MHD modeling of relativistic outflows

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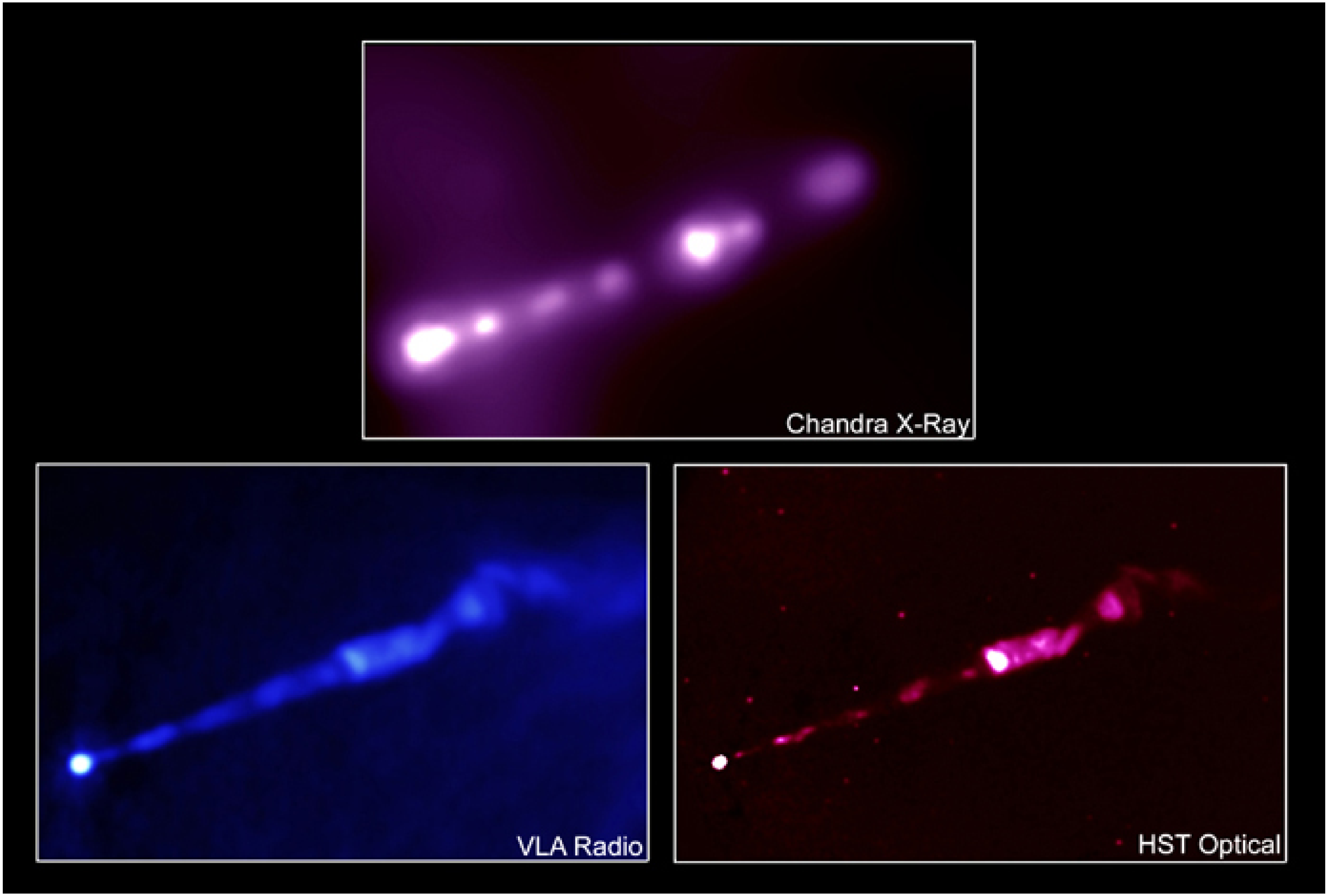
Outline

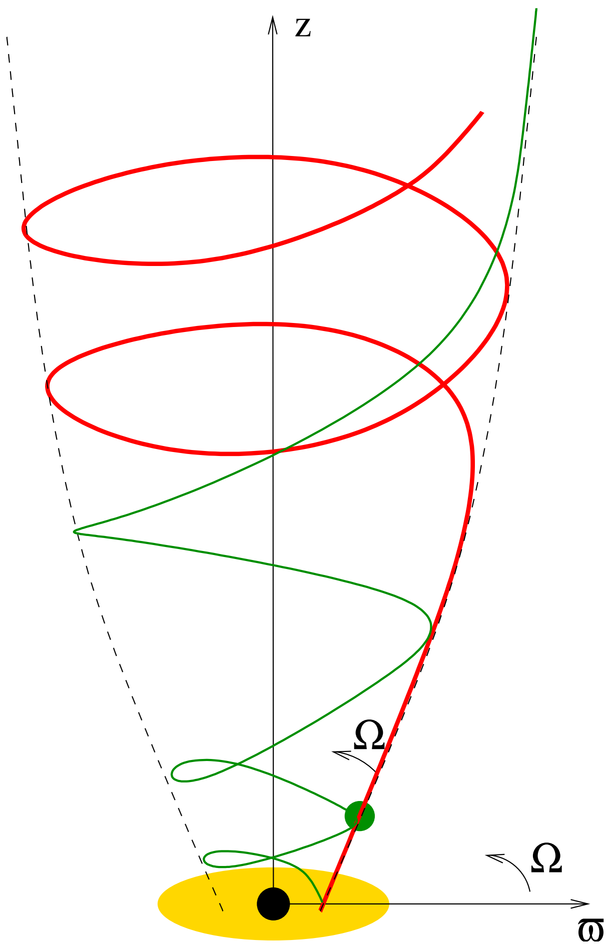
- MHD bulk acceleration and collimation mechanisms
  (general analysis)
- models
  - semi-analytical
  - simulations



collimation at ∼100 Schwarzschild radii, $\gamma_\infty \sim 10$

The question for magnetized outflows

A rotating source (disk or star) creates an axisymmetric outflow



Assume steady-state and ideal magnetohydrodynamics (MHD). Near the source $V_p \ll V_\phi \approx \varpi\Omega$. The energy resides in the electromagnetic field.

- magnetic acceleration $\overset{?}{\rightarrow}$ $\gamma_\infty \sim \mathcal{E}/Mc^2$
- magnetic self-collimation?

Acceleration mechanisms

- thermal (due to ∇P) $\rightarrow$ velocities up to C_s

- magnetocentrifugal (beads on wire - Blandford & Payne)
  $\rightarrow$ velocities up to $\varpi_0 \Omega$

- relativistic thermal (thermal fireball) gives $\gamma \sim \xi_i$,
  where $\xi = \dfrac{\text{enthalpy}}{\text{mass} \times c^2}$.

- magnetic – up to $\gamma_\infty = \mu$? $\left(\text{where } \mu = \dfrac{\frac{dE}{dSdt}}{\frac{dM}{dSdt} c^2} \right)$

The energy integral

All acceleration mechanisms can be seen in the energy conservation equation

$$\mu = \xi \gamma + \frac{\Omega}{\Psi_A c^2} \varpi B_\phi$$

where μ, Ω, Ψ_A(=mass-to-magnetic flux ratio) are constants of motion.

So $\gamma \uparrow$ when $\xi \downarrow$ (thermal, relativistic thermal), or,
$\varpi B_\phi \downarrow \Leftrightarrow I_p \downarrow$ (magnetocentrifugal, magnetic).

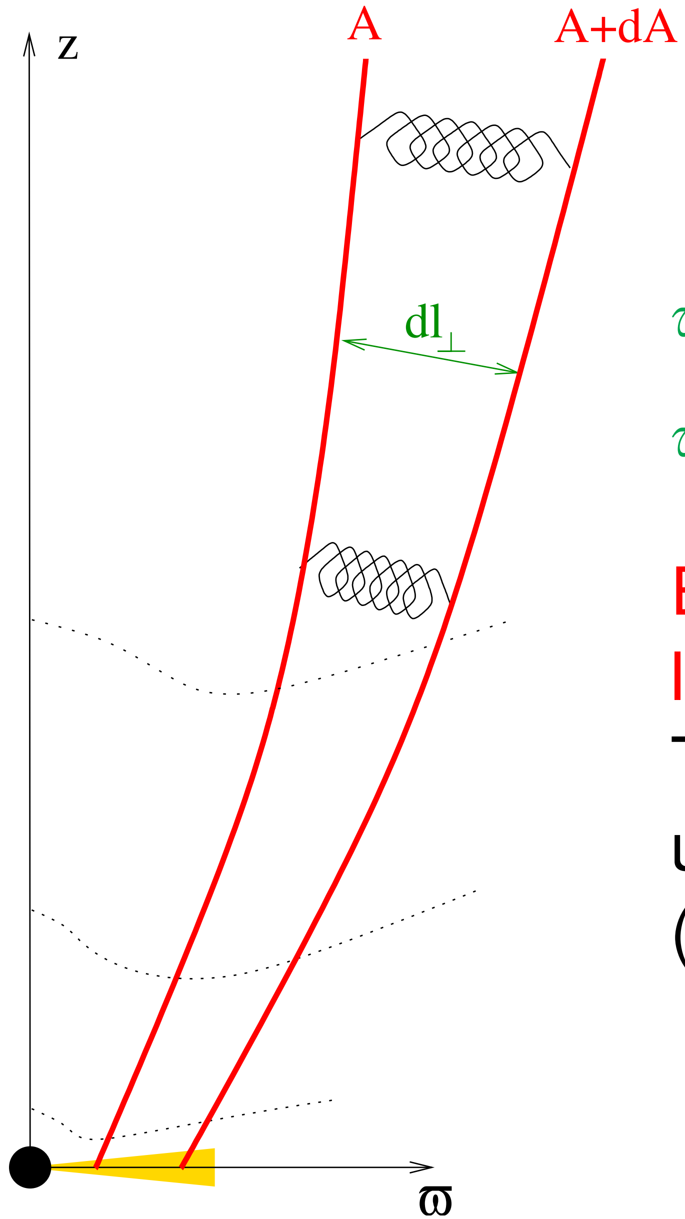
$\gamma_\infty = \mu$ means $\xi = 1$ (its minimum value) and $\varpi B_\phi = 0$.
Is this possible?

Magnetic acceleration *vs* fieldline shape

- From Ferraro's law, $\varpi B_\phi \approx \varpi^2 B_p \Omega / V_p$.
  So, the transfield force-balance determines the acceleration; we are not free to assume a fieldline shape.

- Since $\varpi B_\phi \downarrow \rightarrow$ acceleration,
  $\varpi^2 B_p \downarrow$, or, sufficiently fast expansion $\rightarrow$ acceleration.

- Magnetic flux conservation
  $\frac{1}{2\pi} \iint \boldsymbol{B} \cdot d\boldsymbol{S} = A =$ constant along the flow $\rightarrow$
  lower limit in the asymptotic value of $\varpi^2 B_p \rightarrow$
  acceleration efficiency $< 100\%$.



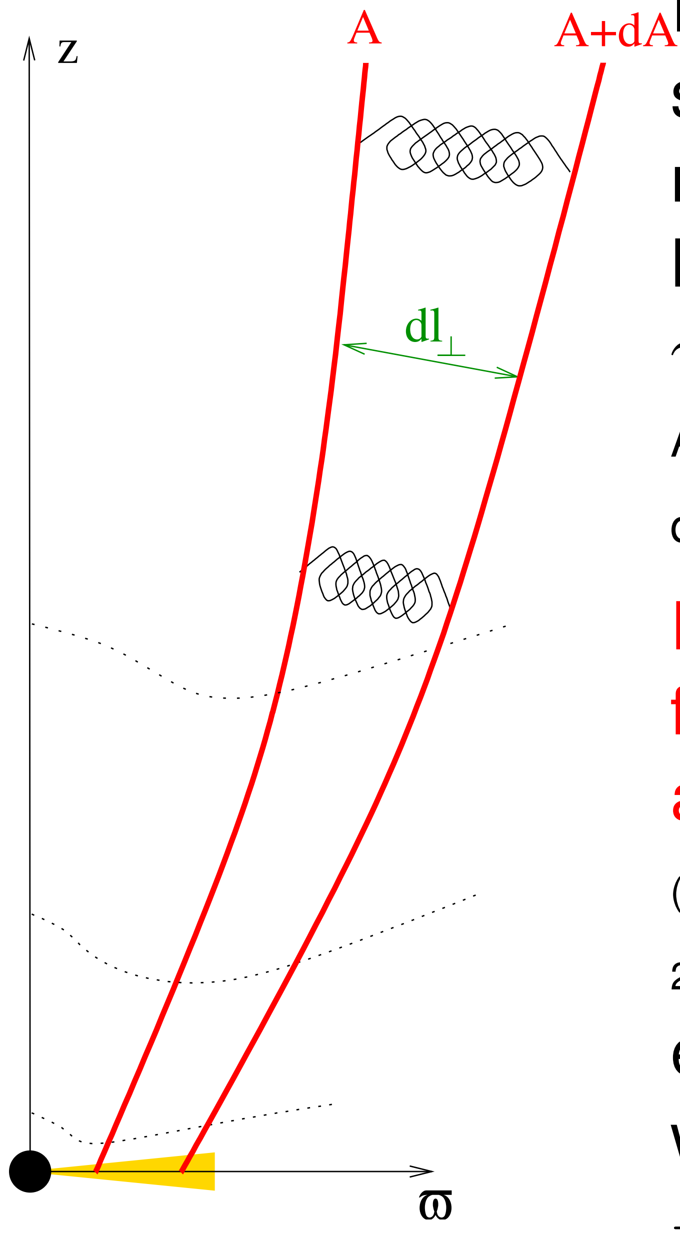
$\varpi B_\phi \downarrow$ for decreasing

$$\varpi^2 B_p = \frac{\varpi^2}{2\pi\varpi dl_\perp}\underbrace{(B_p dS)}_{dA} \propto \frac{\varpi}{dl_\perp}.$$

Expansion with increasing $dl_\perp/\varpi$ leads to acceleration.

The expansion ends in a more-or-less uniform distribution $\varpi^2 B_p \approx A$ (in a quasi-monopolar shape).

Conclusions on the magnetic acceleration



If we assume a quasi-monopolar shape throughout the flow $\rightarrow$ no acceleration. Example: Michel's (1969) solution which gives $\gamma_\infty \approx \mu^{1/3} \ll \mu$.

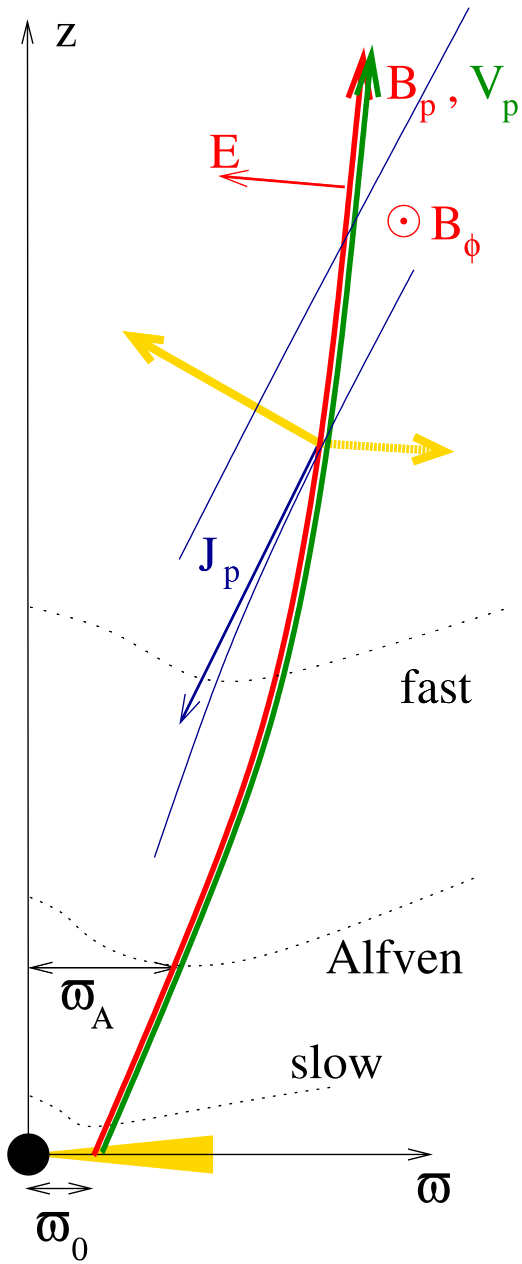
Also Beskin et al (1998); Bogovalov (2001) who found quasi-monopolar solutions.

For any other (more realistic) initial field distribution we have efficient acceleration!

(details and an analytical estimation of the efficiency in Vlahakis 2004, ApSS 293, 67).

example: if we start with $\varpi^2 B_p / A = 2$ we have asymptotically $\varpi^2 B_p / A = 1$ $\rightarrow 50\%$ efficiency

On the collimation



The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force contributes to the collimation (hoop-stress paradigm).
In relativistic flows the electric force plays an opposite role (a manifestation of the high inertia of the flow).

- surrounding medium may play a role (e.g. a slow external wind, or stellar material in the collapsar model for GRBs)
- self-collimation works (mainly at small distances where the velocities are mildly relativistic)

For $\gamma \gg 1$, curvature radius $\mathcal{R} \sim \gamma^2 \varpi \ (\gg \varpi)$.

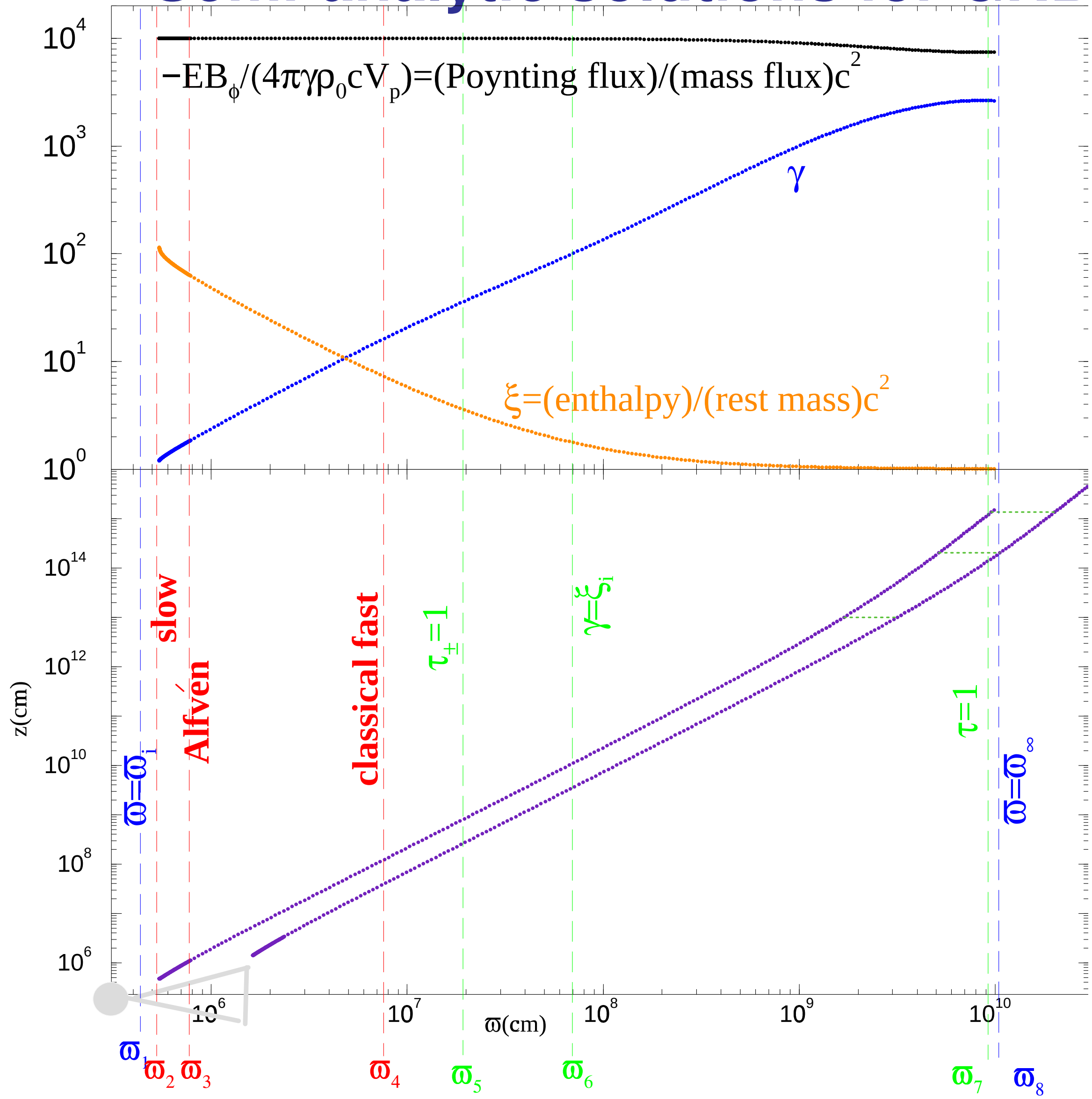
Collimation more difficult, but not impossible!

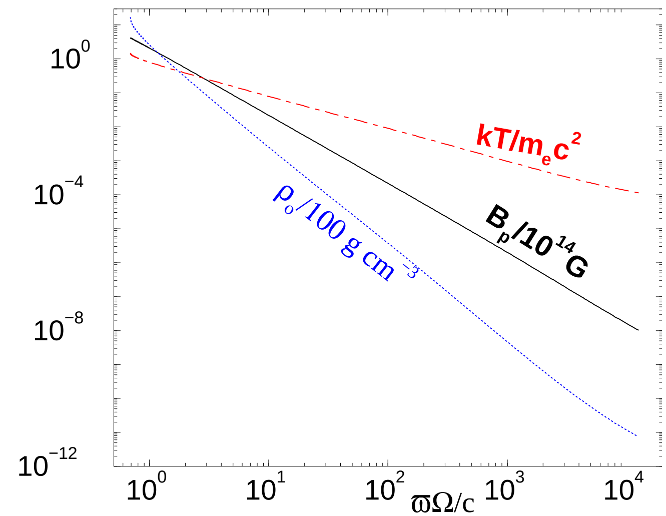
$$\frac{\varpi}{\mathcal{R}} = -\varpi \frac{\partial^2 \varpi}{\partial z^2} \left(\frac{B_z}{B_p} \right)^3 \sim \left(\frac{\varpi}{z} \right)^2$$

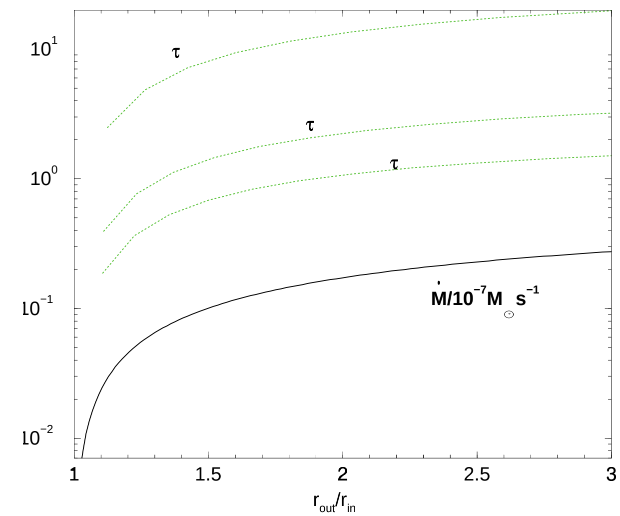
Combining the above, we get $\gamma \sim \dfrac{z}{\varpi}$

The same from $(t =) \ \dfrac{z}{V_z} = \dfrac{\varpi}{V_\varpi} \Leftrightarrow \dfrac{z}{c} = \dfrac{\varpi}{\sqrt{c^2 - V_z^2}} \approx \dfrac{\varpi}{c/\gamma}$

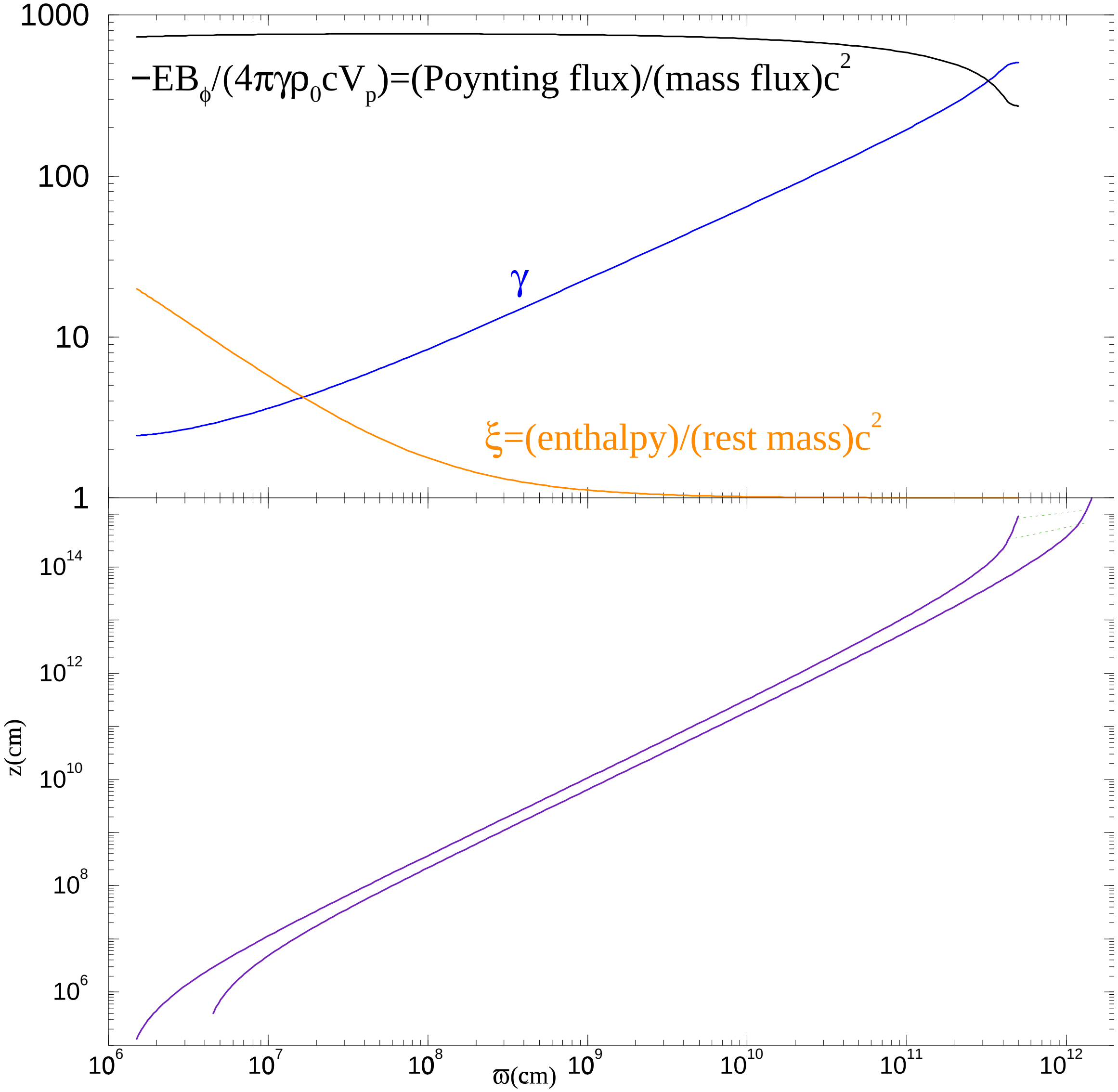
Semi-analytic solutions for GRB Jets (NV & Königl 2001, 2003a,b)

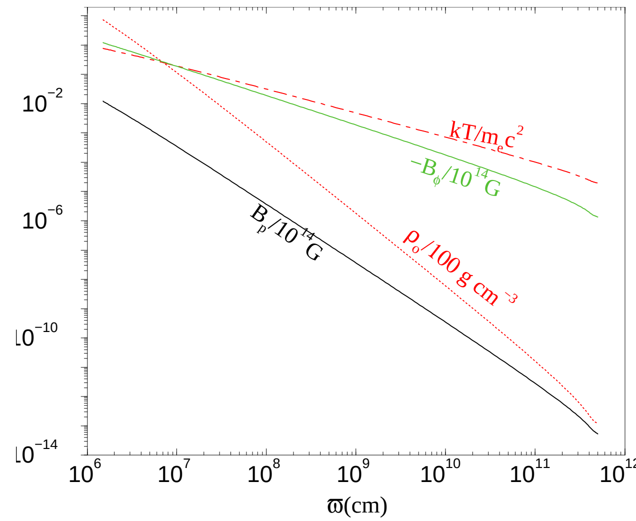


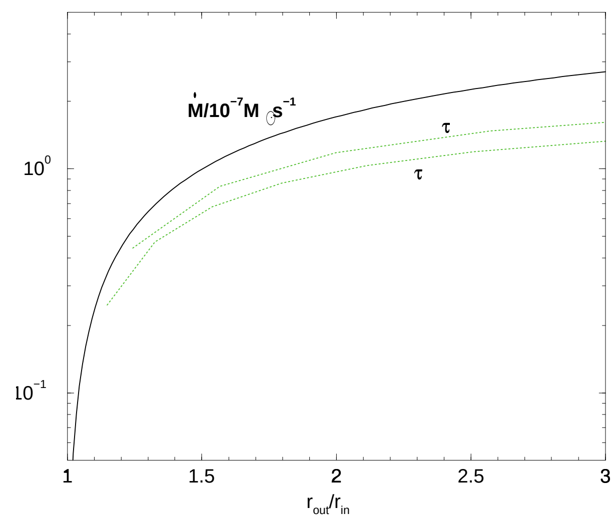




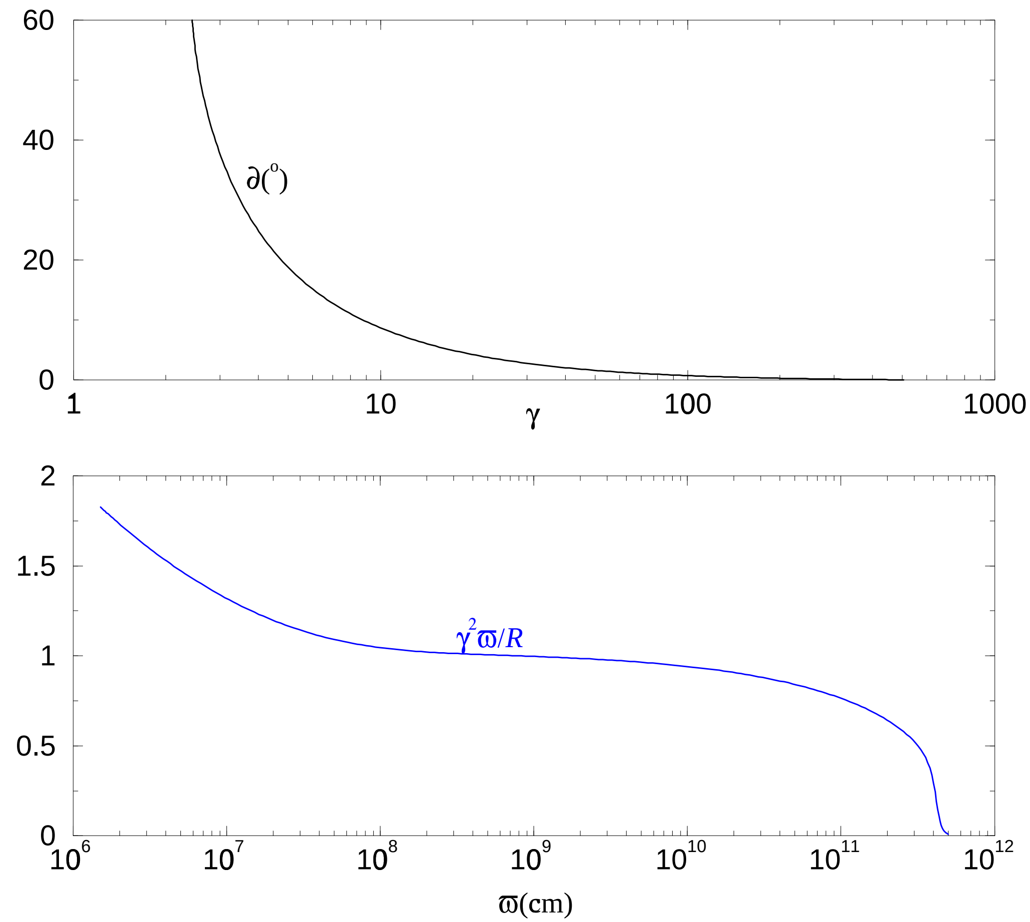
- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$







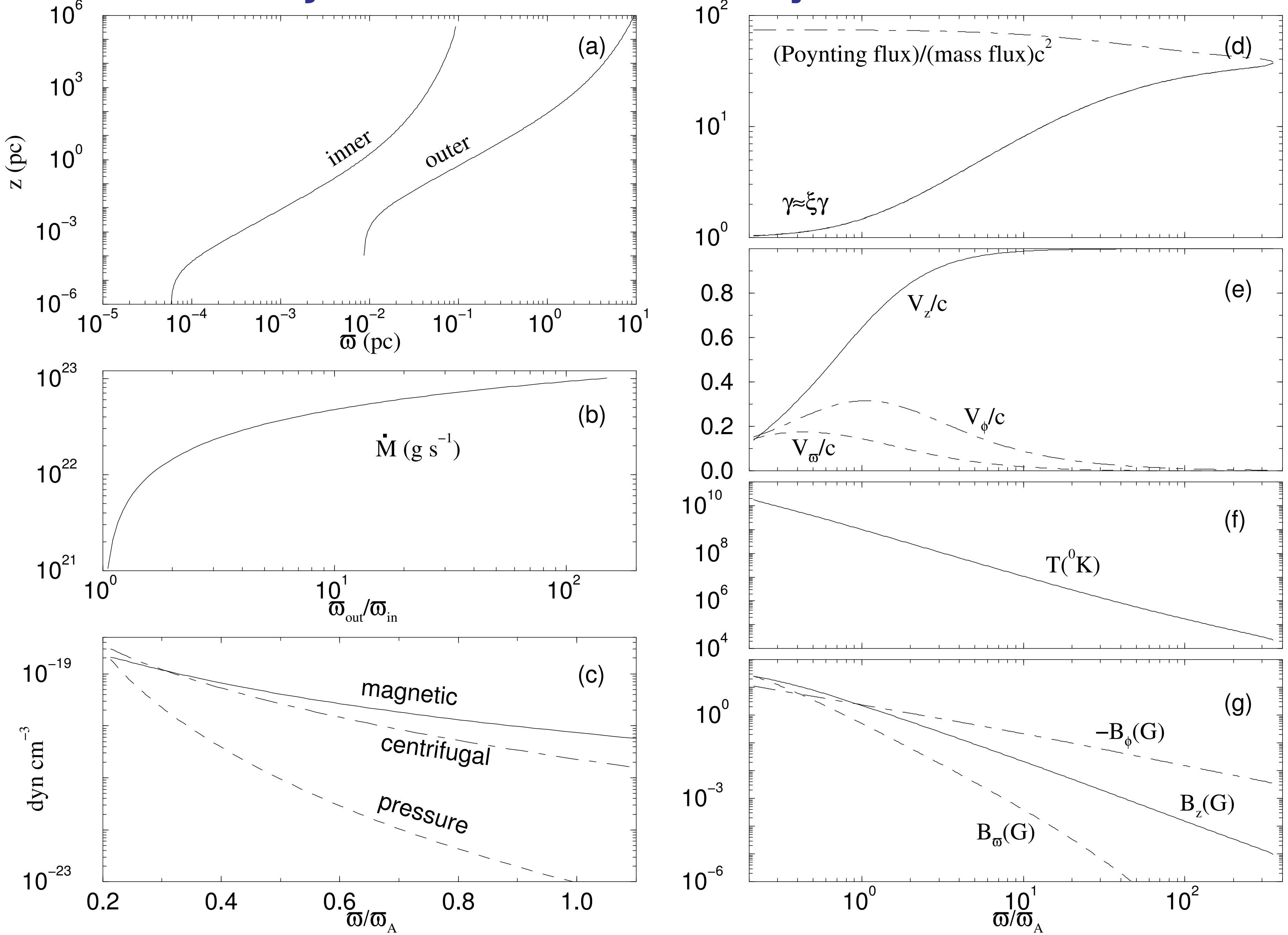
- Thermal acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$, $T \propto \varpi^{-0.8}$, $B_\phi \propto \varpi^{-1}$, $z \propto \varpi^{1.5}$)
- Magnetic acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$)
- cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi / 4\pi\gamma\rho_0 V_p)_\infty$

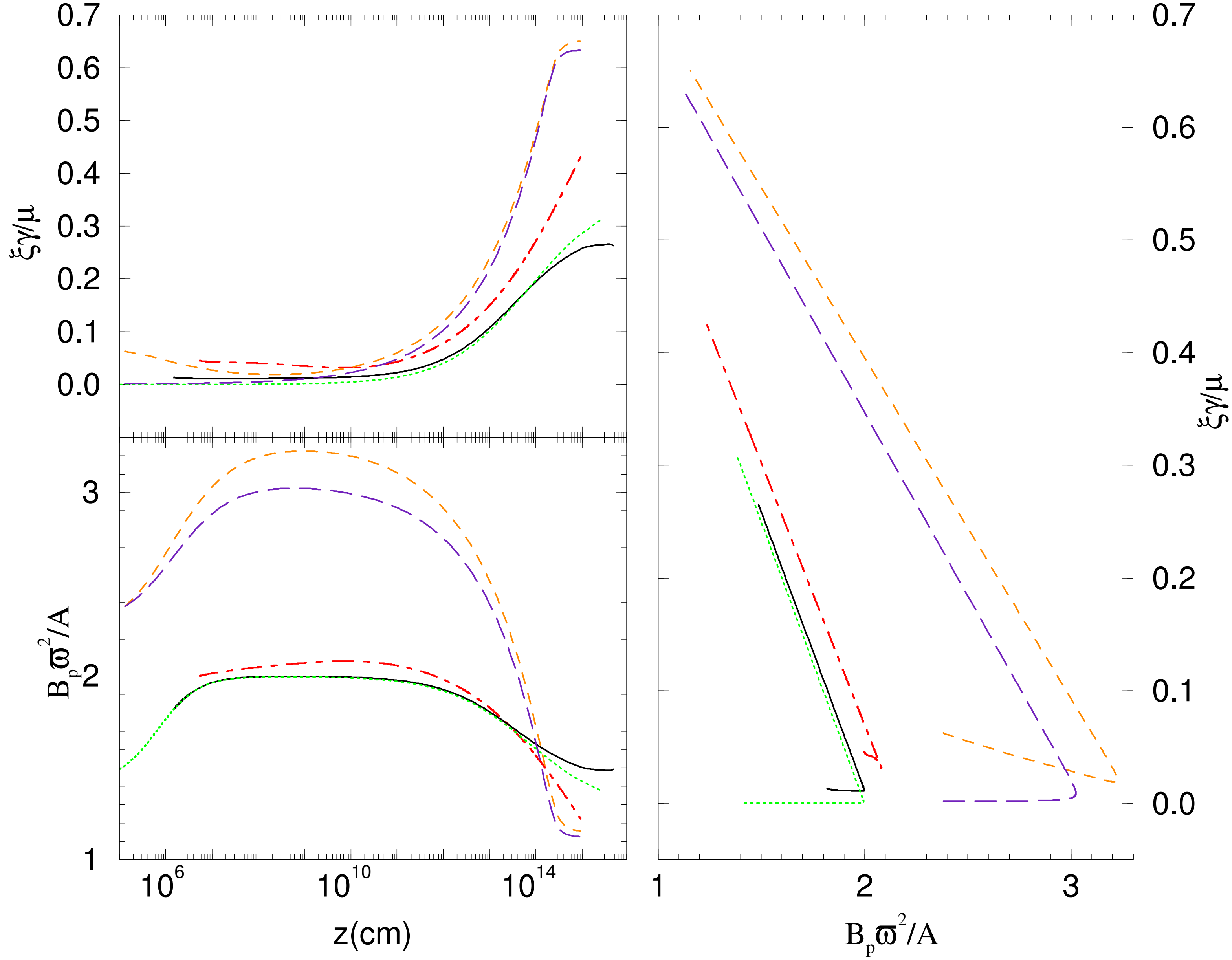


⋆ At $\varpi = 10^8$cm – where $\gamma = 10$ – the opening half-angle is already $\vartheta = 10^o$

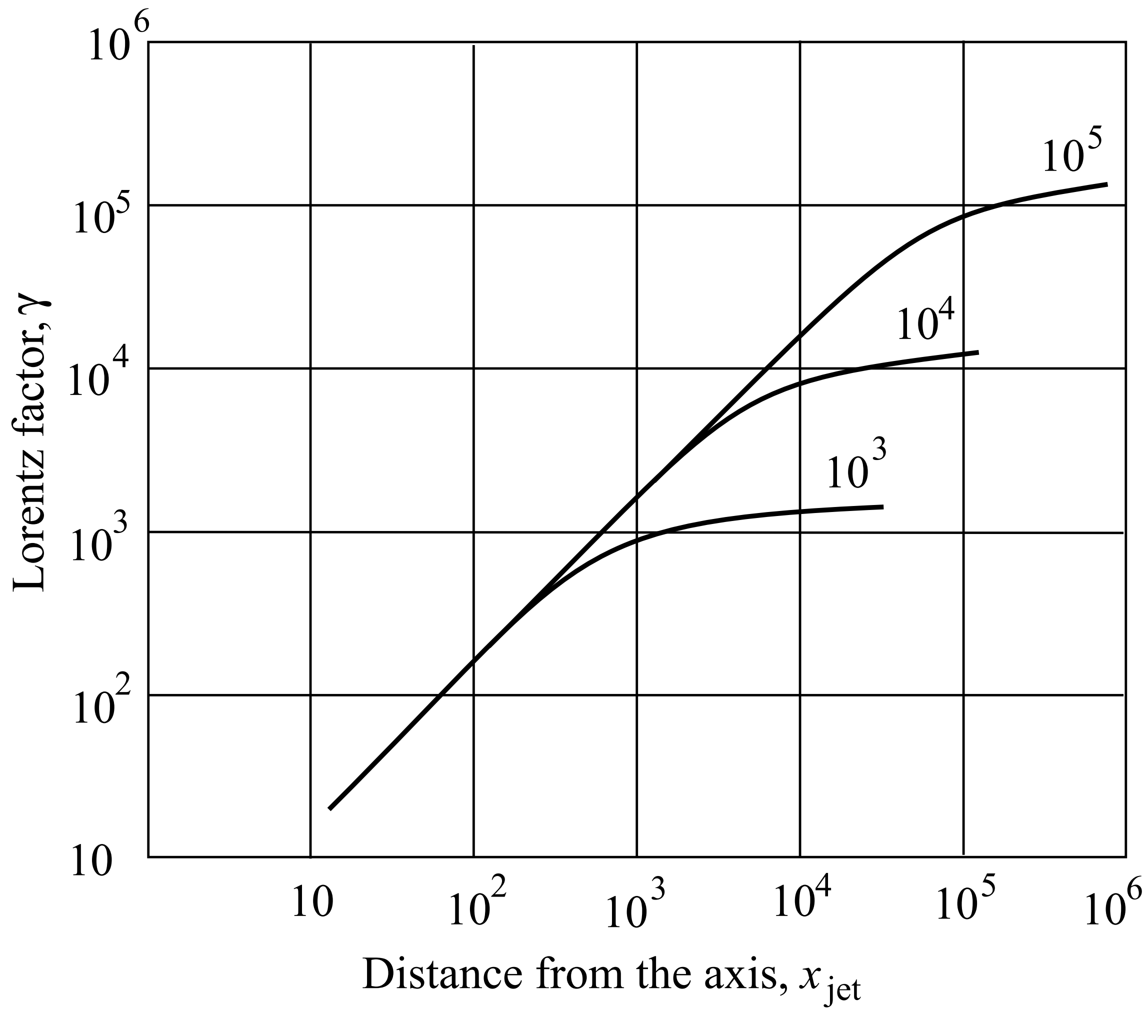
⋆ For $\varpi > 10^8$cm, collimation continues slowly ($\mathcal{R} \sim \gamma^2 \varpi$)

Semi-analytic solutions for AGN jets (Vlahakis & Königl 2004)




0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
ξγ/μ
3
2
1
$B_p\varpi^2/A$
10^6
10^{10}
10^{14}
z(cm)
1
2
3
$B_p\varpi^2/A$
ξγ/μ

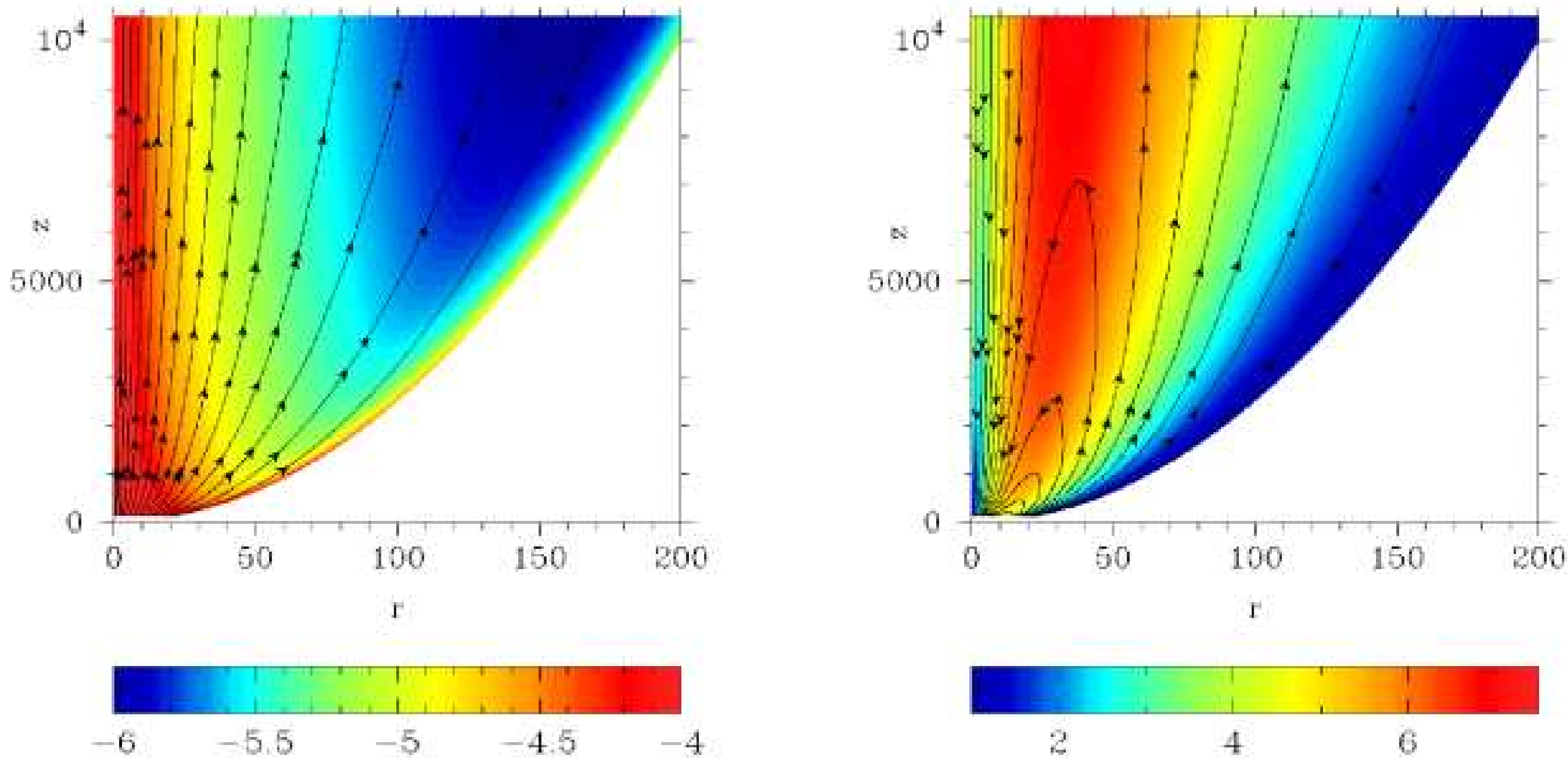
Beskin & Nokhrina (2006)



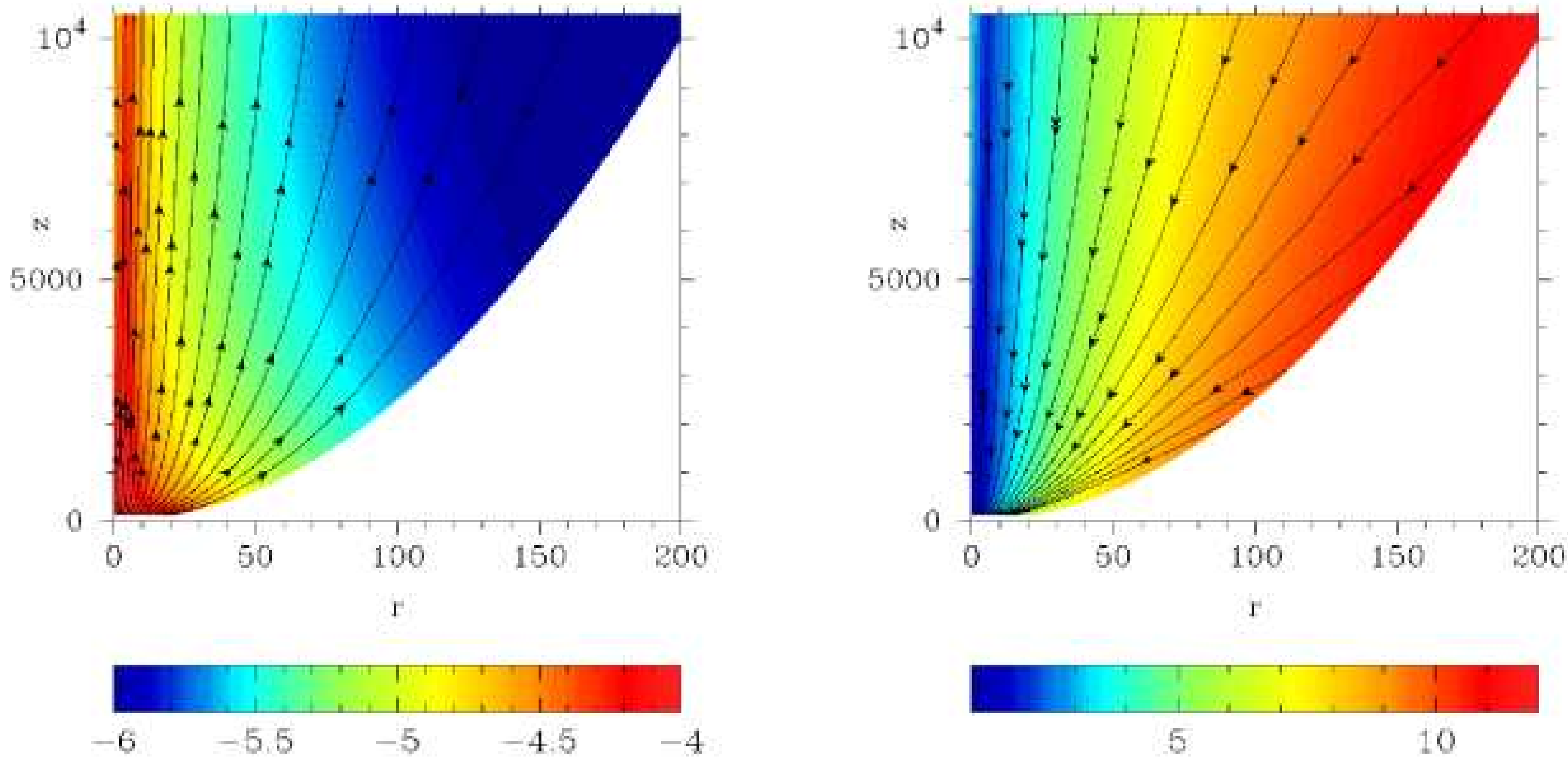
By expanding the equations wrt $2/\mu$ (their $1/\sigma$) they examine a flow with parabolic $z \propto \varpi^2$ shape. The acceleration is efficient, reaching $\gamma_\infty \sim \mu$. The scaling $\gamma \propto \varpi$ is the same as in Vlahakis & Königl (2003a), and in agreement with $\gamma \propto z/\varpi$.

Simulations of relativistic AGN jets

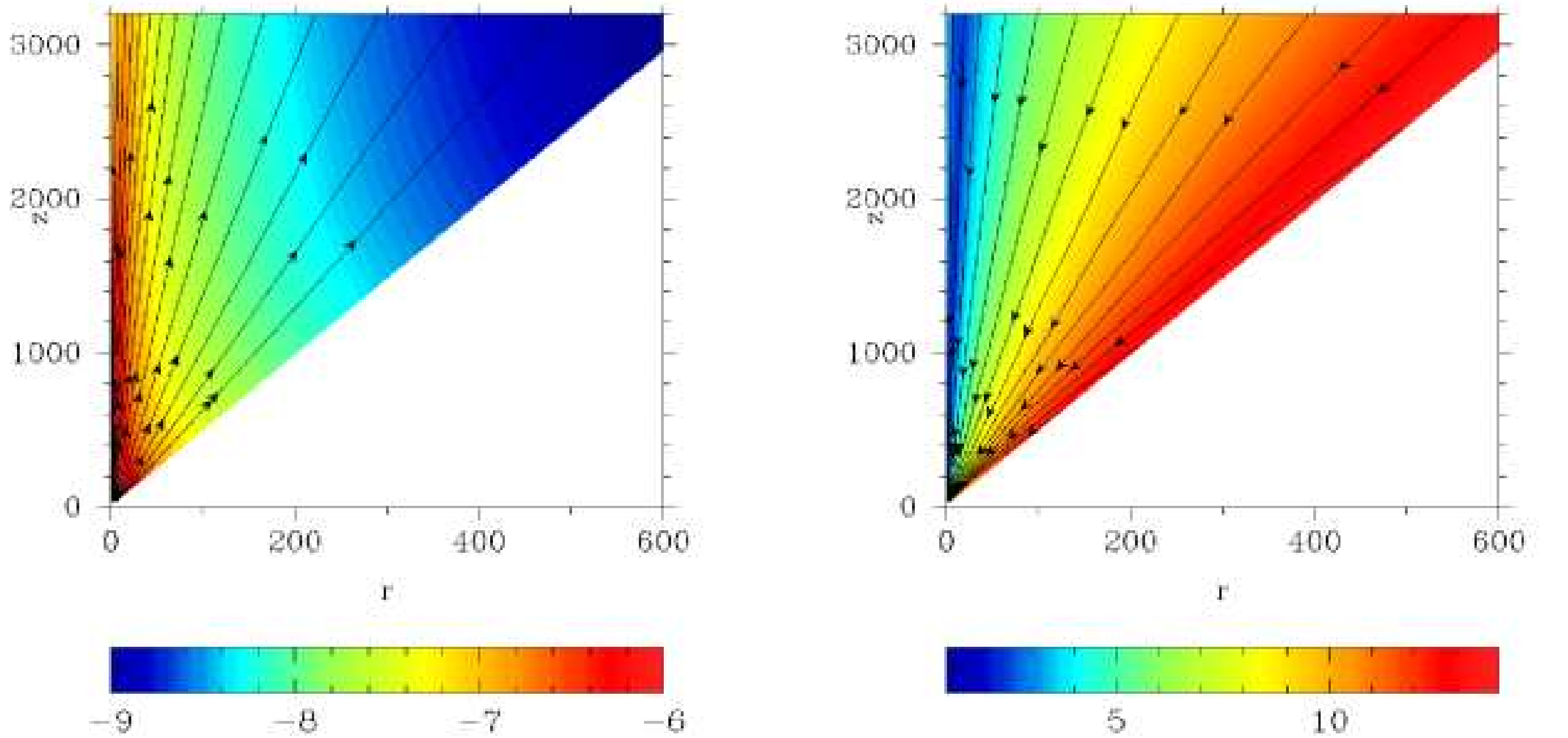
Komissarov, Barkov, Vlahakis, & Königl (2007)

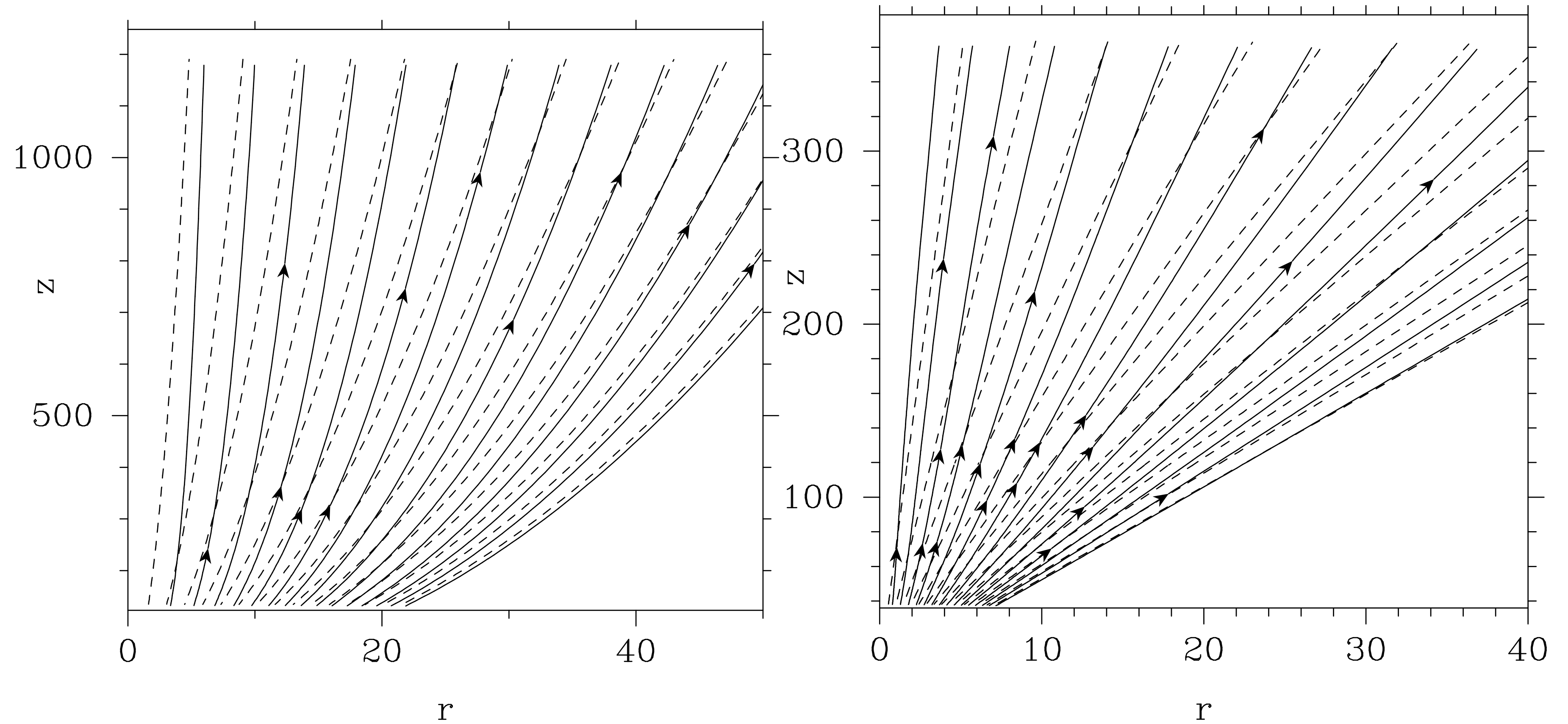


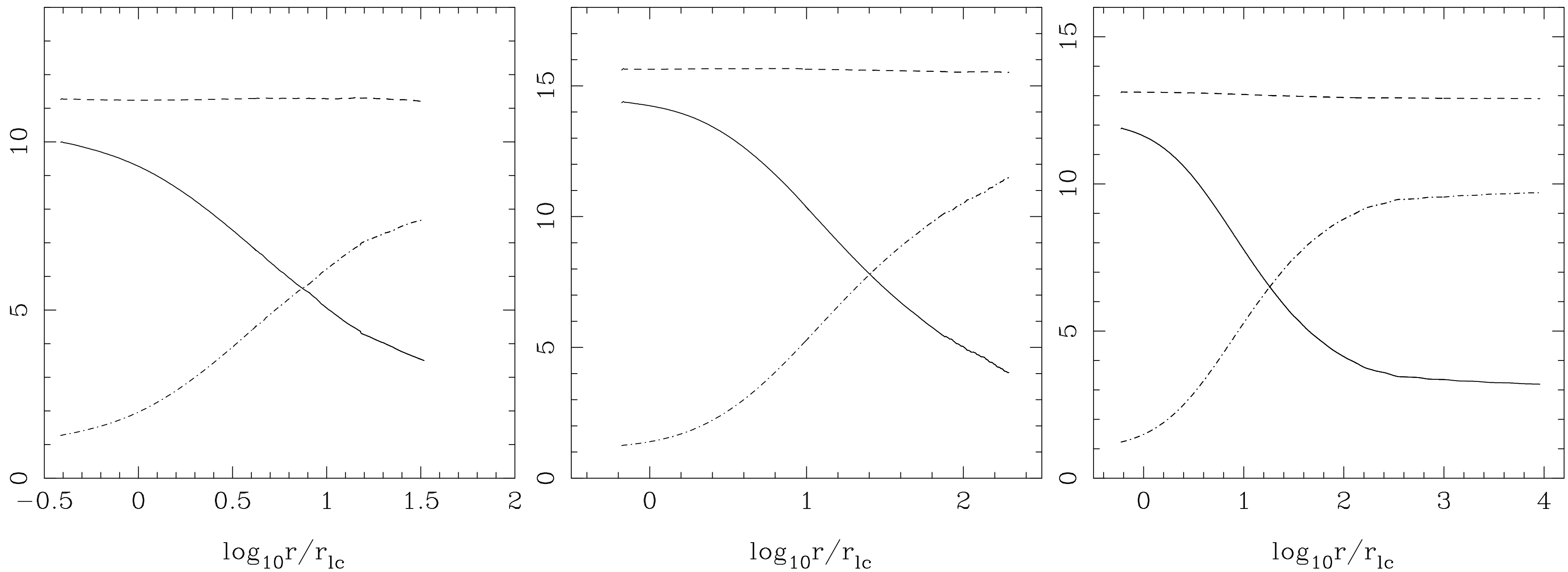
Left panel shows density (colour) and magnetic field lines.
Right panel shows the Lorentz factor (colour) and the current lines.



10^4
z
5000
0
0
50
100
150
200
r
−6
−5.5
−5
−4.5
−4
10^4
z
5000
0
0
50
100
150
200
r
5
10

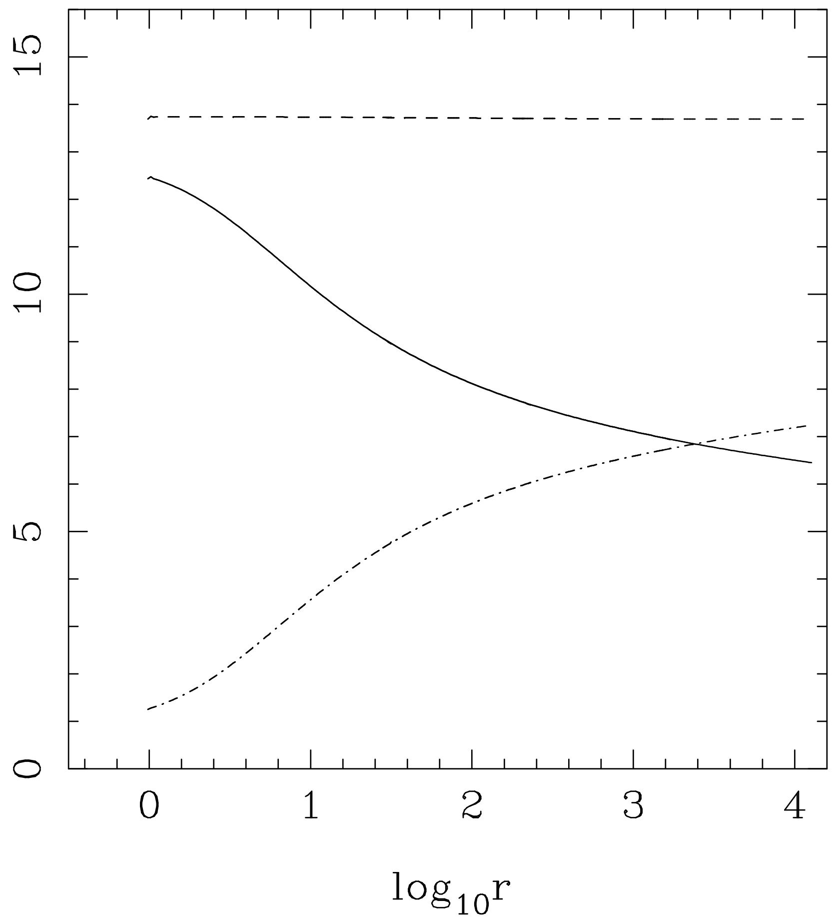

3000
2000
1000
0
z
0
200
400
600
r
−9
−8
−7
−6
3000
2000
1000
0
z
0
200
400
600
r
5
10

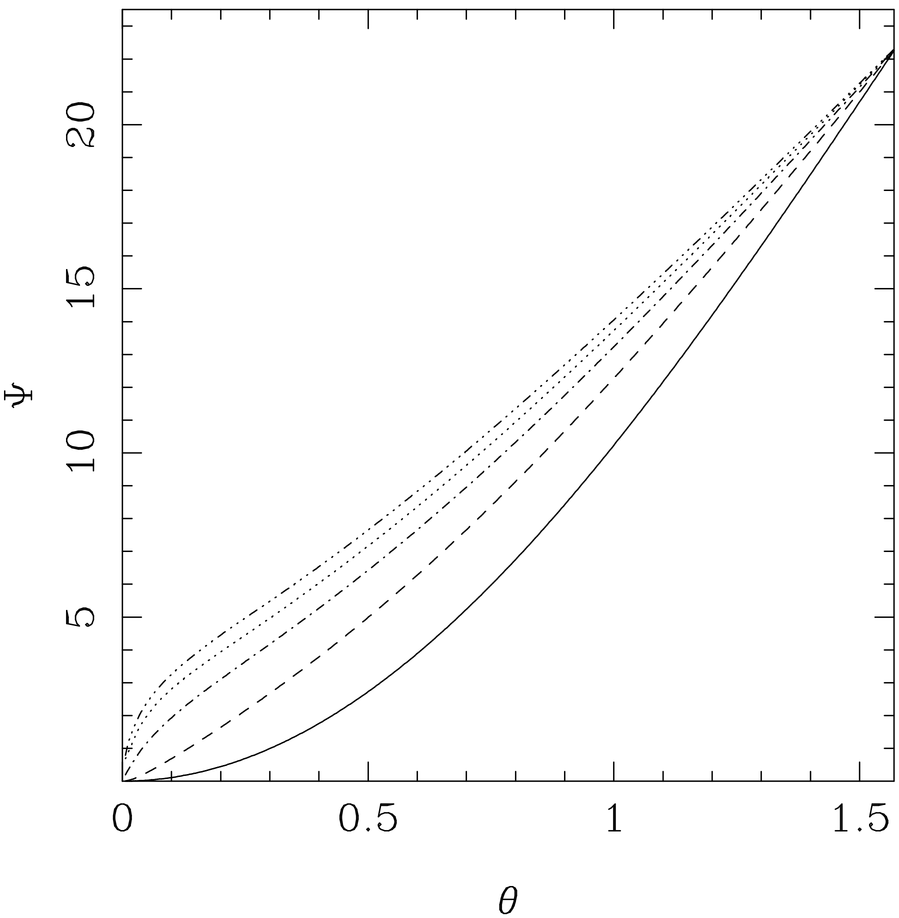

1000
500
z
0
20
40
r
300
200
100
z
0
10
20
30
40
r



$\gamma\sigma$ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius for models C1 (left panel), C2 (middle panel) and A2 (right panel).

(without a wall)

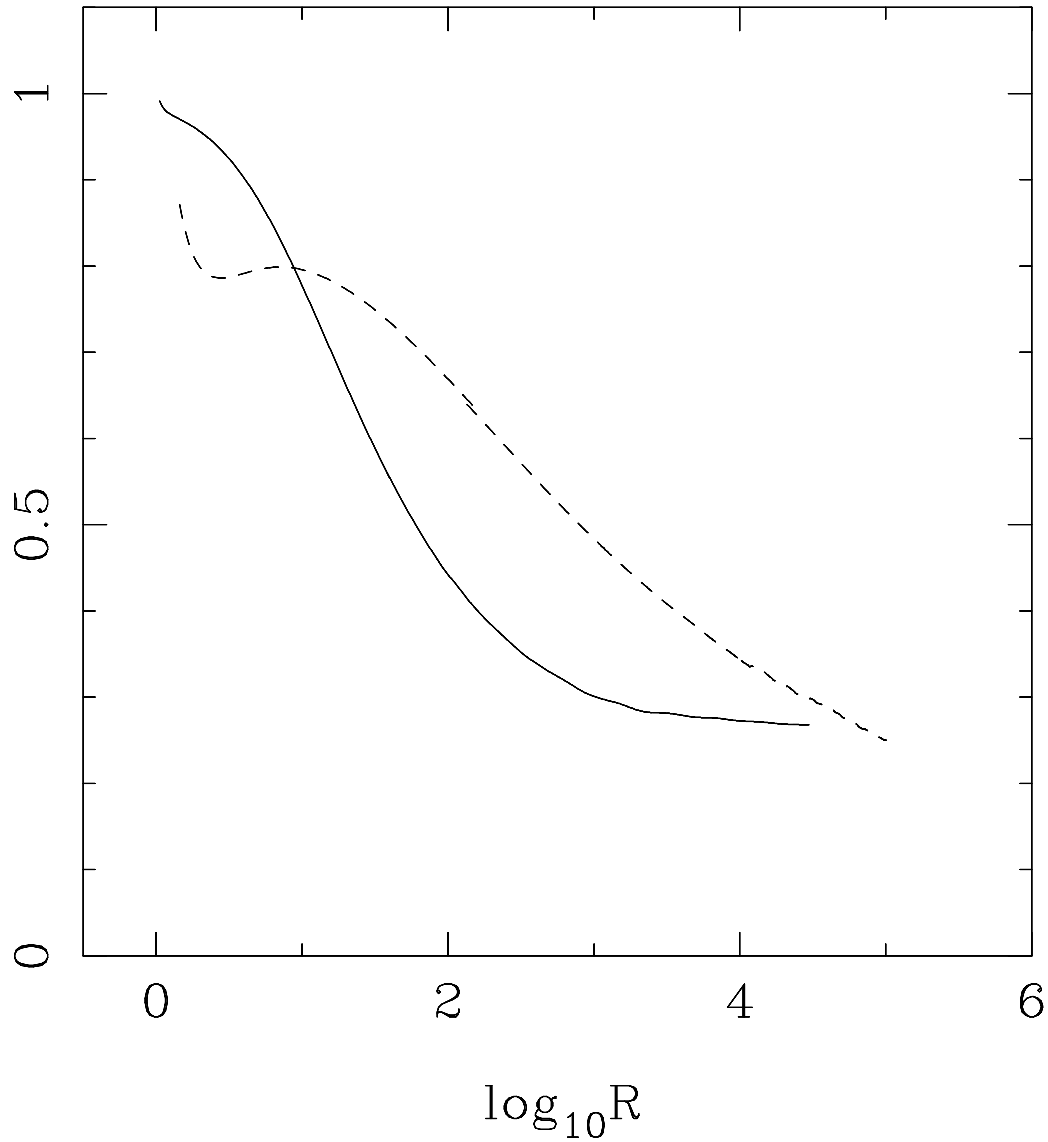




e.g. for $\Psi = 10$, $\vartheta = 57^{\rm o} \rightarrow 40^{\rm o}$

while for $\Psi = 5$, $\vartheta = 40^{\rm o} \rightarrow 15^{\rm o}$

$B_p \varpi^2 / (2A)$



Conclusions

⋆ MHD could explain the dynamics of relativistic jets:

- bulk acceleration: after a possible thermall acceleration phase, the flow is magnetically accelerated up to Lorentz factors of the order of the total energy-to-mass flux ratio,

  $$\gamma_\infty = \underbrace{\text{efficiency}}_{0.5-1} \times \frac{\mathcal{E}}{Mc^2} \text{ — } \gamma \propto \varpi^\beta$$

  The γ_∞ is NOT $= (\mathcal{E}/Mc^2)^{1/3}$, but $\sim \mathcal{E}/Mc^2$

  (σ is NOT constant in MHD flows)

- collimation: parabolic shape consistent with

  $\gamma \sim \dfrac{z}{\varpi} \Leftrightarrow z \propto \varpi^{\beta+1}$ — agrees with $\mathcal{R} \sim \gamma^2 \varpi$

⋆ The paradigm of MHD jets works in a similar way in nonrelativistic (YSO), mildly relativistic (AGN), and highly relativistic (GRB) jets!