The Kelvin-Helmholtz instability in Relativistic Jets

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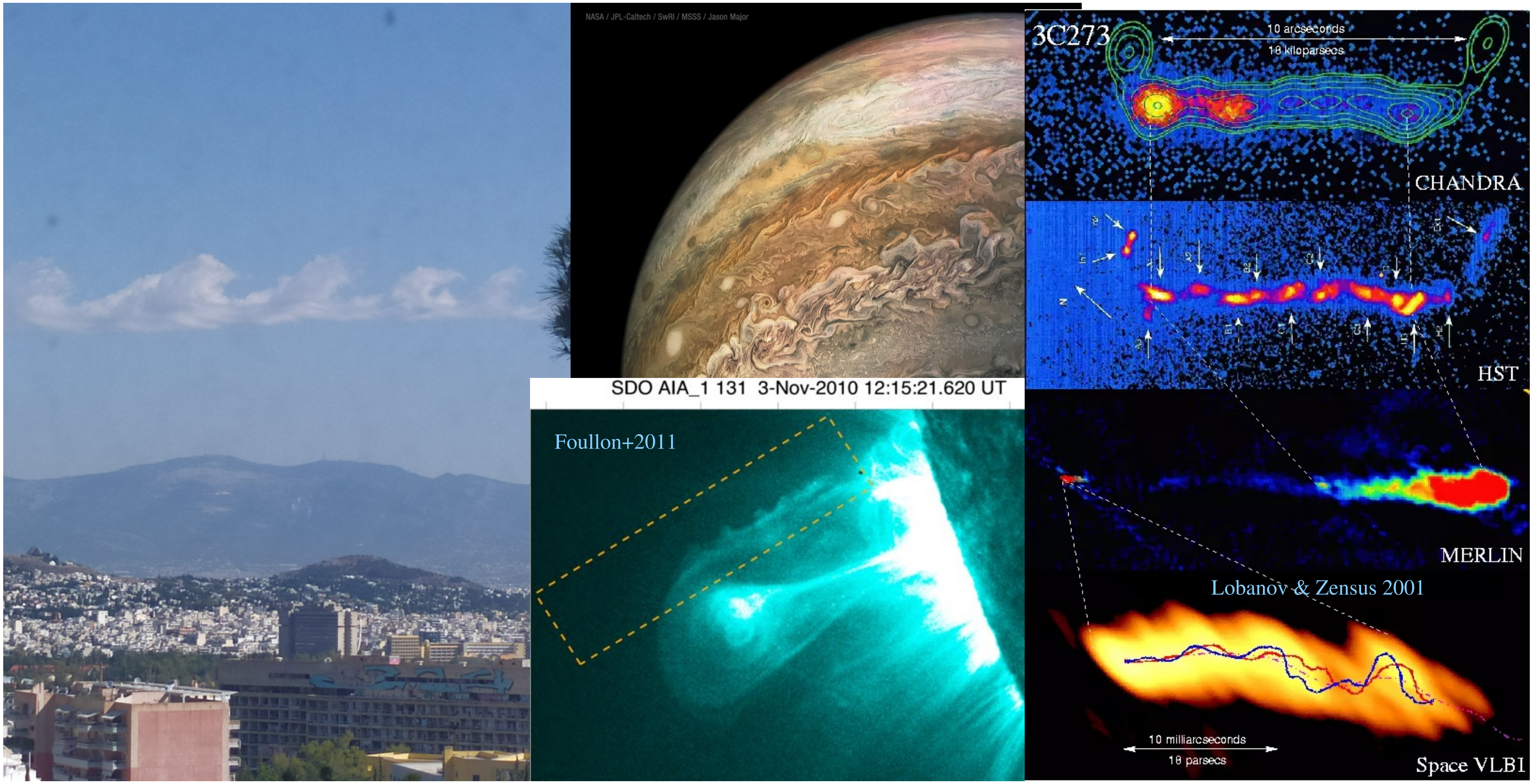
Outline

- introduction (KH basic mechanism)
- linear stability of relativistic magnetized jets
- growth rates (dependence on jet/environment properties)

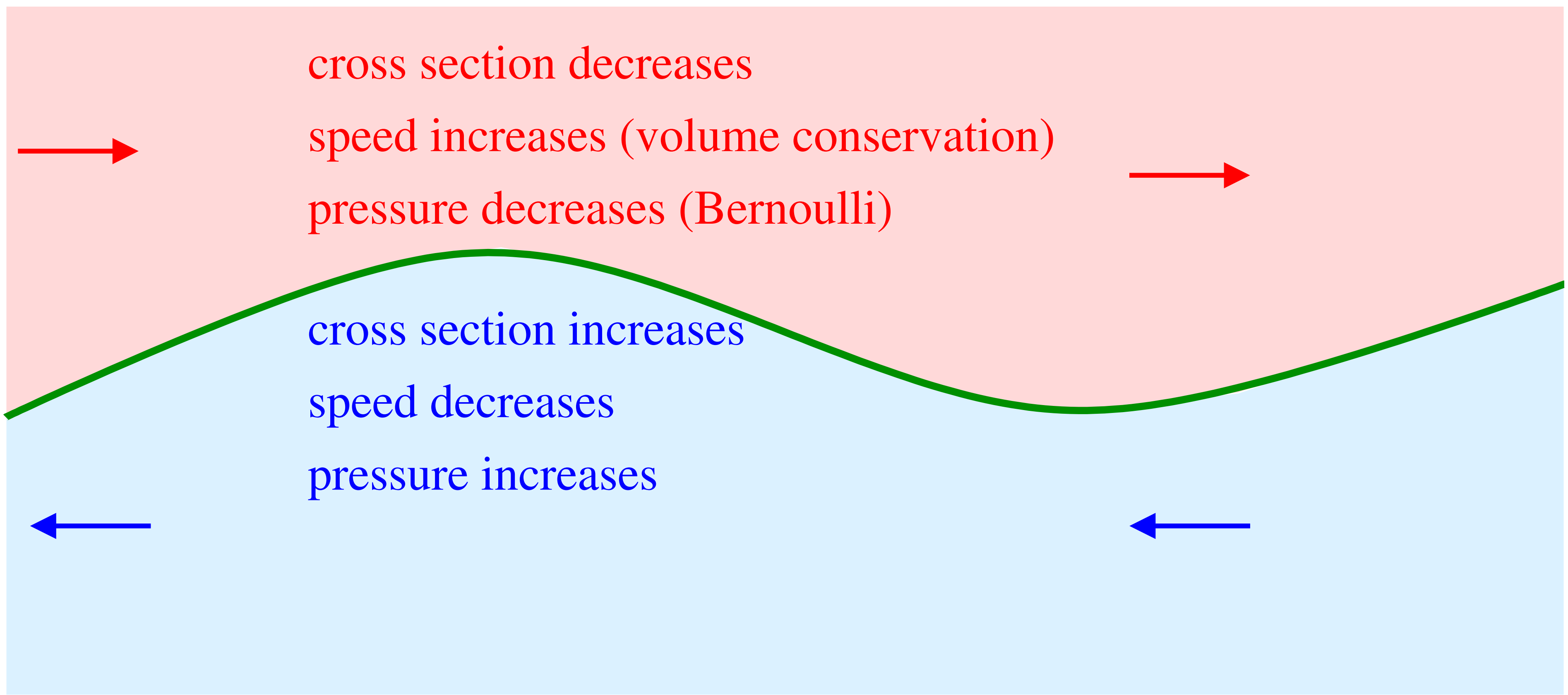
Kelvin-Helmholtz in nature

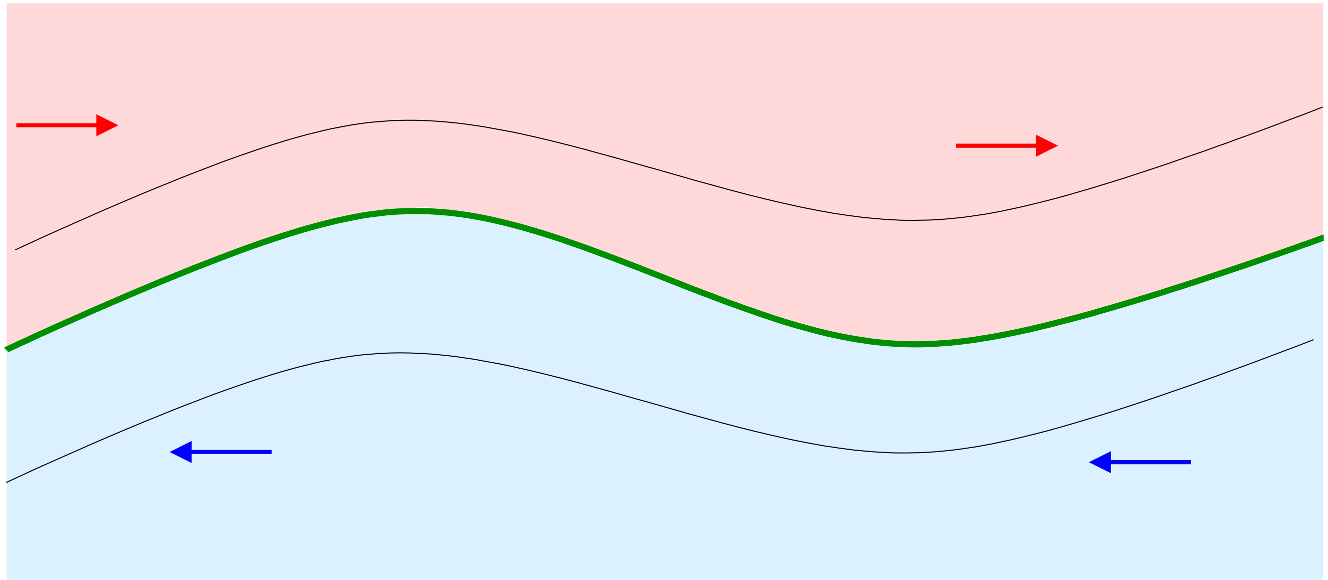


Basic mechanism

Two fluids in relative motion

Simplest variant: incompressible hydrodynamics





- Magnetic field stabilizes (through magnetic tension)

In the non-relativistic case stability when

$$\frac{\rho_1 \, \rho_2}{\rho_1 + \rho_2} \left(\boldsymbol{V}_1 \cdot \boldsymbol{k} - \boldsymbol{V}_2 \cdot \boldsymbol{k} \right)^2 < \frac{(\boldsymbol{B}_1 \cdot \boldsymbol{k})^2 + (\boldsymbol{B}_2 \cdot \boldsymbol{k})^2}{4\pi}$$ (Chandrasekhar 1961)

- Compressibility important

Fast flows (with sufficiently high Mach numbers) are stable

(When the section decreases the density increases – and not the speed as in the incompressible limit – consequently pressure increases)

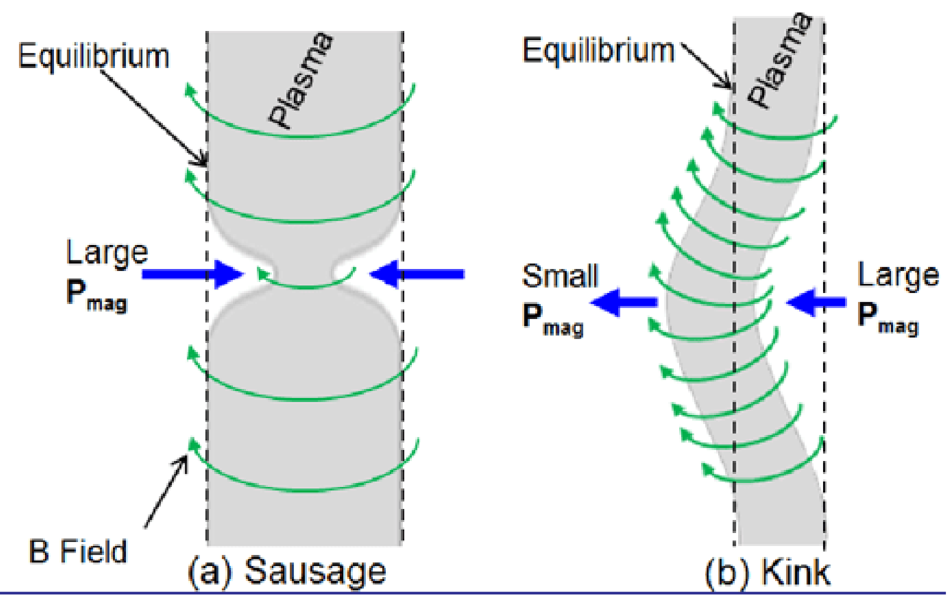
For example, for same fluids without magnetic field, stability when $M > \sqrt{8}$

For astrophysical jets we need to combine

- magnetic field
- compressibility
- relativity (in bulk speed, sound speed, Alfvén speed)
- cylindrical geometry

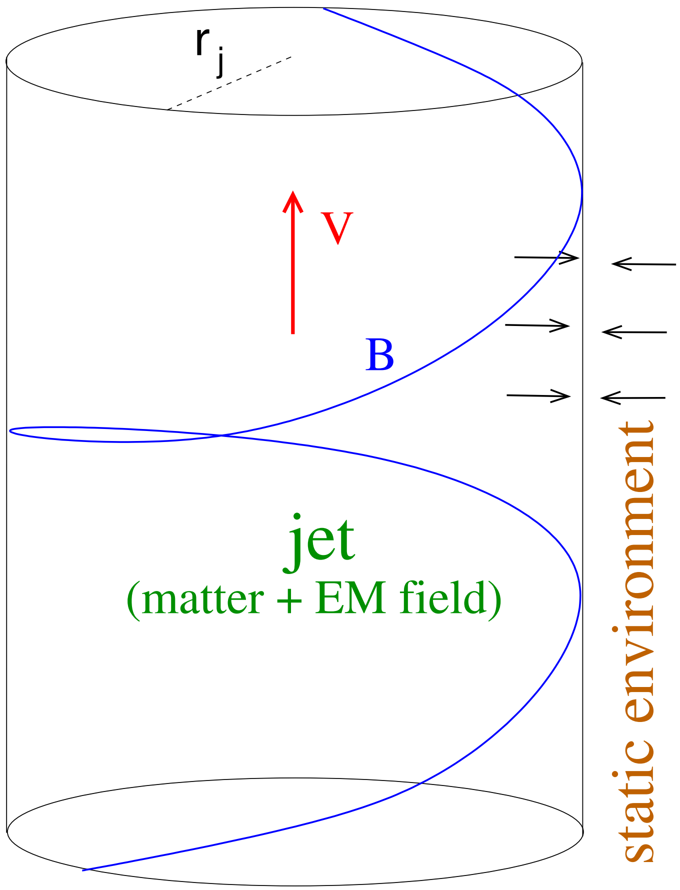
contrary to planar geometry: (i) the jet is inhomogeneous, (ii) we have the spatial scale r_j, (iii) reflections from the symmetry axis may be important

Besides K-H there are also current-driven instabilities
(sketch from Yager-Elorriaga 2017)



Linear stability analysis of jets

Charalampos Sinnis' PhD work



Unperturbed state:

- Cylindrical jet, cold, with constant speed $V_0\hat{z}$, constant density ρ_0, and helical magnetic field

$$B_{0z} = \frac{B_0}{1+(r/r_0)^2}, \quad B_{0\phi} = B_{0z}\gamma\frac{r}{r_0}$$

(satisfying the force balance equation). B_0 controls the magnetization

$\sigma = \frac{B_{\rm co}^2}{4\pi\rho_0 c^2}$, r_0 controls the $\frac{B_\phi}{B_z}$

- Environment: uniform, static, with density $\eta\rho_{0\,jet}$, either hydrodynamic or cold with uniform B_{0z}

• Add perturbations in all quantities

$Q(r\,,z\,,\phi\,,t) = Q_0(r) + Q_1(r)e^{i(kz+m\phi-\omega t)}$

with integer m, real k, and complex ω (temporal approach),

i.e. $Q = Q_0(r) + Q_1(r)e^{\Im\omega t}e^{i(kz+m\phi-\Re\omega t)}$

(instability corresponds to $\Im\omega > 0$)

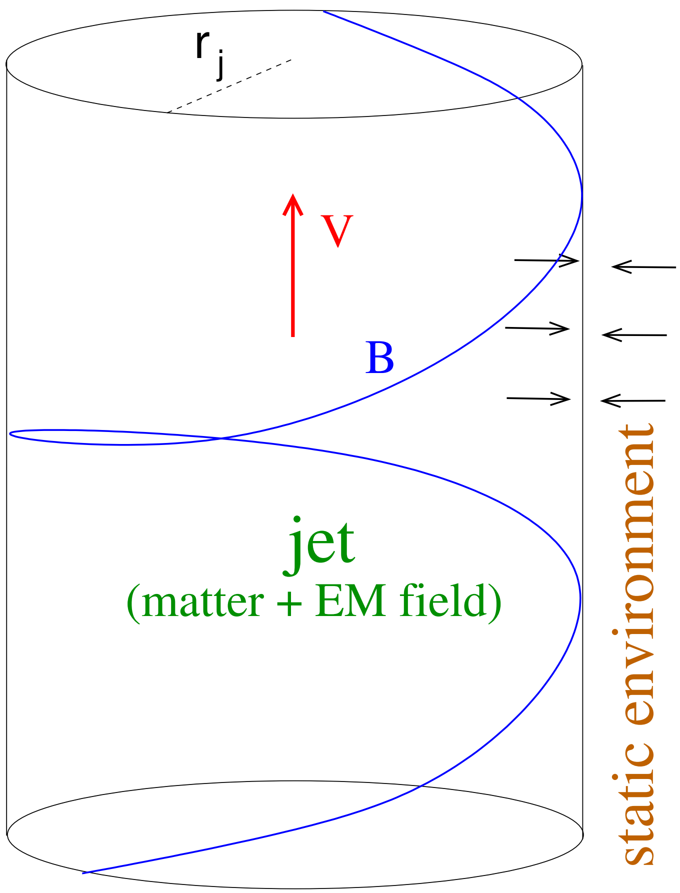
Linearization of the ideal relativistic magnetohydrodynamic equations reduces to two equations with (complex) unknowns:

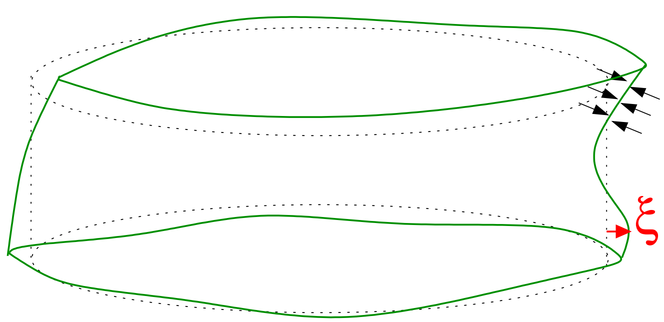
• the Lagrangian displacement of each fluid element in the radial direction y_1

• the perturbation of the total pressure at the displaced position y_2

These should be continuous everywhere (at the interface as well)

Eigenvalue problem

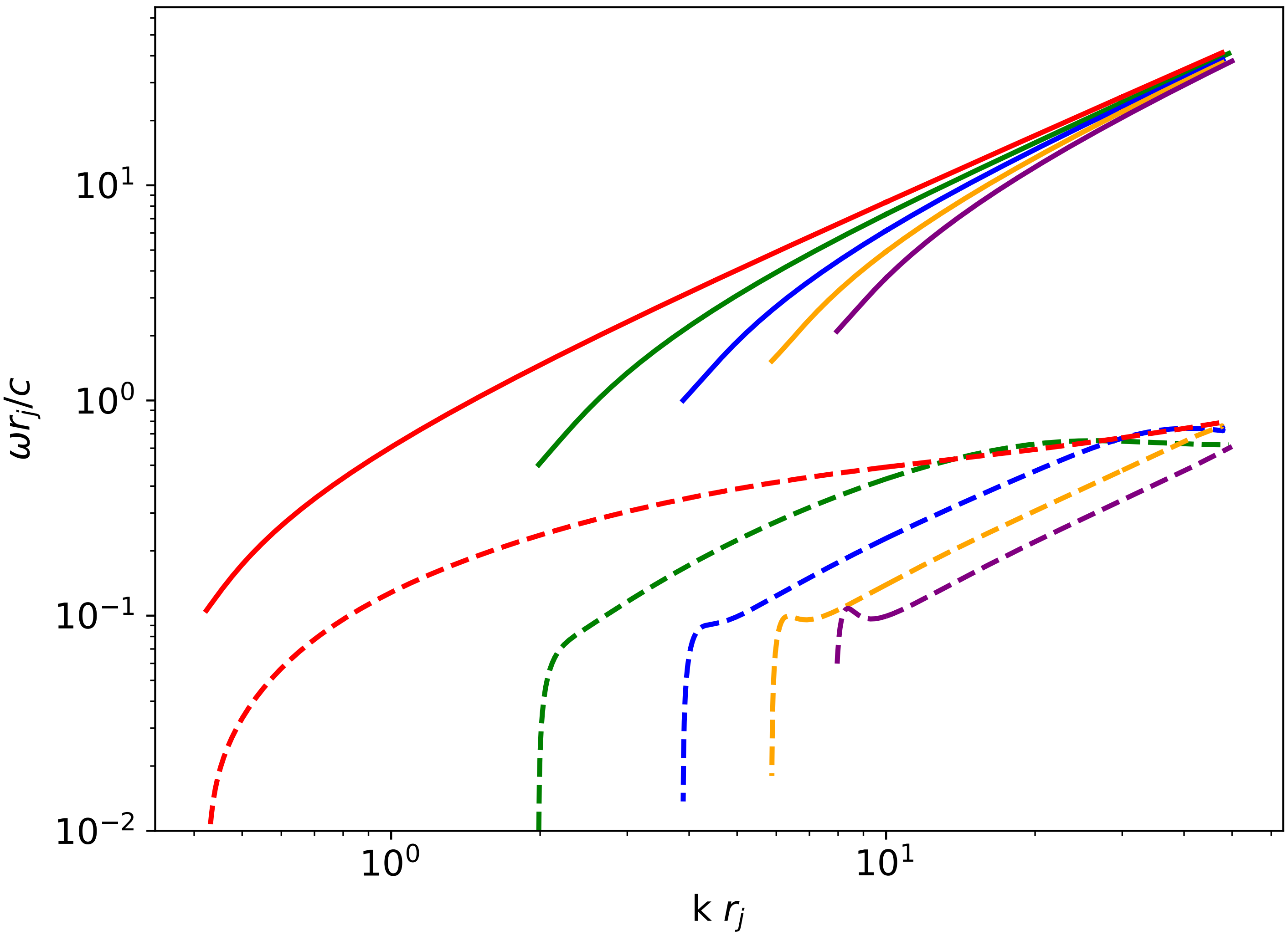




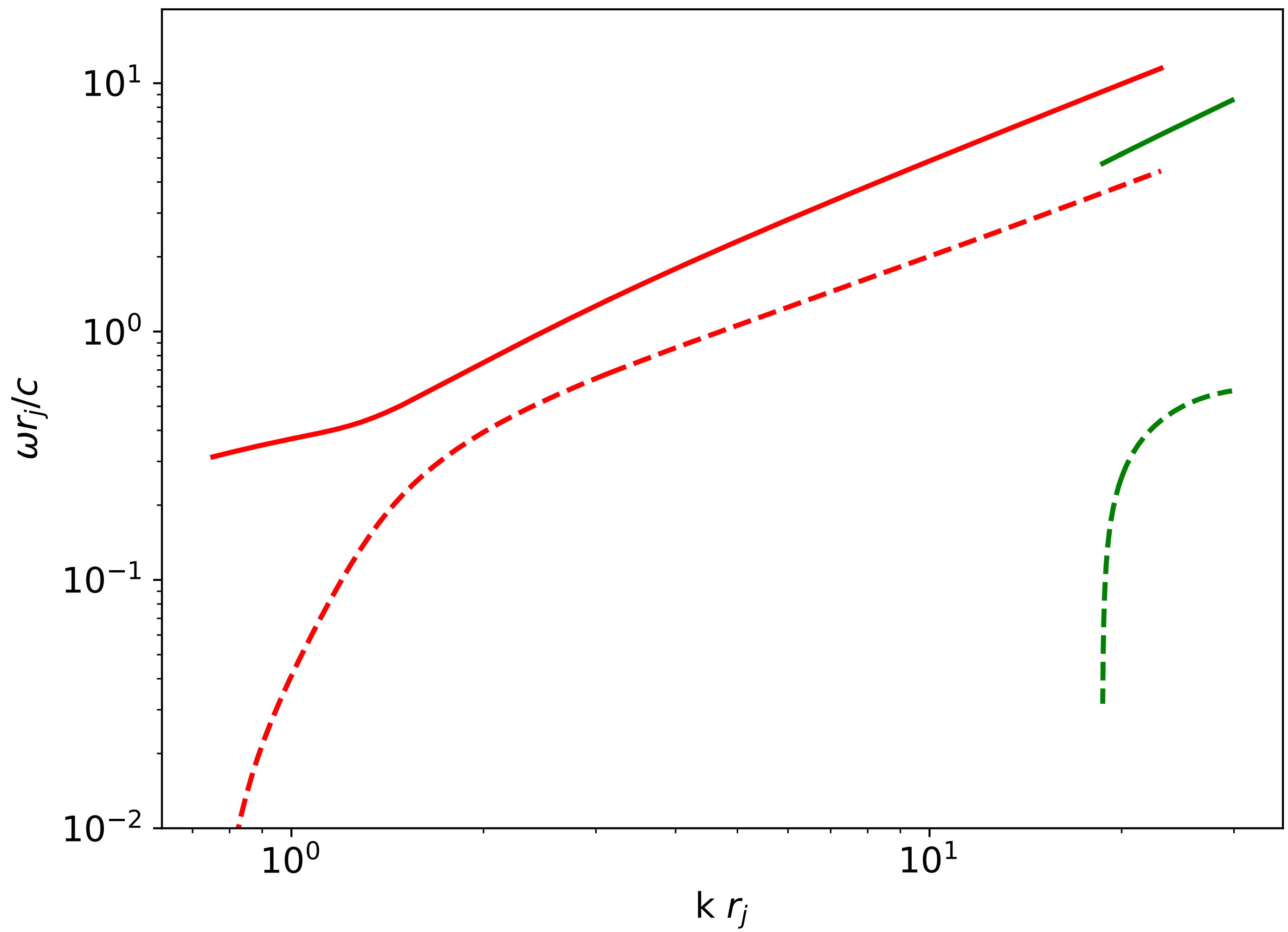
- integrate the equations inside the jet (attention to regularity condition on the axis)
- integrate the equations in the environment (solution vanishes at $r \gg r_j$)
- Match the solutions at r_j:
find ω for which y_1 and y_2 are continuous $\longrightarrow$ dispersion relation
- The solution depends on γ, σ, r_0, η, and the wavenumbers k, m

Results for hydrodynamic environment

Typical result for the dispersion relation (Re=solid, Im=dashed). Here $\gamma = 5$, $\sigma = 1$ (at r_j), $r_0 = 0.1$, $\eta = 10$, and $m = 0$

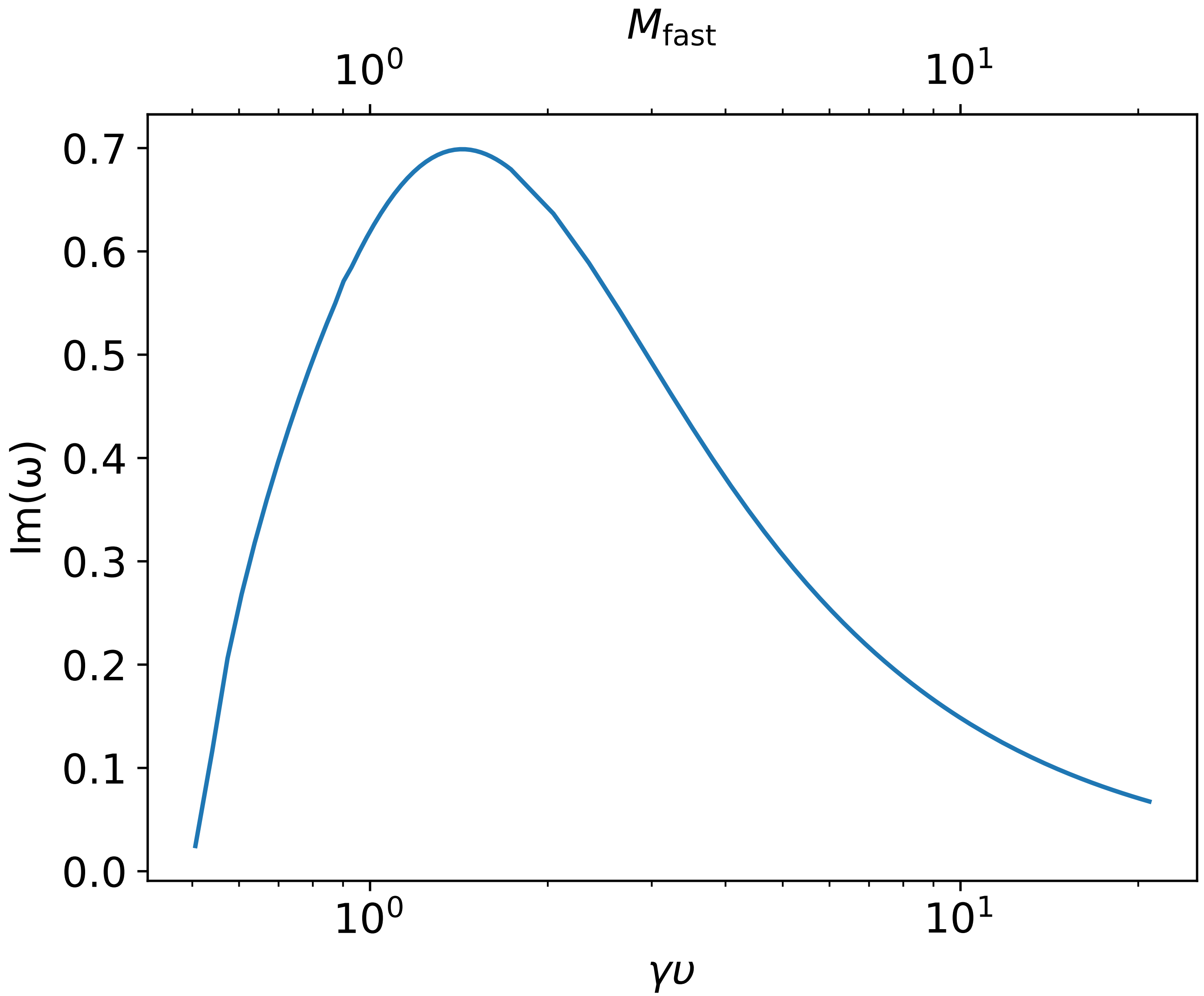


For $\gamma = 2$, $\sigma = 1$ (at r_j), $r_0 = 0.1$, $\eta = 10$ a “hyper-unstable” mode appears! it turns out to be the Kelvin-Helmholtz instability mode



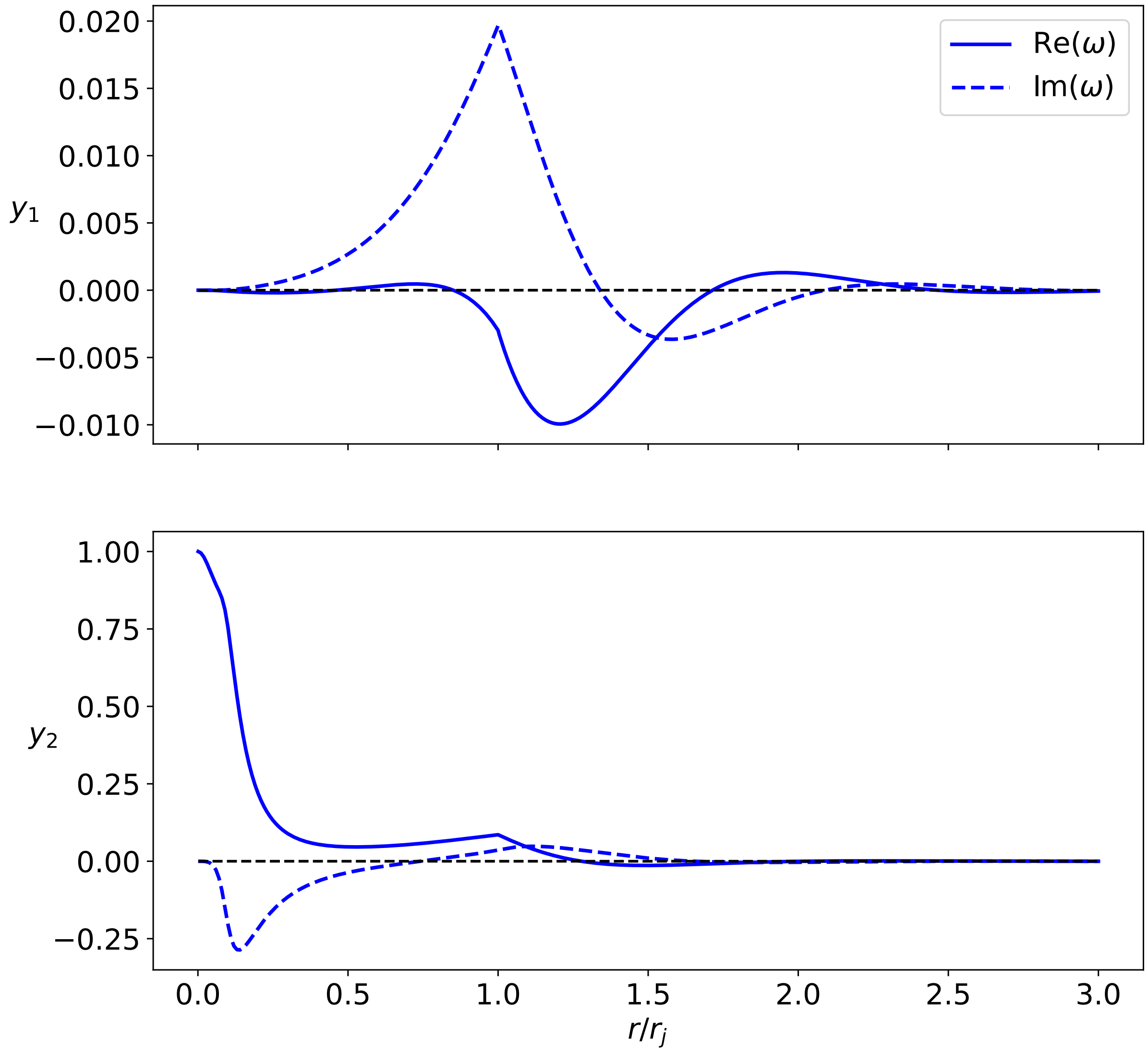
For $k \gtrsim 2$ planar geometry is a good approximation

We explore in the following a fiducial case with $k = \pi$

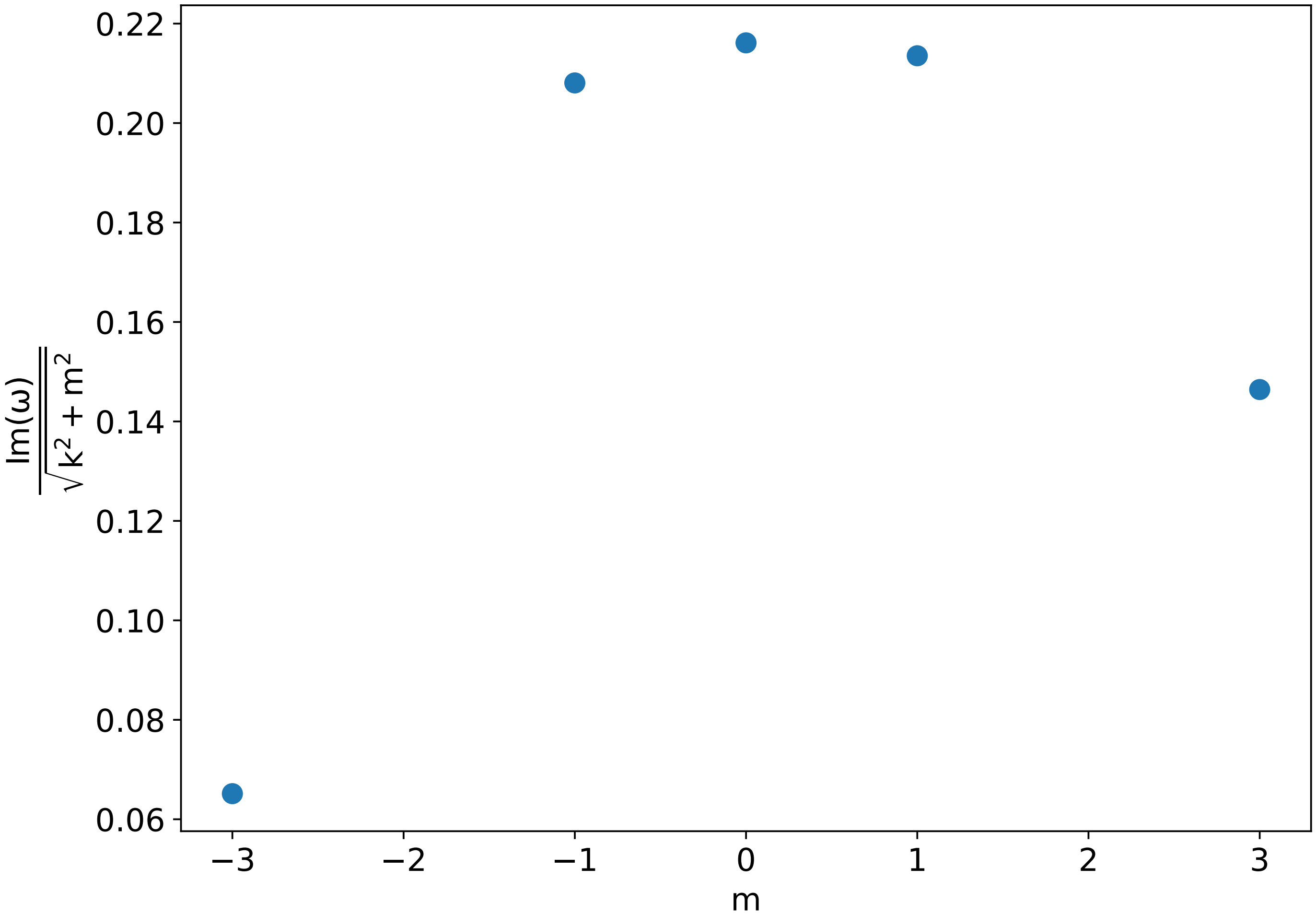


For small speeds $\Im\omega \propto V$ while sufficiently large M_{fast} stabilizes

Locality of the eigenfunction y_1 (Lagrangian displacement)

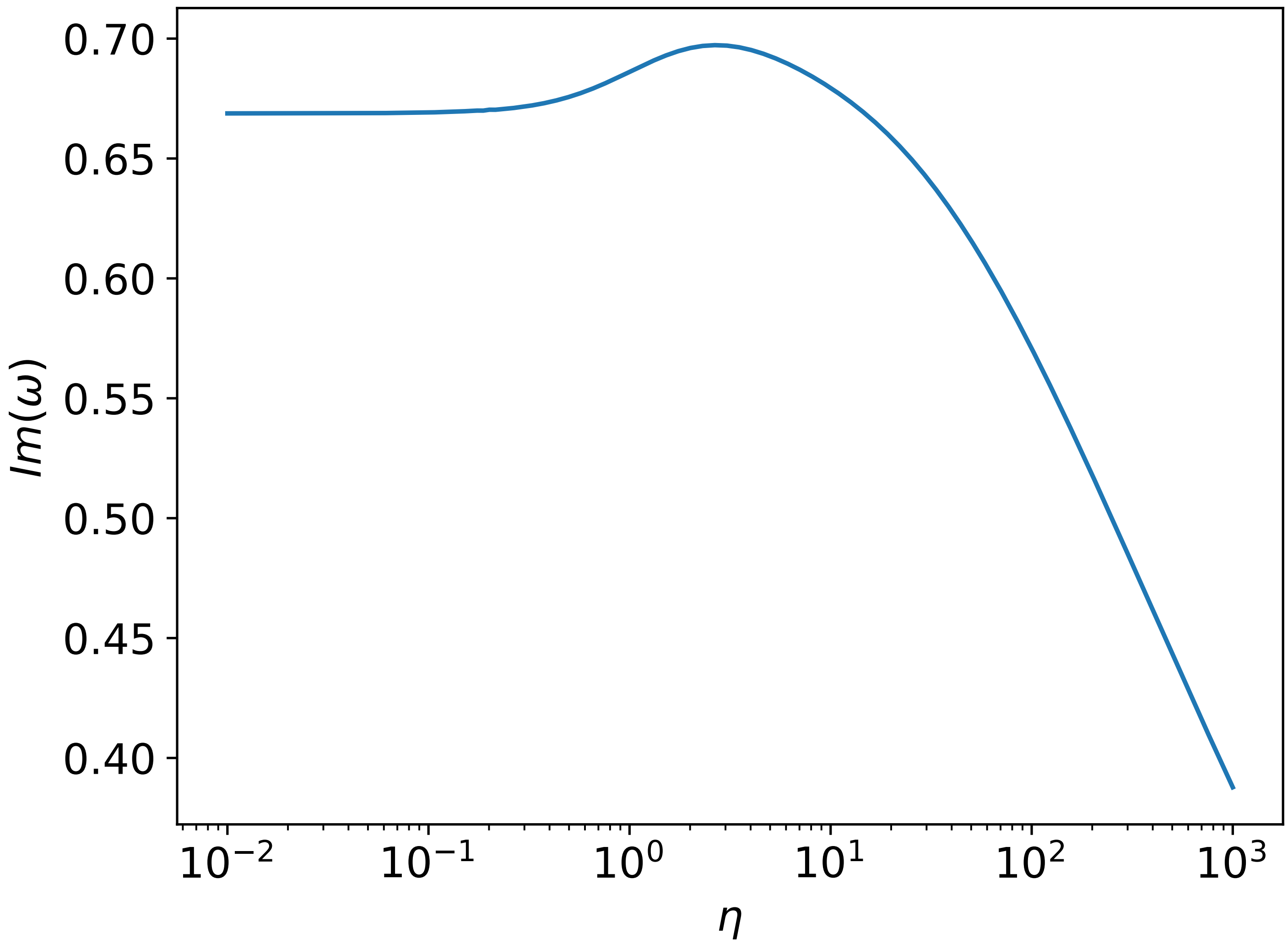


Higher $\boldsymbol{B} \cdot \boldsymbol{k} = B_z k + B_\phi k_\phi$ (through nonzero $k_\phi = m$) stabilizes

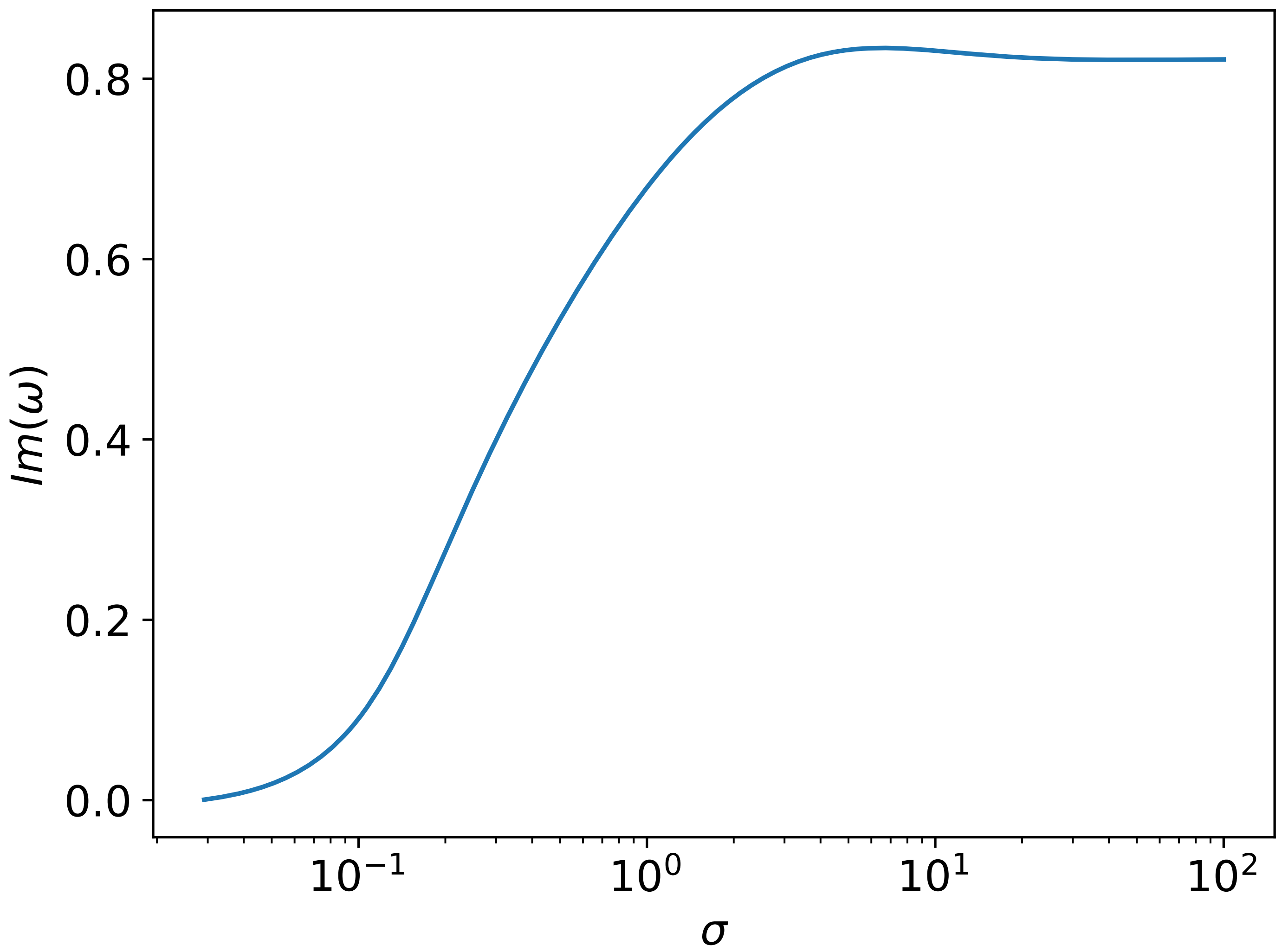


Since the magnetic field is mainly azimuthal (at the jet surface), the most unstable mode is the axisymmetric $m = 0$

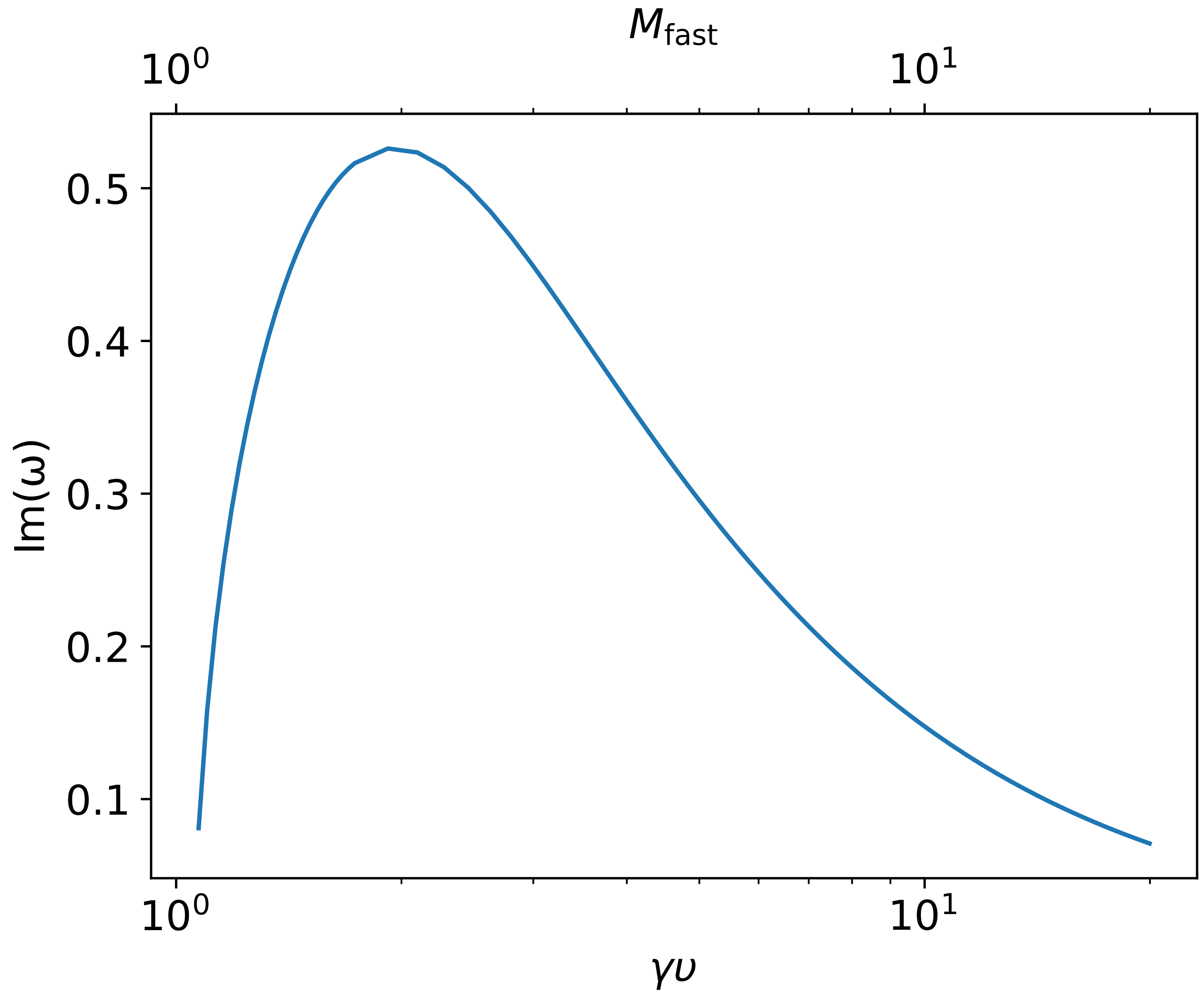
Dependence on the density contrast (which modifies the environment sound speed)



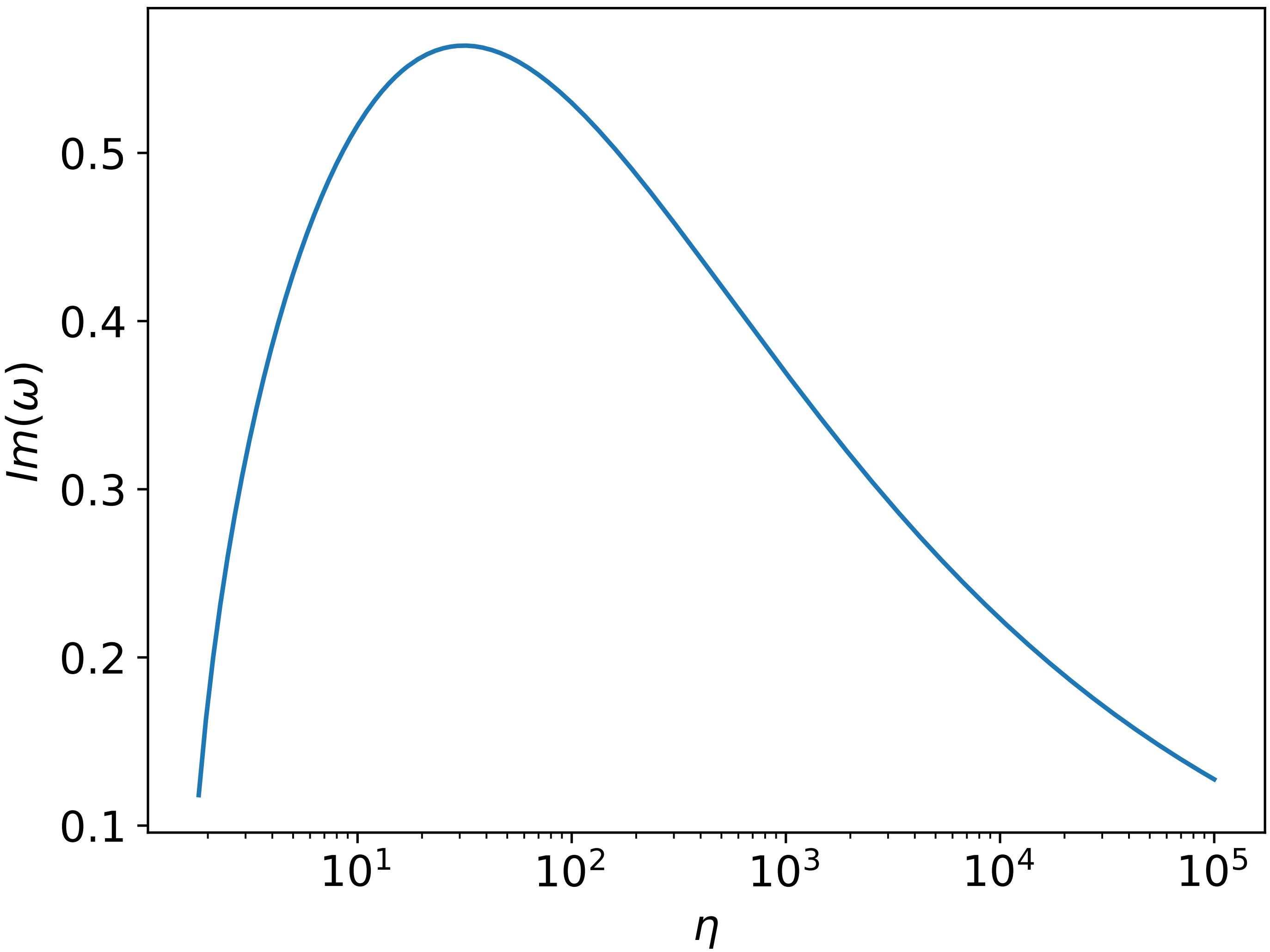
Dependence on the jet magnetization (at r_j)



Results for cold/magnetized environment



Dependence on the density contrast (which modifies the environment Alfvén speed)



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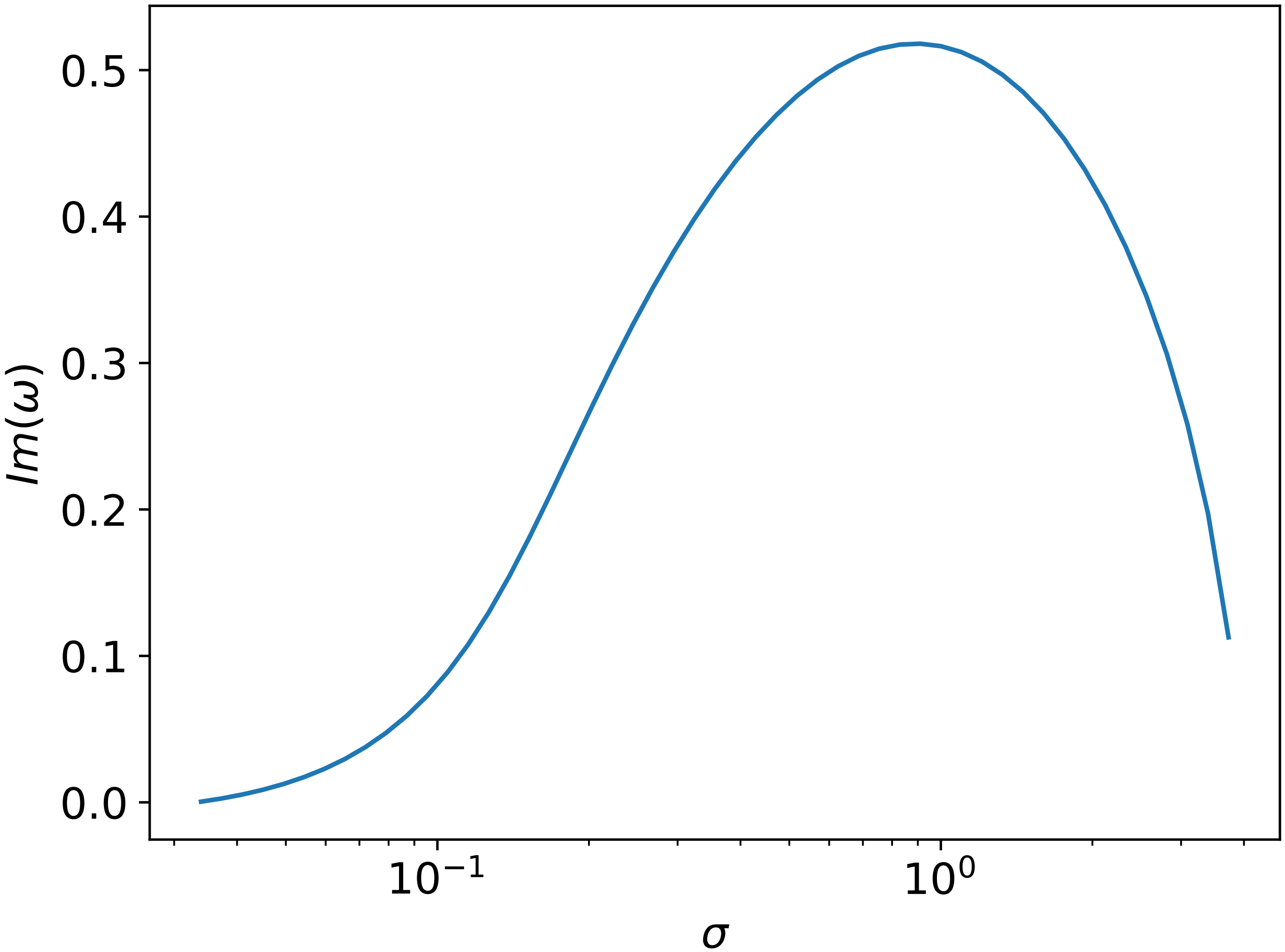

Summary

- ⋆ When KH is present it is the most unstable mode with growth rates comparable to c/r_j (even larger for high k)
- ⋆ The growth rate mostly affected by the fast magnetosonic Mach number $M_{\rm fast} = \gamma/\sqrt{\sigma}$ of the jet (largest growth rates when $M_{\rm fast}$ is in a "window" around ~ 1)
- ⋆ Higher values of $\boldsymbol{B} \cdot \boldsymbol{k}$ (in jet and in environment) stabilize (jets with magnetized environments are in general more stable compared to the ones with hydrodynamic environments)
- ⋆ A sufficiently dense environment stabilizes (acts as solid wall)
- ⋆ For relatively small k the curvature of the jet radius cannot be ignored (planar geometry is not a good approximation)