \sim 1$.

- Michel's solution gives only $\gamma_\infty = \mu^{1/3} \ll \mu \rightarrow$ tiny efficiency, or, $\sigma_\infty \gg 1$!

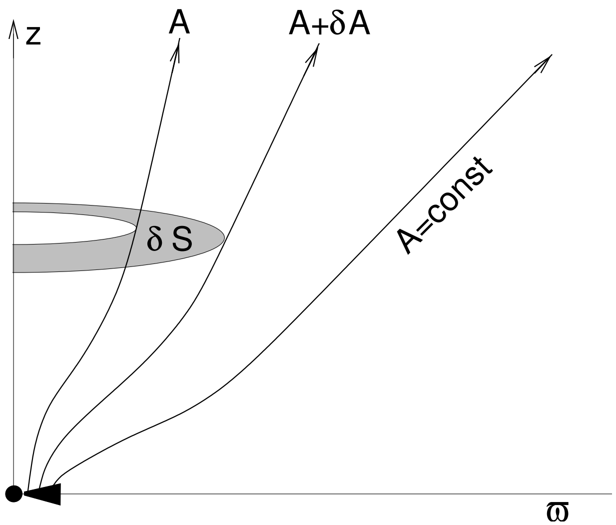
What controls the value of σ_∞?

Basics of ideal MHD:

- Ohm's law $\boldsymbol{E} = -\boldsymbol{V}/c \times \boldsymbol{B}$
- split $\boldsymbol{B}$ into toroidal $\boldsymbol{B}_\phi$ and poloidal $\boldsymbol{B}_p$
- Ferraro's law of isorotation $(\varpi \Omega / c) B_p = E$
  ($\varpi =$ distance from the rotation axis)
- Poynting flux is proportional to the poloidal current
  $I = (c/2)\varpi |B_\phi|$; thus, $\mu - \gamma \propto I$, and $\gamma \uparrow$ when $\varpi |B_\phi| \downarrow$
- define the magnetic flux $\int \boldsymbol{B}_p \cdot d\boldsymbol{S}/2\pi = A$
  - $B_p \varpi^2 \sim A$ means uniformely distributed fieldlines, while
  - $B_p \varpi^2 \gg A$ means bunched fieldlines

  e.g., monopolar field has $B_p \varpi^2 / A = 1$, dipolar has $B_p \varpi^2 / A = 2$

The key function $B_p \varpi^2$



Magnetic flux conservation implies $B_p \delta S = \delta A$.

When $V_\phi \ll V_p$ (superfast flow) Ohm's law $\rightarrow$ $E \approx |B_\phi|$.

Ferraro's law $\rightarrow$ $\varpi B_p \propto E$, or, $\varpi B_p \propto |B_\phi|$.

Thus, $\mu - \gamma \propto I \propto \varpi |B_\phi| \propto \varpi^2 B_p \propto (\varpi^2 / \delta S) \delta A$.

Increasing γ corresponds to expansion of fieldlines such that δS increases faster than ϖ^2.

The poloidal fieldline shape controls the acceleration and we may think the Poynting flux as energy stored in springs connecting the poloidal fieldlines.

Now it is clear why Michel's solution (where $B_p \varpi^2$ is constant by assumption) gives inefficient acceleration.

Quantitative analysis

We proved that, as long as $V_\phi \ll V_p$, i.e., in the superfast regime, $\mu - \gamma \propto \varpi^2 B_p$.

Thus, between r_f and r_∞, $\dfrac{\mu - \gamma_\infty}{\mu - \gamma_f} = \dfrac{(\varpi^2 B_p)_\infty}{(\varpi^2 B_p)_f}$.

Since $\gamma_f \ll \mu$ and $(\varpi^2 B_p)_\infty \approx A$,

$$\gamma_\infty \approx \mu \left(1 - \frac{A}{(\varpi^2 B_p)_f} \right)$$

- The more bunched the fieldlines near the fast surface the higher the acceleration efficiency.

- In the previous shown self-similar solution it happened that $(\varpi^2 B_p)_f \approx 2A$, resulting in equipartition $\gamma_\infty \approx \mu/2$. Efficiencies higher that 50% have been found, corresponding to $(\varpi^2 B_p)_f \gg A$.

Relation between γ_∞ and the top of the disk

How to connect the fast surface with the base?
Easy! The flow is force-free up to the fast, $I_i \approx I_f$. So, (using the relation between B_p and B_ϕ at the fast surface) we find

$$\boxed{\gamma_\infty \approx \mu \left(1 - \frac{A\Omega}{2I_i}\right)}$$

(verified by all solutions that reach an asymptotic stage)

(The Poynting-to-matter energy flux ratio is $\sigma_\infty = (\mu/\gamma_\infty) - 1$)

Summary

- Blazar jets are likely accelerated at relatively large distances from the disk (e.g., a few pc according to Unwin et al for the jet in 3C 345, or, $10^3 - 10^4 r_g$ according to the analysis by Sikora et al.)

- Magnetic driving provides a viable explanation of the jet bulk acceleration. It also
  - ⋆ naturally explains the collimation
  - ⋆ could explain the apparent motion of the jet components

- Although the MHD equations are highly intractable (extremely difficult to solve them even numerically) there is a simple analysis explaining the acceleration efficiency and its dependence on the disk conditions

  (detailed analysis in Vlahakis 2004, Ap&SS, 293, 67)

Discussion

to follow . . .

The ideal MHD equations

Maxwell:

$$\nabla \cdot \mathbf{B} = 0\,, \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{c\partial t}\,, \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{\partial \mathbf{E}}{c\partial t}\,, \nabla \cdot \mathbf{E} = \frac{4\pi}{c}J^0$$

Ohm: $\mathbf{E} + \frac{\mathbf{V}}{c} \times \mathbf{B} = 0$

mass conservation: $\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)(\gamma\rho_0) + \gamma\rho_0 \nabla \cdot \mathbf{V} = 0\,,$

energy $U_\mu T^{\mu\nu}_{,\nu} = 0$: $\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)\left(\frac{P}{\rho_0^\Gamma}\right) dt = 0$

momentum $T^{\nu i}_{,\nu} = 0$:

$$\gamma\rho_0 \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)(\xi\gamma\mathbf{V}) = -\nabla P + \frac{J^0\mathbf{E} + \mathbf{J} \times \mathbf{B}}{c}$$