Formation and Kinematic Properties of Relativistic MHD Jets

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Outline

• ideal MHD in general

• semianalytical modeling
  - \( r \)-self similarity
    * AGN outflows
    * GRB outflows
  - \( z \)-self similarity
    * Crab-like pulsar winds

• summary – meet the observations
• How the jet is collimated and accelerated? Need to examine outflows taking into account

  – matter: velocity $V$, rest density $\rho_0$, pressure $P$, specific enthalpy $\xi c^2$
  – electromagnetic field: $E$, $B$

• ideal MHD equations:

  – Maxwell: $\nabla \cdot B = 0 = \nabla \times E + \frac{\partial B}{c \partial t}$, $\nabla \times B = \frac{\partial E}{c \partial t} + \frac{4\pi}{c} J$, $\nabla \cdot E = \frac{4\pi}{c} J^0$
  – Ohm: $E + \frac{V}{c} \times B = 0$
  – mass conservation: $\frac{\partial (\gamma \rho_0)}{\partial t} + \nabla \cdot (\gamma \rho_0 V) = 0$
  – specific entropy conservation: $\left( \frac{\partial}{\partial t} + V \cdot \nabla \right) \left( \frac{P}{\rho_0^\Gamma} \right) = 0$
  – momentum: $\gamma \rho_0 \left( \frac{\partial}{\partial t} + V \cdot \nabla \right) (\xi \gamma V) = -\nabla P + \frac{J^0 E + J \times B}{c}$
Integration

• assume
  – axisymmetry ($\partial / \partial \phi = 0$, $E_\phi = 0$)
  – steady state ($\partial / \partial t = 0$)

• introduce the magnetic flux function $A$
  ($A = \text{const}$ is a poloidal field-streamline)

• the full set of ideal MHD equations can be partially integrated to yield five fieldline constants:
  ① the mass-to-magnetic flux ratio (continuity equation)
  ② the field angular velocity (Faraday + Ohm)
  ③ the specific angular momentum ($\phi$ component of momentum equation)
  ④ the total energy-to-mass flux ratio (momentum equation along $\mathbf{V}$)
  ⑤ the adiabat (entropy equation)

• two integrals remain to be performed, involving the Bernoulli and transfield force-balance

• boundary conditions?
If the boundary conditions on the conical disk surface $\theta = \theta_i$ are power laws:

\[
B_r = -C_1 r^{F-2}, \quad B_\phi = -C_2 r^{F-2}, \\
V_r/c = C_3, \quad V_\theta/c = -C_4, \quad V_\phi/c = C_5, \\
\rho_0 = C_6 r^{2(F-2)}, \quad P = C_7 r^{2(F-2)},
\]

then the variables $r, \theta$ are separable and the system reduces to ODEs. The solution should cross the Alfvén and the modified fast singular points.

[Blandford & Payne – (nonrelativistic)
Li, Chiuheh, & Begelman (1992) and Contopoulos (1994) – (cold)
Vlahakis & Königl (2003, astro-ph/0303482,0303483) – (including thermal/radiation effects)]

$F$ (the only parameter of the model) controls the current distribution:

$I \propto \varpi B_\phi \propto r^{F-1}$

- $F > 1$: current-carrying jet (near the rotation axis)
- $F < 1$: return-current (possibly at large $\varpi$)
AGN outflows (Vlahakis & Königl in preparation)  
(modeling the sub-pc-scale jet in NGC 6251)

VLBI measurements show sub-pc scale acceleration of the radio-jet in NGC 6251, from $V(r = 0.53\text{pc}) = 0.13c$ to $V(r = 1\text{pc}) = 0.42c$ [Sudou, H., et al. 2000, PASJ, 52, 989]


- **temperature:** $T = 10^{12} \left[ \frac{r(\text{pc})}{0.026} \right]^{-4/3} \text{K}$
- **sound speed:** $\frac{c_s}{c} = 0.5573 \left[ \frac{r(\text{pc})}{0.026} \right]^{-2/3}$
- **specific enthalpy:** $\xi = 1 + 0.466 \left[ \frac{r(\text{pc})}{0.026} \right]^{-4/3}$

Thus, for $0.53\text{pc} < r < 1\text{pc}$ the flow is supersonic and the quantity $\xi \gamma - 1$ changes from 0.01562 at $r = 0.53\text{pc}$ to 0.106 at $r = 1\text{pc}$.

As for hydrodynamic flows $\xi \gamma - 1 = \text{const.}$, the conclusion is that the flow is not hydrodynamically accelerated. We propose the magnetic acceleration as a plausible explanation of the observations.
\[\frac{-\sigma \Omega B_{\phi}}{\Psi_A c^2} = \frac{\text{Poynting flux}}{\text{mass flux}} c^2\]

\[\xi = \frac{\text{enthalpy+kinekinetic}}{\text{mass flux}} c^2\]

\[\gamma = \text{Lorentz factor}\]

\[\xi \gamma = \frac{\text{enthalpy+kinetic}}{\text{mass flux}} c^2\]

\[\zeta = \frac{\text{Poynting flux}}{\text{mass flux}} c^2\]

\[\xi \gamma = \xi \gamma c^2\]

\[\frac{-\sigma \Omega B_{\phi}}{\Psi_A c^2} = \frac{\text{Poynting flux}}{\text{mass flux}} c^2\]

\[\xi \gamma = \frac{\text{enthalpy+kinetic}}{\text{mass flux}} c^2\]

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\[\xi \gamma = \frac{\text{enthalpy+kinetic}}{\text{mass flux}} c^2\]
GRB outflows (including time dependence, $e^\pm$, radiation)

- $\omega_1 < \omega < \omega_6$: Thermal acceleration - force free magnetic field
  ($\gamma \propto \omega$, $\rho_0 \propto \omega^{-3}$, $T \propto \omega^{-1}$, $\omega B_\phi = \text{const}$, parabolic shape of fieldlines: $z \propto \omega^2$)
- $\omega_6 < \omega < \omega_8$: Magnetic acceleration ($\gamma \propto \omega$, $\rho_0 \propto \omega^{-3}$)
- $\omega = \omega_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0V_p)_\infty$

Relativistic MHD Jets

Mayschoss / May 13, 2003
Collimation – Acceleration

• The flow is centrifugally accelerated for $V_\phi \gtrsim V_p \Rightarrow V_p \lesssim \frac{c}{\sqrt{2}}$.

• Thermal acceleration is important for $\gamma \lesssim \xi_i$.

• For $\gamma \gtrsim \xi_i$, $\xi \approx 1$ and the magnetic acceleration takes over.

• How efficient is the magnetic acceleration? ($\sigma_\infty = ?$)
  
  – For $F > 1$ the flow reaches asymptotically a rough equipartition between kinetic and Poynting fluxes ($\sigma_\infty \approx 1$). The Lorentz force is capable of collimating the flow reaching cylindrical asymptotics (the collimation is possible for $\gamma \lesssim$ a few $\times 10$, following $\gamma^2 \varpi \sim R$).
  
  – For $F < 1$, the acceleration is more efficient. The collimation is not so strong and the flow eventually approaches conical asymptotics.

• Is the 100% acceleration efficiency possible ($\sigma_\infty = 0$)? Super-Alfvénic asymptotic solutions show that it is!
Crab-like pulsar winds

Integrate transfield force-balance equation under the $z$ self-similar ansatz $z = \mathcal{F}_1(A)\mathcal{F}_2(\varpi)$.

\[ J_{\parallel} < 0 \rightarrow J_{\parallel} > 0 \]
Summary of the previous results

- The shape is determined close to the source ($J_\parallel < 0$)
- Collimation is possible
- The acceleration continues at larger distances ($J_\parallel > 0$)
- The magnetic acceleration is efficient
- $r$ self-similar: does not cover both ($J_\parallel \leq 0$) cases ($F > 1$ is preferable)
- Alternatives:
  - $z$ self similar (captures both cases)
  - $\theta$ self-similar: applies to thermally driven flows near the axis (inside the light cylinder)
  - Fully numerical studies
Meet the observations

boundary conditions on the disk $B, \, T, \, \xi(e^{\pm} \text{ or } e^{-}p^{+}?), \, V \sim C_s, \, V_{\phi} = \varpi \Omega$, size $(M_{BH}), \, \dot{M}$

line shape $z = z(\varpi)$, $V, \, B, \, \rho_0, \, P$ as function of distance along each fieldline

- bulk flow
- synchrotron emission (knowing $B$ in space)
- positions of the shocks $\sim \gamma^2 c \Delta t$ (knowing $\gamma$) (Source variability $\Delta t$?)
- final value of $\sigma$ (asymptotic $B \rightarrow B$ in shocks)
- polarization
- asymptotic width – opening angle