The magnetic acceleration and collimation paradigm for relativistic jets

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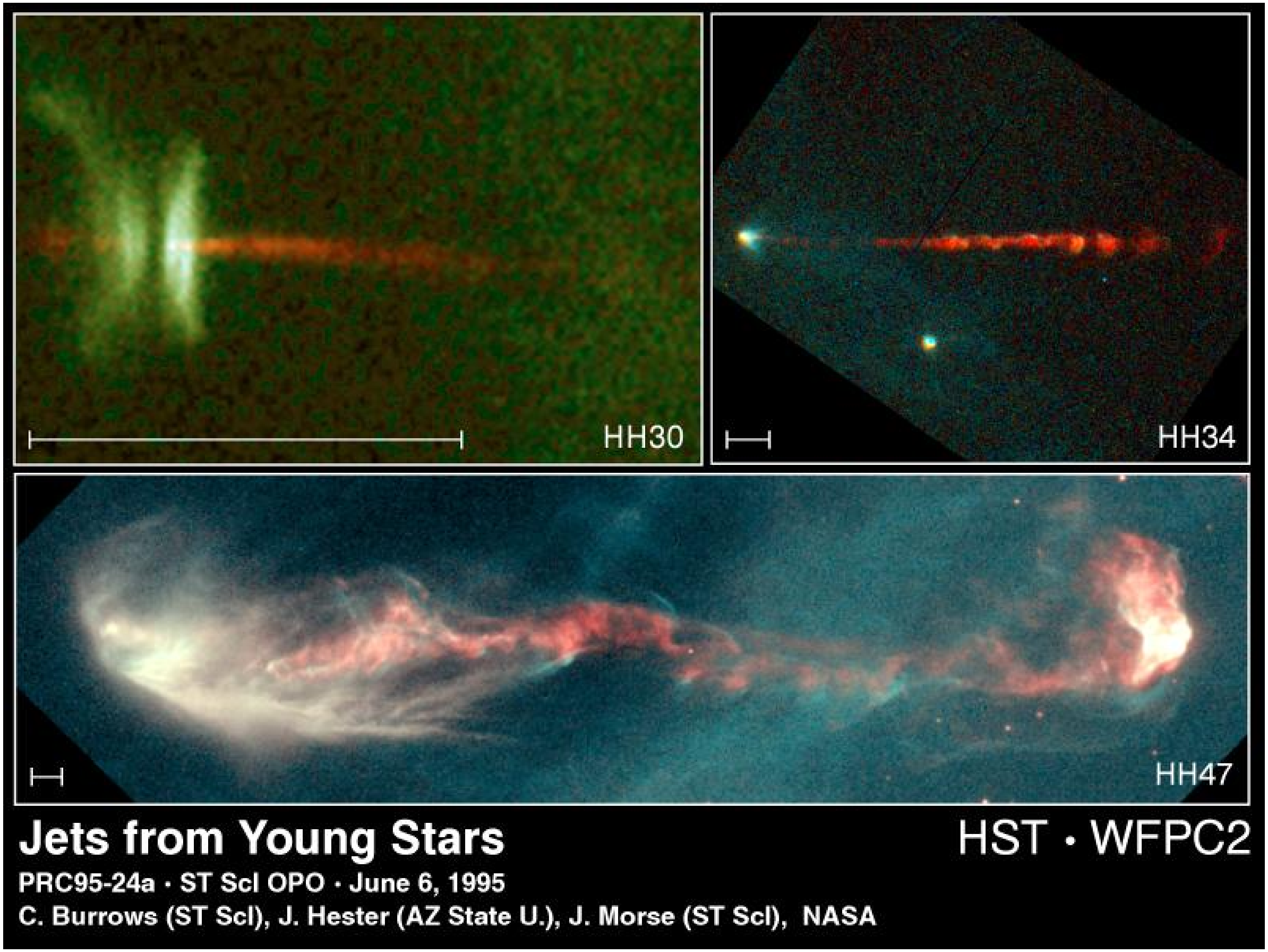
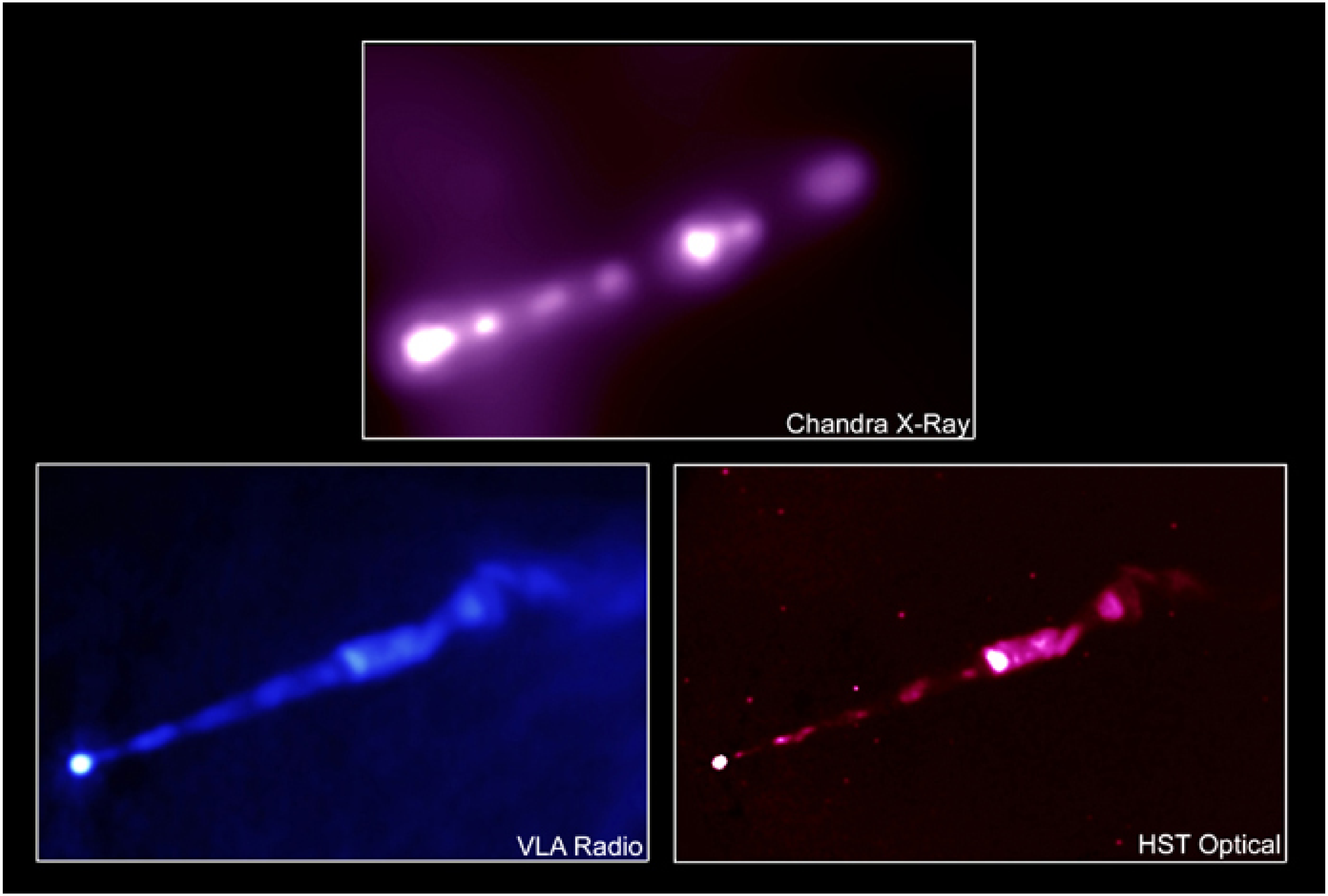
Outline

- MHD acceleration and collimation mechanisms
  - general analysis
- exact solutions
  - semi-analytical models
  - simulations



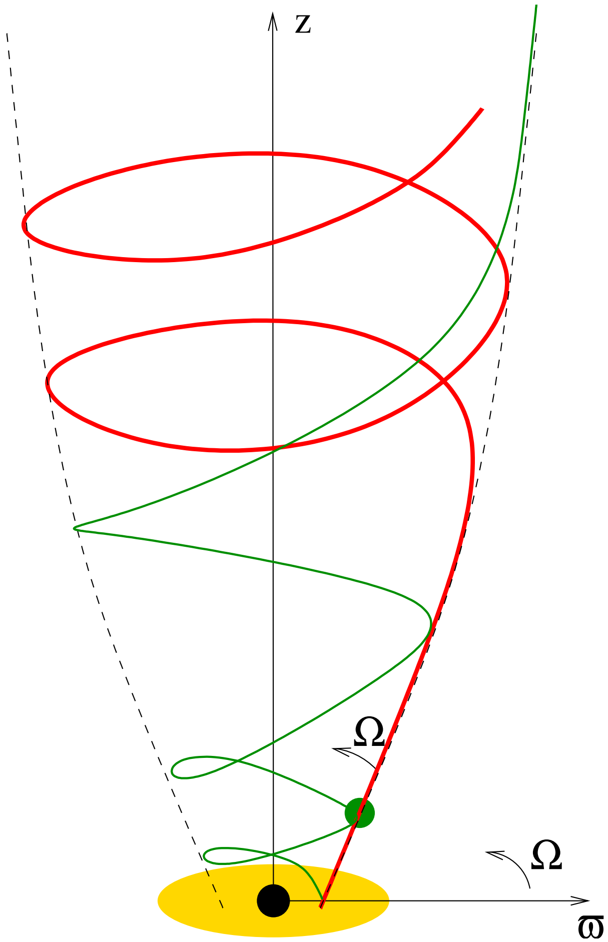
(scale =1000 AU, $V_\infty = a few 100$km/s)



collimation at ∼100 Schwarzschild radii, $\gamma_\infty \sim 10$

The structure of a magnetized outflow

A rotating source (disk or star) creates an axisymmetric outflow



Assume steady-state and ideal magnetohydrodynamics (MHD):

- Initially $V_\phi = \varpi\Omega \gg V_p$, $B_p \gtrsim B_\phi$
  The energy resides in the electromagnetic field
- Flux freezing: velocity $\parallel \boldsymbol{B}$ plus $\boldsymbol{E} \times \boldsymbol{B}$ drift $\rightarrow \boldsymbol{V}_p \parallel \boldsymbol{B}_p$.
- $\boldsymbol{B}_p \propto 1/\varpi^2$, $\boldsymbol{B}_\phi \propto 1/\varpi$

Acceleration mechanisms

- thermal (due to ∇P) $\rightarrow$ velocities up to C_s
- magnetocentrifugal (beads on wire - Blandford & Payne)
  - in reality due to magnetic pressure
  - initial half-opening angle $\vartheta > 30^o$
  - the $\vartheta > 30^o$ not necessary for nonnegligible P
  - velocities up to $\varpi_0 \Omega$
- relativistic thermal (thermal fireball) gives $\gamma \sim \xi_i$, where $\xi = \dfrac{\text{enthalpy}}{\text{mass} \times c^2}$.
- magnetic – up to $\gamma_\infty = \mu$, where $\mu = \dfrac{\dfrac{dE}{dSdt}}{\dfrac{dM}{dSdt}c^2}$?

All acceleration mechanisms can be seen in the energy conservation equation

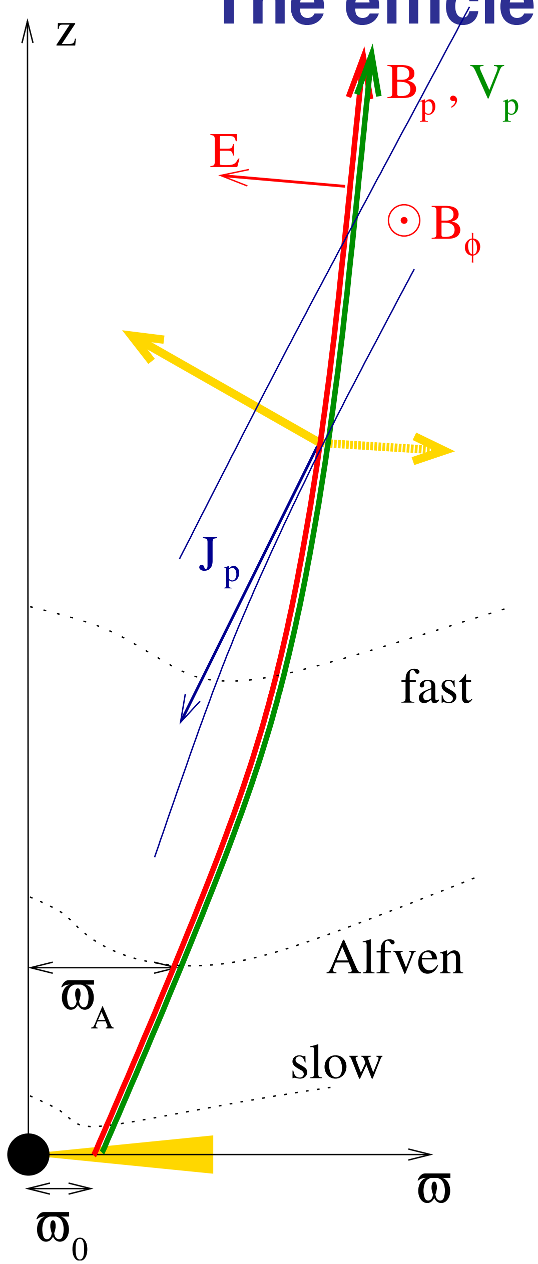
$$\mu = \xi\gamma + \frac{\Omega}{\Psi_A c^2}\varpi B_\phi$$

where μ, Ω, Ψ_A(=mass-to-magnetic flux ratio) are constants of motion.

So $\gamma \uparrow$ when $\xi \downarrow$ (thermal, relativistic thermal), or, $\varpi B_\phi \downarrow \Leftrightarrow I_p \downarrow$ (magnetocentrifugal, magnetic).

$\gamma_\infty = \mu$ means $\xi = 1$ (its minimum value) and $\varpi B_\phi = 0$.
Is this possible?

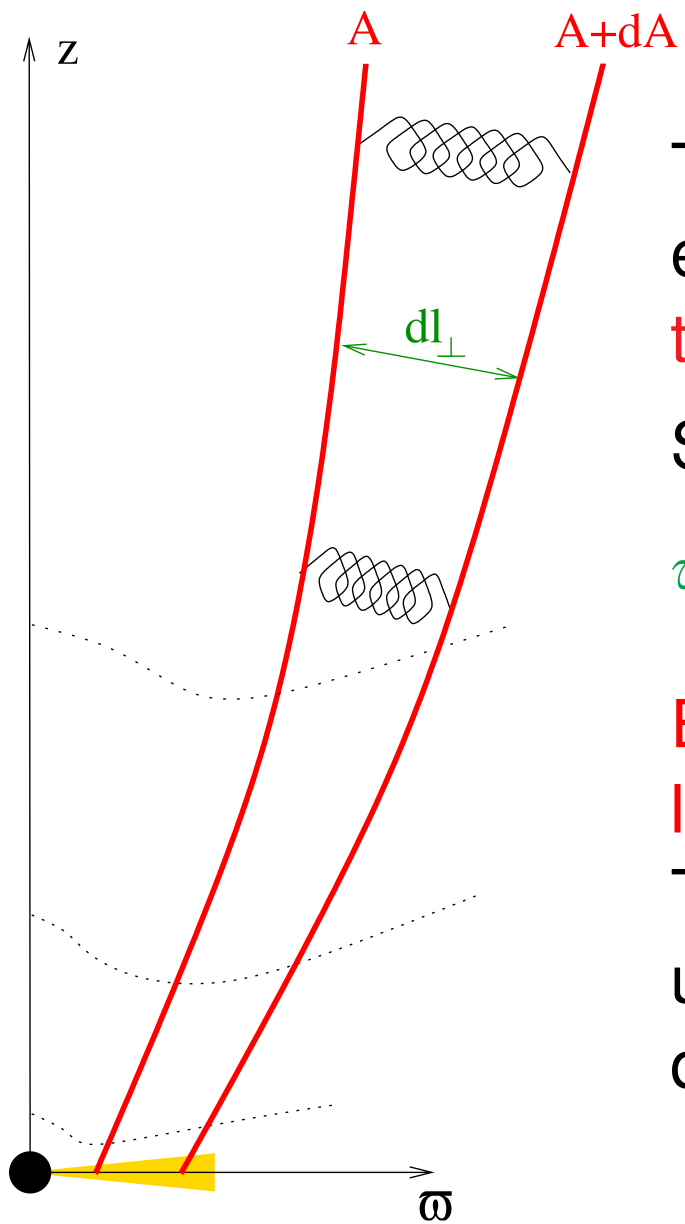
The efficiency of the magnetic acceleration



The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force strongly depends on the angle between field-lines and current-lines.

Are we free to choose these two lines? NO! All MHD quantities are related to each other and should be found by solving the full system of equations.

From Ferraro's law, $\varpi B_\phi \approx \varpi^2 B_p \Omega / V_p$. So, the transfield force-balance determines the acceleration.



The magnetic field minimizes its energy under the condition of keeping the magentic flux constant.

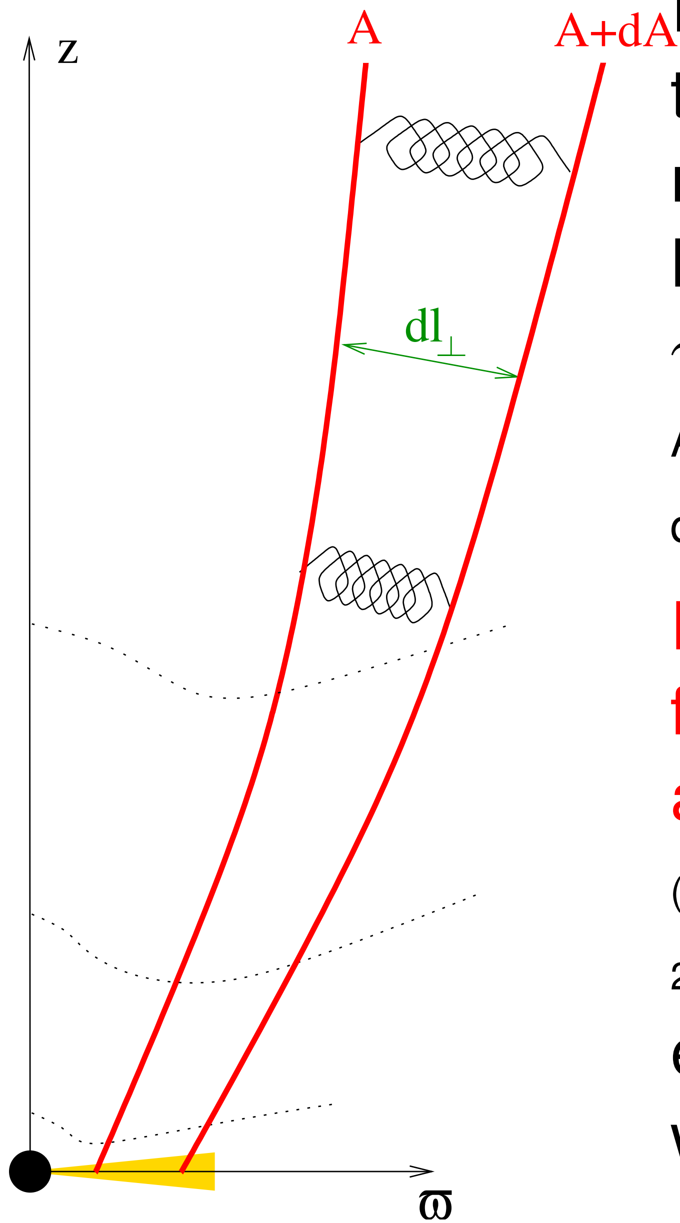
So, $\varpi B_\phi \downarrow$ for decreasing

$$\varpi^2 B_p = \frac{\varpi^2}{2\pi\varpi dl_\perp}\underbrace{(B_p dS)}_{dA} \propto \frac{\varpi}{dl_\perp}.$$

Expansion with increasing $dl_\perp/\varpi$ leads to acceleration.

The expansion ends in a more-or-less uniform distribution $\varpi^2 B_p \approx A$ (in a quasi-monopolar shape).

Conclusions on the magnetic acceleration



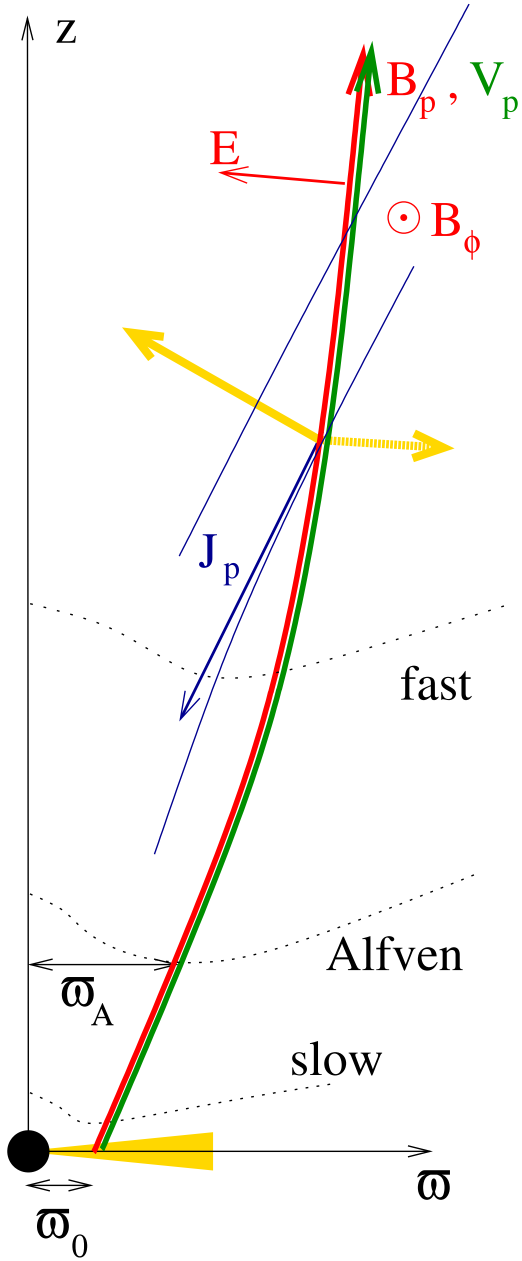
If we start with a uniform distribution the magnetic energy is already minimum $\rightarrow$ no acceleration. Example: Michel's (1969) solution which gives $\gamma_\infty \approx \mu^{1/3} \ll \mu$.

Also Beskin et al (1998); Bogovalov (2001) who found quasi-monopolar solutions.

For any other (more realistic) initial field distribution we have efficient acceleration!

(details and an analytical estimation of the efficiency in Vlahakis 2004, ApSS 293, 67).

example: if we start with $\varpi^2 B_p / A = 2$ we have asymptotically $\varpi^2 B_p / A = 1$ $\rightarrow 50\%$ efficiency



On the collimation

The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force contributes to the collimation (hoop-stress paradigm).
In relativistic flows the electric force plays an opposite role (a manifestation of the high inertia of the flow).

- collimation by an external wind (Bogovalov & Tsinganos 2005, for AGN jets)
- surrounding medium may play a role (in the collapsar model)
- self-collimation mainly works at small distances where the velocities are mildly relativistic (Vlahakis & Königl 2003)

For $\gamma \gg 1$, curvature radius $\mathcal{R} \sim \gamma^2 \varpi \ (\gg \varpi)$.

Collimation more difficult, but not impossible!

$$\frac{\varpi}{\mathcal{R}} = -\varpi \frac{\partial^2 \varpi}{\partial z^2} \left(\frac{B_z}{B_p}\right)^3 \sim \left(\frac{\varpi}{z}\right)^2$$

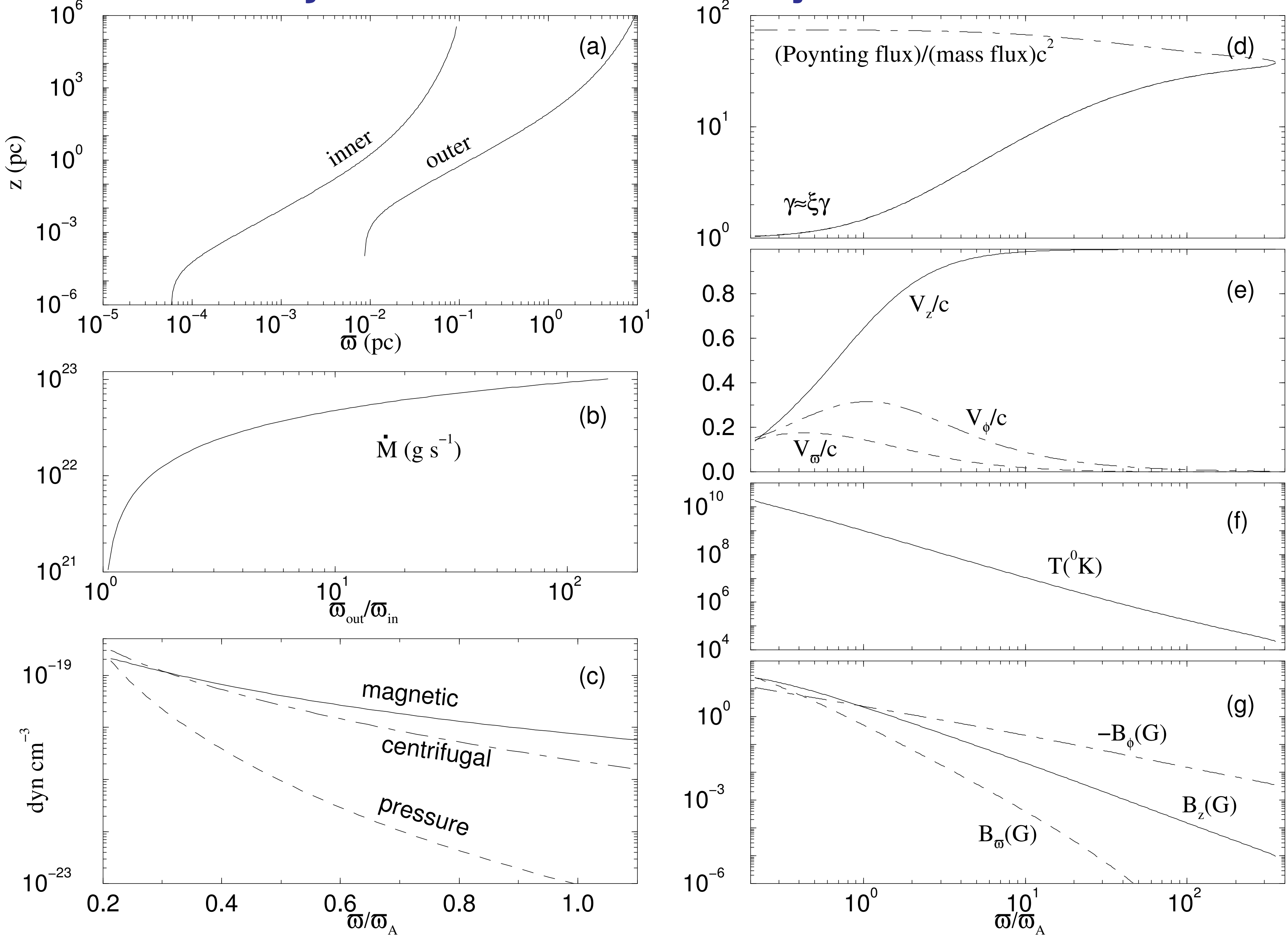
Combining the above, we get

$$\gamma \sim \frac{z}{\varpi}$$

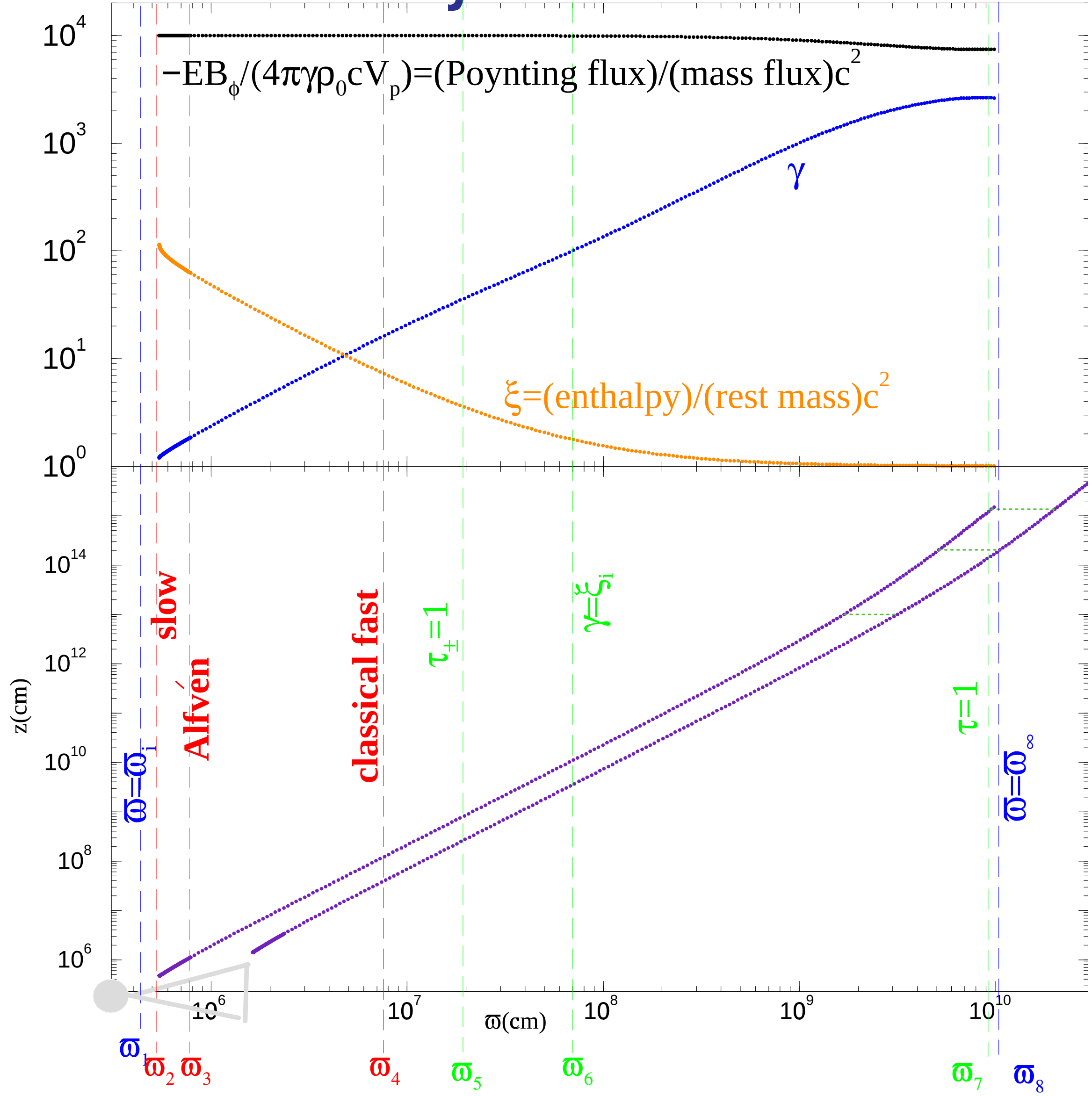
The same from

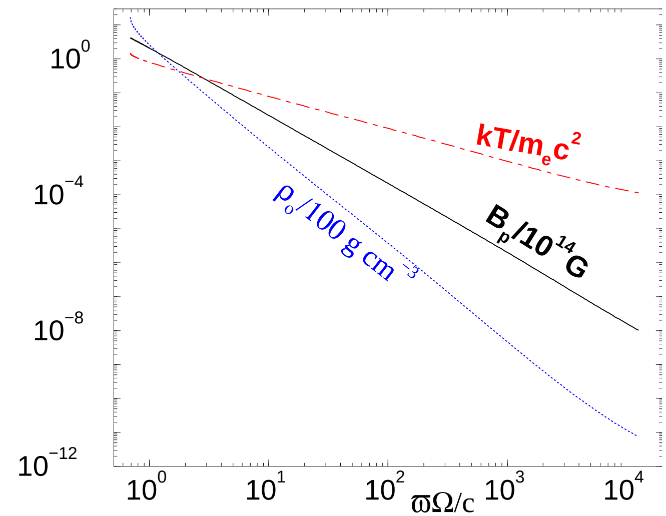
$$(t =) \, \frac{z}{V_z} = \frac{\varpi}{V_\varpi} \Leftrightarrow \frac{z}{c} = \frac{\varpi}{\sqrt{c^2 - V_z^2}} \approx \frac{\varpi}{c/\gamma}$$

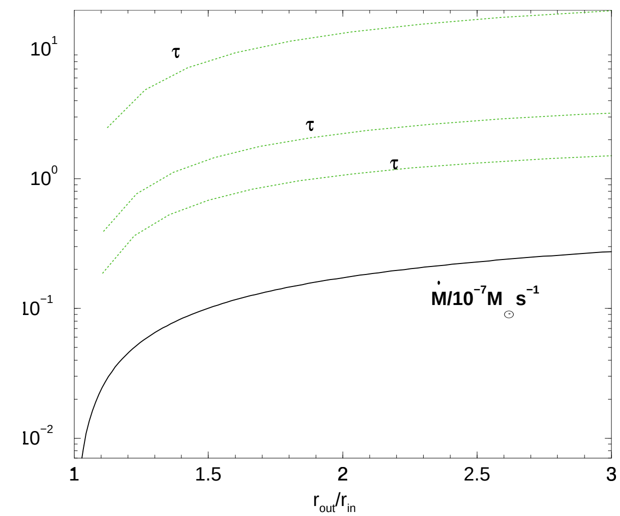
Semi-analytic solutions for AGN jets (Vlahakis & Königl 2004)



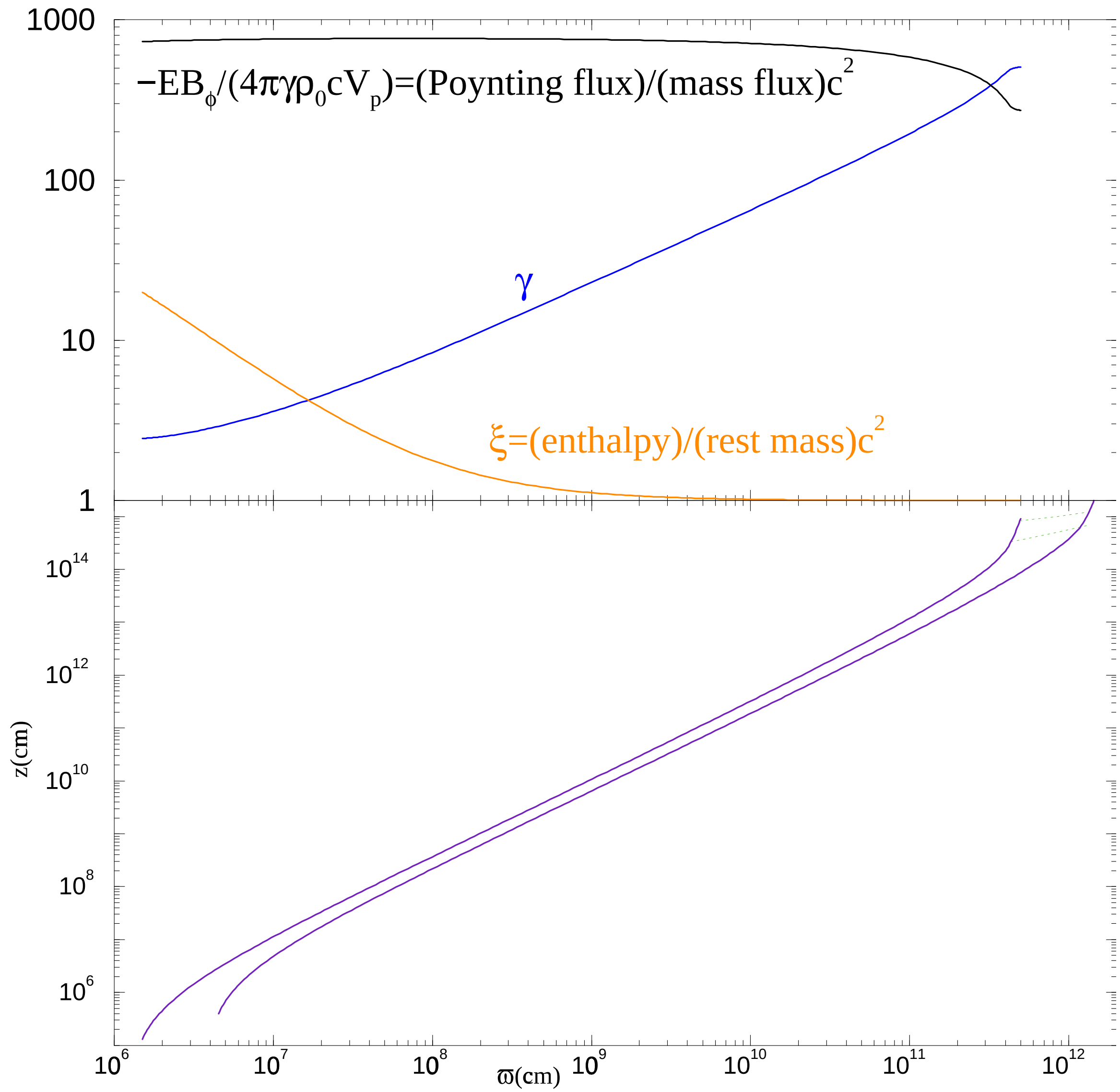
Semi-analytic solutions for GRB Jets (NV & Königl 2001, 2003a,b)

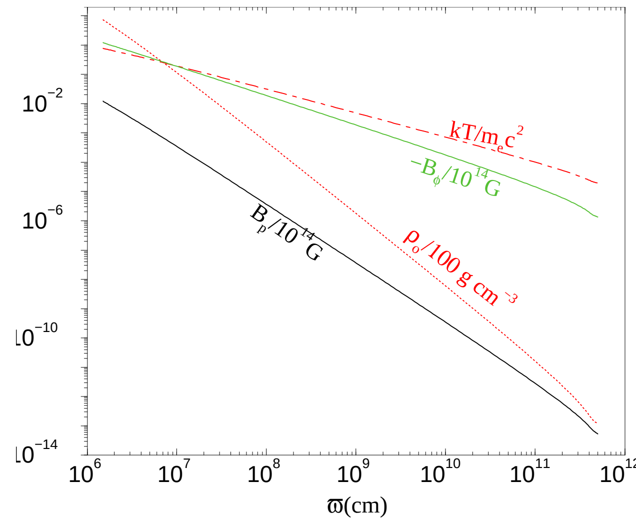


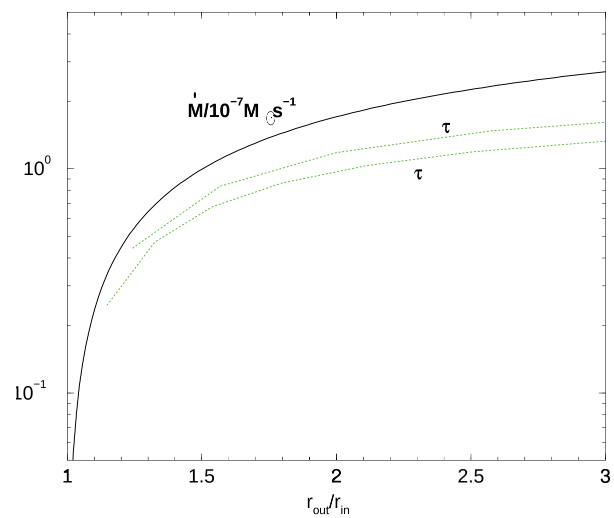




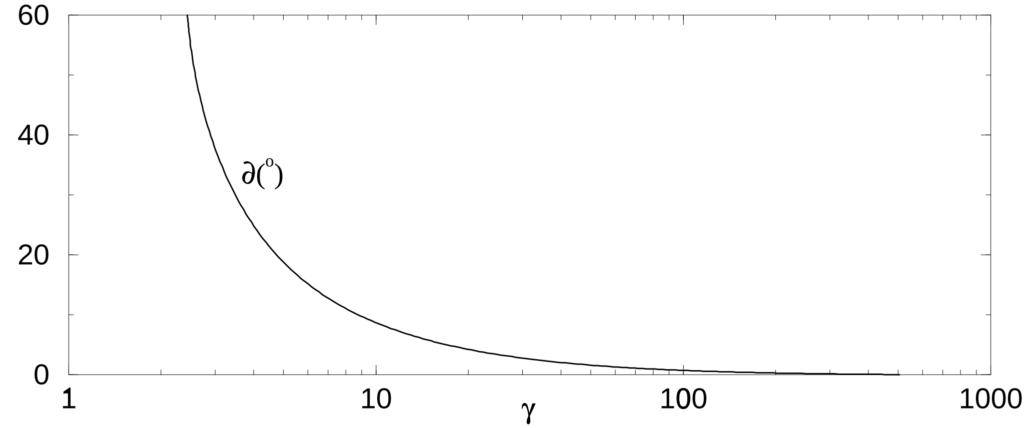
- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

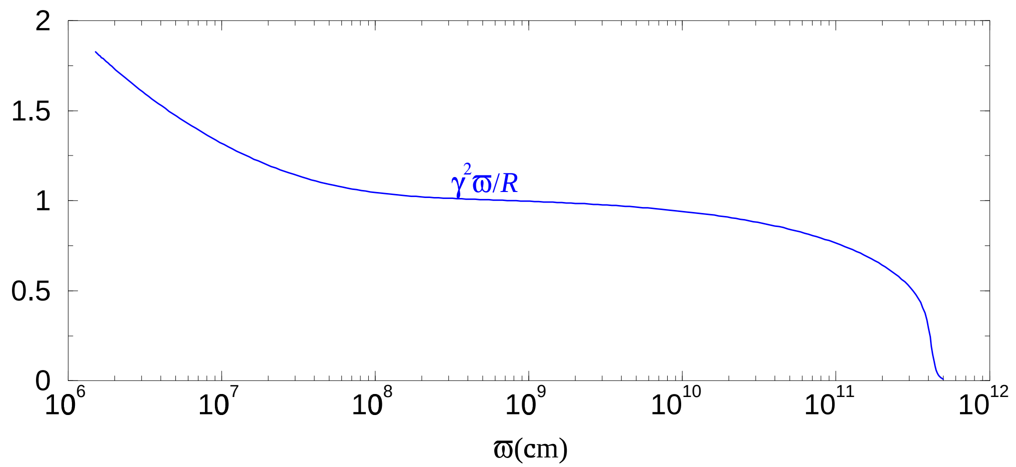






- Thermal acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$, $T \propto \varpi^{-0.8}$, $B_\phi \propto \varpi^{-1}$, $z \propto \varpi^{1.5}$)
- Magnetic acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$)
- cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

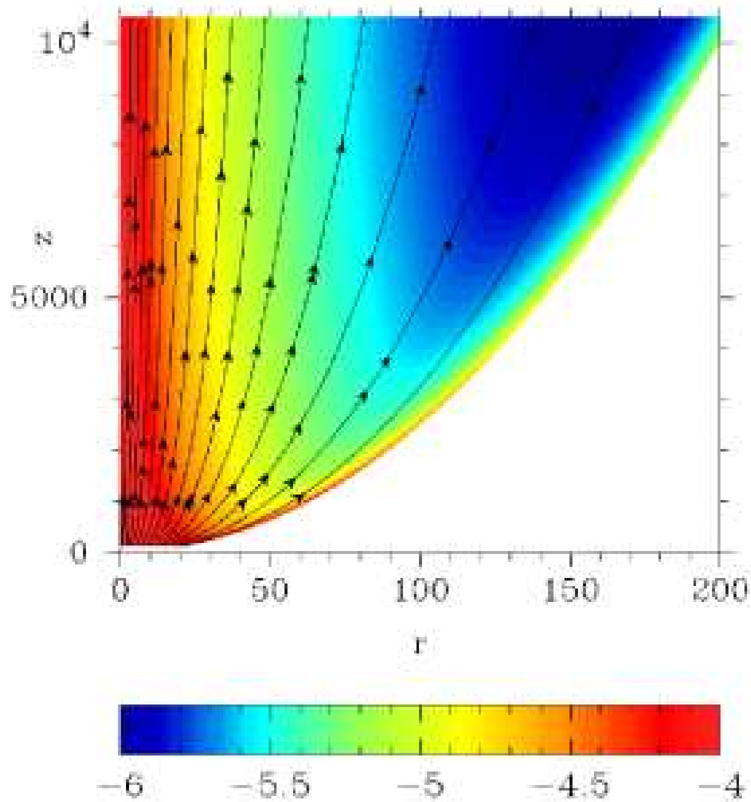


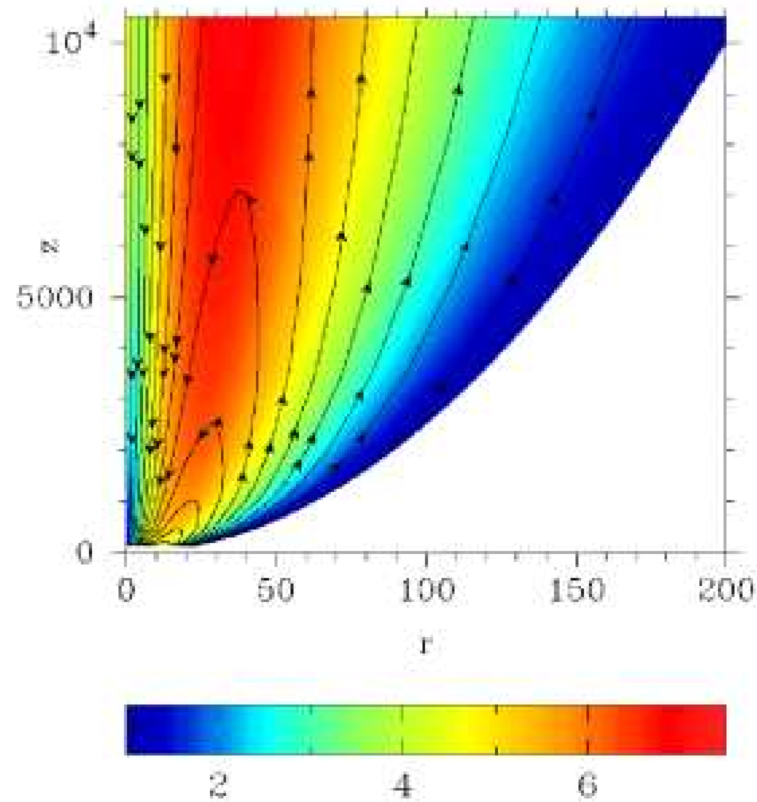


- ⋆ At $\varpi = 10^8$cm – where $\gamma = 10$ – the opening half-angle is already $\vartheta = 10^o$
- ⋆ For $\varpi > 10^8$cm, collimation continues slowly ($\mathcal{R} \sim \gamma^2 \varpi$)

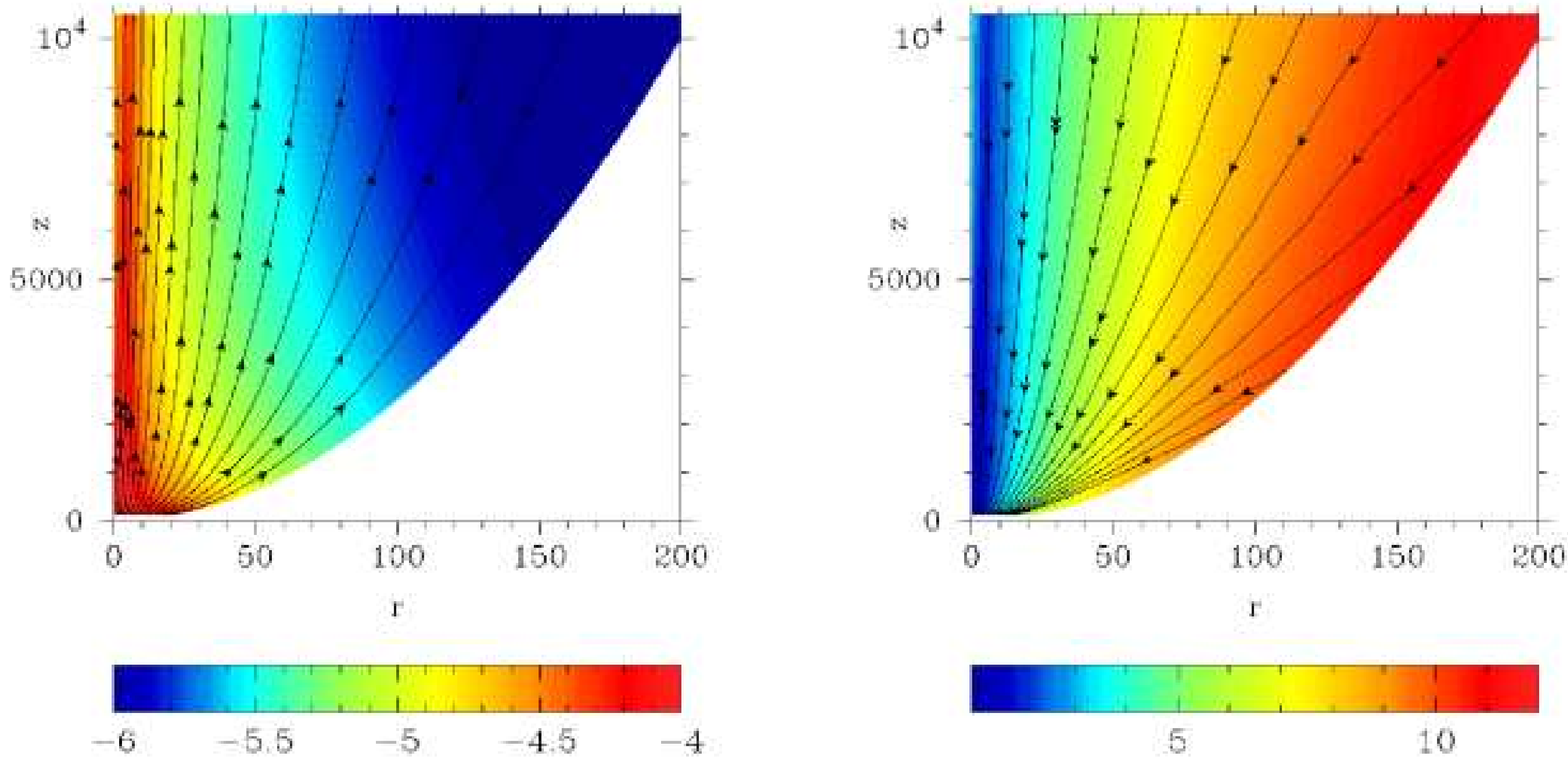
Simulations of relativistic AGN jets

Komissarov, Barkov, Vlahakis, & Königl (2007)

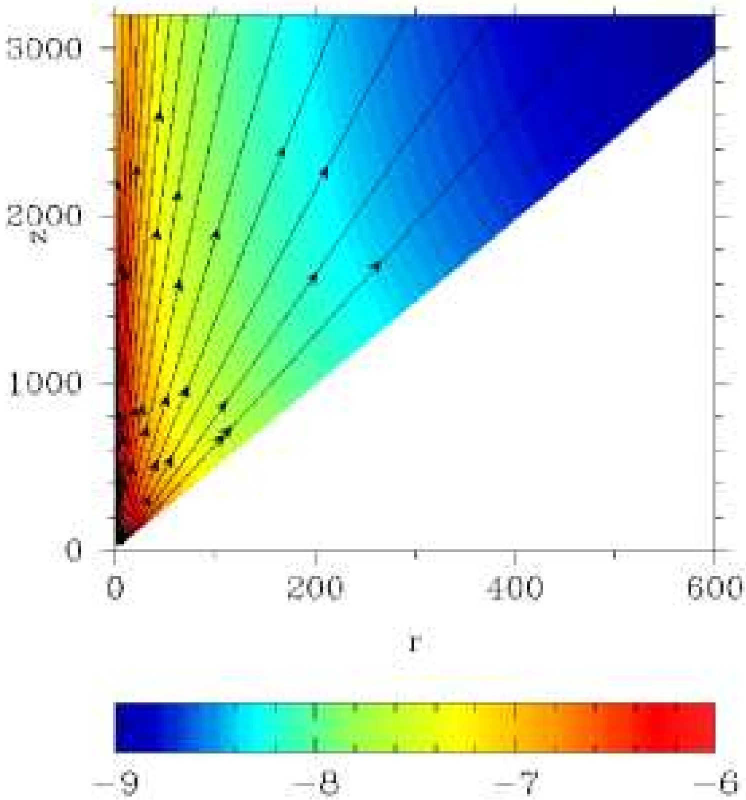


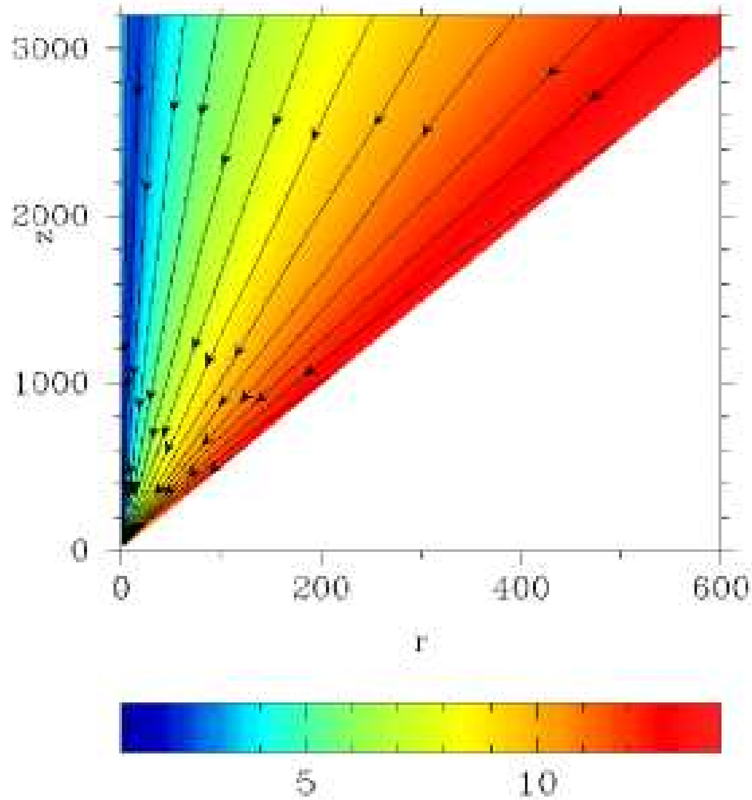


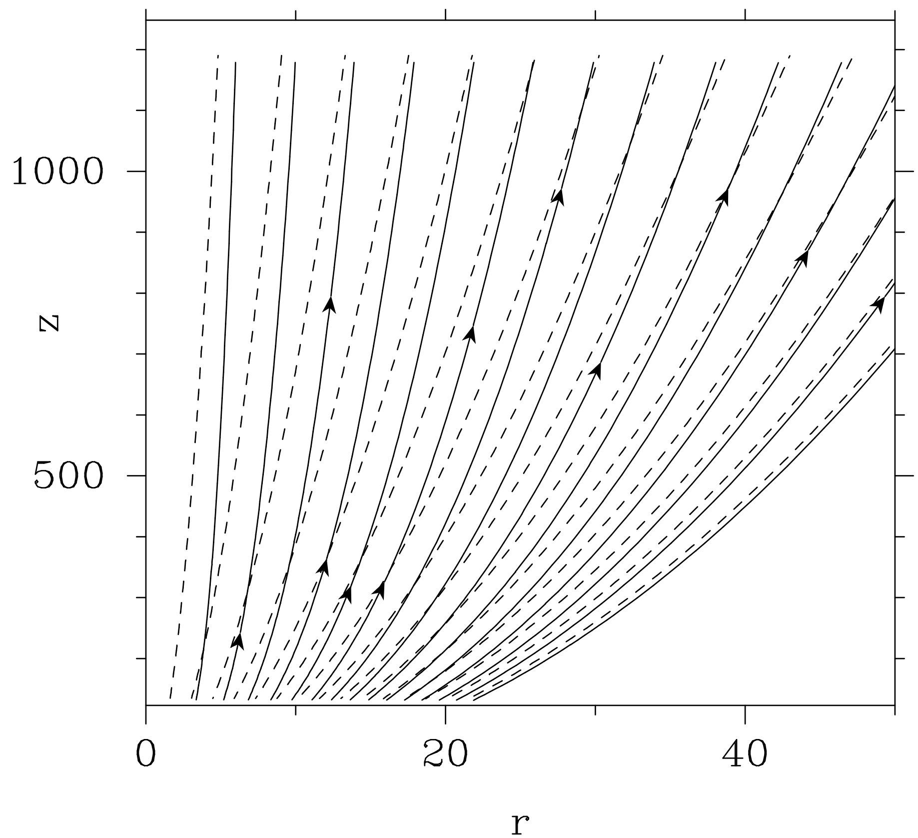
Left panel shows density (colour) and magnetic field lines.
Right panel shows the Lorentz factor (colour) and the current lines.



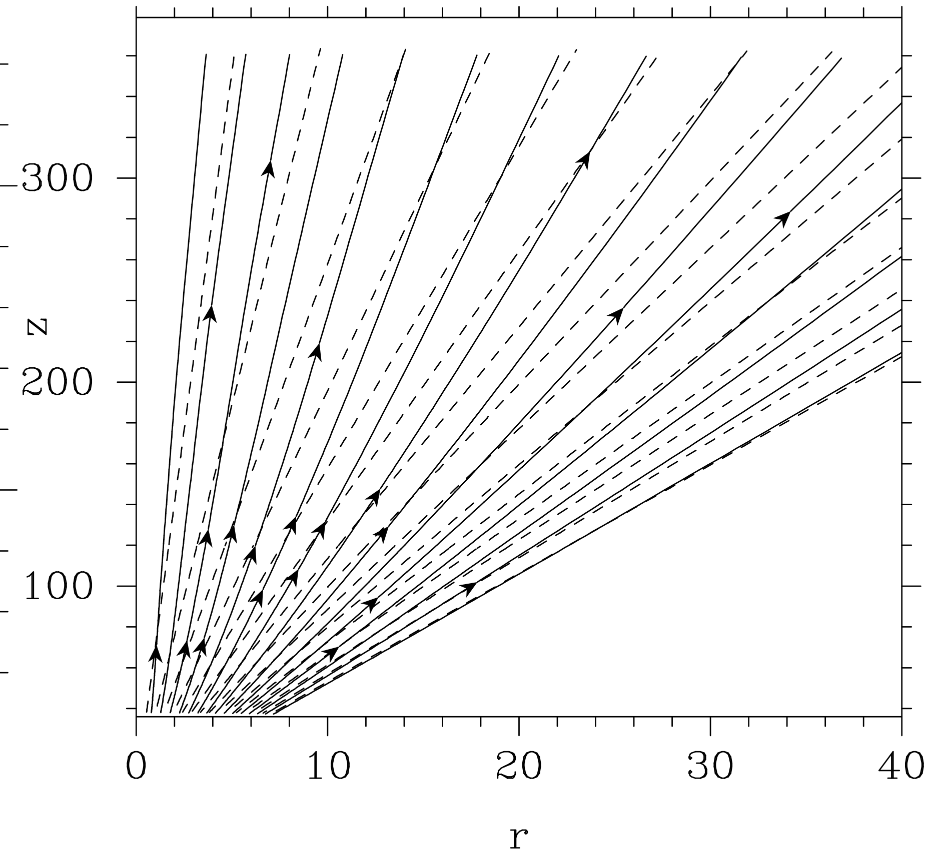
10^4
z
5000
0
0
50
100
150
200
r
−6
−5.5
−5
−4.5
−4
10^4
z
5000
0
0
50
100
150
200
r
5
10


3000
2000
z
1000
0
0
200
400
600
r
−9
−8
−7
−6

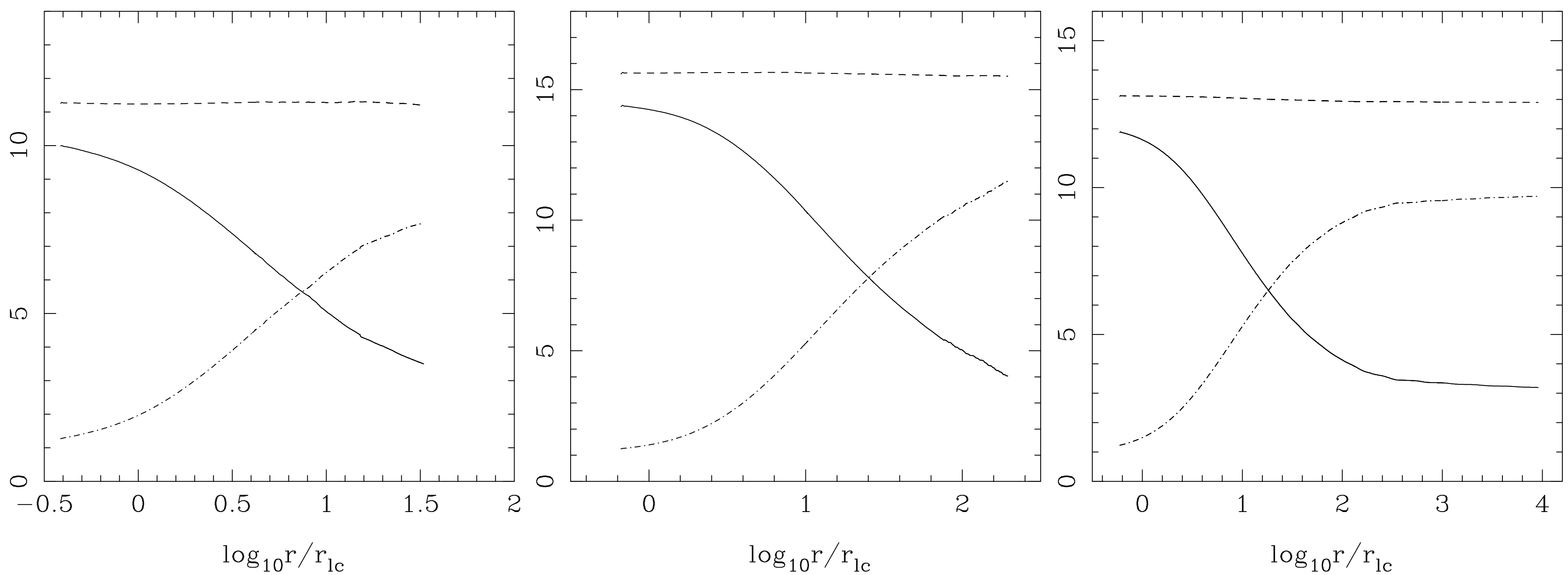

3000
2000
z
1000
0
0
200
400
600
r
5
10



1000
500
z
0
20
40
r

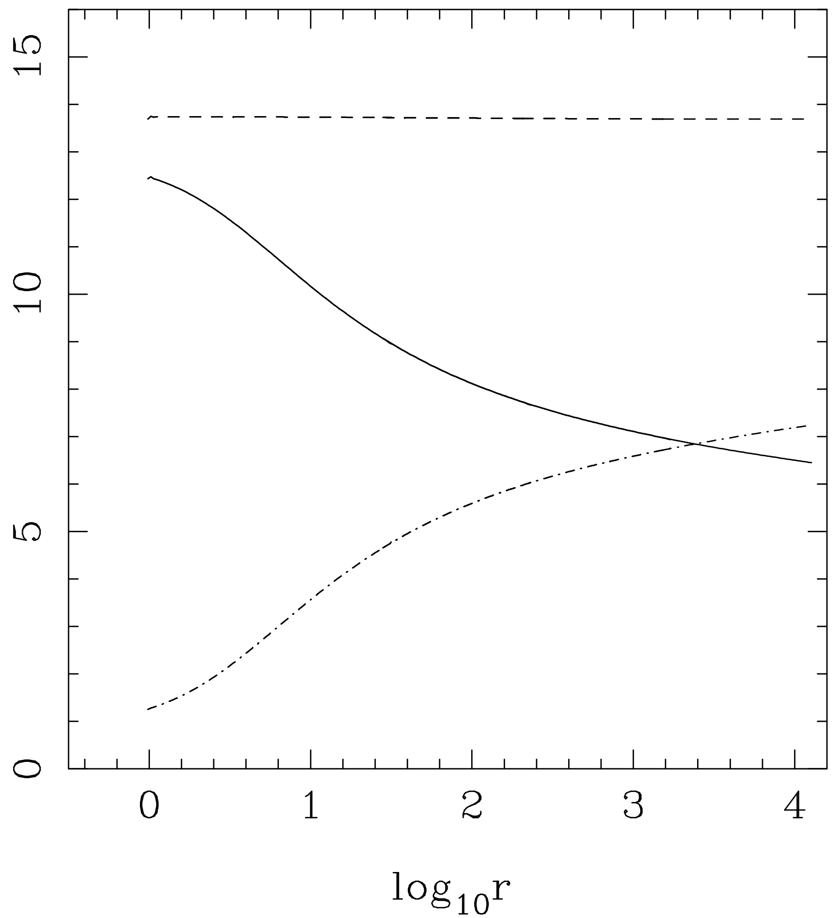


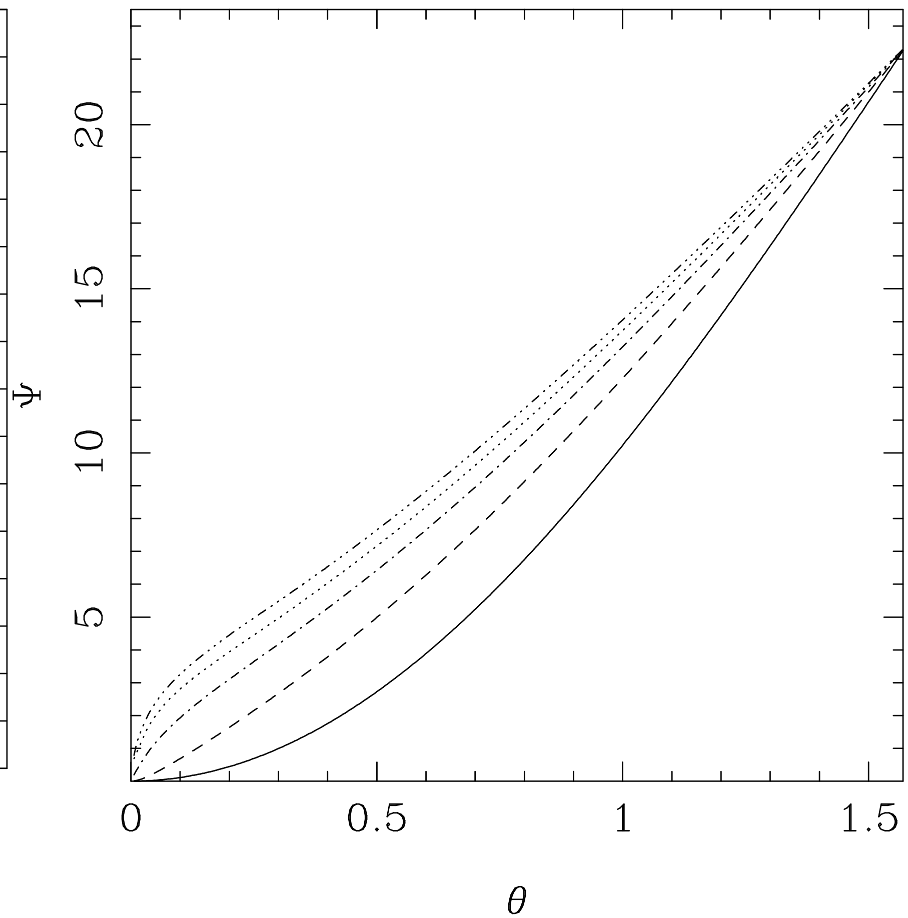
300
200
100
z
0
10
20
30
40
r



$\gamma\sigma$ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius for models C1 (left panel), C2 (middle panel) and A2 (right panel).

(without a wall)



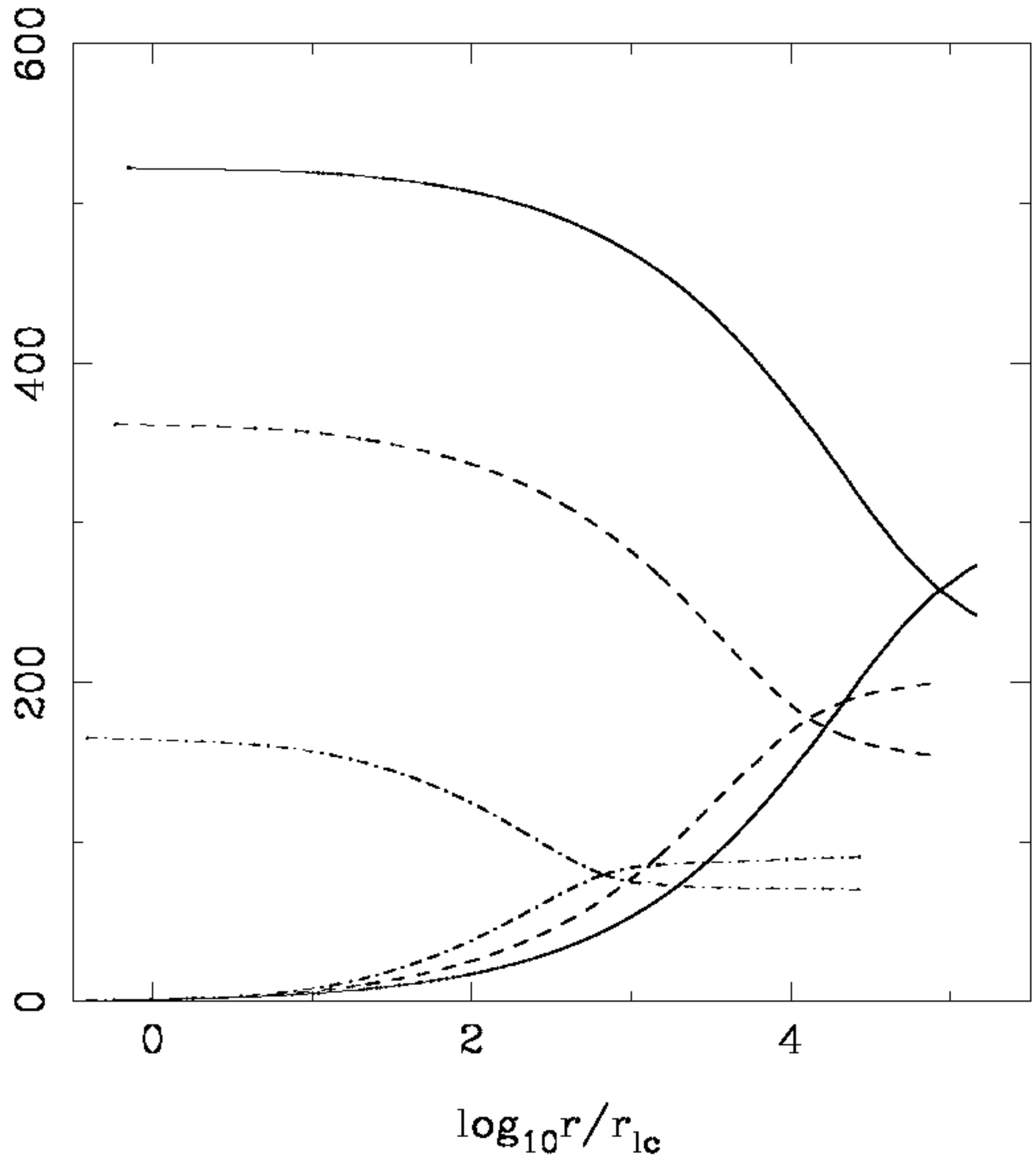


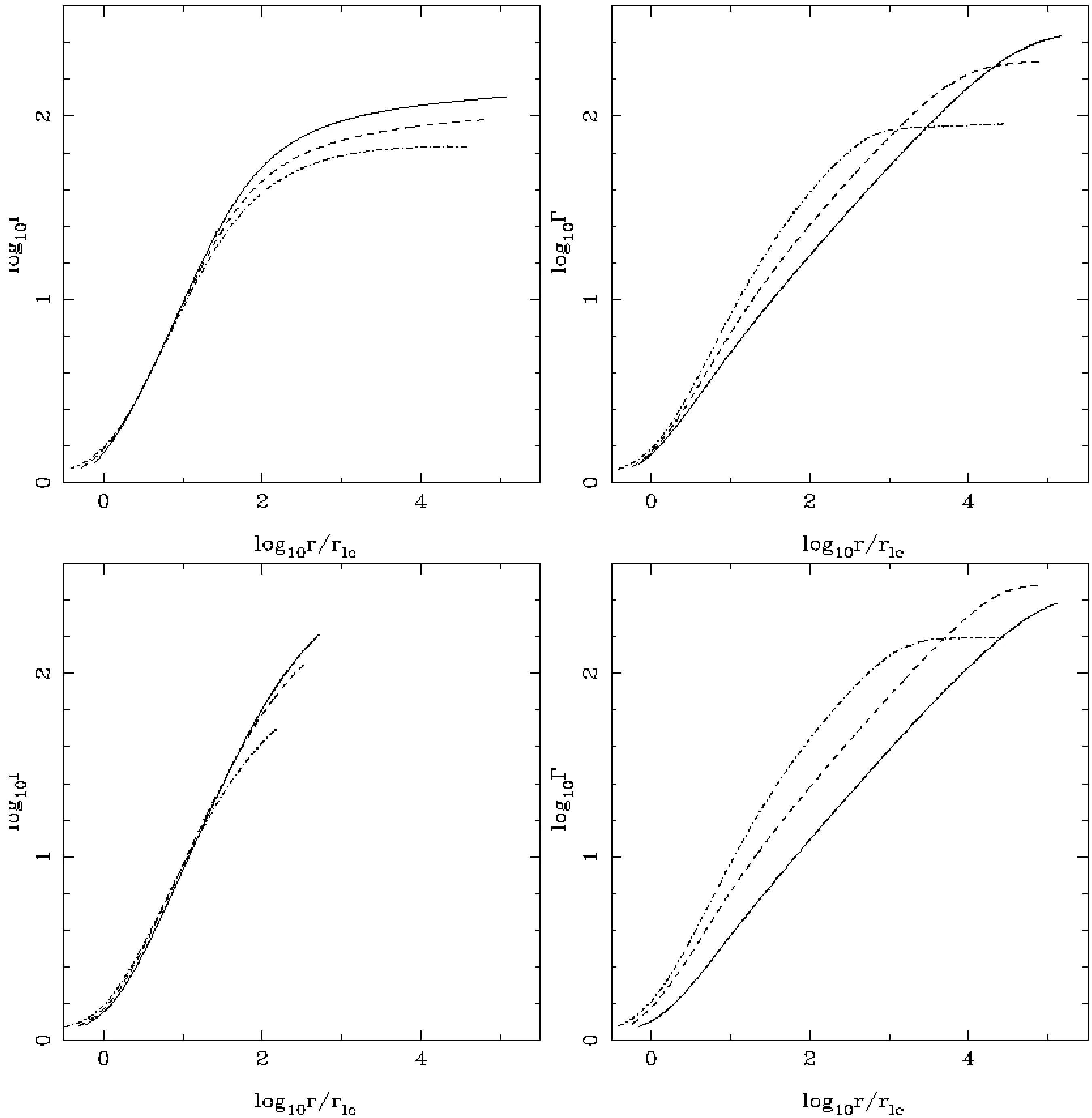
e.g. for $\Psi = 10$, $\vartheta = 57^{\rm o} \to 40^{\rm o}$

while for $\Psi = 5$, $\vartheta = 40^{\rm o} \to 15^{\rm o}$

Simulations of relativistic GRB jets

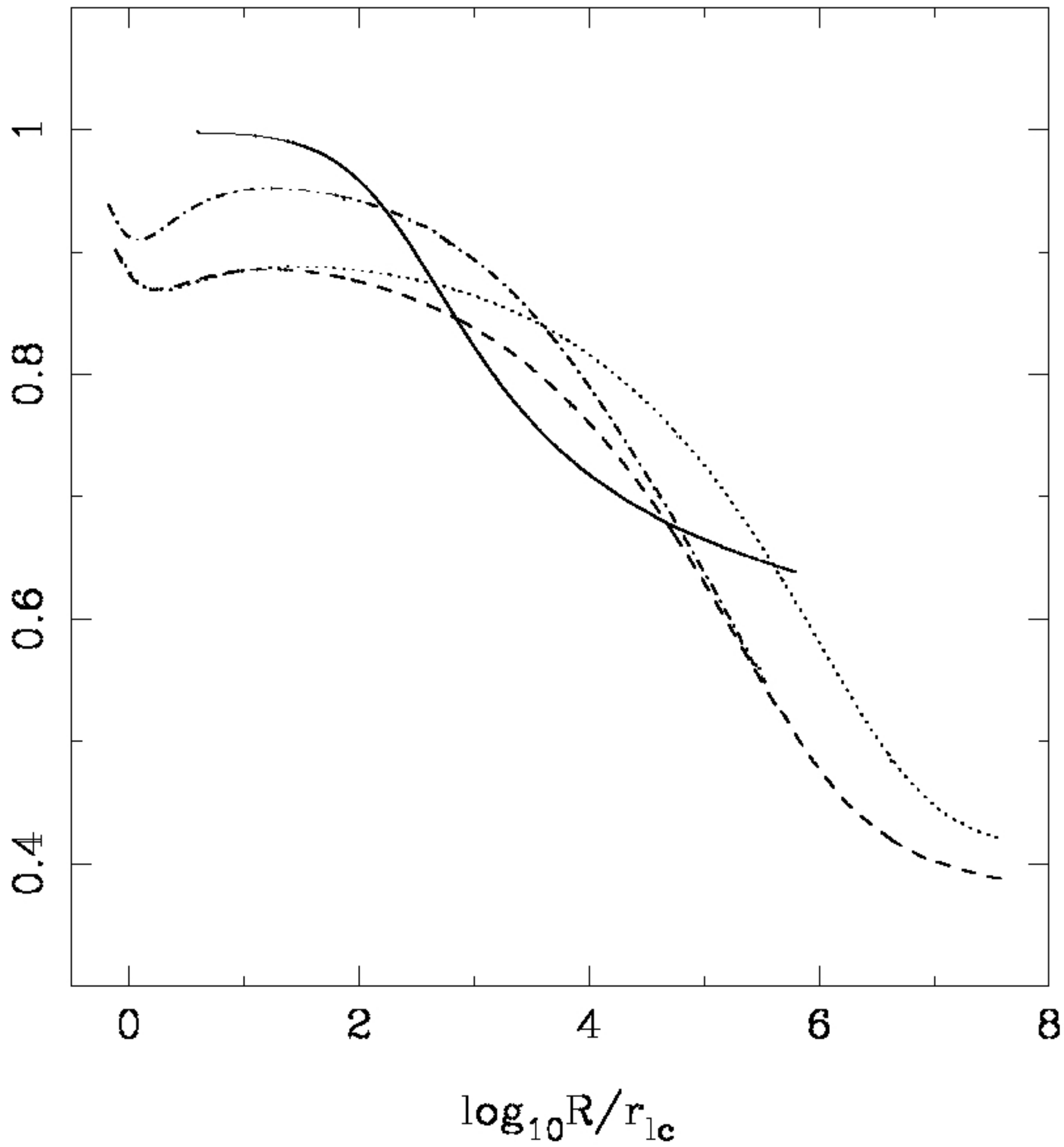
Komissarov, Barkov, Vlahakis, & Königl, in preparation





log10Γ
log10r/rlc
0
1
2
0
2
4

$B_p \varpi^2/(2A)$



Conclusions

⋆ MHD could explain the dynamics of relativistic jets:

- acceleration (the flow is initially thermally, and subsequently magnetically accelerated up to Lorentz factors corresponding to rough equipartition between kinetic and Poynting fluxes) – $\gamma \propto \varpi^{\beta}$
- collimation (parabolic shape consistent with $\gamma \sim z/\varpi \Leftrightarrow z \propto \varpi^{\beta+1}$) agrees with $\mathcal{R} \sim \gamma^2 \varpi$

⋆ The paradigm of MHD jets works in a similar way in YSOs, AGN, GRBs!