Formation and Kinematic Properties of Relativistic MHD Jets

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Outline

- ideal MHD in general
- semianalytical modeling
  - r-self similarity
    - AGN outflows
    - GRB outflows
  - z-self similarity
    - Crab-like pulsar winds
- summary – meet the observations

Ideal Magneto-Hydro-Dynamics

- How the jet is collimated and accelerated? Need to examine outflows taking into account
  - matter: velocity $\mathbf{V}$, rest density ρ_0, pressure P, specific enthalpy ξc^2
  - electromagnetic field: $\mathbf{E}\,,\mathbf{B}$

- ideal MHD equations:
  - Maxwell: $\nabla \cdot \mathbf{B} = 0 = \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{c\partial t}\,,\ \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{c\partial t} + \frac{4\pi}{c}\mathbf{J}\,,\ \nabla \cdot \mathbf{E} = \frac{4\pi}{c} J^0$
  - Ohm: $\mathbf{E} + \frac{\mathbf{V}}{c} \times \mathbf{B} = 0$
  - mass conservation: $\frac{\partial(\gamma\rho_0)}{\partial t} + \nabla \cdot (\gamma\rho_0\mathbf{V}) = 0$
  - specific entropy conservation: $\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)\left(\frac{P}{\rho_0^\Gamma}\right) = 0$
  - momentum: $\gamma\rho_0\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)(\xi\gamma\mathbf{V}) = -\nabla P + \frac{J^0\mathbf{E} + \mathbf{J} \times \mathbf{B}}{c}$

Integration

- assume
  - axisymmetry $(\partial/\partial\phi = 0\,,\ E_\phi = 0)$
  - steady state $(\partial/\partial t = 0)$

- introduce the magnetic flux function A
  ($A = const$ is a poloidal field-streamline)

- the full set of ideal MHD equations can be partially integrated to yield five fieldline constants:
  ① the mass-to-magnetic flux ratio (continuity equation)
  ② the field angular velocity (Faraday + Ohm)
  ③ the specific angular momentum ($\hat{\phi}$ component of momentum equation)
  ④ the total energy-to-mass flux ratio (momentum equation along $\mathbf{V}$)
  ⑤ the adiabat (entropy equation)

- two integrals remain to be performed, involving the Bernoulli and transfield force-balance

- boundary conditions?

r self-similarity

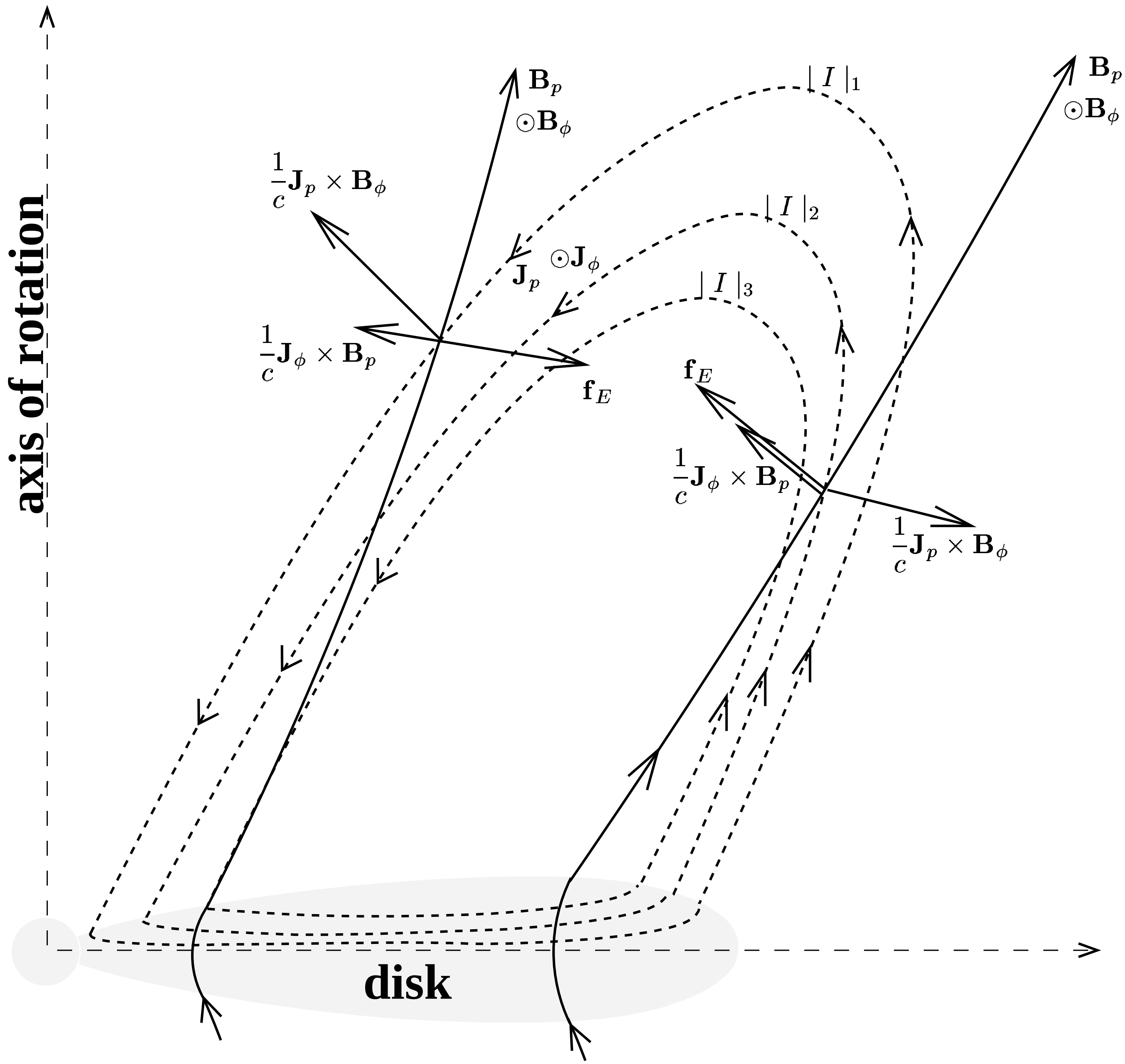
If the boundary conditions on the conical disk surface $\theta = \theta_i$ are power laws:

$$B_r = -\mathcal{C}_1 r^{F-2}\,,\ B_\phi = -\mathcal{C}_2 r^{F-2}\,,$$
$$V_r/c = \mathcal{C}_3\,,\ V_\theta/c = -\mathcal{C}_4\,,\ V_\phi/c = \mathcal{C}_5\,,$$
$$\rho_0 = \mathcal{C}_6 r^{2(F-2)}\,,\ P = \mathcal{C}_7 r^{2(F-2)}\,,$$

then the variables $r\,,\theta$ are separable and the system reduces to ODEs.
The solution should cross the Alfvén and the modified fast singular points.
[Blandford & Payne – (nonrelativistic)
Li, Chiueh, & Begelman (1992) and Contopoulos (1994) – (cold)
Vlahakis & Königl (2003, astro-ph/0303482,0303483) – (including thermal/radiation effects)]

F (the only parameter of the model) controls the current distribution:
$I \propto \varpi B_\phi \propto r^{F-1}$

- $F > 1$: current-carrying jet (near the rotation axis)
- $F < 1$: return-current (possibly at large ϖ)


axis of rotation
$\mathbf{B}_p$
$\odot\mathbf{B}_\phi$
$\frac{1}{c}\mathbf{J}_p \times \mathbf{B}_\phi$
$\mathbf{J}_p$
$\odot\mathbf{J}_\phi$
$\frac{1}{c}\mathbf{J}_\phi \times \mathbf{B}_p$
$\mathbf{f}_E$
$|I|_1$
$|I|_2$
$|I|_3$
$\mathbf{B}_p$
$\odot\mathbf{B}_\phi$
$\mathbf{f}_E$
$\frac{1}{c}\mathbf{J}_\phi \times \mathbf{B}_p$
$\frac{1}{c}\mathbf{J}_p \times \mathbf{B}_\phi$
disk

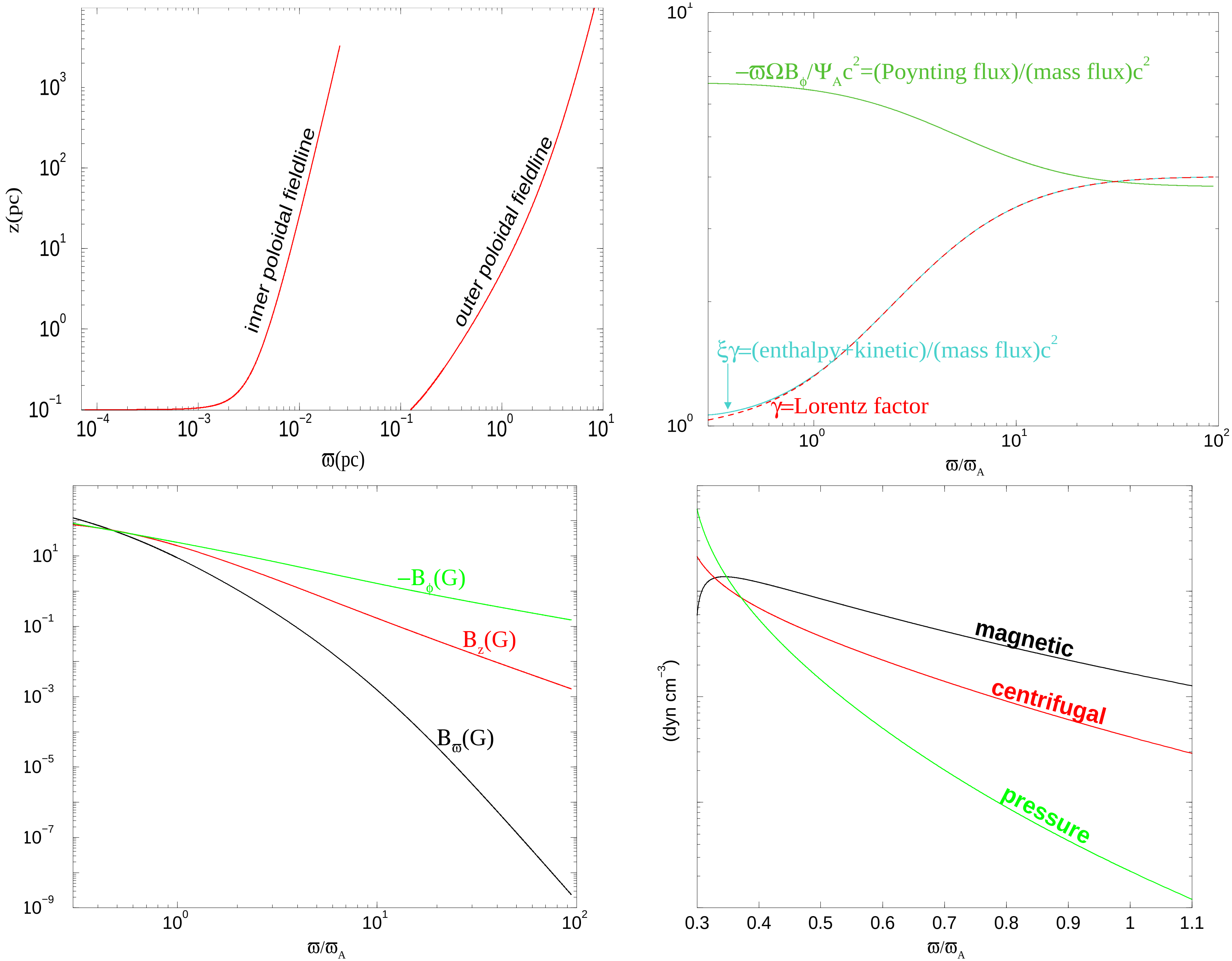
AGN outflows (Vlahakis & Königl in preparation) (modeling the sub-pc-scale jet in NGC 6251)

VLBI measurements show sub-pc scale acceleration of the radio-jet in NGC 6251, from $V(r = 0.53\text{pc}) = 0.13c$ to $V(r = 1\text{pc}) = 0.42c$ [Sudou, H., et al. 2000, PASJ, 52, 989]

Adopting the best fit model of Melia et al. 2002, ApJ, 567, 811 (consistent with the limits set by Jones et al. 1986, ApJ, 305, 684) and assuming $n \propto r^{-2}$ we find

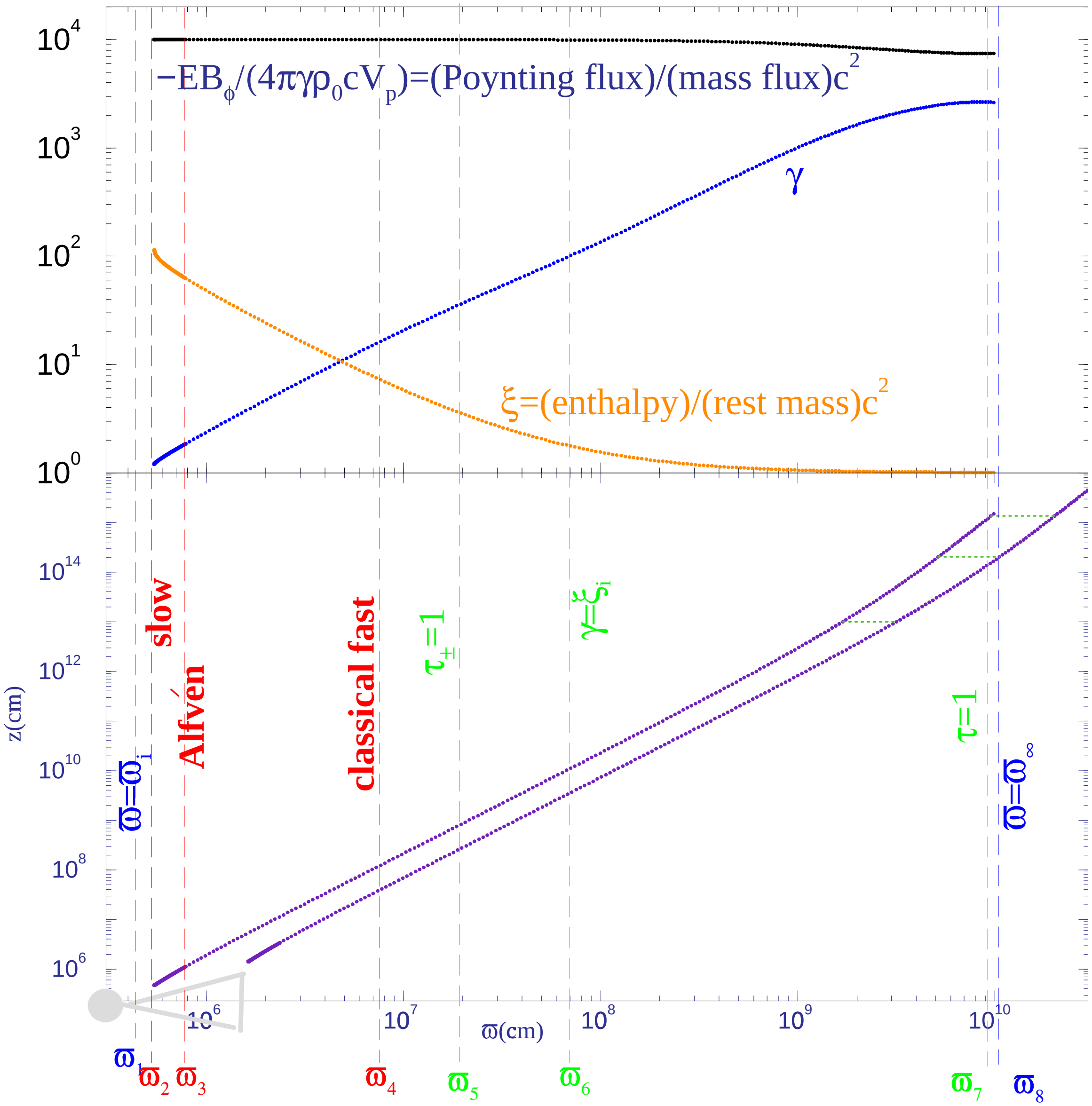
- temperature: $T = 10^{12} \left[\frac{r(pc)}{0.026}\right]^{-4/3}$ °K
- sound speed: $\frac{C_s}{c} = 0.5573 \left[\frac{r(pc)}{0.026}\right]^{-2/3}$
- specific enthalpy: $\xi = 1 + 0.466 \left[\frac{r(pc)}{0.026}\right]^{-4/3}$

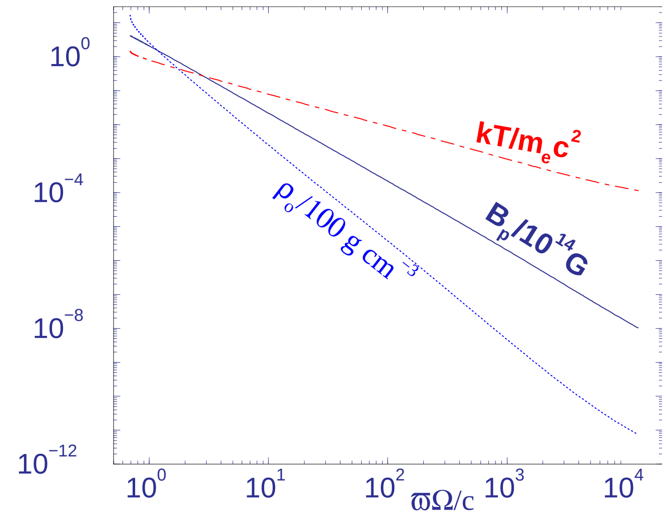
Thus, for 0.53pc$< r < 1$pc the flow is supersonic and the quantity $\xi\gamma - 1$ changes from 0.01562 at $r = 0.53$pc to 0.106 at $r = 1$pc.
As for hydrodynamic flows $\xi\gamma - 1 = const.$, the conclusion is that the flow is not hydrodynamically accelerated. We propose the magnetic acceleration as a plausible explanation of the observations.

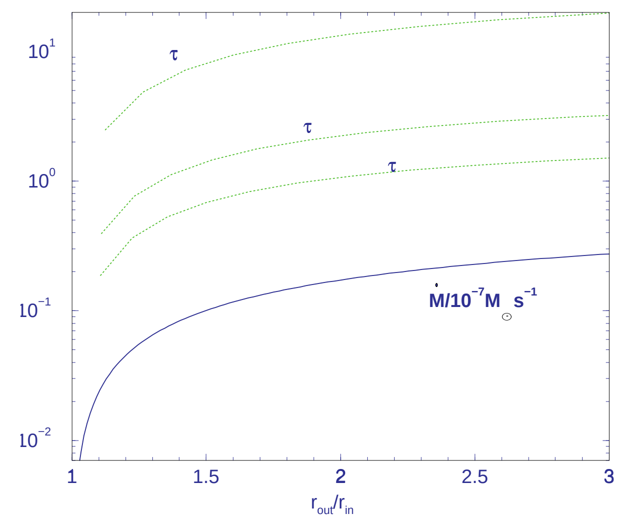


inner poloidal fieldline
outer poloidal fieldline
z(pc)
ϖ(pc)
$-\varpi\Omega B_\phi/\Psi_A c^2$=(Poynting flux)/(mass flux)c^2
$\xi\gamma$=(enthalpy+kinetic)/(mass flux)c^2
γ=Lorentz factor
ϖ/ϖ_A
$-B_\phi$(G)
B_z(G)
B_ϖ(G)
ϖ/ϖ_A
magnetic
centrifugal
pressure
(dyn cm^{-3})
ϖ/ϖ_A

GRB outflows (including time dependence, $e^{\pm}$, radiation)







- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

Collimation – Acceleration

- The flow is centrifugally accelerated for $V_\phi \gtrsim V_p \Rightarrow V_p \lesssim \frac{c}{\sqrt{2}}$.

- Thermal acceleration is important for $\gamma \lesssim \xi_i$.

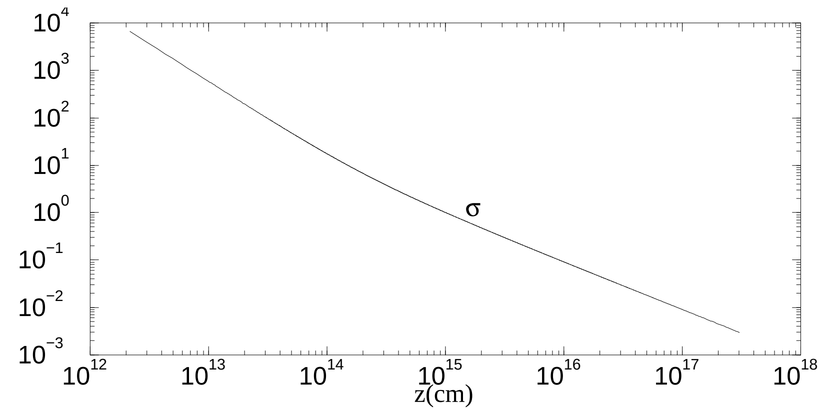
- For $\gamma \gtrsim \xi_i$, $\xi \approx 1$ and the magnetic acceleration takes over.

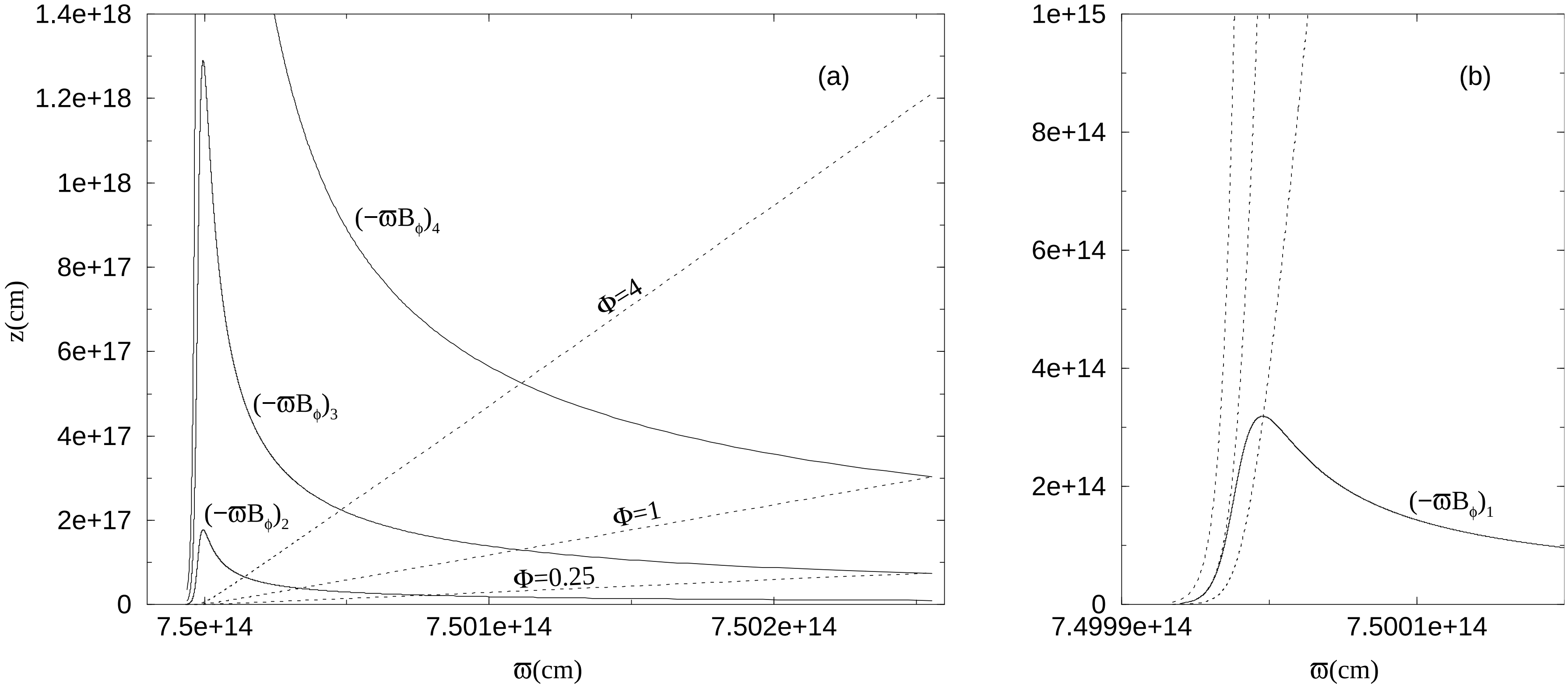
- How efficient is the magnetic acceleration? ($\sigma_\infty =$?)
  - For $F > 1$ the flow reaches asymptotically a rough equipartition between kinetic and Poynting fluxes ($\sigma_\infty \approx 1$). The Lorentz force is capable of collimating the flow reaching cylindrical asymptotics (the collimation is possible for $\gamma \lesssim$ a few $\times 10$, following $\gamma^2 \varpi \sim \mathcal{R}$).
  - For $F < 1$, the acceleration is more efficient. The collimation is not so strong and the flow eventually approaches conical asymptotics.

- Is the 100% acceleration efficiency possible ($\sigma_\infty = 0$)?
  Super-Alfvénic asymptotic solutions show that it is!

Crab-like pulsar winds

Integrate transfield force-balance equation under the z self-similar ansatz $z = \mathcal{F}_1(A)\mathcal{F}_2(\varpi)$.





$J_{\parallel} < 0 \rightarrow J_{\parallel} > 0$

Summary of the previous results

- The shape is determined close to the source ($J_{\|} < 0$)
- Collimation is possible
- The acceleration continues at larger distances ($J_{\|} > 0$)
- The magnetic acceleration is eficient

- r self-similar: does not cover both ($J_{\|} \lesseqgtr 0$) cases ($F > 1$ is preferable)
- Alternatives:
  - z self similar (captuers both cases)
  - θ self-similar: applies to thermally driven flows near the axis (inside the light cylinder)
  - Fully numerical studies

Meet the observations

boundary conditions on the disk		line shape $z = z(\varpi)$,
$\mathbf{B}$, T, $\xi(e^{\pm}$ or $e^{-}p^{+}$?), $V \sim C_s$,	$\longrightarrow$	$\mathbf{V}$, $\mathbf{B}$, ρ_0, P
$V_\phi = \varpi\Omega$, size ($M_{\rm BH}$), $\dot{M}$		as function of distance along each fieldline

- bulk flow
- synchrotron emission (knowing $\mathbf{B}$ in space)
- positions of the shocks $\sim \gamma^2 c \Delta t$ (knowing γ) (Source variability Δt?)
- final value of σ (asymptotic $\mathbf{B} \rightarrow \mathbf{B}$ in shocks)
- polarizarion
- asymptotic width – opening angle