Output from MHD models

Nektarios Vlahakis

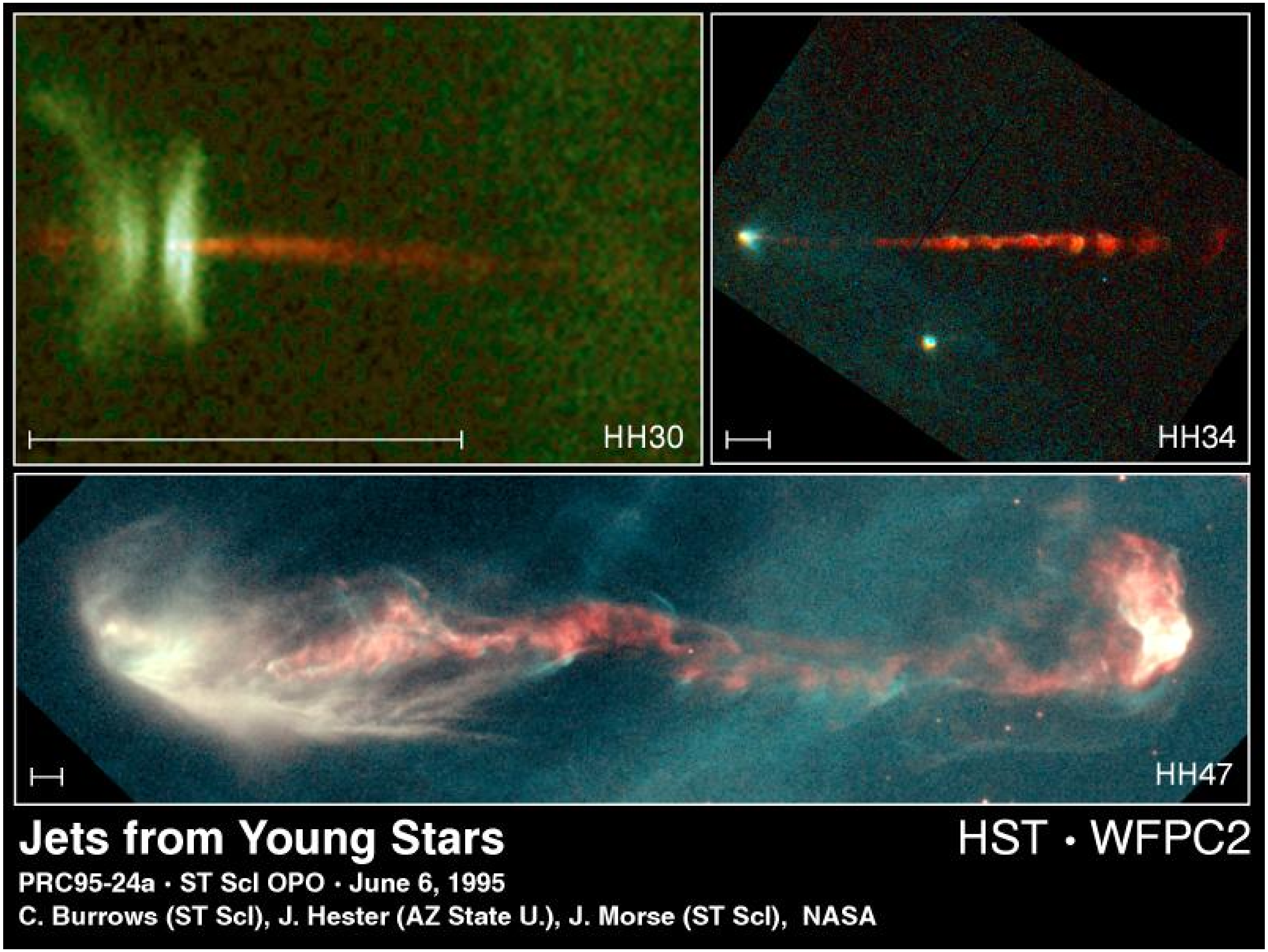
Outline

- MHD formalism

  acceleration and collimation mechanisms

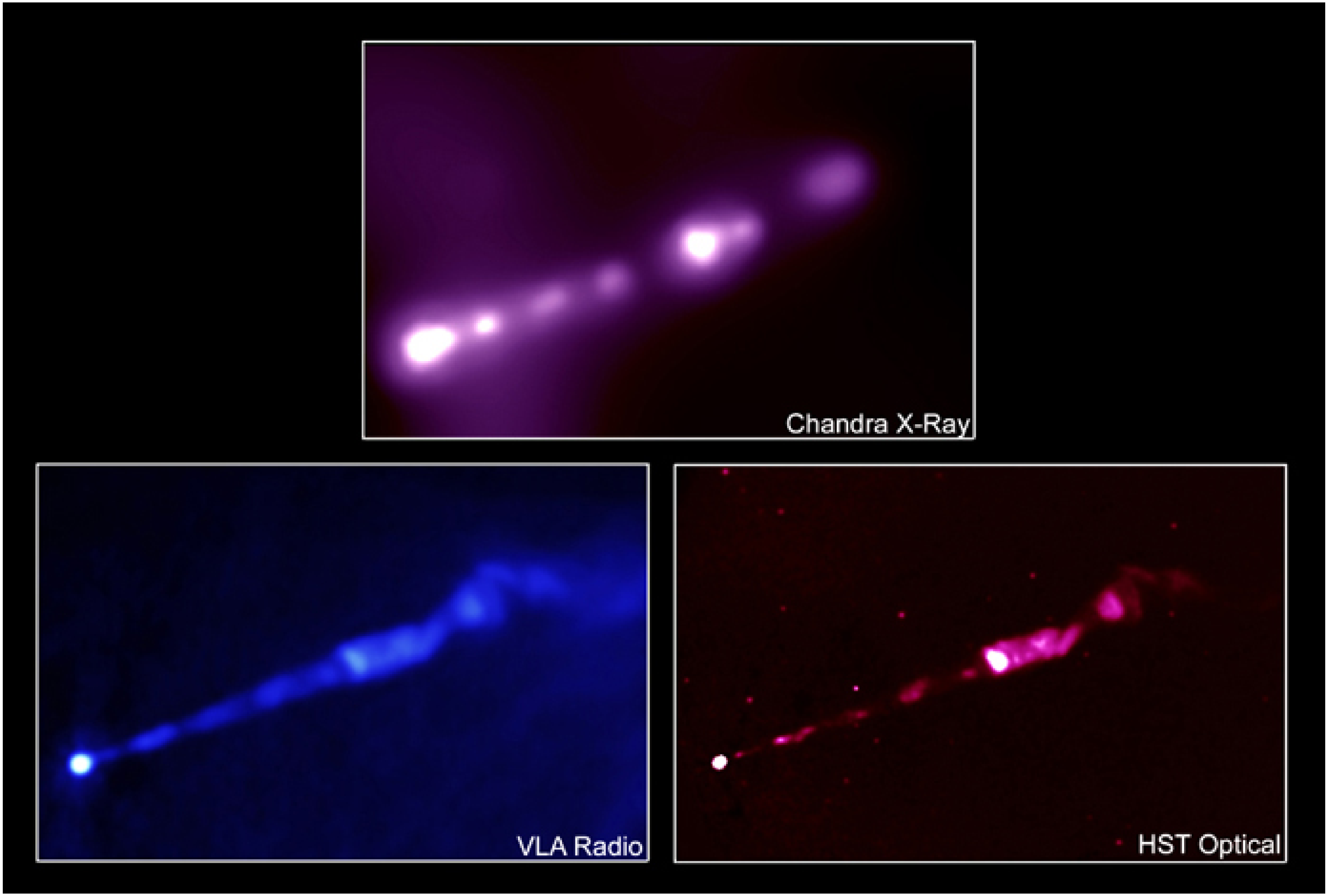
- analytical models

- simulations



(scale =1000 AU, $V_\infty = a few 100$km/s)

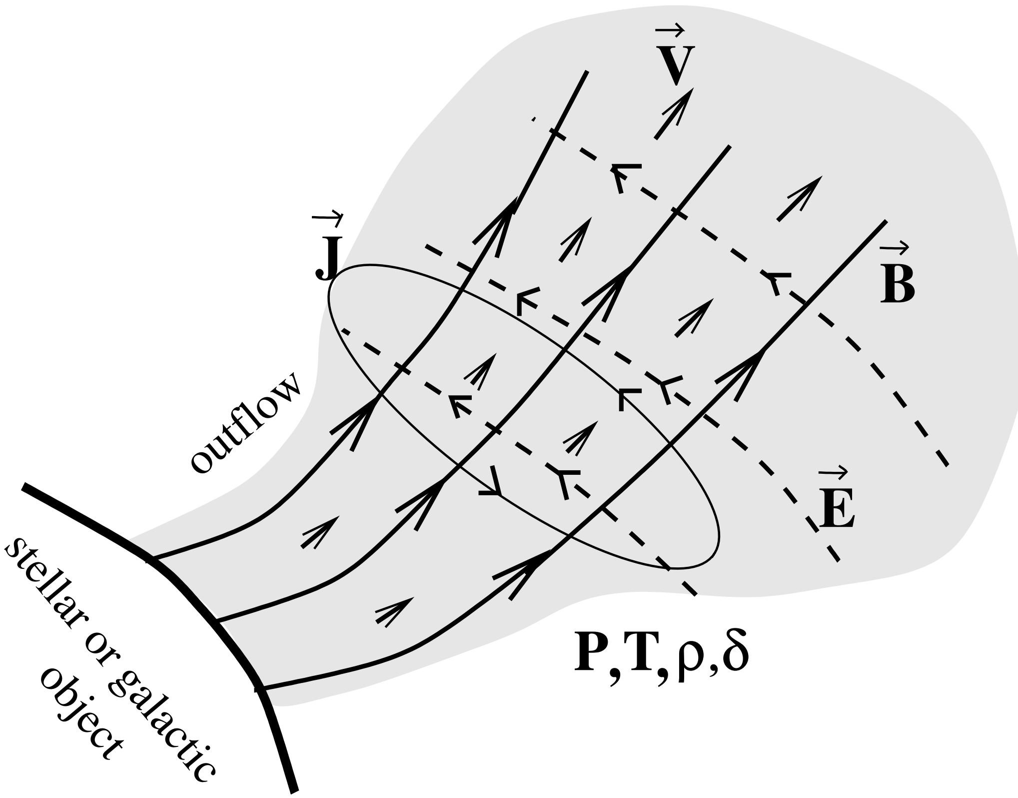

Herbig-Haro 49/50
NASA / JPL-Caltech / J. Bally (Univ. of Colo.)
Spitzer Space Telescope • IRAC
sig06-002



collimation at $\sim$100 Schwarzschild radii, $\gamma_\infty \sim 10$

MHD (Magneto-Hydro-Dynamics)

- matter: velocity $\boldsymbol{V}$, density ρ, pressure P
- electromagnetic field: $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{J}$, δ



The flow can be relativistic wrt

- bulk velocity $\boldsymbol{V} \approx c$, $\gamma \gg 1$
- random motion in comoving frame $k_\mathrm{B}T/m/c^2 \gg 1$, or, $P \gg \rho_0 c^2$, where $\rho_0 = \rho/\gamma$ the density in comoving frame
  Define specific enthalpy
  $\xi c^2 = \frac{\mathrm{mass}\times c^2 + \mathrm{internal\ energy} + P\times \mathrm{volume}}{\mathrm{mass}} = c^2 + \frac{1}{\Gamma-1}\frac{P}{\rho_0} + \frac{P}{\rho_0}$, or,
  $\xi = 1 + \frac{\Gamma}{\Gamma-1}\frac{P}{\rho_0 c^2} \gg 1$
- gravity $r \sim \mathcal{GM}/c^2$
  Define lapse function $h = \sqrt{1 - \frac{2\mathcal{GM}}{c^2 r}}$

In nonrelativistic flows, γ, ξ, h, all are ≈ 1

- Ohm: $\boldsymbol{E} + \dfrac{\boldsymbol{V}}{c} \times \boldsymbol{B} = 0$
- Maxwell:
$$\nabla \cdot \boldsymbol{B} = 0 = \nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{c\partial t}\,, \quad \boldsymbol{J} = \frac{c}{4\pi}\nabla \times \boldsymbol{B} - \frac{1}{4\pi}\frac{\partial \boldsymbol{E}}{\partial t}\,, \quad \delta = \frac{1}{4\pi}\nabla \cdot \boldsymbol{E}$$
- mass conservation: $\dfrac{\partial(\gamma\rho_0)}{\partial t} + \nabla \cdot (\gamma\rho_0 \boldsymbol{V}) = 0$
- momentum:
$$\gamma\rho_0 \left(\frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla\right)(\xi\gamma\boldsymbol{V}) = -\nabla P + \delta\boldsymbol{E} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c} - \gamma^2\rho_0\xi\frac{\mathcal{GM}}{r^2}\hat{r}$$
- energy: $\left(\dfrac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla\right)\left(\dfrac{1}{\Gamma - 1}\dfrac{P}{\rho_0}\right) + P\left(\dfrac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla\right)\dfrac{1}{\rho_0} = \dfrac{q}{\rho_0}$

On the energy equation

- no heating/cooling (adiabatic):
$$\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\left(\frac{1}{\Gamma-1}\frac{P}{\rho_0}\right)+P\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)\frac{1}{\rho_0}=0$$
$\Leftrightarrow (\partial/\partial t+\boldsymbol{V}\cdot\nabla)\left(P/\rho_0^{\Gamma}\right)=0$ (entropy conservation)

- with $q\neq 0$ two approaches:
  - ⋆ polytropic: Give appropriate q such that the energy eq has a similar to the adiabatic form $(\partial/\partial t+\boldsymbol{V}\cdot\nabla)\left(P/\rho_0^{\Gamma_{\rm eff}}\right)=0$ (in steady-state $\frac{q}{\rho_0}=\frac{\Gamma-\Gamma_{\rm eff}}{\Gamma-1}P\boldsymbol{V}\cdot\nabla\frac{1}{\rho_0}$) like adiabatic with $\frac{2}{\Gamma_{\rm eff}-1}$ degrees of freedom
  - ⋆ non-polytropic: ignore the energy equation, use it only a posteriori to find q (after solving for the dynamics)

    In essence, the pressure is also eliminated in this method, by replacing the poloidal momentum eq with $\nabla\times\nabla P=0$ (e.g., in nonrelativistic steady-state, with $\nabla\times\left[-\rho(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}+\boldsymbol{J}\times\boldsymbol{B}/c-\rho\mathcal{G}\mathcal{M}/r^2\right]=0$)

Assumptions

- ideal MHD
  zero resistivity: $0 = \boldsymbol{E}_{\rm co} = \gamma \left(\boldsymbol{E} + \frac{\boldsymbol{V}}{c} \times \boldsymbol{B}\right) - (\gamma - 1)\left(\boldsymbol{E} \cdot \frac{\boldsymbol{V}}{V}\right) \frac{\boldsymbol{V}}{V}$
  $\Leftrightarrow \boldsymbol{E} = -\frac{\boldsymbol{V}}{c} \times \boldsymbol{B}$

- one fluid approximation
  $\boldsymbol{V}_+ \approx \boldsymbol{V}_-$, or, $J \ll \frac{\rho}{m}|e|V$ with $\boldsymbol{J} = n_+ q_+ \boldsymbol{V}_+ + n_- q_- \boldsymbol{V}_-$
  quasi-neutrality $\delta = n_+ q_+ + n_- q_- \ll \frac{\rho}{m}|e|$
  $n_+ \approx n_- \approx \rho / m_p$ for $e^- p$ plasma

- further assume steady-state $\partial / \partial t = 0$

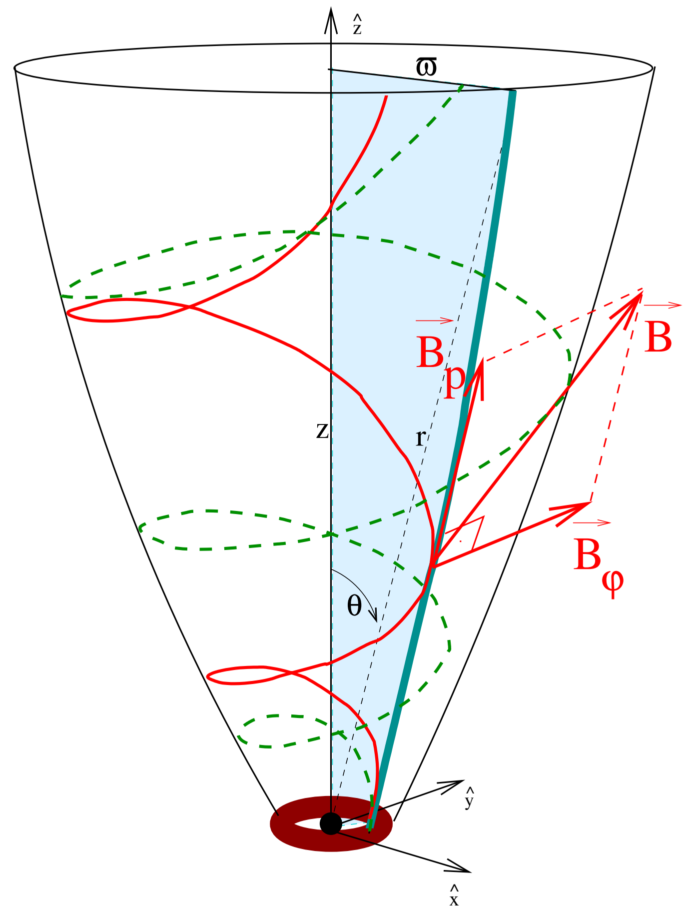
- axisymmetry $\partial / \partial \phi = 0$ in cylindrical $(z \,, \varpi \,, \phi)$ or spherical $(r \,, \theta \,, \phi)$ coordinates

Integrals of motion

From $\nabla \cdot \boldsymbol{B} = 0$

$$\boldsymbol{B}_p = \frac{\nabla A \times \hat{\phi}}{\varpi}, \text{ or, } \boldsymbol{B}_p = \nabla \times \left(\frac{A\, \hat{\phi}}{\varpi} \right)$$

$$A = \frac{1}{2\pi} \iint \boldsymbol{B}_p \cdot d\boldsymbol{S}$$



From $\nabla \times \boldsymbol{E} = 0$, $\boldsymbol{E} = -\nabla \Phi$
Because of axisymmetry $E_\phi = 0$.
Combining with Ohm's law
$(\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B}/c)$ we find $\boldsymbol{V}_p \parallel \boldsymbol{B}_p$.

Because $\boldsymbol{V}_p \parallel \boldsymbol{B}_p$ we can write

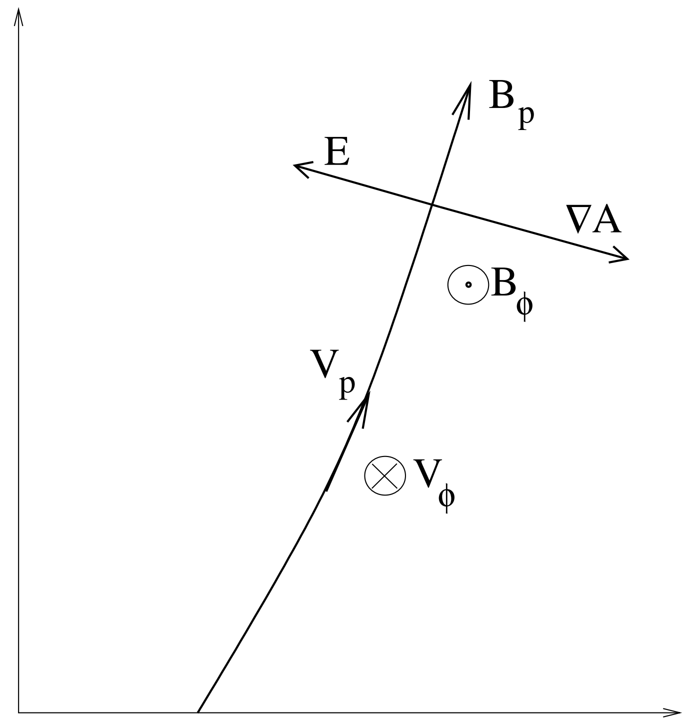
$$\boldsymbol{V} = \frac{\Psi_A}{4\pi\gamma\rho_0}\boldsymbol{B} + \varpi\Omega\hat{\boldsymbol{\phi}}\,, \quad \frac{\Psi_A}{4\pi\gamma\rho_0} = \frac{V_p}{B_p}\,.$$

$$\left(V_\phi = \frac{B_\phi}{B_p}V_p + \varpi\Omega\right)$$

The Ω and Ψ_A are constants of motion, $\Omega = \Omega(A)$, $\Psi_A = \Psi_A(A)$.

- $\Omega =$ angular velocity at the base
- $\Psi_A =$ mass-to-magnetic flux ratio

The electric field $\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B}/c = -(\varpi\Omega/c)\hat{\boldsymbol{\phi}} \times \boldsymbol{B}_p$ is a poloidal vector, normal to $\boldsymbol{B}_p$. Its magnitude is $E = \frac{\varpi\Omega}{c}B_p$.



So far, we've used Maxwell's eqs, Ohm's law and the continuity.

The energy and momentum equations remain. The latter:

$$\gamma\rho_0 \left(\boldsymbol{V}\cdot\nabla\right)\left(\xi\gamma\boldsymbol{V}\right) = -\nabla P + \delta\boldsymbol{E} + \frac{\boldsymbol{J}\times\boldsymbol{B}}{c} - \gamma^2\rho_0\xi\frac{\mathcal{G}\mathcal{M}}{r^2}\hat{r}$$

, or,

$$\gamma\rho_0 \left(\boldsymbol{V}\cdot\nabla\right)\left(\xi\gamma\boldsymbol{V}\right) = -\nabla P + \frac{(\nabla\cdot\boldsymbol{E})\boldsymbol{E} + (\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi} - \gamma^2\rho_0\xi\frac{\mathcal{G}\mathcal{M}}{r^2}\hat{r}$$

Due to axisymmetry, the toroidal component can be integrated to give the total angular momentum-to-mass flux ratio:

$$\xi\gamma\varpi V_\phi \quad \underbrace{-\frac{\varpi B_\phi}{\Psi_A}}_{\dfrac{-\varpi B_p B_\phi/4\pi}{\rho V_p}} \quad = L(A)$$

For polytropic flows:

- the energy eq gives $P/\rho_0^{\Gamma_{\rm eff}} = Q(A)$
  (the effective entropy is a constant of motion).

- the momentum along $\boldsymbol{V}_p$ gives $(h\xi\gamma - 1)c^2 - h\varpi\Omega B_\phi/\Psi_A = \mathcal{E}(A)$

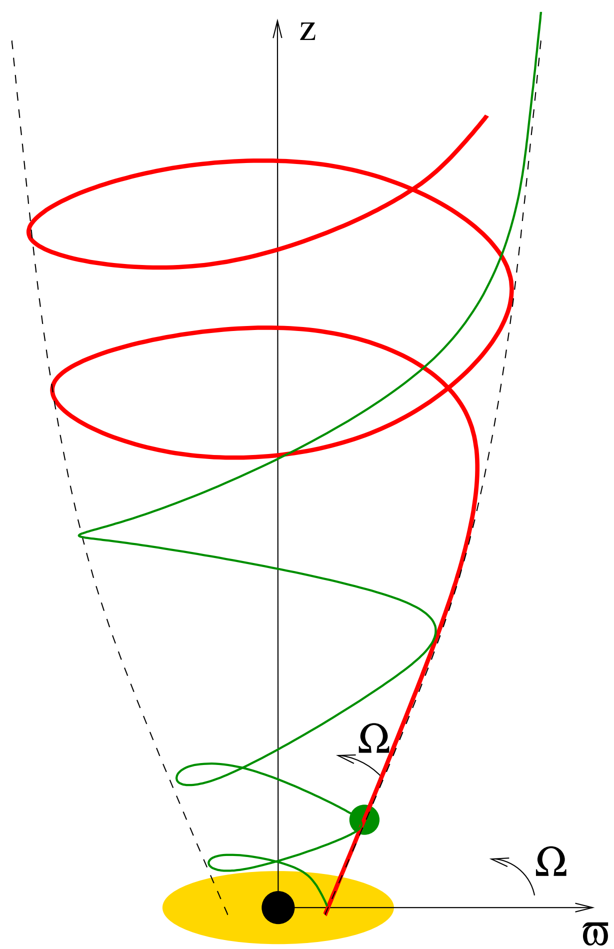
  ⋆ In relativistic MHD,

$$h\xi\gamma - h\varpi\Omega B_\phi/\Psi_A c^2 = 1 + \mathcal{E}/c^2 \equiv \mu(A) \text{ = maximum } \gamma$$

  ⋆ In nonrelativistic, expansion wrt $(\ldots)/c^2$ gives

$$\frac{V^2}{2} + \frac{\Gamma_{\rm eff}}{\Gamma_{\rm eff}-1}\frac{P}{\rho} - \frac{\mathcal{G}\mathcal{M}}{r} \underbrace{-\frac{\varpi\Omega B_\phi}{\Psi_A}}_{\frac{(c/4\pi)E|B_\phi|}{\rho V_p}} = \mathcal{E}(A)$$

Info from integrals (nonrelativistic case)



- – near the source,
  $V_\phi = V_p B_\phi / B_p + \varpi\Omega \to V_\phi \approx \varpi\Omega$
  for disk-driven flows $\Omega \approx \Omega_{\rm K}$
  solid-body rotation (rotating wires up to Alfvén surface – $|B_\phi|/B_p$ increases)
  – at large distances
  $\varpi V_\phi - \frac{\varpi B_\phi}{\Psi_A} = L(A) \to V_\phi \approx L/\varpi$
- $A =$const $\to B_p \propto 1/\varpi^2$
  $\rho V_p \propto B_p \to \rho V_p \propto 1/\varpi^2$
- near the source, for Poynting-dominated flows $\mathcal{E} \approx -\varpi\Omega B_{\phi i}/\Psi_A \to B_{\phi i} \approx \frac{-\Psi_A \mathcal{E}}{\varpi_i \Omega}$.
- Alfvén surface: combination of integrals $\to$
  $B_\phi = -\frac{L\Psi_A}{\varpi}\frac{1-\varpi^2\Omega/L}{1-M^2} \to \varpi_A = \sqrt{L/\Omega}$
  ($M = V_p\sqrt{4\pi\rho}/B_p =$ Alfvén Mach)
  $L = \Omega\varpi_A^2$, ϖ_A= lever arm for L-extraction
- maximum asymptotic velocity $V_\infty = \sqrt{2\mathcal{E}} \approx \sqrt{2L\Omega} = \varpi_A\Omega\sqrt{2} = \frac{\varpi_A}{\varpi_i}\sqrt{2}V_{\phi i}$
- $L \approx \varpi_\infty V_{\phi\infty}$, so $V_\infty \approx \sqrt{2\varpi_\infty V_{\phi\infty}\Omega}$
  (connection between V_∞ , $V_{\phi\infty}$, $\varpi_\infty - \varpi_i$)

The partial integration of the system, the relations between the integrals, and the definition of $M = V_p/(B_p/\sqrt{4\pi\rho})$, yield

$$\boldsymbol{B} = \frac{\nabla A \times \hat{\phi}}{\varpi} - \frac{L\Psi_A}{\varpi}\frac{1-\varpi^2\Omega/L}{1-M^2}\hat{\phi}\,,$$

$$\boldsymbol{V} = \frac{M^2}{\Psi_A}\frac{\nabla A \times \hat{\phi}}{\varpi} + \frac{L}{\varpi}\frac{\varpi^2\Omega/L - M^2}{1-M^2}\hat{\phi}\,,$$

$$\rho = \frac{\Psi_A^2}{4\pi M^2}$$

Thus, we have only two unknowns, the A and M, given by the two poloidal components of the momentum equation

~~M comes from momentum along the flow and A from the transfield~~

these two eqs are coupled!

Poloidal components of the momentum eq

$$\gamma\rho_0 \left(\boldsymbol{V}\cdot\nabla\right)\left(\xi\gamma\boldsymbol{V}\right) = -\nabla P + \frac{(\nabla\cdot\boldsymbol{E})\boldsymbol{E} + (\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi} \Leftrightarrow$$

$$\boldsymbol{f}_G + \boldsymbol{f}_T + \boldsymbol{f}_C + \boldsymbol{f}_I + \boldsymbol{f}_P + \boldsymbol{f}_E + \boldsymbol{f}_B = 0$$

$$\left.\begin{array}{lll}
\boldsymbol{f}_G = -\gamma\rho_0\xi\left(\boldsymbol{V}\cdot\nabla\gamma\right)\boldsymbol{V} & & \\
\boldsymbol{f}_T = -\gamma^2\rho_0\left(\boldsymbol{V}\cdot\nabla\xi\right)\boldsymbol{V} & : & \text{“temperature” force} \\
\boldsymbol{f}_C = \hat{\boldsymbol{\varpi}}\gamma^2\rho_0\xi V_\phi^2/\varpi & : & \text{centrifugal force} \\
\boldsymbol{f}_I = -\gamma^2\rho_0\xi\left(\boldsymbol{V}\cdot\nabla\right)\boldsymbol{V} - \boldsymbol{f}_C & &
\end{array}\right\} \text{inertial force}$$

$$\begin{array}{lll}
\boldsymbol{f}_P = -\nabla P & : & \text{pressure force} \\
\boldsymbol{f}_E = \left(\nabla\cdot\boldsymbol{E}\right)\boldsymbol{E}/4\pi & : & \text{electric force} \\
\boldsymbol{f}_B = \left(\nabla\times\boldsymbol{B}\right)\times\boldsymbol{B}/4\pi & : & \text{magnetic force}
\end{array}$$

Acceleration mechanisms

- thermal (due to ∇P) $\longrightarrow$ velocities up to C_s
- magnetocentrifugal (beads on wire)
  - ⋆ initial half-opening angle $\vartheta > 30^o$ (only for cold flows)
  - ⋆ velocities up to $\lesssim \varpi_i \Omega$
  - ⋆ in reality due to magnetic pressure: the constancy of the integral L gives $f_{C\parallel} = -\rho V_\phi \partial V_\phi / \partial \ell + (V_\phi / V_p)(B_p / |B_\phi|) f_{B\parallel}$
- relativistic thermal (thermal fireball) gives $\gamma \sim \xi_i$, where $\xi = \dfrac{\text{enthalpy}}{\text{mass} \times c^2}$.
- magnetic due to $f_{B\parallel} \propto$ gradient of $\varpi B_\phi \longrightarrow$ velocities up to complete matter domination (not always the case)

All acceleration mechanisms can be seen in

$$\frac{V^2}{2} + \frac{\Gamma_{\rm eff}}{\Gamma_{\rm eff}-1}\frac{P}{\rho} - \frac{\mathcal{GM}}{r} - \frac{\Omega}{\Psi_A}\varpi B_\phi = \mathcal{E} \text{ , or, } \xi\gamma - \frac{\Omega}{\Psi_A c^2}\varpi B_\phi = \mu$$

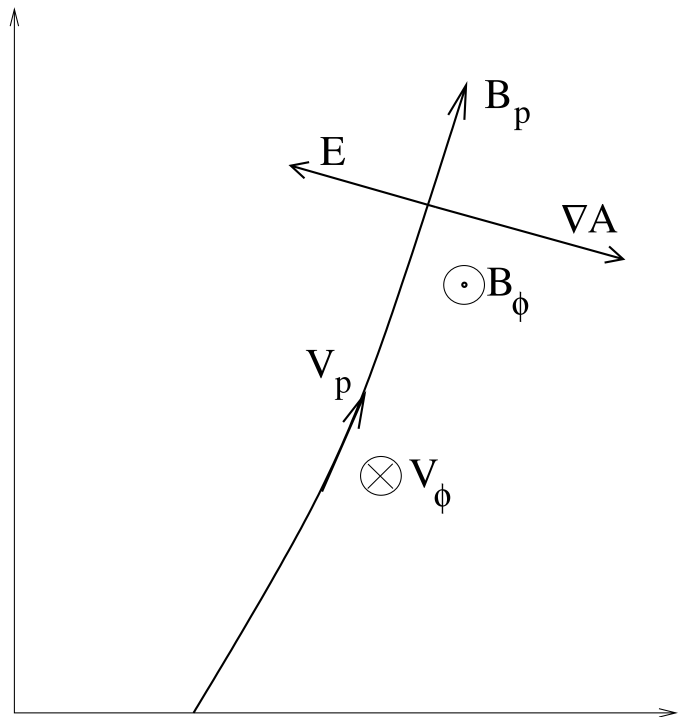
So $V, \gamma \uparrow$ when $P/\rho_0 \, , \xi \downarrow$ (thermal, relativistic thermal), or, $\varpi |B_\phi| \downarrow$ (magnetocentrifugal, magnetic).

On the magnetic acceleration

$$(\nabla\times\boldsymbol{B})\times\boldsymbol{B}=\underbrace{(\nabla\times\boldsymbol{B}_p)\times\boldsymbol{B}_p}_{\perp\boldsymbol{B}_p}+\underbrace{(\nabla\times\boldsymbol{B}_p)\times\boldsymbol{B}_\phi}_{0}+\underbrace{(\nabla\times\boldsymbol{B}_\phi)\times\boldsymbol{B}_p}_{\parallel\hat{\phi}}+\underbrace{(\nabla\times\boldsymbol{B}_\phi)\times\boldsymbol{B}_\phi}_{-\frac{B_\phi^2}{\varpi}\hat{\varpi}-\nabla\frac{B_\phi^2}{2}=-\frac{B_\phi}{\varpi}\nabla(\varpi B_\phi)}$$

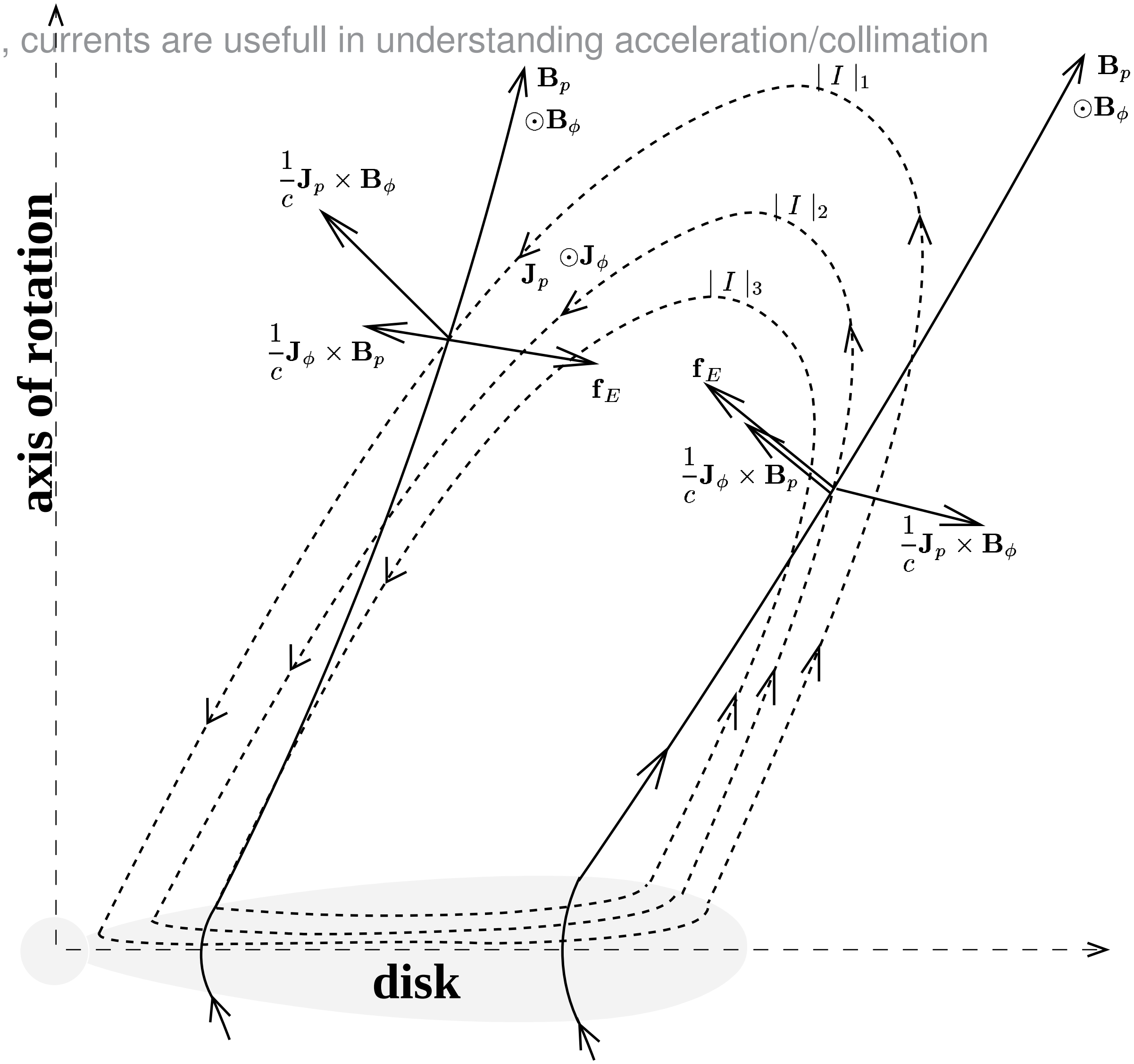
$$f_{B\parallel}=-\frac{B_\phi}{4\pi\varpi}\frac{\partial}{\partial\ell}(\varpi B_\phi)$$

ϖB_ϕ is related to the poloidal current
$\boldsymbol{J}_p=\frac{c}{4\pi}\nabla\times\boldsymbol{B}_\phi=\frac{1}{2\pi\varpi}\nabla I\times\hat{\phi}$, with
$I=\iint\boldsymbol{J}_p\cdot d\boldsymbol{S}=\frac{c}{2}\varpi B_\phi$

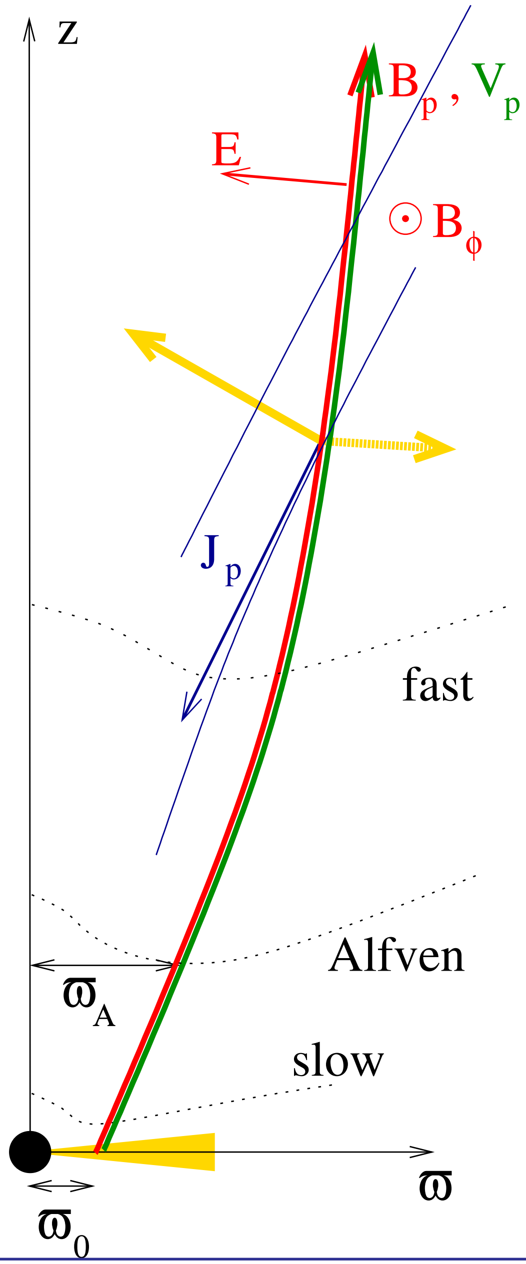


Currents or magnetic fields?

Although in MHD $\boldsymbol{B}$ drive the currents and not the opposite (since $\boldsymbol{J} = ne(\boldsymbol{V}_+ - \boldsymbol{V}_-)$ with $|\boldsymbol{V}_+ - \boldsymbol{V}_-| \ll V$), currents are usefull in understanding acceleration/collimation



The efficiency of the magnetic acceleration



The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force strongly depends on the angle between field-lines and current-lines.

Are we free to choose these two lines? NO! All MHD quantities are related to each other and should be found by solving the full system of equations.

At classical fast surface $V_p \approx B/\sqrt{4\pi\rho}$,

$$\frac{\text{kinetic}}{\text{Poynting}} = \frac{V_p^2/2}{-\varpi\Omega B_\phi/\Psi_A} \approx \frac{1}{2}\left(1 - \frac{V_\phi}{\varpi\Omega}\right)\frac{B^2}{B_\phi^2} \sim \frac{1}{2}$$

(using $\Psi_A = 4\pi\rho V_p/B_p$ and $V_\phi = \varpi\Omega + V_p B_\phi/B_p$)

(For relativistic flows this ratio is $\mu^{-2/3} \ll 1$!)

$V_p = (M^2/\Psi_A)B_p$ (from $M^2 = 4\pi\rho V_p^2/B_p^2$ and $\Psi_A = 4\pi\rho V_p/B_p$)

At $M \gg 1$, $\varpi \gg \varpi_{\rm A}$, $-\dfrac{\varpi\Omega B_\phi}{\Psi_A} \approx \dfrac{\Omega^2 B_p \varpi^2}{\Psi_A V_p}$

(from $B_\phi = -\dfrac{L\Psi_A}{\varpi}\dfrac{1-\varpi^2\Omega/L}{1-M^2} \approx -\dfrac{L\Psi_A}{\varpi}\dfrac{\varpi^2\Omega/L}{M^2}$ at large distances)

The $\frac{\text{kinetic}}{\text{Poynting}} = 1/2$ at fast gives

$$V_f = \left(\frac{\Omega^2 B_p \varpi^2}{\Psi_A}\right)^{1/3}$$

(The relativistic version is $\gamma_f = \mu^{1/3}$.)

Acceleration continues after the fast (crucial especially for relativistic flows)

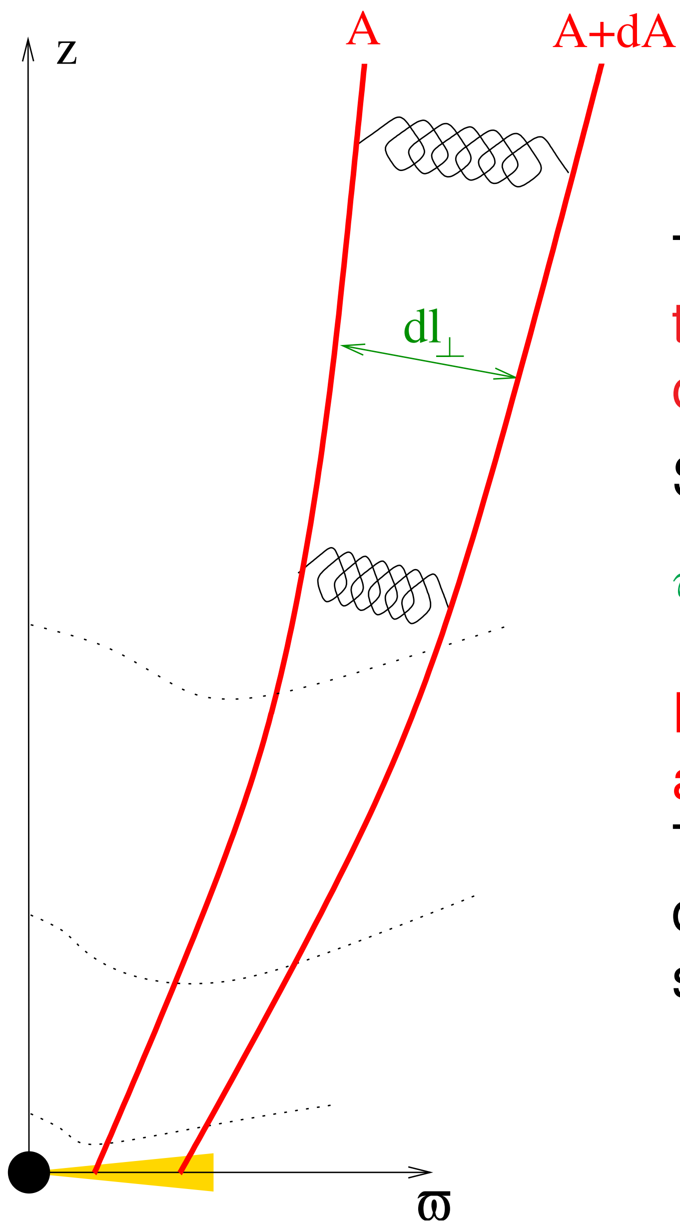
Defining the constant of motion

$$\sigma_m = \frac{A\Omega^2}{\Psi_A \mathcal{E} V_\infty} = \frac{A\Omega^2(1+\mathcal{E}/c^2)}{\Psi_A \mathcal{E}^{3/2}\sqrt{2+\mathcal{E}/c^2}}$$

we find (by combining the integral relations)

$$\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi\Omega}\right) \frac{V_\infty}{V_p} \frac{B_p \varpi^2}{A} \propto \frac{B_p \varpi^2}{A}$$

So, the transfield force-balance determines the acceleration through $B_p\varpi^2/A$.



The magnetic field minimizes its energy under the condition of keeping the magentic flux constant.

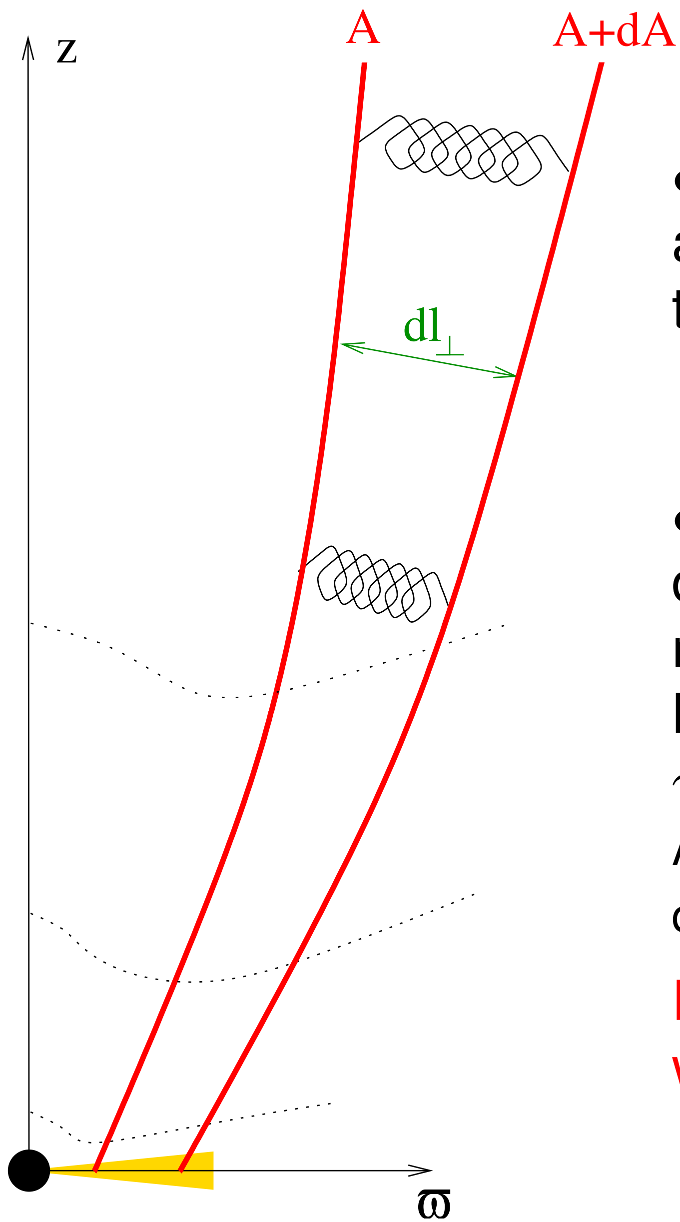
So, $\varpi B_\phi \downarrow$ for decreasing

$$\varpi^2 B_p = \frac{\varpi^2}{2\pi\varpi dl_\perp}\underbrace{(B_p dS)}_{dA} \propto \frac{\varpi}{dl_\perp}.$$

Expansion with increasing $dl_\perp/\varpi$ leads to acceleration

The expansion ends in a more-or-less uniform distribution $\varpi^2 B_p \approx A$ (in a quasi-monopolar shape).

Conclusions on the magnetic acceleration



• In nonrelativistic flows efficiency $\sim 1/3$ already at classical fast (increases further – the final value depends on the fieldline shape).

• in relativistic flows: If we start with a uniform distribution the magnetic energy is already minimum $\rightarrow$ no acceleration. Example: Michel's (1969) solution which gives $\gamma_\infty \approx \mu^{1/3} \ll \mu$.

Also Beskin et al (1998); Bogovalov (2001) who found quasi-monopolar solutions.

For any other (more realistic) field distribution we have efficient acceleration!

The acceleration efficiency in nonrelativistic flows

Applying $$\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi\Omega}\right) \frac{V_\infty}{V_p} \frac{B_p \varpi^2}{A}$$

at fast, we get $\sigma_m \approx (2/3^{3/2})(A/B_p\varpi^2)_{\text{fast}}$ (since at fast Poynting / total = 2/3, kinetic / total =1/3 $\Leftrightarrow V_f = \sqrt{2\mathcal{E}/3}$ and $V_\infty = \sqrt{2\mathcal{E}}$)

Applying the same relation at infinity – where $B_p\varpi^2 \approx A$ – yields

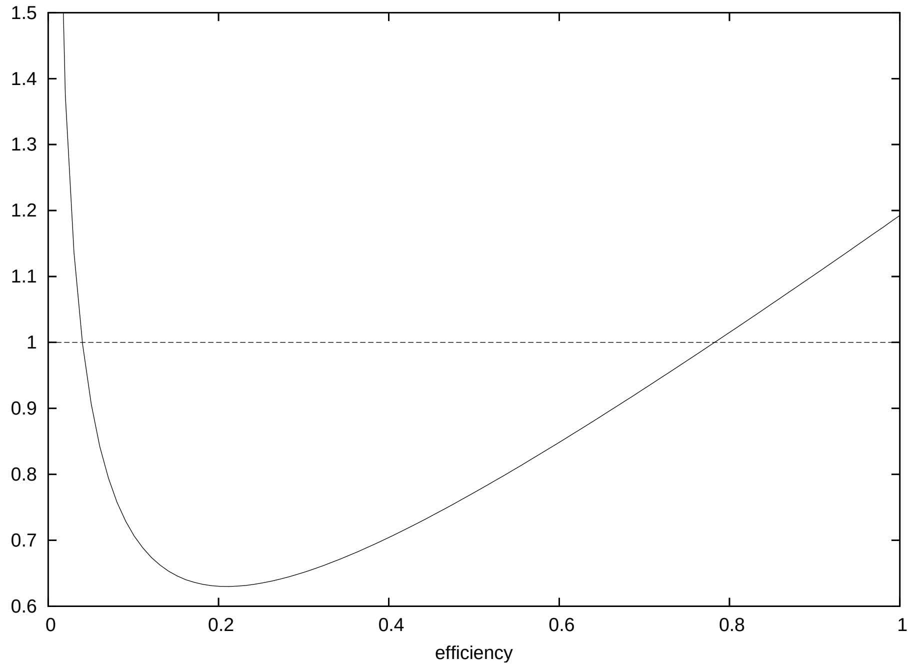
$(\text{Poynting/total energy flux})_\infty = \sigma_m \sqrt{2\mathcal{E}}/V_p$, or, $\dfrac{\mathcal{E} - V_p^2/2}{\mathcal{E}} = \sigma_m \sqrt{2\mathcal{E}}/V_p \rightarrow$

$\zeta + \dfrac{\sigma_m}{\zeta^{1/2}} = 1$, where $\zeta = \dfrac{V_p^2}{2\mathcal{E}}$ the efficiency at ∞, and $\sigma_m \approx \dfrac{2}{3^{3/2}} \left(\dfrac{A}{B_p\varpi^2}\right)_{\text{fast}}$

$$\boxed{\zeta + \frac{\sigma_m}{\zeta^{1/2}} = 1, \text{ where } \zeta = \frac{V_p^2}{2\mathcal{E}} \text{ the efficiency at } \infty, \text{ and } \sigma_m \approx \frac{2}{3^{3/2}}\left(\frac{A}{B_p\varpi^2}\right)_{\rm fast}}$$

The function $\zeta + \sigma_m/\zeta^{1/2}$ has a minimum $= (\sigma_m 3^{3/2}/2)^{2/3}$ at $\zeta = (\sigma_m/2)^{2/3}$

- If $\sigma_m = 2/3^{3/2} (\Leftrightarrow (B_p\varpi^2)_{\rm fast} = A)$ then $\zeta = 1/3$ and the fast is at ∞
- If $\sigma_m < 2/3^{3/2} (\Leftrightarrow (B_p\varpi^2)_{\rm fast} > A)$ then $\zeta > 1/3$ (the fast at finite distance)



For $(B_p\varpi^2)_{\rm fast}/A = 2$,
$\sigma_m = 0.19$ and $\zeta \approx 0.8$ (figure)

For $(B_p\varpi^2)_{\rm fast}/A = 4$,
$\sigma_m = 0.096$ and $\zeta \approx 0.9$

$\sigma_m = \frac{A\Omega^2}{\sqrt{2}\Psi_A \mathcal{E}^{3/2}}$ (or equivalently the $(B_p\varpi^2/A)_{\rm fast}$) determines the efficiency.
(Only for $\sigma_m \approx 0$ we have 100% efficiency.)

The acceleration efficiency in relativistic flows

Applying $$\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi \Omega}\right) \frac{V_\infty}{V_p} \frac{B_p \varpi^2}{A}$$

at fast, we get $\sigma_m \approx (A/B_p \varpi^2)_{\rm fast}$

Applying the same relation at infinity – where $B_p \varpi^2 \approx A$ – yields (Poynting/total energy flux)$_\infty = \sigma_m$, or, $\dfrac{\mathcal{E} - \gamma_\infty c^2}{\mathcal{E}} = \sigma_m \rightarrow$

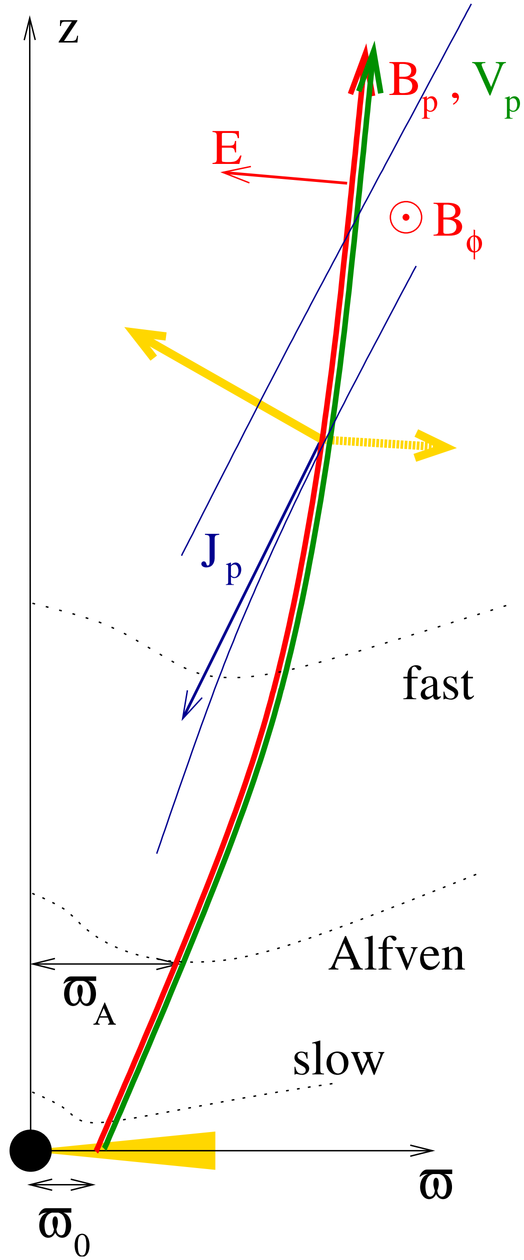
$$\boxed{\gamma_\infty \approx \frac{\mathcal{E}}{c^2} (1 - \sigma_m) \approx \frac{\mathcal{E}}{c^2} \left(1 - \frac{A}{(\varpi^2 B_p)_f}\right)}$$

The more bunched the fieldlines near the fast surface the higher the acceleration efficiency.

$I = c\varpi B_\phi/2 \approx -c\varpi E/2 \approx -c\varpi(\varpi \Omega B_p/c)/2 = (A\Omega/2)(\varpi^2 B_p/A)$.
So, $\gamma_\infty \approx (\mathcal{E}/c^2)\left(1 - A\Omega/2I_f\right)$

Since the flow is force-free up to the fast, $I_i \approx I_f$ and we connect the acceleration efficiency with conditions at the base of the flow $\gamma_\infty \approx (\mathcal{E}/c^2)\left(1 - A\Omega/2I_i\right)$

On the collimation



The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force contributes to the collimation (hoop-stress paradigm).
In nonrelativistic flows works fine.
In relativistic flows the electric force plays an opposite role (a manifestation of the high inertia of the flow).

- collimation by an external wind (Bogovalov & Tsinganos 2005)
- surrounding medium may play a role
- self-collimation mainly works at small distances where the velocities are mildly relativistic (Vlahakis & Königl 2003)

$$\frac{\gamma^2 \varpi}{\mathcal{R}} \approx \left[\frac{\left(\frac{2I}{\Omega B_p \varpi^2} \right)^2 \varpi \nabla \ln \left| \frac{I}{\gamma} \right|}{1 + \frac{4\pi \rho \gamma^2 V_p^2}{B_p^2} \frac{\varpi_{lc}^2}{\varpi^2}} - \gamma^2 \frac{\varpi_{lc}^2}{\varpi^2} \hat{\varpi} \right] \cdot \frac{\nabla A}{|\nabla A|}$$

For $\gamma \gg 1$, curvature radius $\mathcal{R} \sim \gamma^2 \varpi \ (\gg \varpi)$.

Collimation more difficult, but not impossible!

$$\frac{\varpi}{\mathcal{R}} = -\varpi \frac{\partial^2 \varpi}{\partial z^2} \left(\frac{B_z}{B_p} \right)^3 \sim \left(\frac{\varpi}{z} \right)^2$$

Combining the above, we get

$$\gamma \sim \frac{z}{\varpi}$$

The same from

$$(t =) \, \frac{z}{V_z} = \frac{\varpi}{V_\varpi} \Leftrightarrow \frac{z}{c} = \frac{\varpi}{\sqrt{c^2 - V_z^2}} \approx \frac{\varpi}{c/\gamma}$$

Asymptotic flow-shape

From
$$\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi\Omega}\right) \frac{V_\infty}{V_p} \frac{B_p \varpi^2}{A}$$

we find
$$\sigma_m \left(1 - \frac{V_\phi}{\varpi\Omega}\right) \frac{B_p \varpi^2}{A} = \frac{V_p}{V_\infty} \frac{\text{Poynting}}{\text{total energy flux}} < 1\,.$$

For $V_\phi \ll \varpi\Omega$, this gives $B_p \varpi^2 / A < 1/\sigma_m$ (same as in Heyvaerts & Norman asymptotic analysis).

Assume two lines that cross a given cylinder $\varpi =$const, one reference line A_i and another A that crosses the cylinder at higher z (so $A < A_i$).

The $\hat{\varpi}$ component of $\boldsymbol{B}_p = \nabla A \times \hat{\phi}/\varpi$ is $B_\varpi = -(1/\varpi)\partial A/\partial z$, or, $\partial z/\partial A = -1/(\varpi B_\varpi)$ and can be integrated along the cylinder to give $z(A,\varpi) = z(A_i,\varpi) + \int_A^{A_i} dA/(\varpi B_\varpi)$.

But, $B_\varpi < B_p < A/(\varpi^2 \sigma_m)$, and thus,

$$z(A,\varpi) > z(A_i,\varpi) + \varpi \int_A^{A_i} \frac{\sigma_m}{A} dA$$

lines cannot bend towards equator, they are situtated above some minimal cone

From
$$\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi \Omega}\right) \frac{V_\infty}{V_p} \frac{B_p \varpi^2}{A}$$
with Poynting $= -\varpi \Omega B_\phi / \Psi_A = -2 I \Omega / (c \Psi_A)$ and $B_p / V_p = 4\pi\rho / \Psi_A$ we find

$$\frac{I}{\gamma} = -2\pi c \frac{\Omega}{\Psi_A} \rho_0 \varpi^2$$

- if $\rho_0 \varpi^2 \stackrel{\varpi\to\infty}{\longrightarrow} f(A)$ (conical lines $z/\varpi \stackrel{\varpi\to\infty}{\longrightarrow}$const) then
  $I_\infty(A)/\gamma_\infty(A) =$const
  this constant is independent of A, from

$$\frac{\gamma^2 \varpi}{\mathcal{R}} \approx \left[\frac{\left(\frac{2I}{\Omega B_p \varpi^2}\right)^2 \varpi \nabla \ln \left|\frac{I}{\gamma}\right|}{1 + \frac{4\pi\rho\gamma^2 V_p^2}{B_p^2} \frac{\varpi_{lc}^2}{\varpi^2}} - \gamma^2 \frac{\varpi_{lc}^2}{\varpi^2} \hat{\varpi} \right] \cdot \frac{\nabla A}{|\nabla A|} \text{ with } \mathcal{R}/\varpi \stackrel{\varpi\to\infty}{\longrightarrow} \infty$$

- if $\rho_0 \varpi^2 \stackrel{\varpi\to\infty}{\longrightarrow} 0$ (parabolic lines $z/\varpi \stackrel{\varpi\to\infty}{\longrightarrow} \infty$) then $I_\infty/\gamma_\infty = 0$
  (100% acceleration efficiency)

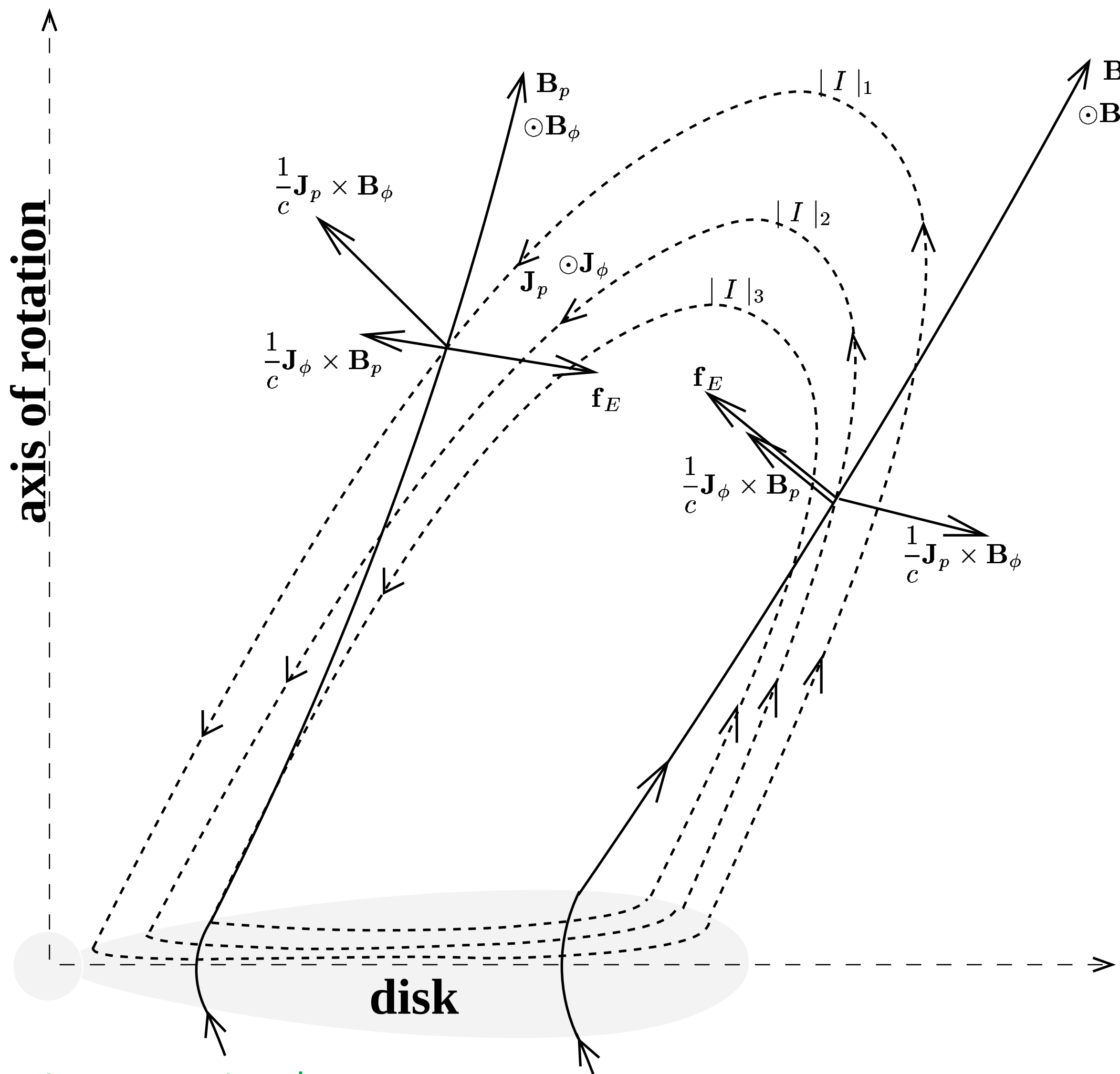
$I_\infty(A)/\gamma_\infty(A) =$const: solvability condition at infinity

(Heyvaerts & Norman 1989; Chiueh, Li, & Begelman 19991; see also Vlahakis 2004 for generalized analysis)

- nonrelativistic case: $I_\infty(A) =$const
  - ⋆ no current $\boldsymbol{J}_p$ flows between lines
  - ⋆ If the flow carries some finite Poynitng flux at infinity, the corresponding $\boldsymbol{J}_p$ flows inside a cylindrical core (this is the only way to have I smoothly varying from zero on the axis to I_∞ at the edge of the core).
  - ⋆ Note that for cylindrical lines the previous analysis (based on $\varpi \rightarrow \infty$) doesn't hold.

- relativistic case:
  again the cylindrical core is the only way to have $I_\infty(A=0)=0$ and $I_\infty(A)/\gamma_\infty(A) =$const $\neq 0$ at larger A

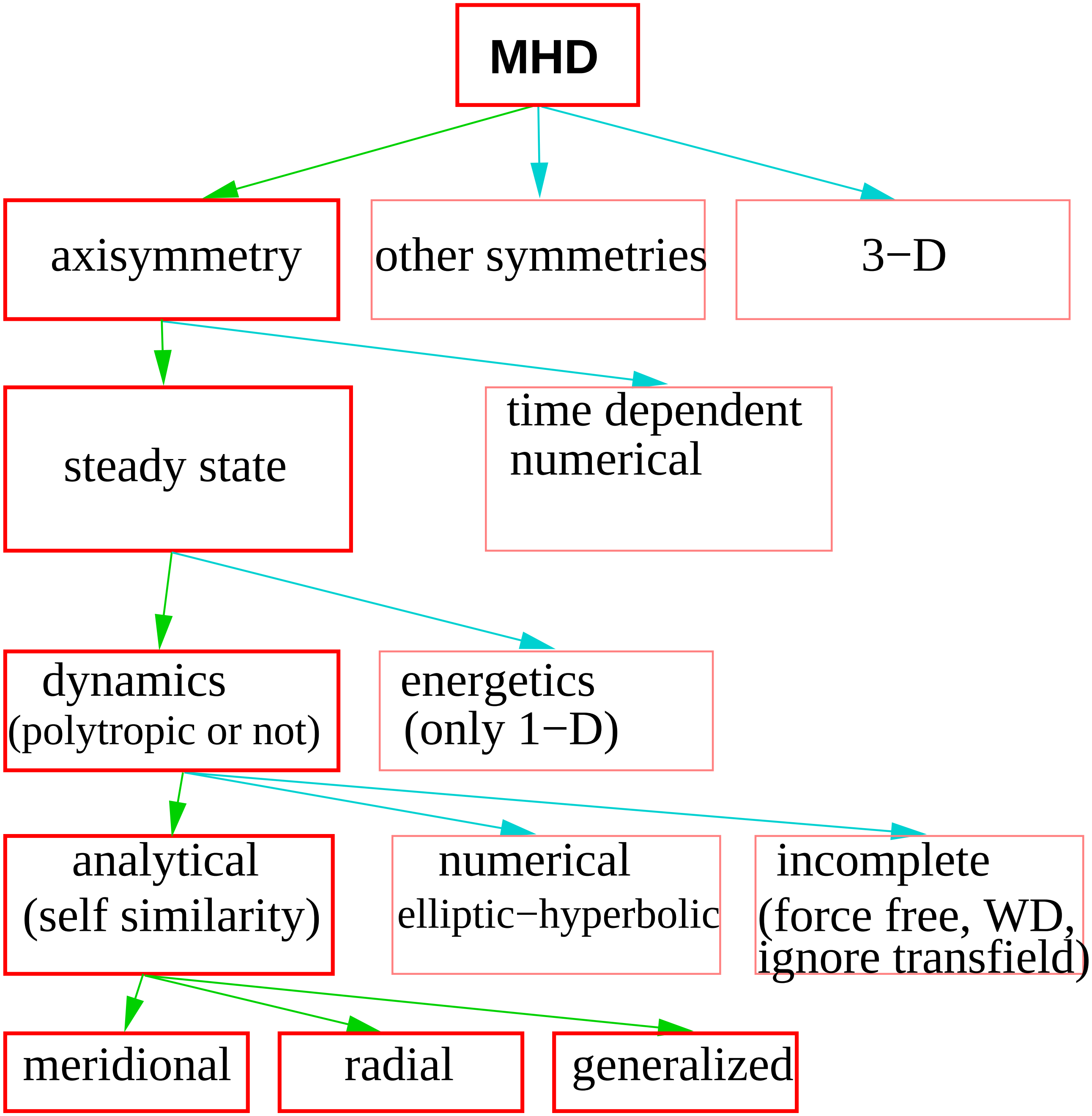
On Okamoto's anti-collimation theorem



De-collimation in the return-current regime.

But: (i) density higher near the axis, (ii) the return-current regime may not be distributed in the volume of the jet (current-sheet at the jet-boundary), (iii) in relativistic flows $\boldsymbol{E}$ collimates for $\nabla\gamma \cdot \nabla A < 0$.

Semi-analytical outflow models



Self-similarity

- simple 1-D models (e.g., Weber & Davis, or sperically summetric, monopole-like) cannot describe jets

- giving the flow-shape and solve for velocity is incomplete (solving the transfield is crucial for the acceleration)

- self-similarity: 1-D from mathematical point of view (ODEs), 2-D from physical
  we need an algorithm to produce all lines from a reference one
  Example: $A = r^x f(\theta)$:
  If a line A starts from r_0 at $\theta = \pi/2$, then $A = r_0^x f(\pi/2)$ and so,
  $r = r_0 \left[f(\pi/2)/f(\theta)\right]^{1/x}$.
  If we know one line – we know $f(\theta)$ – from the eq above we find all other lines.
  choose a coordinate system on the poloidal plane
  give the dependence on the one coordinate
  solve for the dependence on the other

The r self-similar nonrelativistic polytropic model

Blandford & Payne, Contopoulos & Lovelace, Rosso & Pelletier, Ferreira & Pelletier, Ostriker, ...,

Vlahakis et al 2000, Ferreira & Casse 2004

Using the flux function A, the integrals $\Psi_A(A)$, $L(A)$, $\Omega(A)$ and the definitions $\varpi_{\rm A} = \sqrt{L/\Omega}$, $G^2 = \varpi^2/\varpi_{\rm A}^2$, $\rho_{\rm A} = \Psi_A^2/4\pi$, $M^2 = \rho_{\rm A}/\rho$ we write

$$\boldsymbol{B} = \frac{\nabla A \times \hat{\phi}}{\varpi} - \frac{L\Psi_A}{\varpi}\frac{1-G^2}{1-M^2}\hat{\phi}\,, \boldsymbol{V} = \frac{M^2}{\Psi_A}\frac{\nabla A \times \hat{\phi}}{\varpi} + \frac{L}{\varpi}\frac{G^2-M^2}{1-M^2}\hat{\phi}\,, \rho = \frac{\Psi_A^2}{4\pi M^2}$$

For polytropic flows $P = Q(A)\rho^\Gamma = Q\left(\frac{\Psi_A^2}{4\pi}\right)^\Gamma \frac{1}{M^{2\Gamma}}$

Unknowns: M, A

Equations:

– Bernoulli (momentum along $\boldsymbol{V}$): $\frac{V_p^2}{2} + \frac{V_\phi^2}{2} + \frac{\Gamma}{\Gamma-1}\frac{P}{\rho} - \frac{\mathcal{GM}}{r} - \frac{\varpi\Omega B_\phi}{\Psi_A} = \mathcal{E}(A)$

– transfield: ...

Idea: choose $(r\,,\theta)$, give the r-dependence trying to find eqs with θ alone.

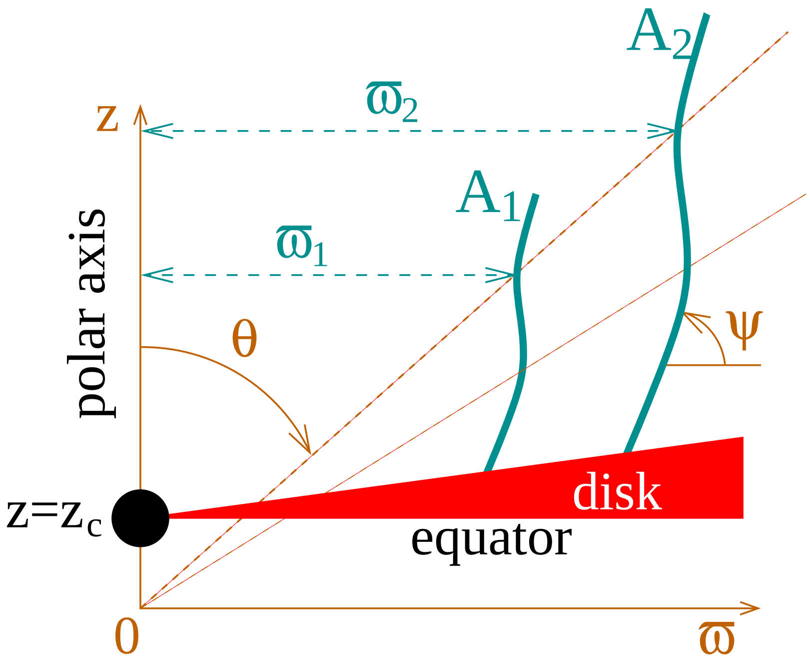
$M=M(\theta)$, $G=G(\theta)$, $A=r^x f(\theta)$

In the Bernoulli eq, give r-dependence to all terms such that they become proportional to gravity $(\propto 1/r)$:

- $\mathcal{E}(A)\propto 1/r$. Since $r=(A/f)^{1/x}$ this gives $\mathcal{E}=c_1 A^{-1/x}$
- $V_\phi^2/2\propto 1/r$. So $L\propto r^{1/2}$, or $L=c_2 A^{1/2x}$
- $-\varpi\Omega B_\phi/\Psi_A=L\Omega(1-G^2)/(1-M^2)$. So $L\Omega\propto A^{-1/x}$, or, $\Omega=c_3 A^{-3/2x}$
- $|\nabla A|^2/A^2=\left(x^2+f'^2/f^2\right)/r^2$ and
$V_p^2/2=(M^4/\Psi_A^2)(|\nabla A|^2/A^2)\propto(M^4/\sin^2\theta)(A^2/r^4\Psi_A^2)$.
This must be $\propto 1/r\Rightarrow\Psi_A=c_4 A^{1-3/2x}$
- Similarly $Q=c_5 A^{2-2\Gamma+(3\Gamma-4)/x}$
- With these forms, transfield gives an equation with θ alone!

So, we have unknowns $M(\theta)$, $G(\theta)$ and two ODEs wrt θ
($\varpi_{\rm A}=\sqrt{L/\Omega}$ and $G(\theta)\equiv\varpi/\varpi_{\rm A}$ give $f=\left(\frac{c_3}{c_2}\frac{\sin^2\theta}{G^2}\right)^{x/2}$)

This model corresponds to boundary conditions at a cone $\theta = \theta_i$:
$V_r = D_1 r^{-1/2}$, $V_\theta = D_2 r^{-1/2}$, $V_\phi = D_3 r^{-1/2}$, $\rho = D_4 r^{2x-3}$, $P = D_5 r^{2x-4}$,
$B_\phi = D_6 r^{x-2}$, $B_r = D_7 r^{x-2}$.



self-similar ansatz $r = \mathcal{F}_1(A)\, \mathcal{F}_2(\theta)$

For points on the same cone $\theta = const$,
$$\frac{\varpi_1}{\varpi_2} = \frac{r_1}{r_2} = \frac{\mathcal{F}_1(A_1)}{\mathcal{F}_1(A_2)}.$$

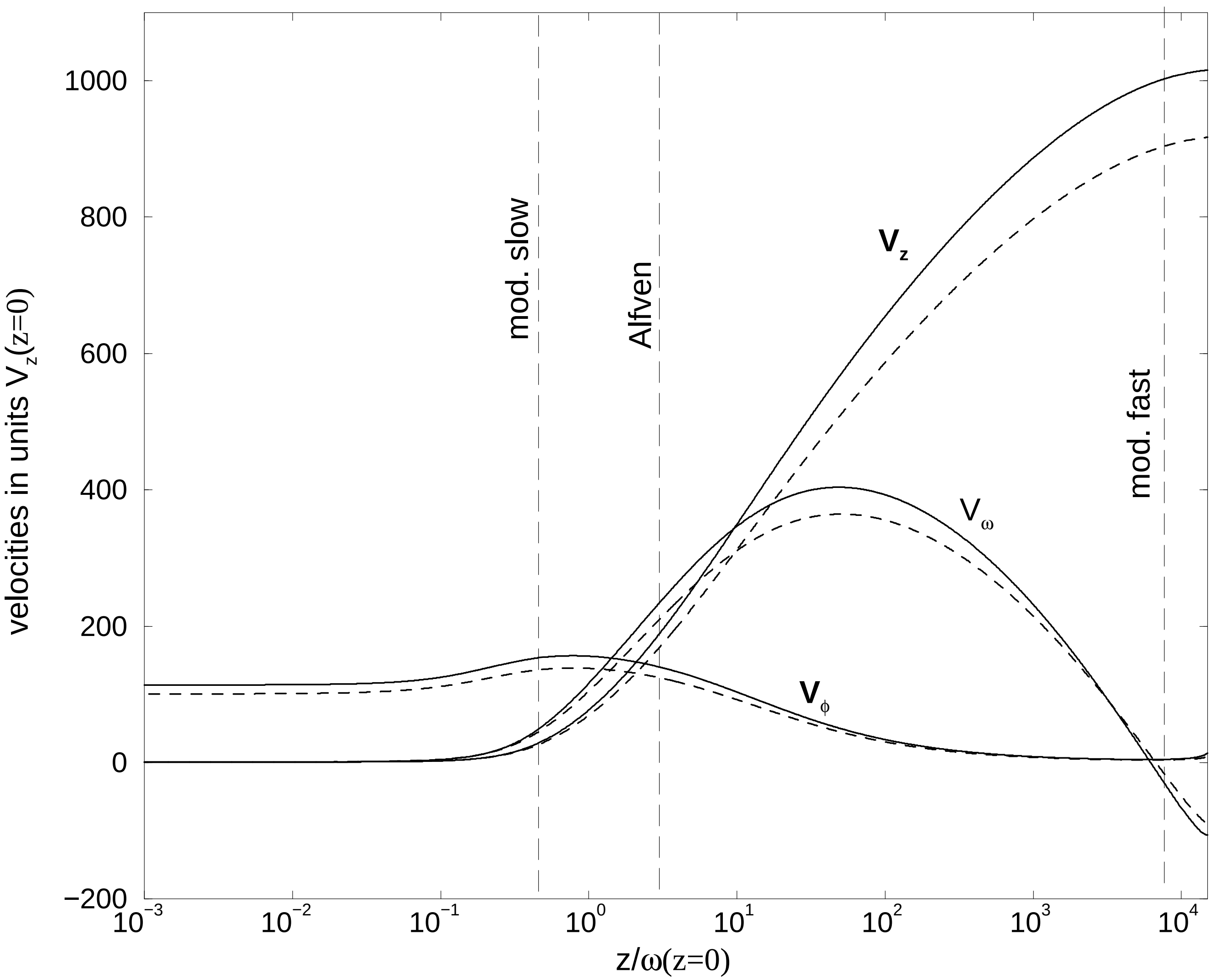
ODEs
$$\left\{ \begin{array}{l} \psi = \psi(G\,, M\,, \theta)\,, \text{(Bernoulli)} \\ \dfrac{dG}{d\theta} = \mathcal{N}_0(G\,, M\,, \psi\,, \theta)\,, \text{(definition of } \psi) \\ \dfrac{dM}{d\theta} = \dfrac{\mathcal{N}(G\,, M\,, \psi\,, \theta)}{\mathcal{D}(G\,, M\,, \psi\,, \theta)}\,, \text{(transfield)} \end{array} \right\}$$

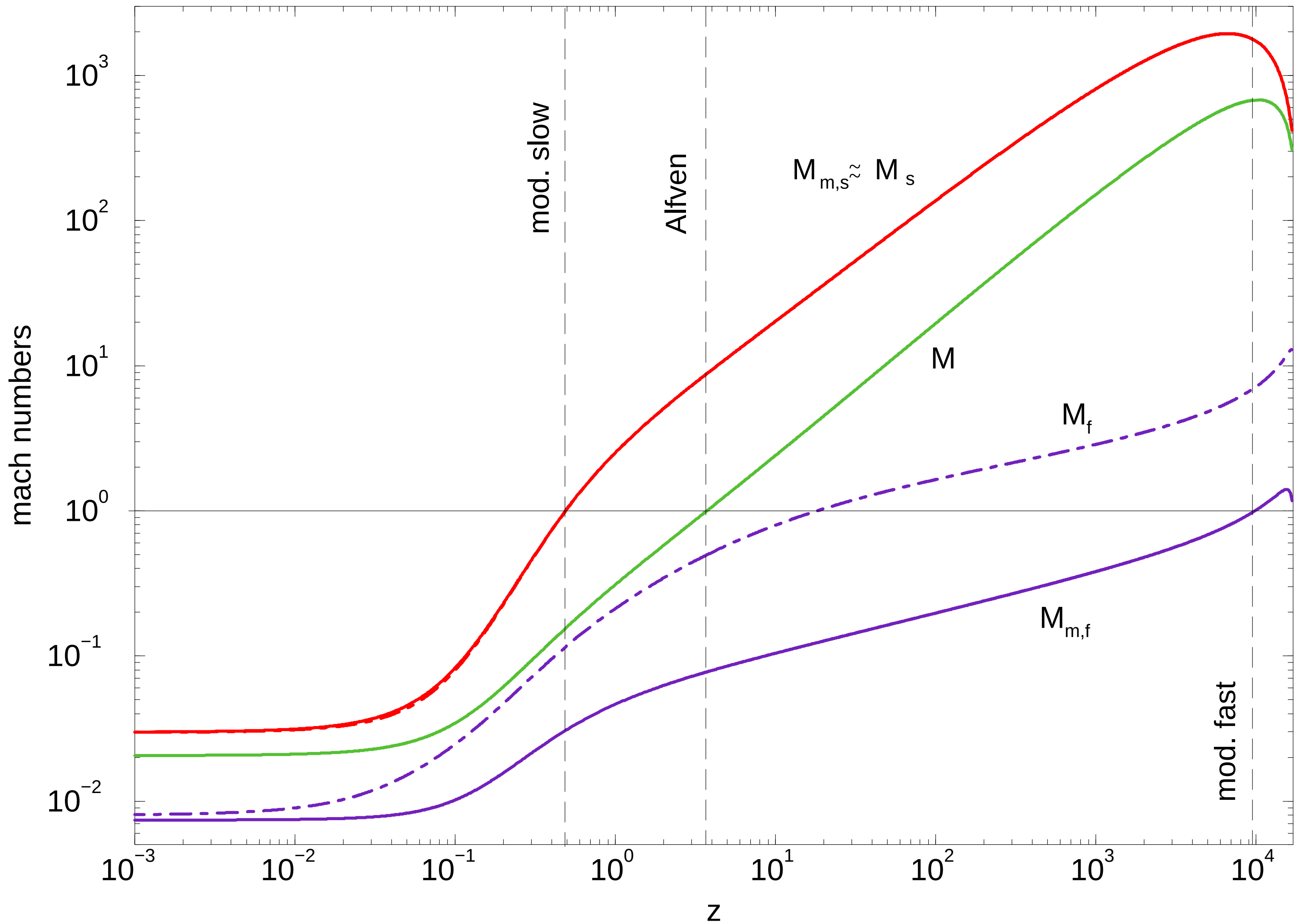
$\mathcal{D} = 0$: singular points
(Alfvén, modified slow - fast)

conical Alfvén surface

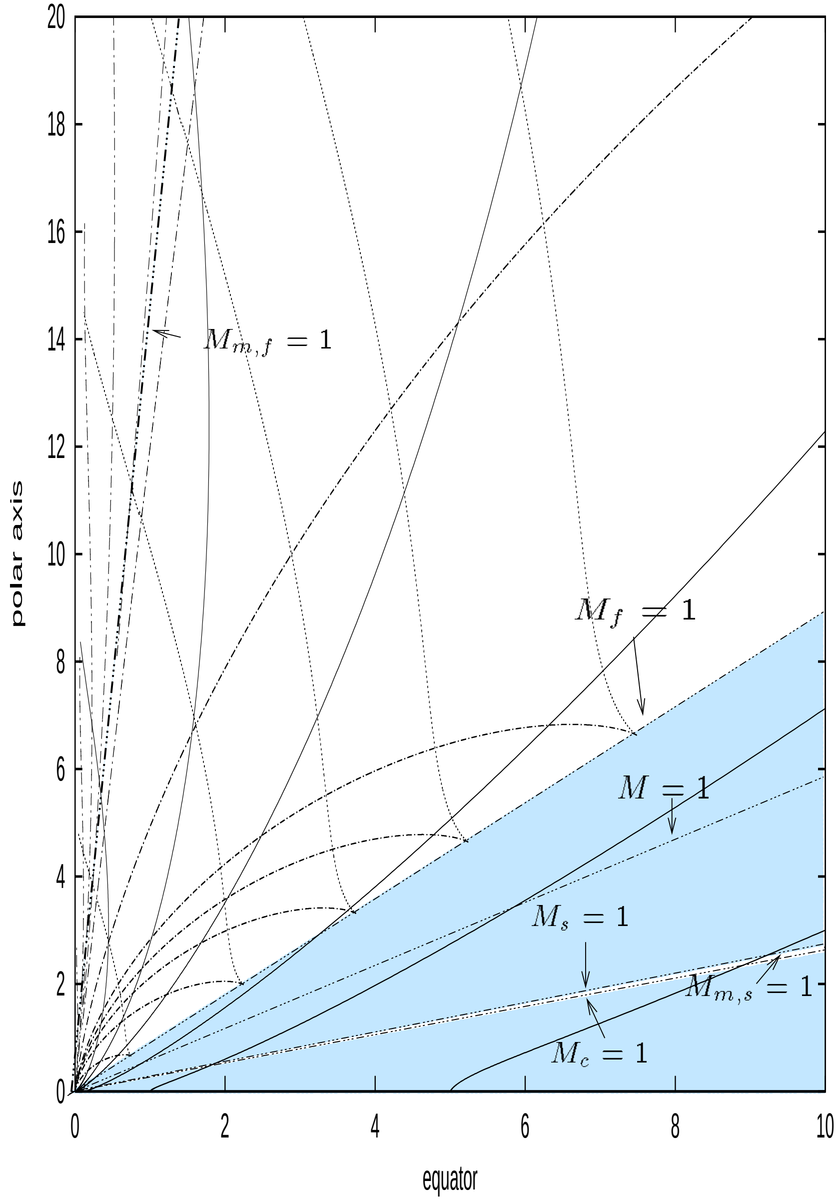
caveats: singular at $A = 0$, absence of scale (need to give $A_{\rm in}$, $A_{\rm out}$)

Vlahakis et al 2000

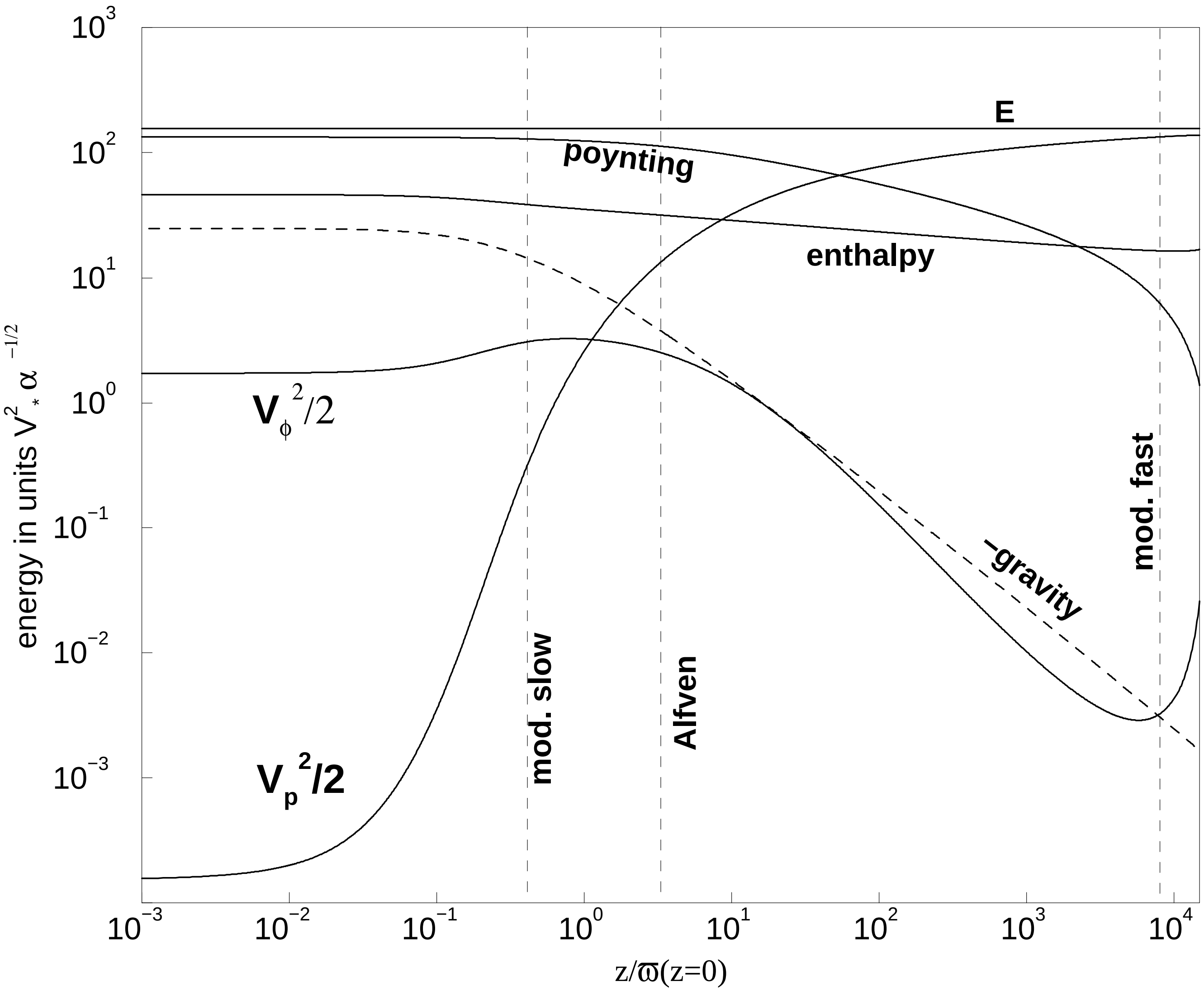




mod. slow
Alfven
mod. fast
$M_{m,s} \approx M_s$
M
M_f
$M_{m,f}$
mach numbers
z
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{-3}
10^{4}



polar axis
equator
$M_{m,f} = 1$
$M_f = 1$
$M = 1$
$M_s = 1$
$M_{m,s} = 1$
$M_c = 1$
0
2
4
6
8
10
12
14
16
18
20

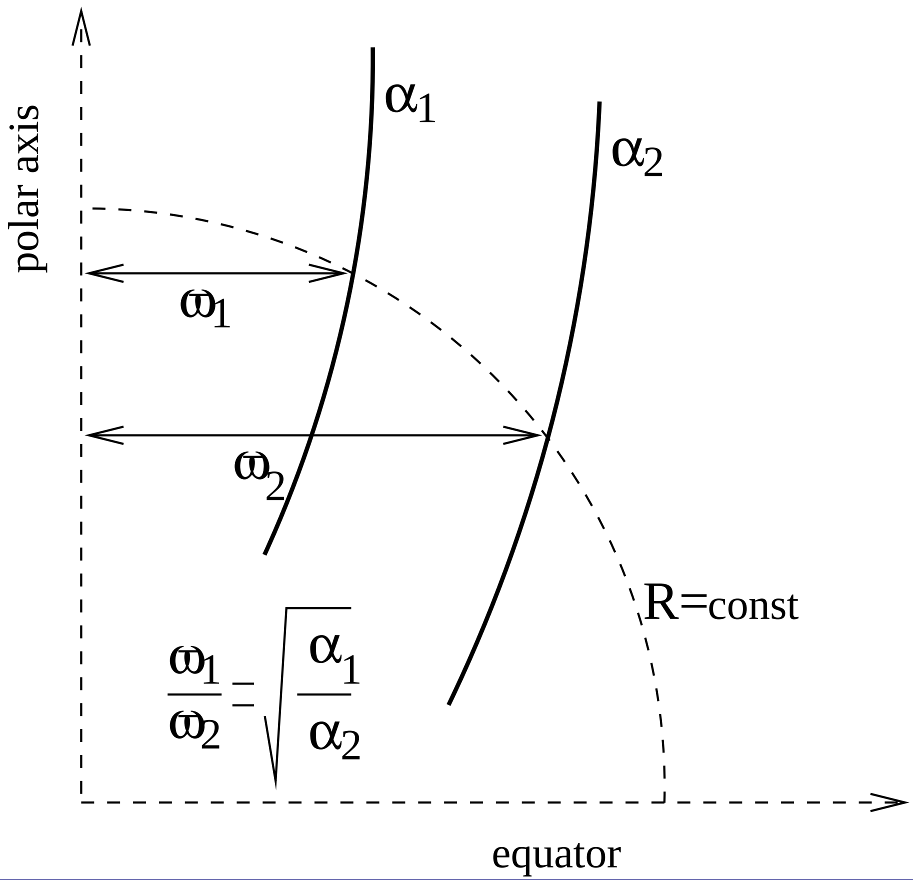


E
poynting
enthalpy
$V_\phi{}^2/2$
$V_p{}^2/2$
mod. slow
Alfven
mod. fast
−gravity
energy in units $V_*^2\,\alpha^{-1/2}$
$z/\varpi(z=0)$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4}

non-polytropic θ self-similar models

$\boxed{M = M(R)}$, $R = r/r_{\rm A}$, $M(1) = 1$

$\boxed{G = G(R)}$, so $\alpha \equiv \dfrac{\varpi_{\rm A}^2}{r_{\rm A}^2} = \dfrac{R^2}{G^2(R)} \sin^2\theta$.



Systematic Construction

$(r\,,\theta) \longrightarrow ({\color{red}R}\,,{\color{blue}\alpha})$

$\hat{\theta}\cdot$ momentum equation $\Leftrightarrow$

$$\frac{\partial}{\partial\alpha}\left(P+\frac{B^2}{8\pi}\right)=\colorbox{red}{1}\cdot\colorbox{blue}{1}+\colorbox{red}{2}\cdot\colorbox{blue}{2}\Rightarrow$$

$$P+\frac{B^2}{8\pi}=\sum_{i=1}^{2}\colorbox{red}{i}\cdot\colorbox{blue}{i}+\colorbox{red}{0}$$

Substitute in $\hat{r}\cdot$ momentum equation $\rightarrow \displaystyle\sum_{i=1}^{7}\colorbox{red}{i}\cdot\colorbox{blue}{i}=0$

The same if we substitute the poloidal momentum eq with

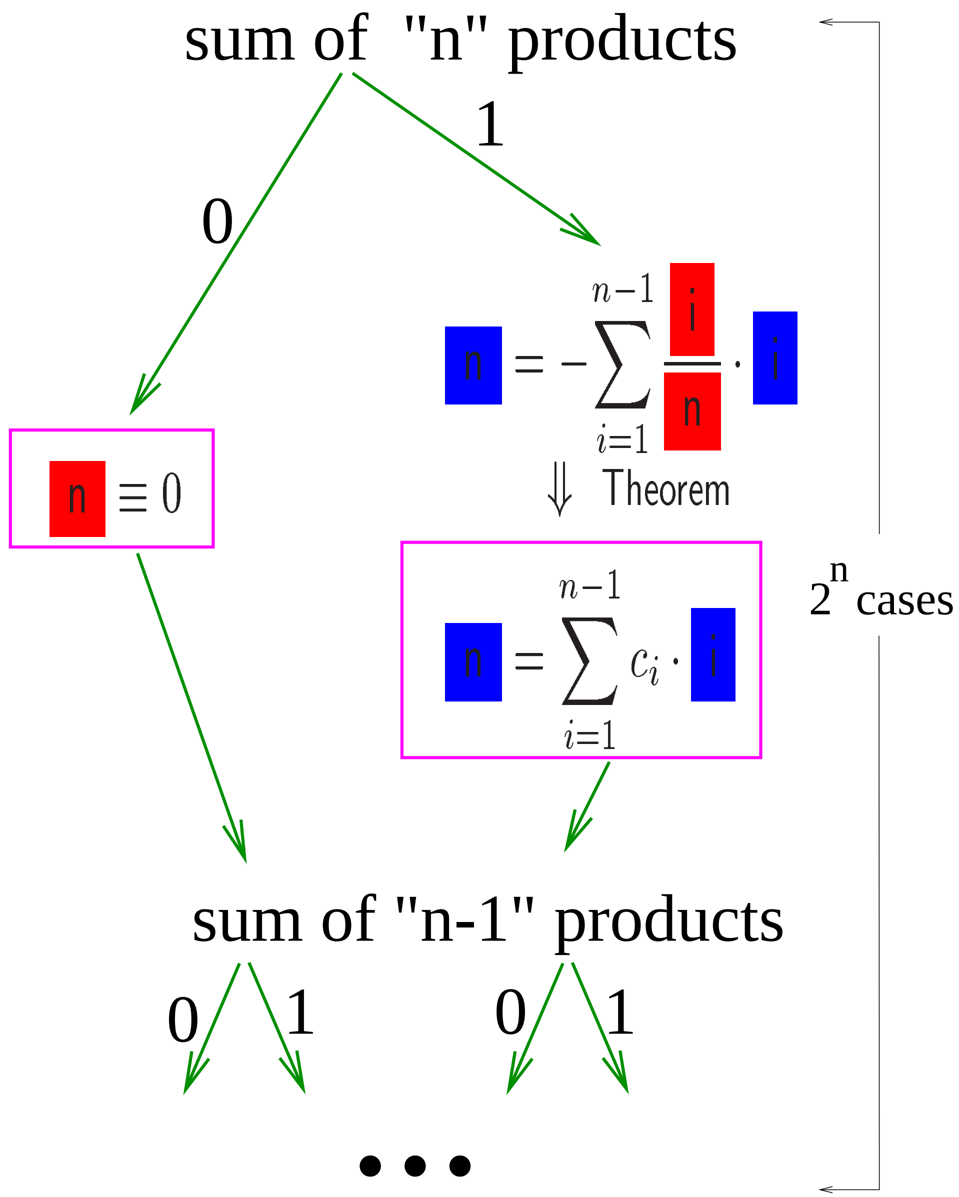
$$\nabla\times\nabla P=0\Leftrightarrow\nabla\times\left[-\rho(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}+\boldsymbol{J}\times\boldsymbol{B}/c-\rho\mathcal{G}\mathcal{M}/r^2\right]=0.$$

Theorem.

$$\colorbox{blue}{}=\sum_{i=1}^{n}\colorbox{red}{i}\cdot\colorbox{blue}{i}\Rightarrow\exists\ c_i\ :\ \colorbox{blue}{}=\sum_{i=1}^{n}c_i\cdot\colorbox{blue}{i}$$

(when $\displaystyle\sum_{i=1}^{n}\left(\colorbox{red}{i}-c_i\right)\cdot\colorbox{blue}{i}=0$*)*

$$\sum_{i=1}^{n} \boxed{i} \cdot \boxed{i} = 0$$



resulting in:

• 9 cases (including the model by Sauty, Tsinganos, & Trussoni)

• 5 cases with radial fieldlines (including Parker's solution and Lima, Tsinganos, & Priest solution)

– all non-polytropic – heating important

– thermally driven

– confinement could be due to pressure or magnetic self-collimation

Sauty, Tsinganos, & Trussoni 2004

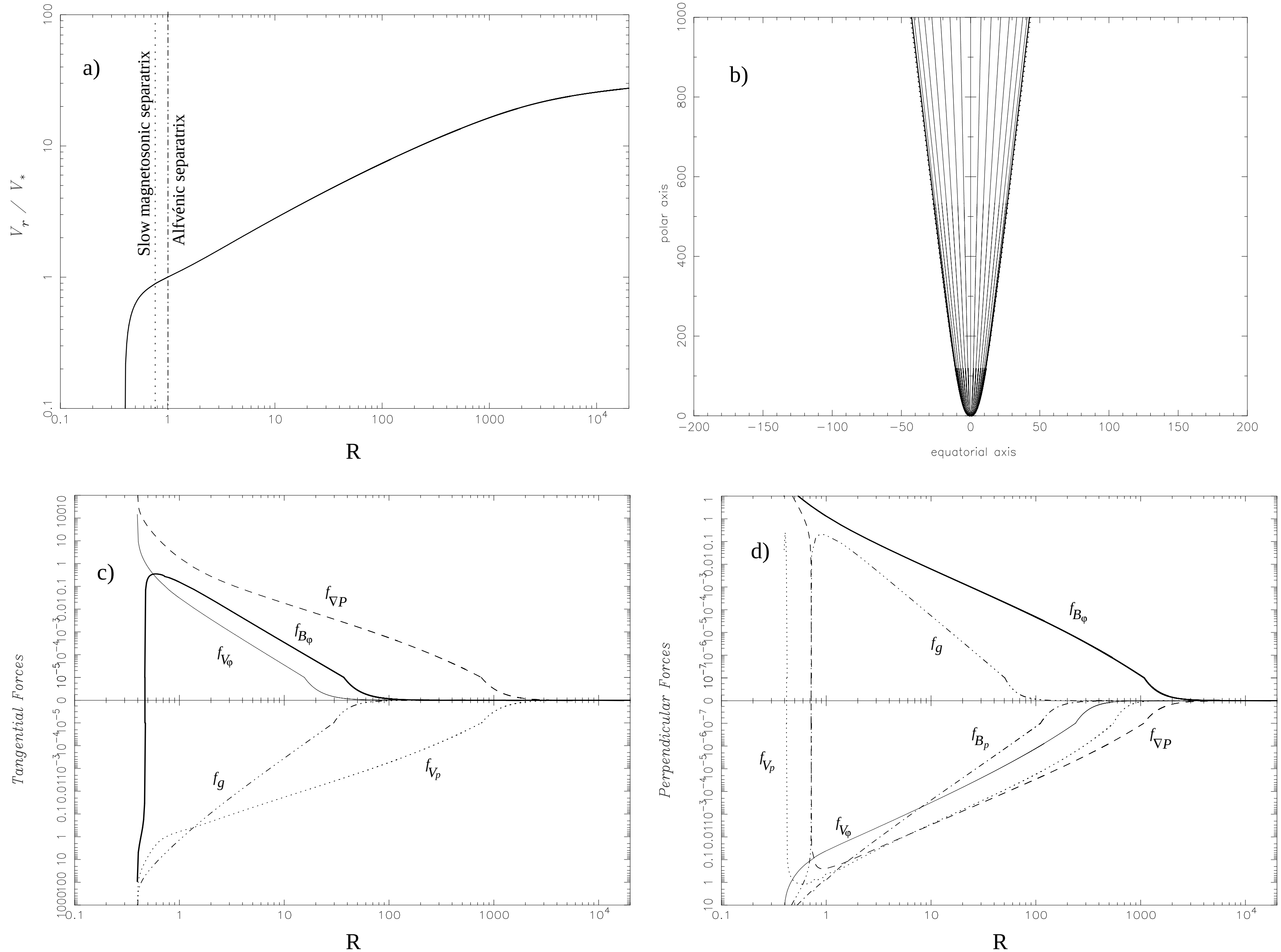
$$P = \frac{\rho_\star V_\star^2}{2}\Pi(R)(1+\kappa\alpha) + P_0\,,$$

$$\rho = \frac{\rho_\star}{M^2}\left(1+\delta\alpha\right),$$

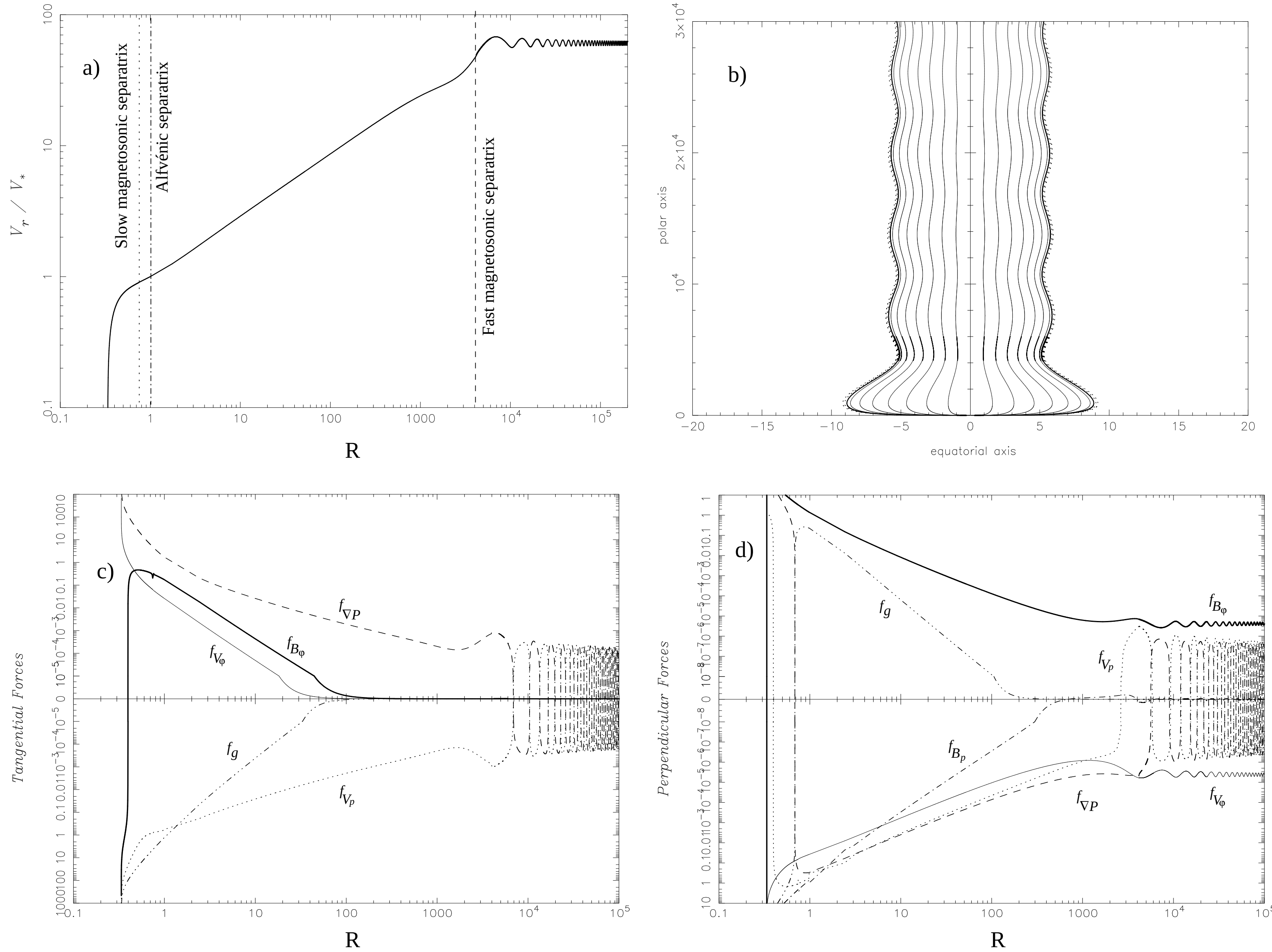
$$\vec{B} = B_\star\left(\frac{\cos\theta}{G^2}\hat{r} - \frac{F\sin\theta}{2G^2}\hat{\theta} - \lambda\sqrt{\alpha}\frac{1-G^2}{G\left(1-M^2\right)}\hat{\phi}\right),$$

$$\vec{V} = \frac{\Psi_A}{4\pi\rho}\vec{B} + \Omega r\sin\theta\hat{\phi}\,,$$

(expansion around the axis)

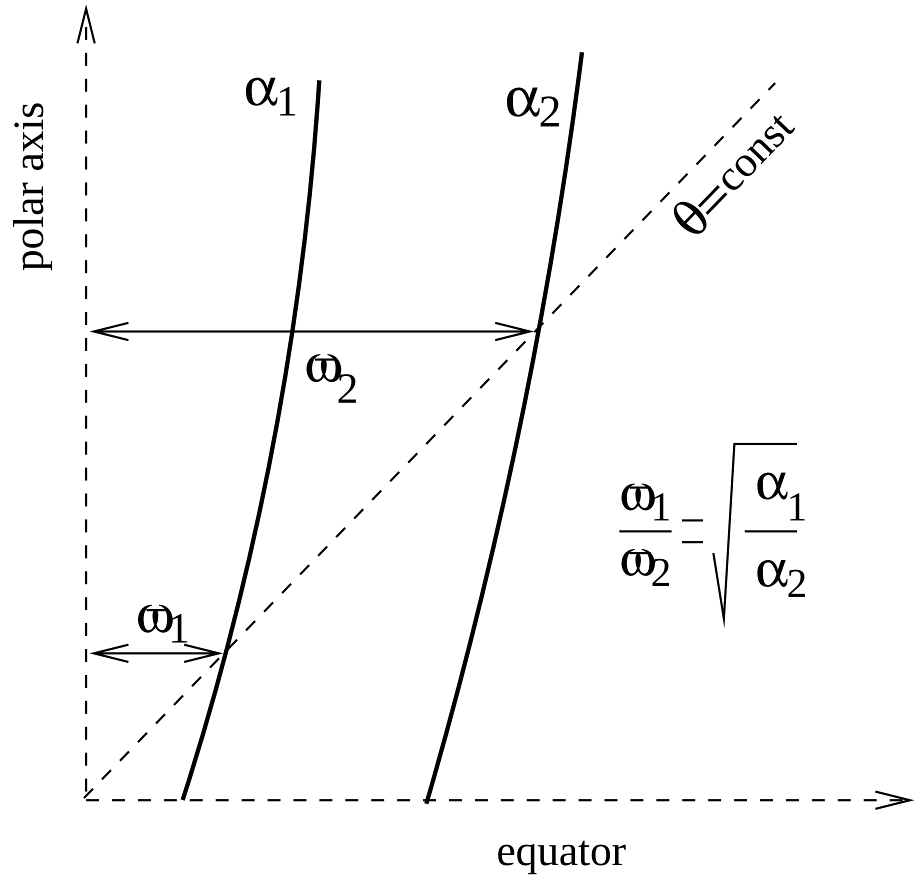


a)
Slow magnetosonic separatrix
Alfvénic separatrix
$V_r \ / \ V_*$
R
b)
polar axis
equatorial axis
c)
$f_{\nabla P}$
f_{B_φ}
f_{V_φ}
f_g
f_{V_p}
Tangential Forces
R
d)
f_{B_φ}
f_g
f_{B_p}
$f_{\nabla P}$
f_{V_p}
f_{V_φ}
Perpendicular Forces
R


a)
Slow magnetosonic separatrix
Alfvénic separatrix
Fast magnetosonic separatrix
$V_r\ /\ V_*$
R
b)
polar axis
equatorial axis
c)
$f_{\nabla P}$
f_{B_φ}
f_{V_φ}
f_g
f_{V_p}
Tangential Forces
R
d)
f_g
f_{B_φ}
f_{V_p}
f_{B_p}
$f_{\nabla P}$
f_{V_φ}
Perpendicular Forces
R

non-polytropic r self-similar models

$\boxed{M = M(\theta)}$ $\boxed{G = G(\theta)}$, so $\alpha \equiv \dfrac{\varpi_A^2}{\varpi_0^2} = \left(\dfrac{r}{\varpi_0}\right)^2 \dfrac{\sin^2\theta}{G^2(\theta)}$.

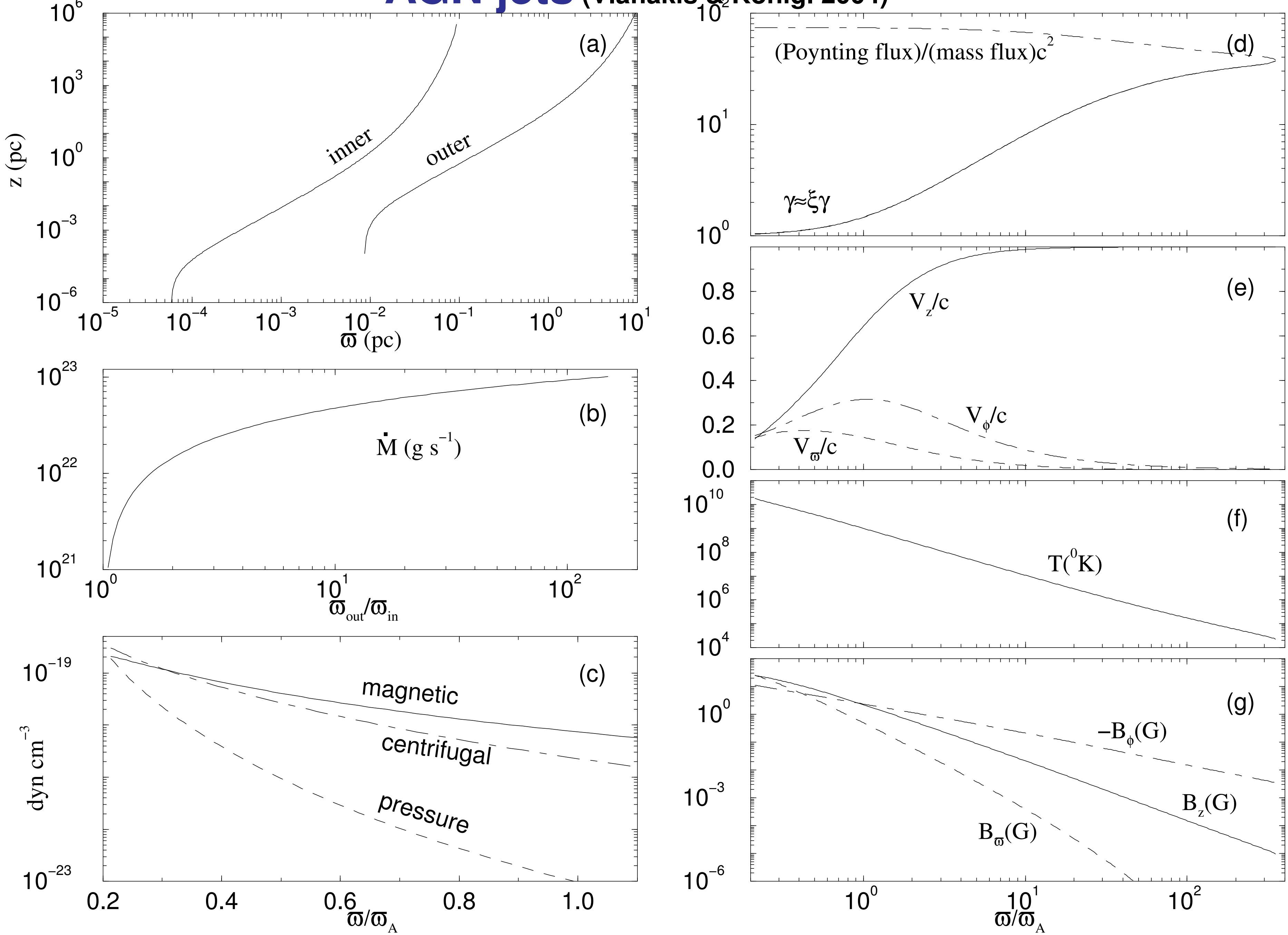


6 cases including the generalized Blandford & Payne model (the only polytropic)

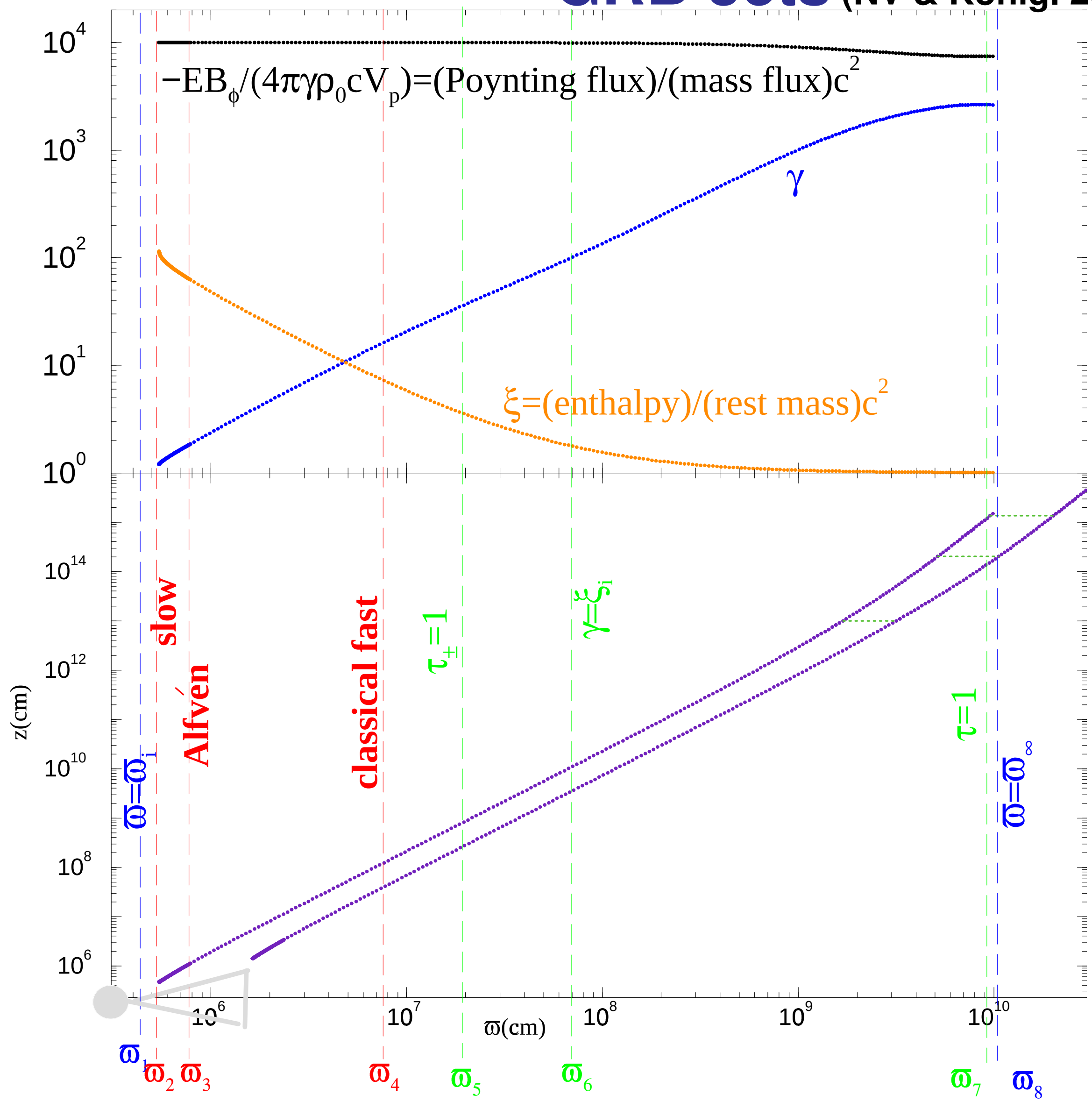
relativistic polytropic r self-similar models

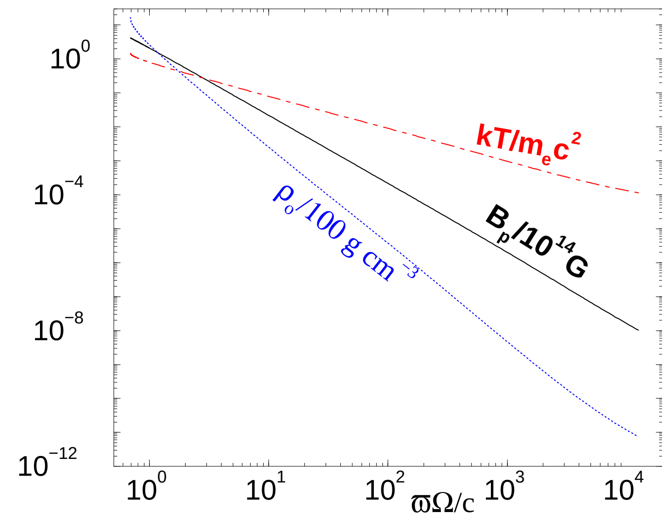
- special relativistic
- the cold version found by Li, Chiueh, & Begelman (1992) and Contopoulos (1994); Vlahakis & Königl (2003) included thermal effects
- reference velocity c (and not $V_{\rm K} \propto r^{-1/2}$)
- corresponds to boundary conditions on a cone $\theta = \theta_i$: $B_\theta = -\mathcal{C}_1 r^{F-2}$, $B_\phi = -\mathcal{C}_2 r^{F-2}$, $V_r/c = \mathcal{C}_3$, $V_\theta/c = -\mathcal{C}_4$, $V_\phi/c = \mathcal{C}_5$, $\rho_0 = \mathcal{C}_6 r^{2(F-2)}$, and $P = \mathcal{C}_7 r^{2(F-2)}$, where F = parameter.

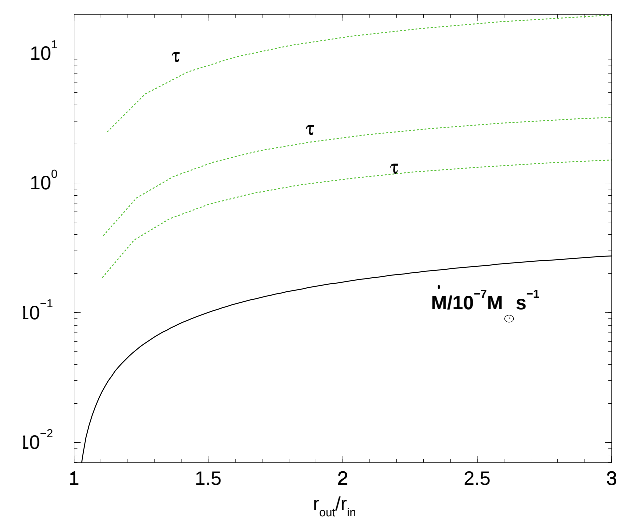
AGN jets (Vlahakis & Königl 2004)



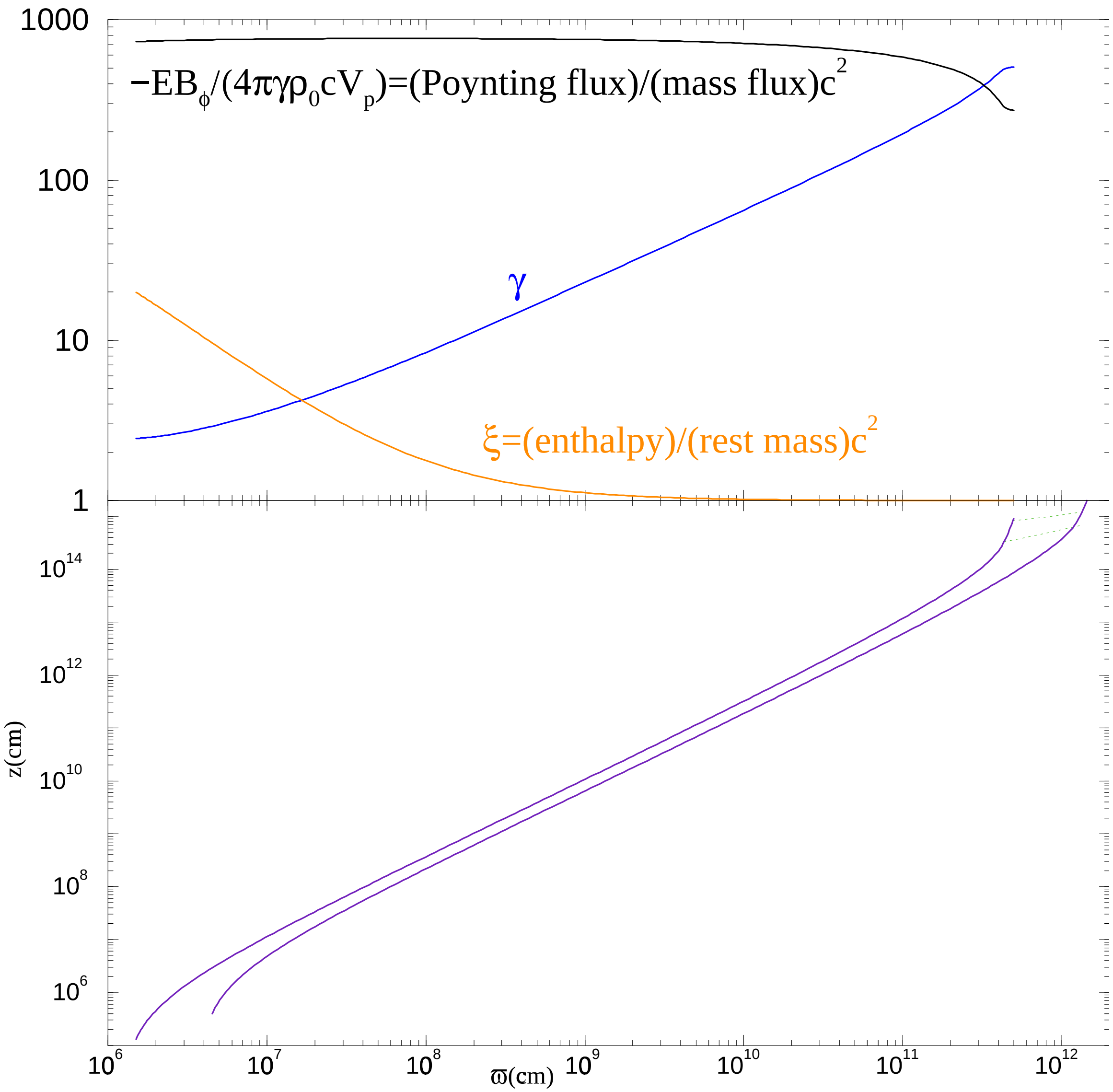
GRB Jets (NV & Königl 2001, 2003a,b)

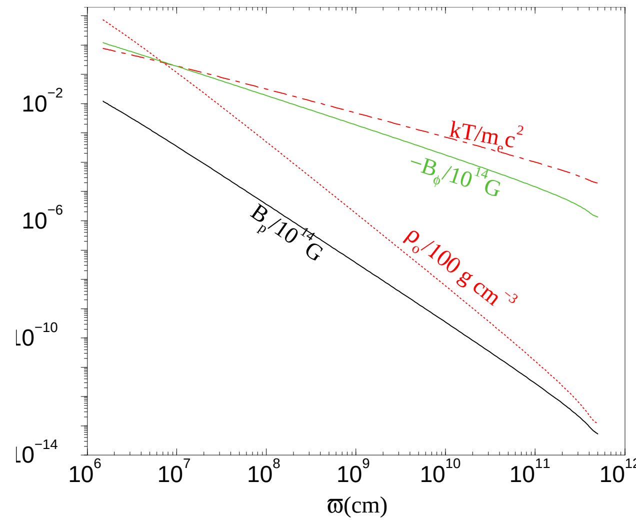


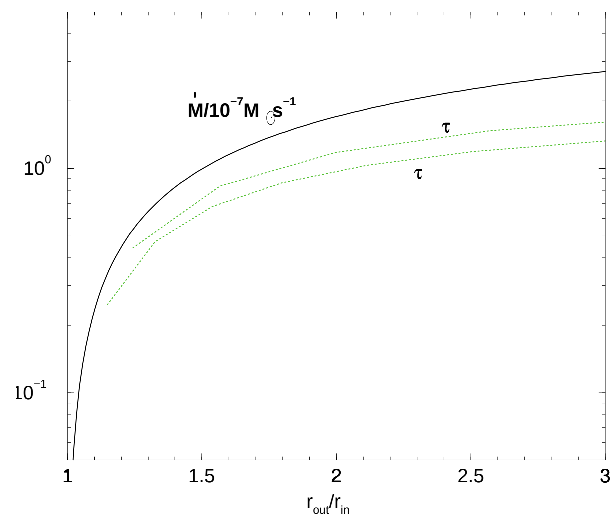




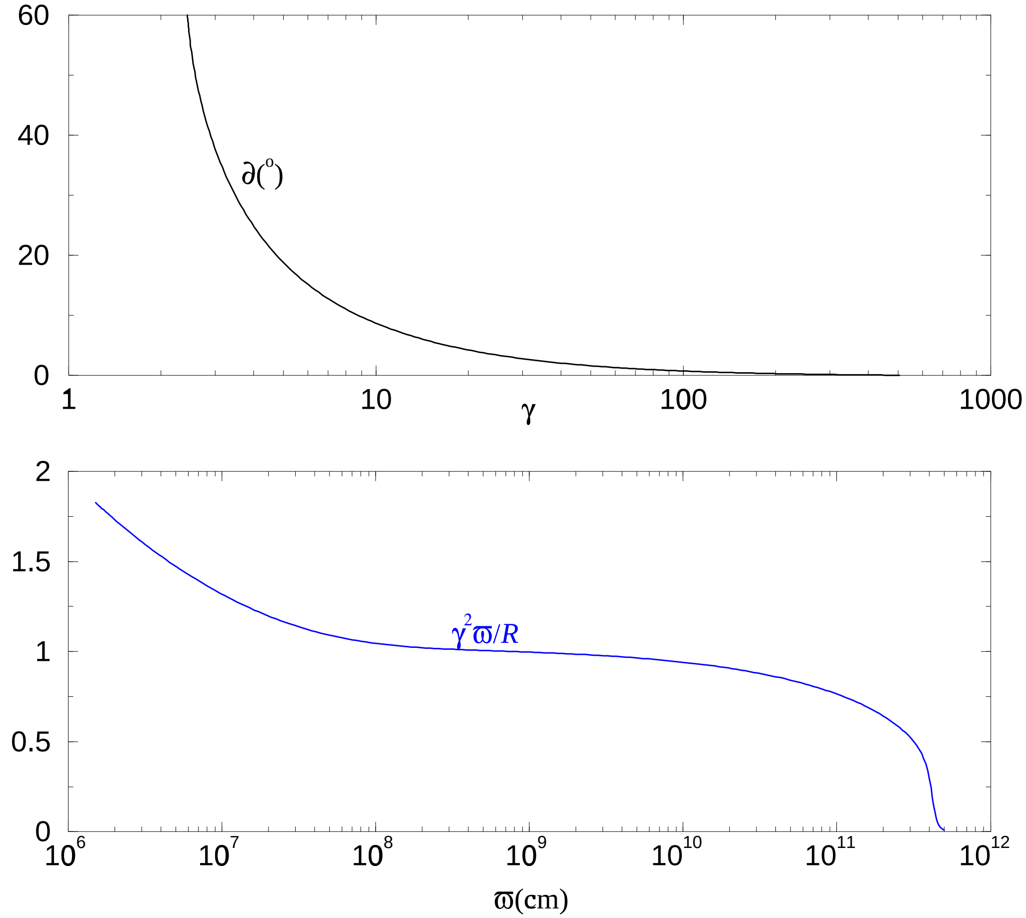
- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$







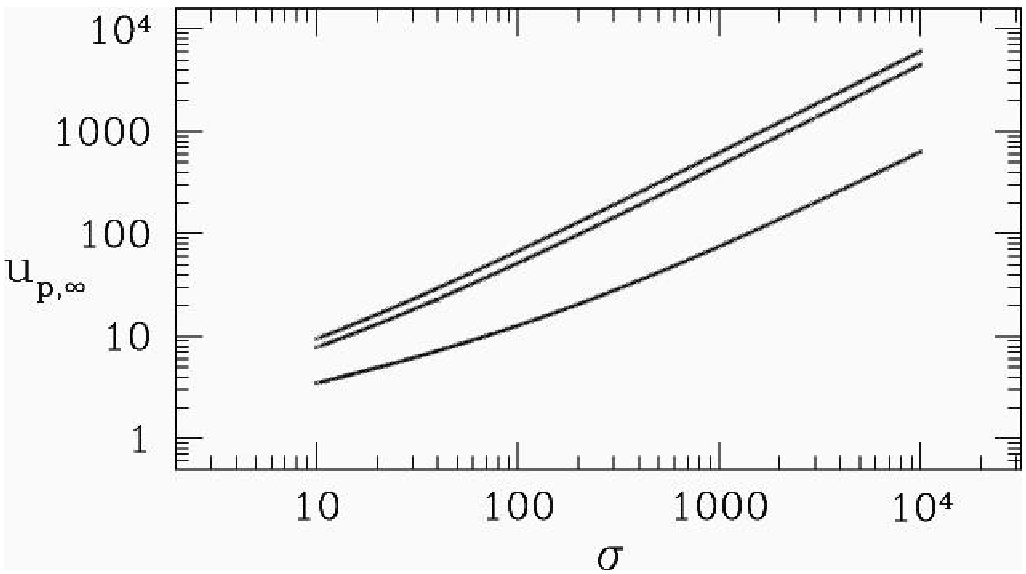
- Thermal acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$, $T \propto \varpi^{-0.8}$, $B_\phi \propto \varpi^{-1}$, $z \propto \varpi^{1.5}$)
- Magnetic acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$)
- cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$

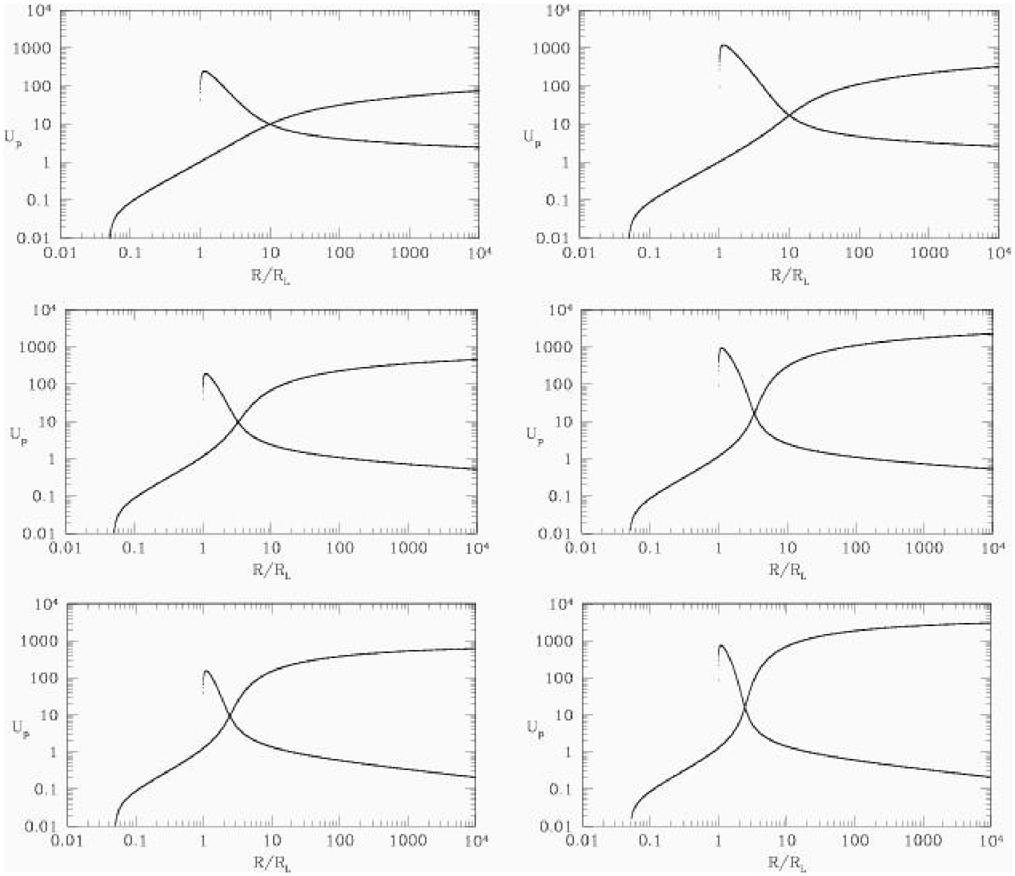


⋆ At $\varpi = 10^8$cm – where $\gamma = 10$ – the opening half-angle is already $\vartheta = 10^o$

⋆ For $\varpi > 10^8$cm, collimation continues slowly ($\mathcal{R} \sim \gamma^2 \varpi$)

Fendt & Ouyed (2004)

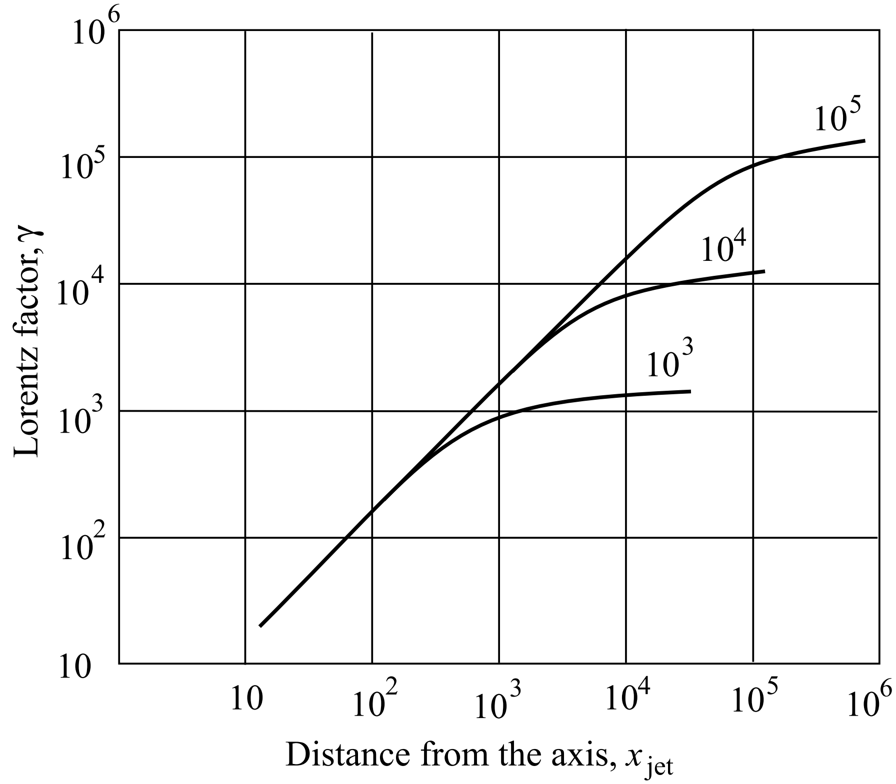




They used prescribed fieldlines (with $\varpi^2 B_p/A \propto \varpi^{-q}$) and found efficient acceleration with γ_∞ (their $u_{p,\infty}$) $\sim \mu$ (their σ).

Although the analysis is not complete (the transfield is not solved), the results show the relation between line-shape and efficiency.

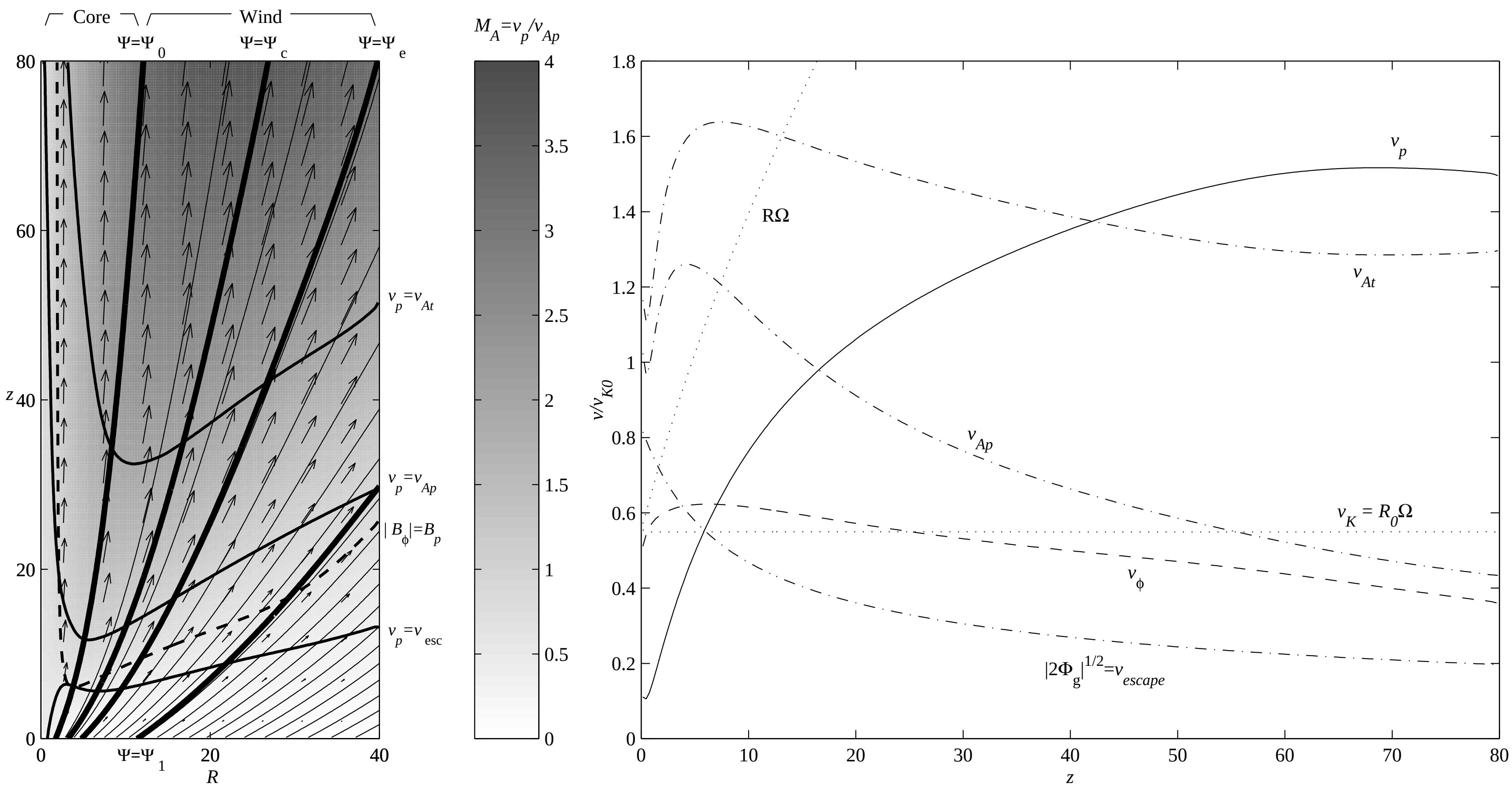
Beskin & Nokhrina (2006)



By expanding the equations wrt $2/\mu$ (their $1/\sigma$) they found a parabolic solution. The acceleration in the superfast regime is efficient, reaching $\gamma_\infty \sim \mu$.
The scaling $\gamma \propto \varpi$ is the same as in Vlahakis & Königl (2003a).

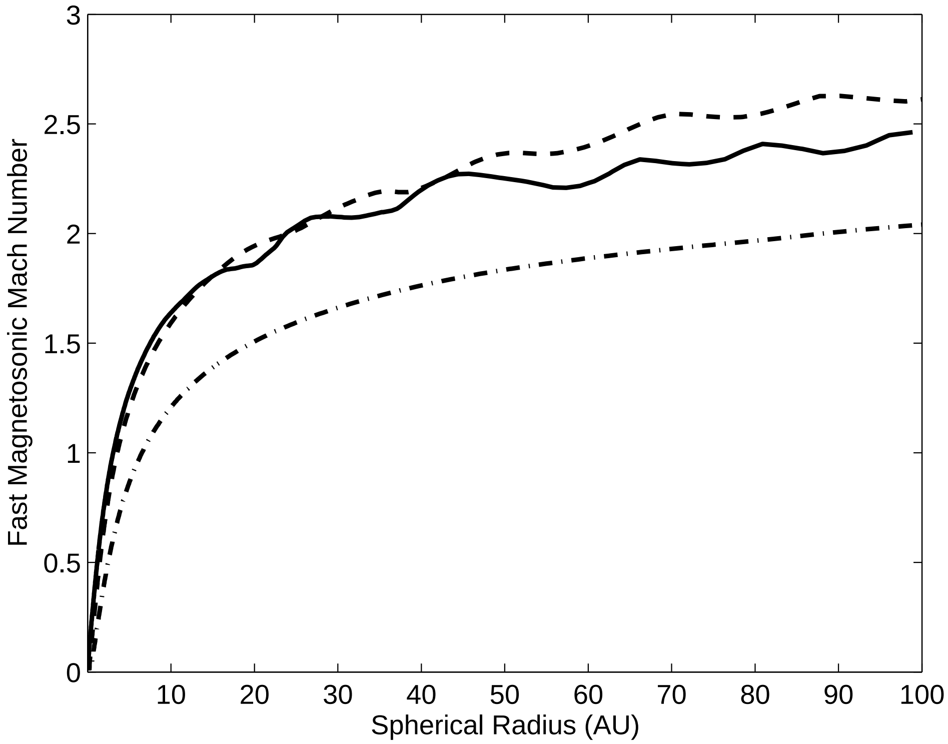
Examples of outflow simulations

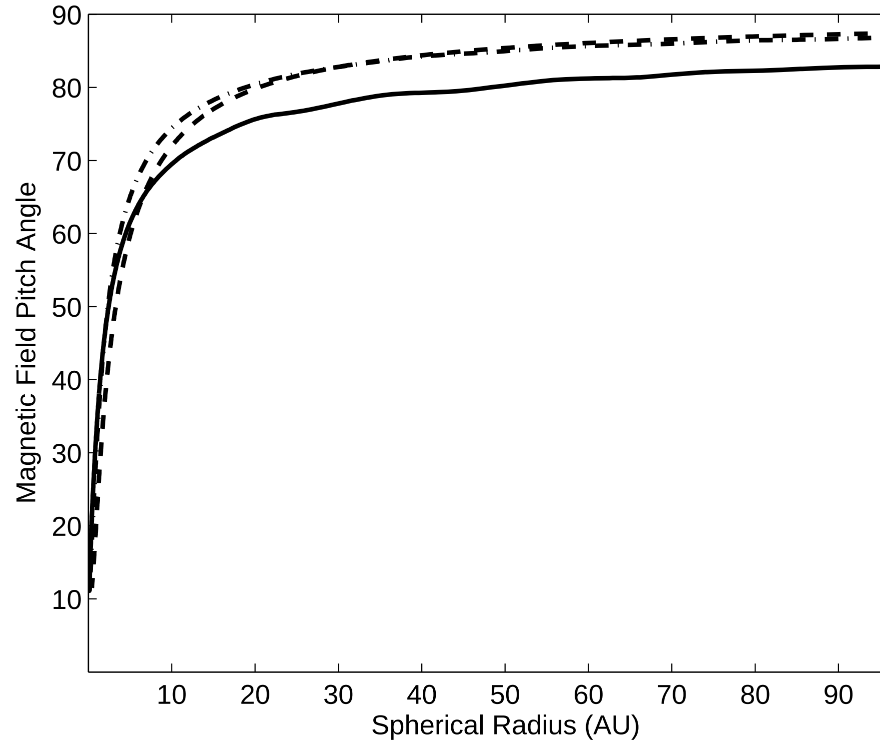
Krasnopolsky et al (1999)

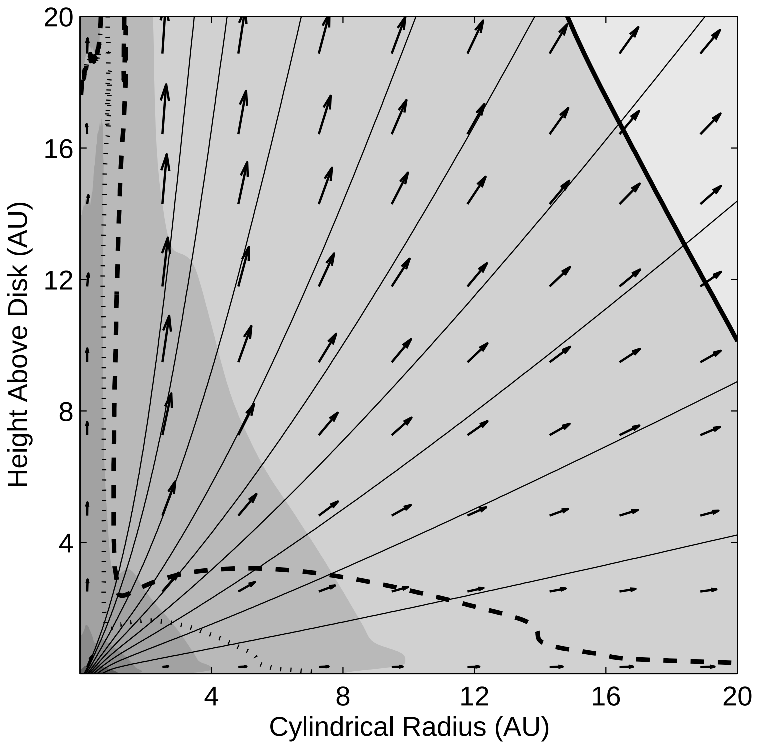


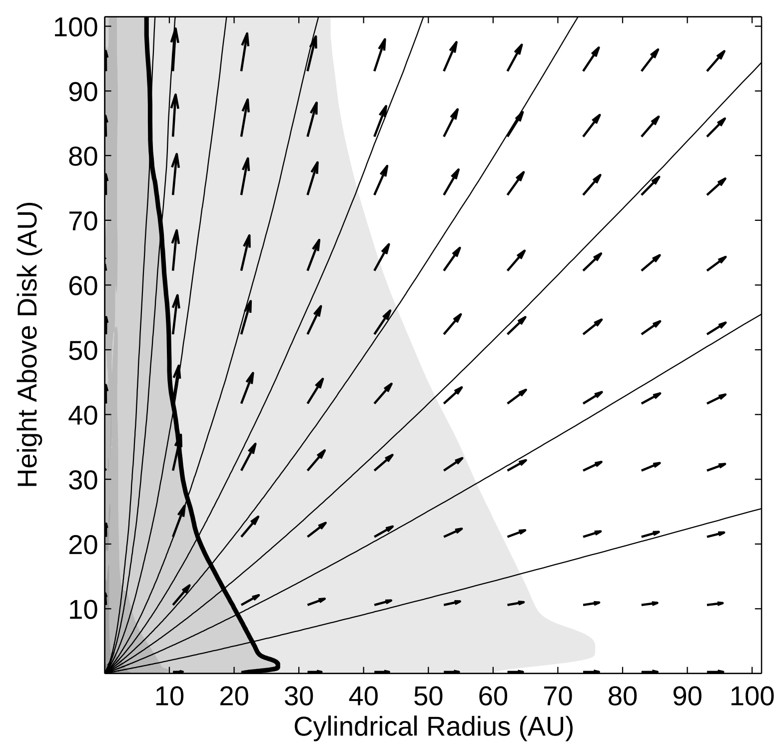
boundary conditions mimic the r self-similar solution

Anderson et al (2005)



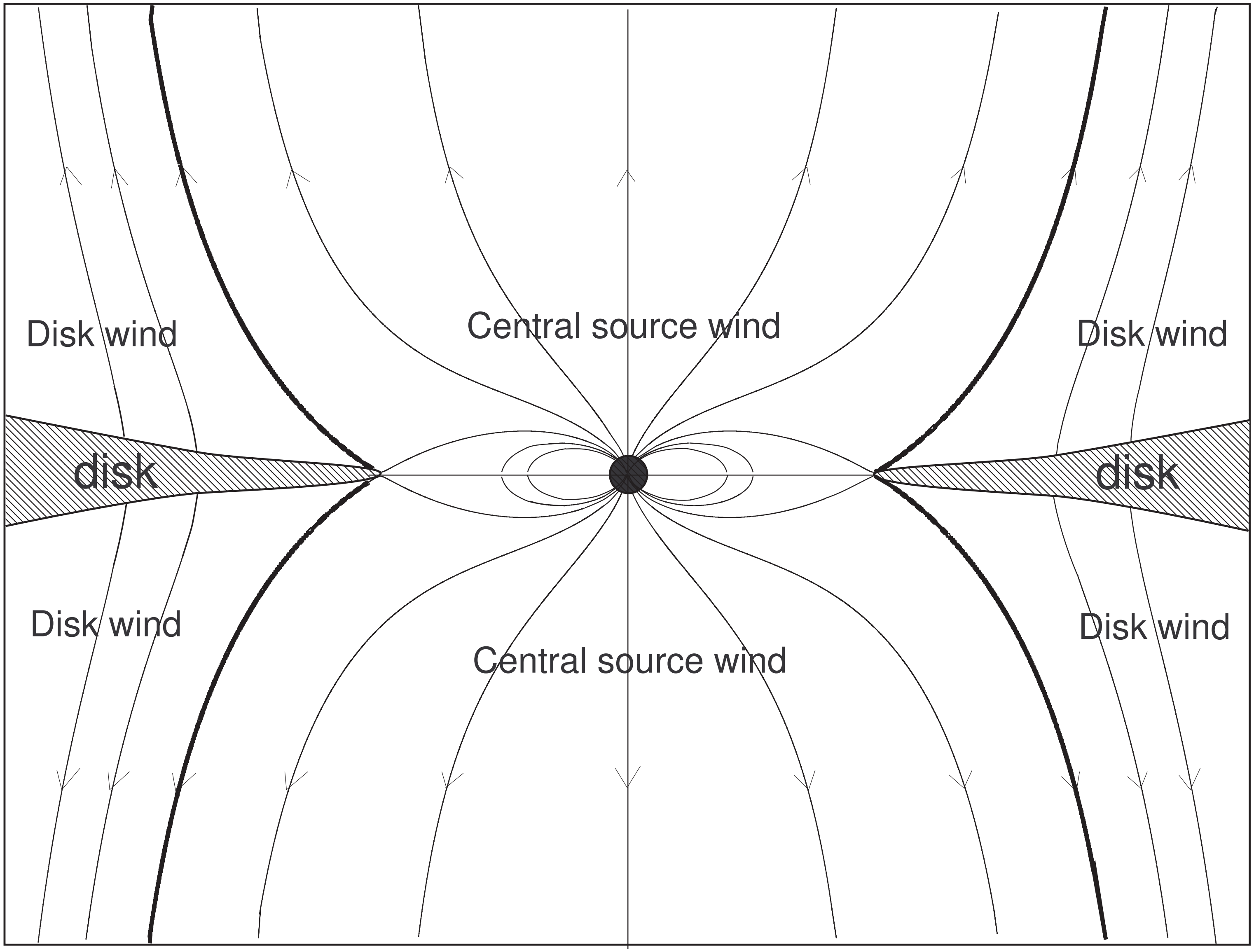


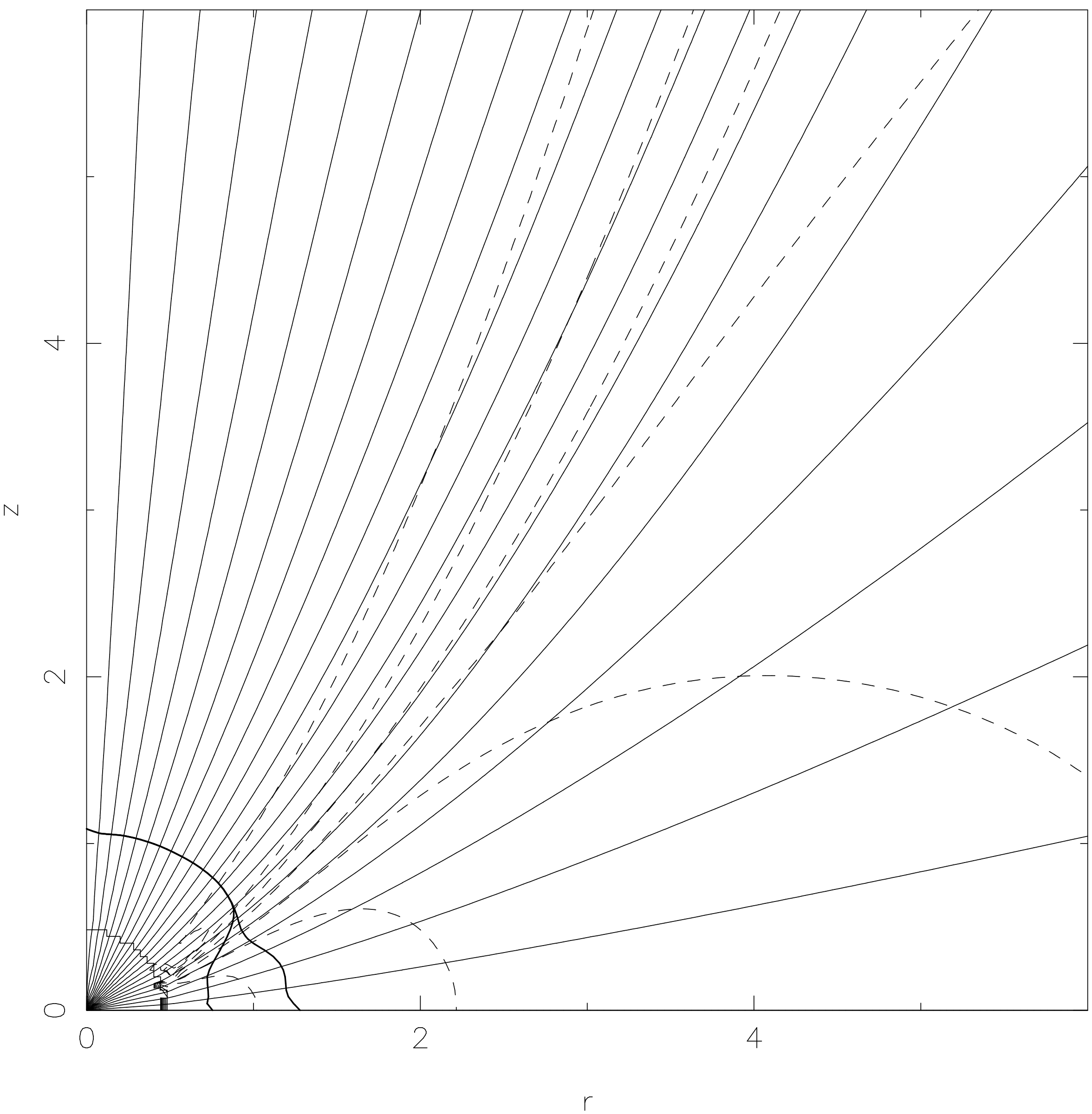




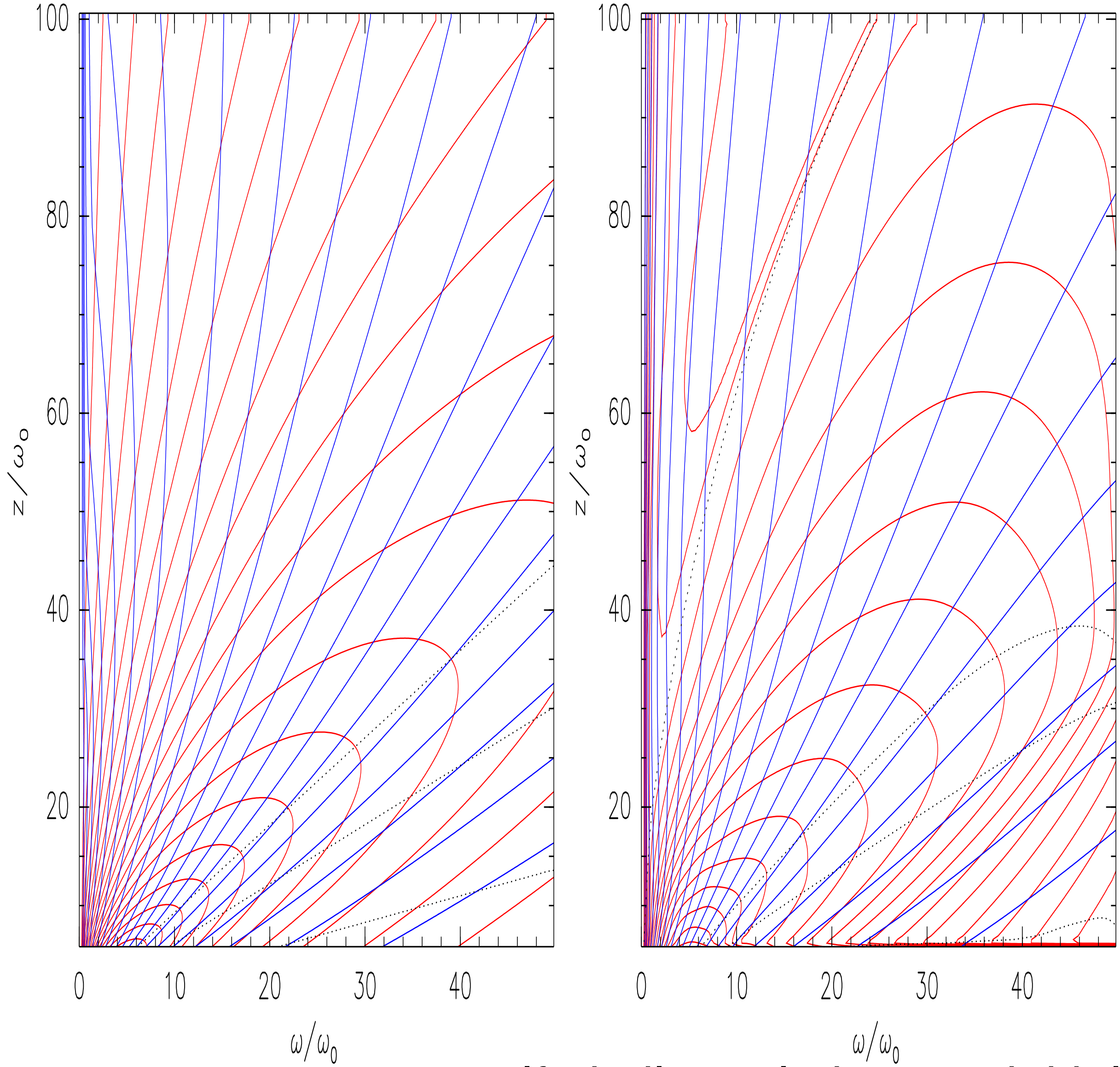
with equatorial line collimation less effective

Bogovalov & Tsinganos (2001)




0
2
4
r
z

Gracia et al (2006)



blue: flow-lines

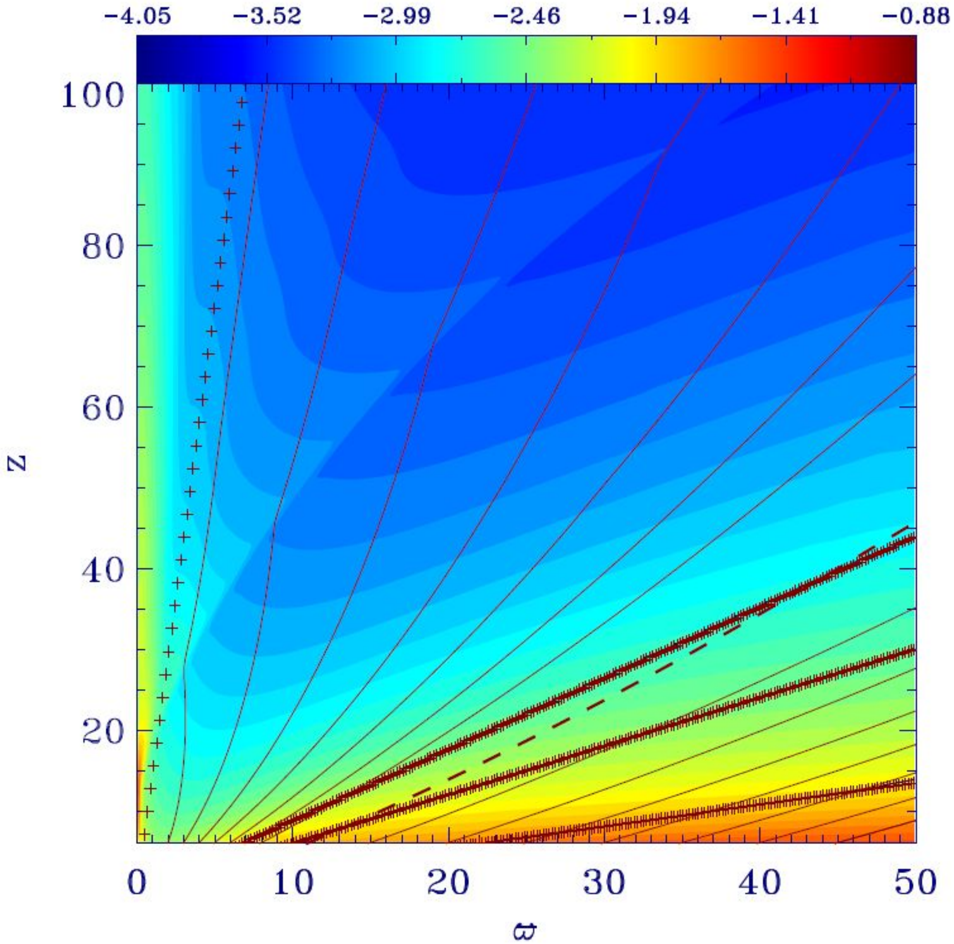
red: current-lines

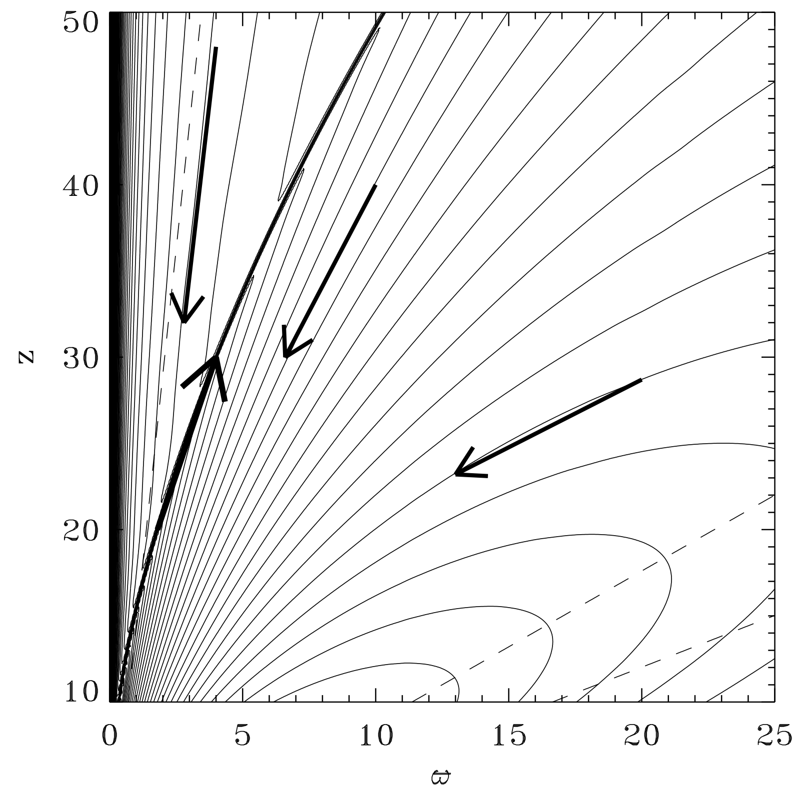
(left: self-similar,

right: simulation)

use r self-similar solutions as initial conditions

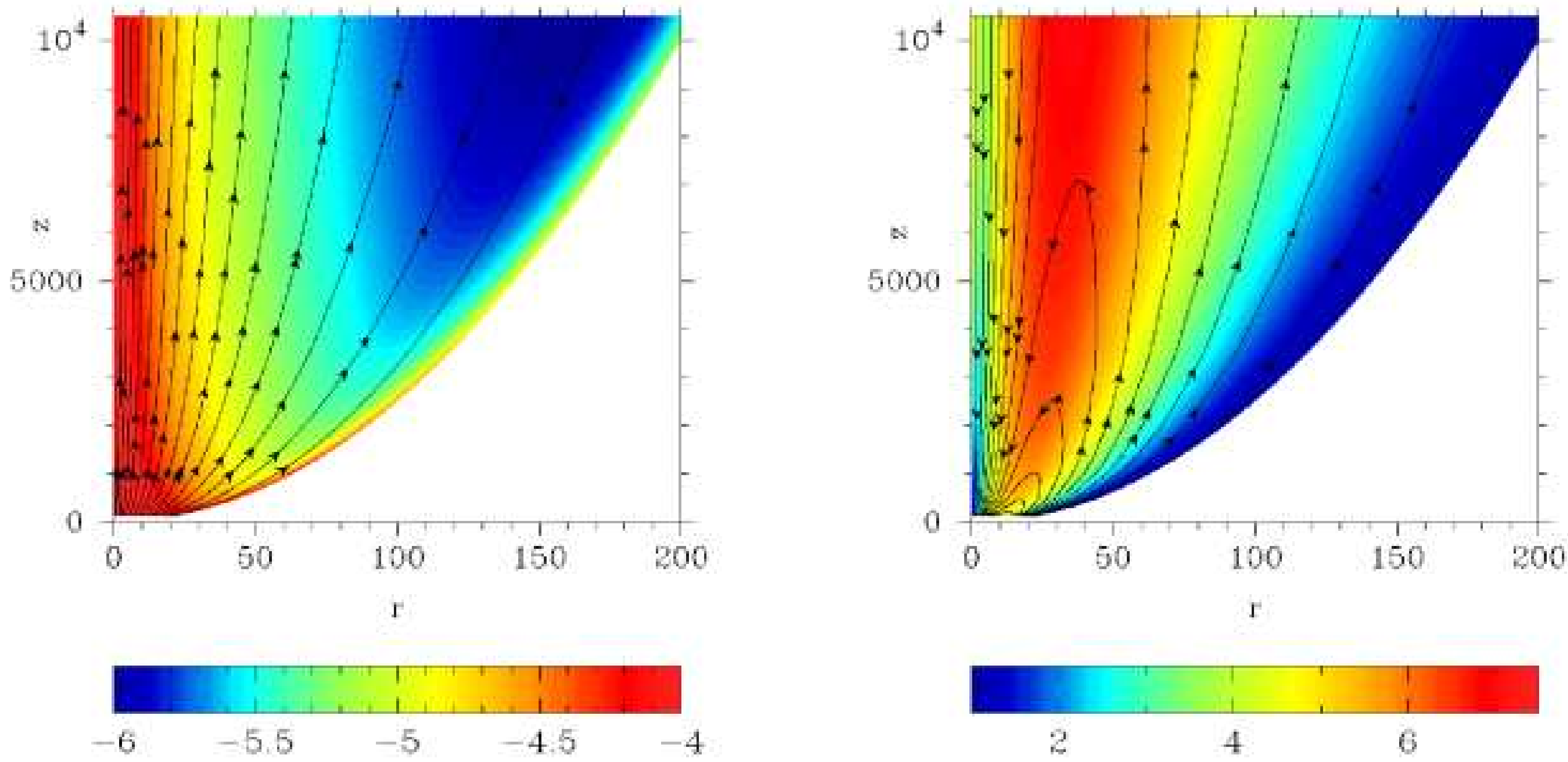
Matsakos et al (2007 submitted)



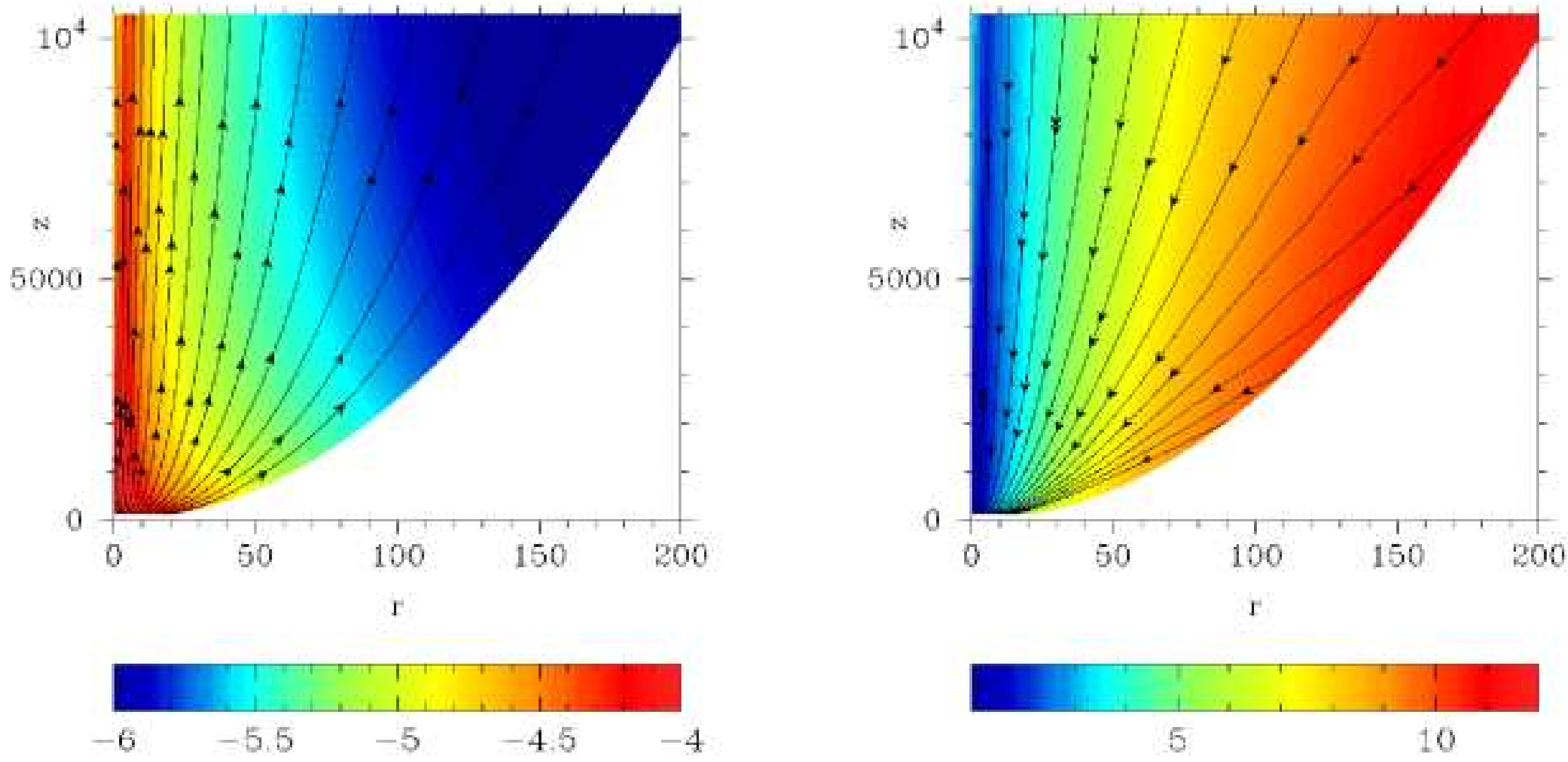


Simulations of relativistic jets

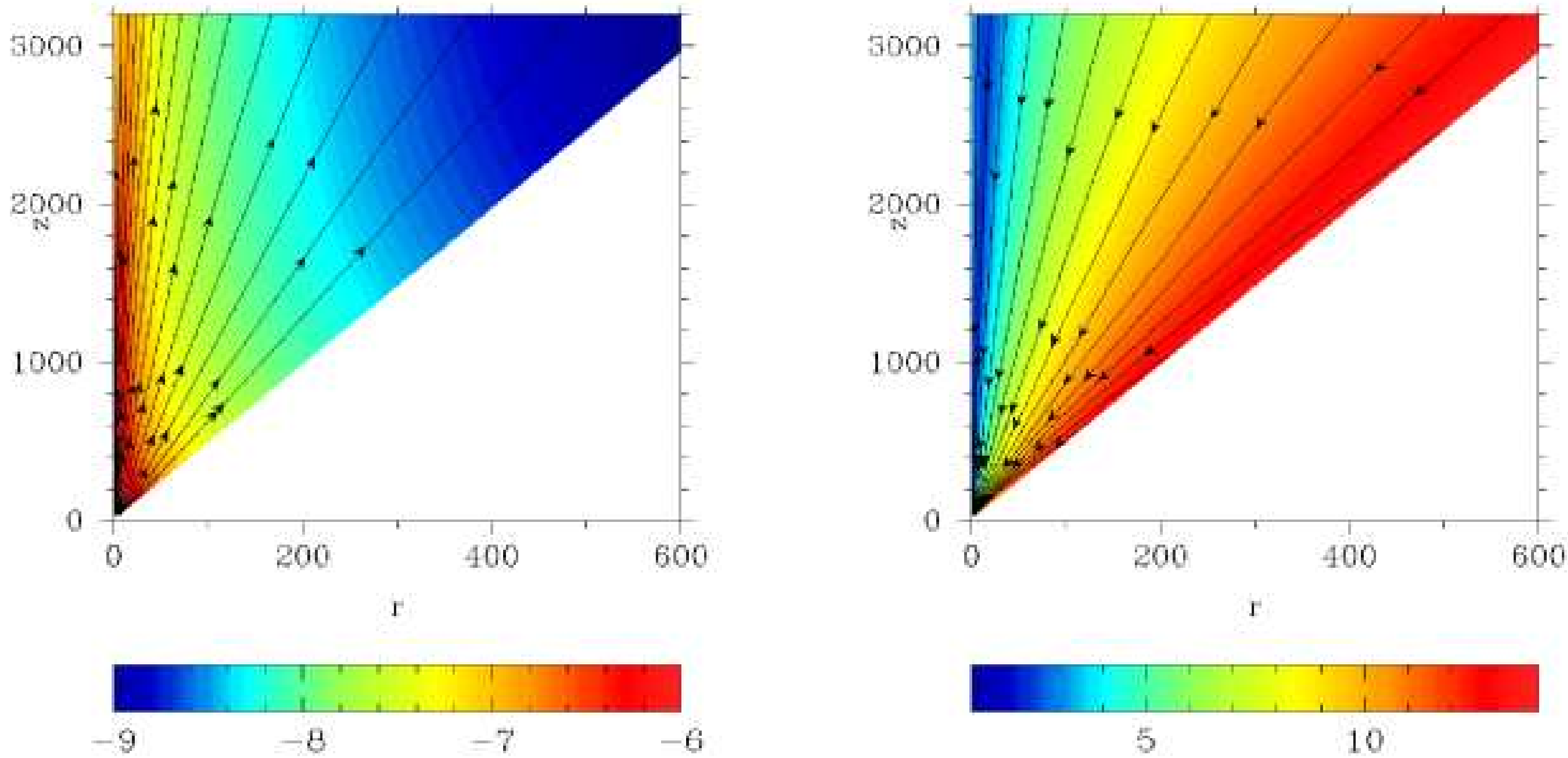
Komissarov et al (2007)



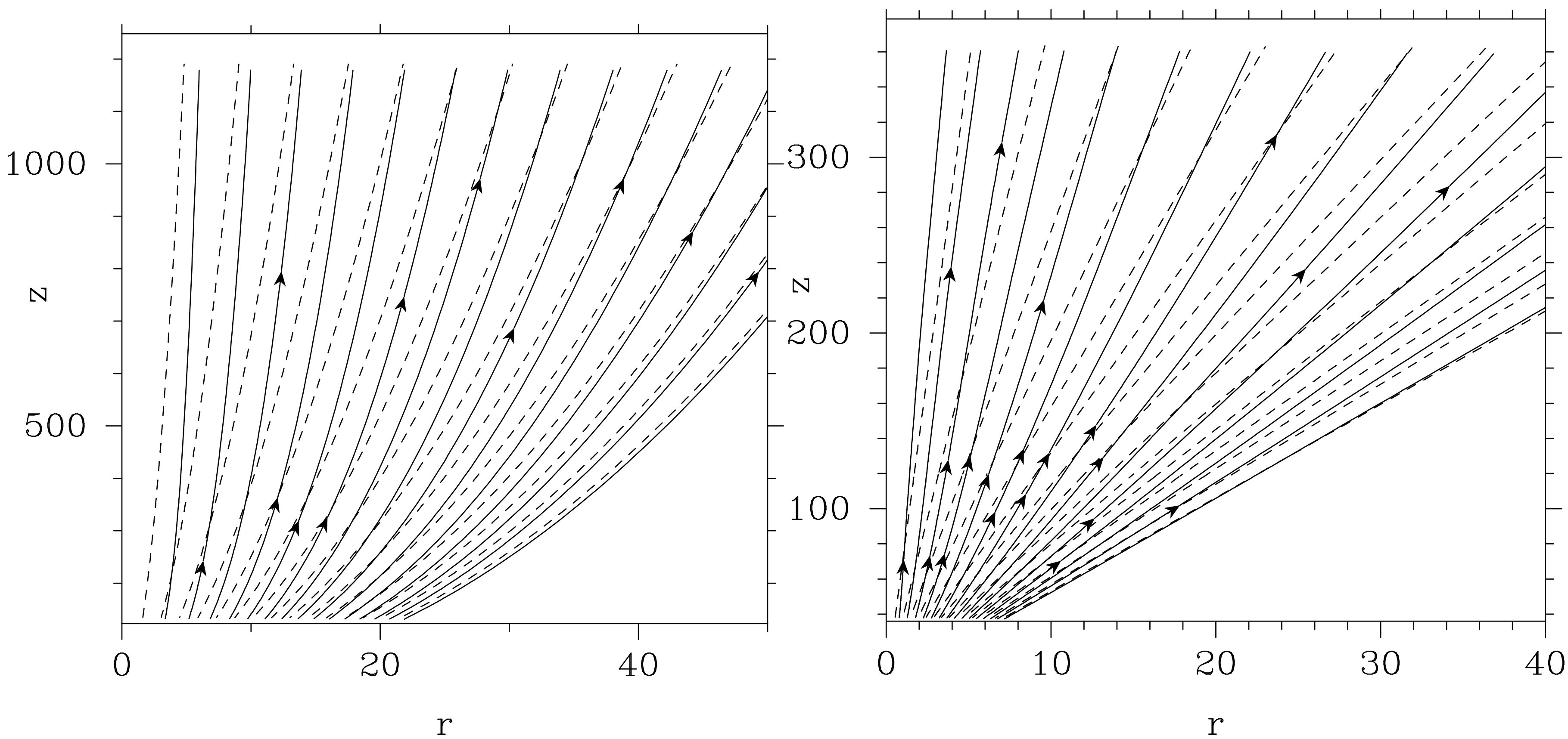
Left panel shows density (colour) and magnetic field lines.
Right panel shows the Lorentz factor (colour) and the current lines.



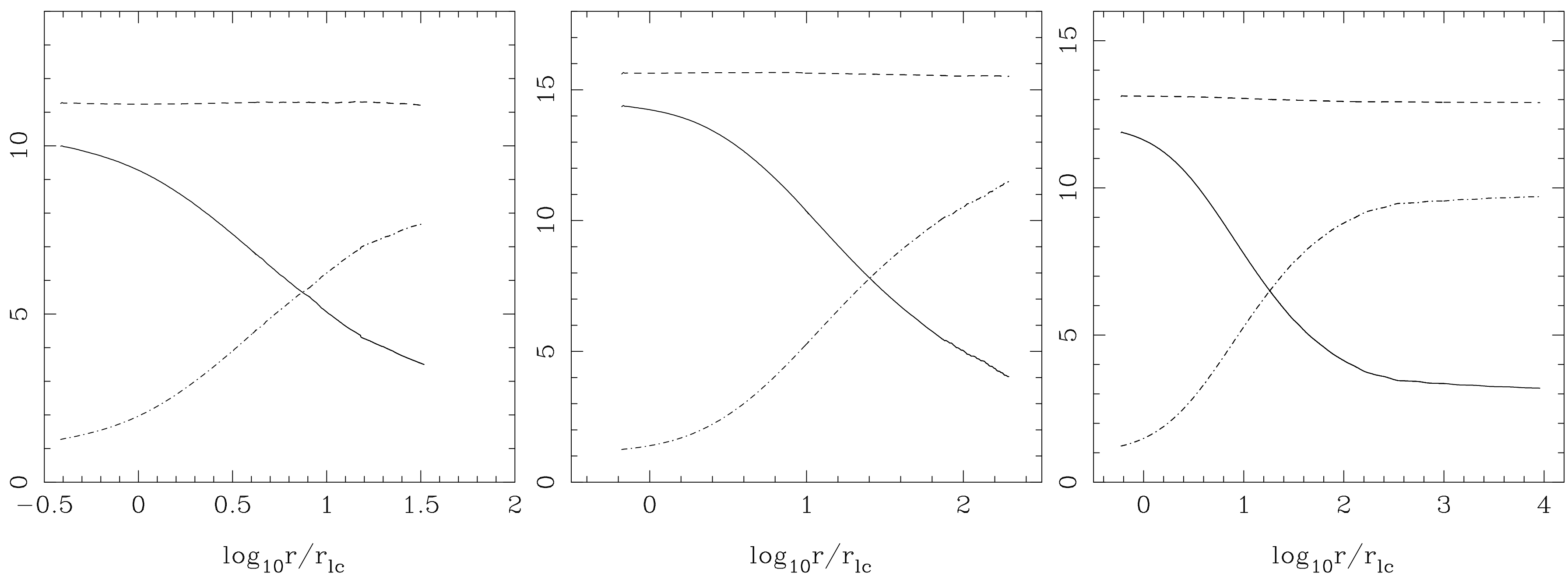
10^4
z
5000
0
0 50 100 150 200
r
−6 −5.5 −5 −4.5 −4
10^4
z
5000
0
0 50 100 150 200
r
5 10



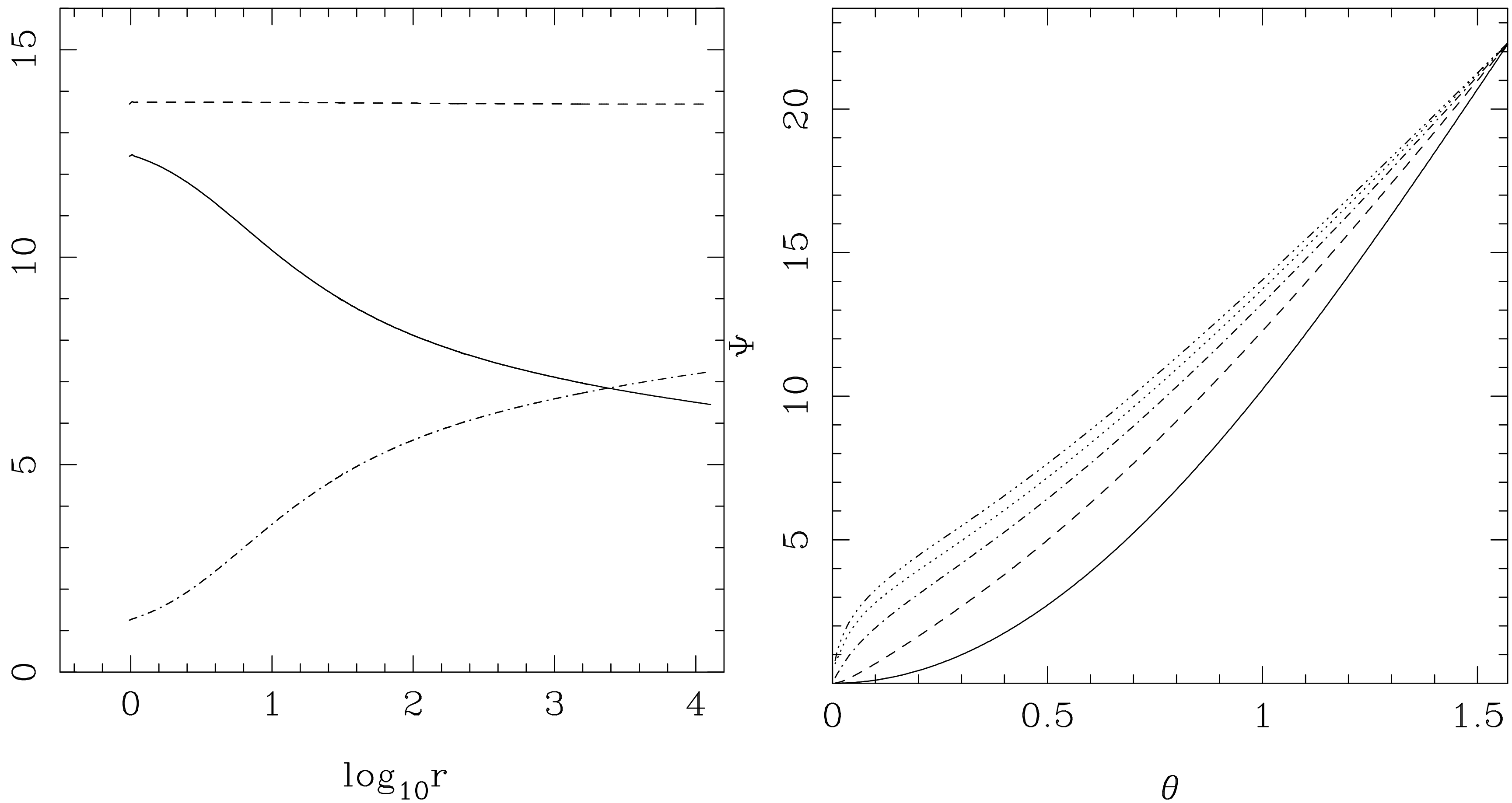
3000
2000
1000
0
z
0
200
400
600
r
−9
−8
−7
−6
3000
2000
1000
0
z
0
200
400
600
r
5
10



1000
500
z
0
20
40
r
300
200
100
z
0
10
20
30
40
r



$\gamma\sigma$ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius for models C1 (left panel), C2 (middle panel) and A2 (right panel).



(without a wall)

Open problems – works in progress

- 3-D simulations – stability
- effects of finite resistivity in outflows
- disk-jet connection
- include heating/cooling
- stellar or disk driven flows (or combination)
  star/disk interaction