Acceleration and collimation in relativistic astrophysical jets

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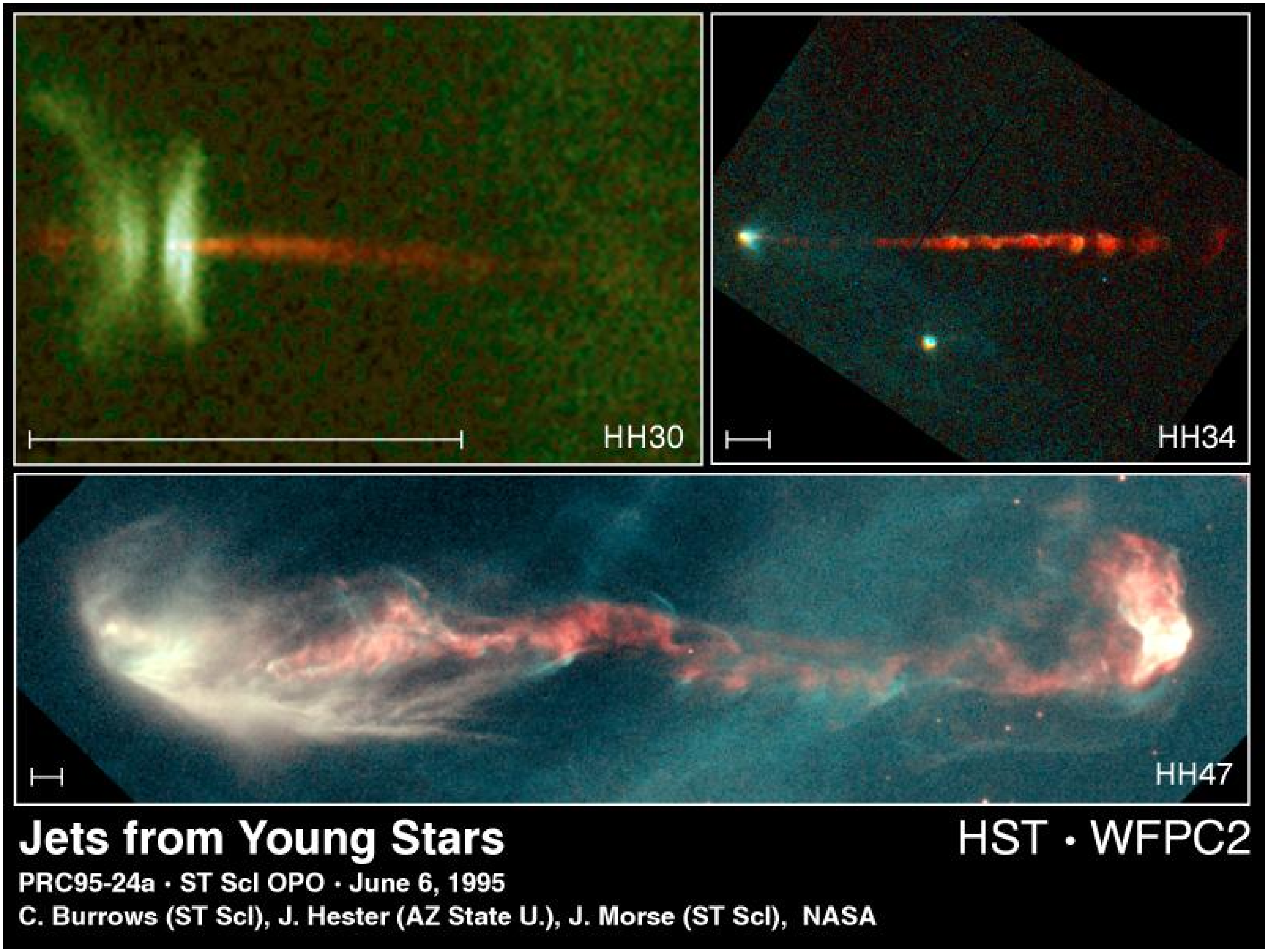
outline

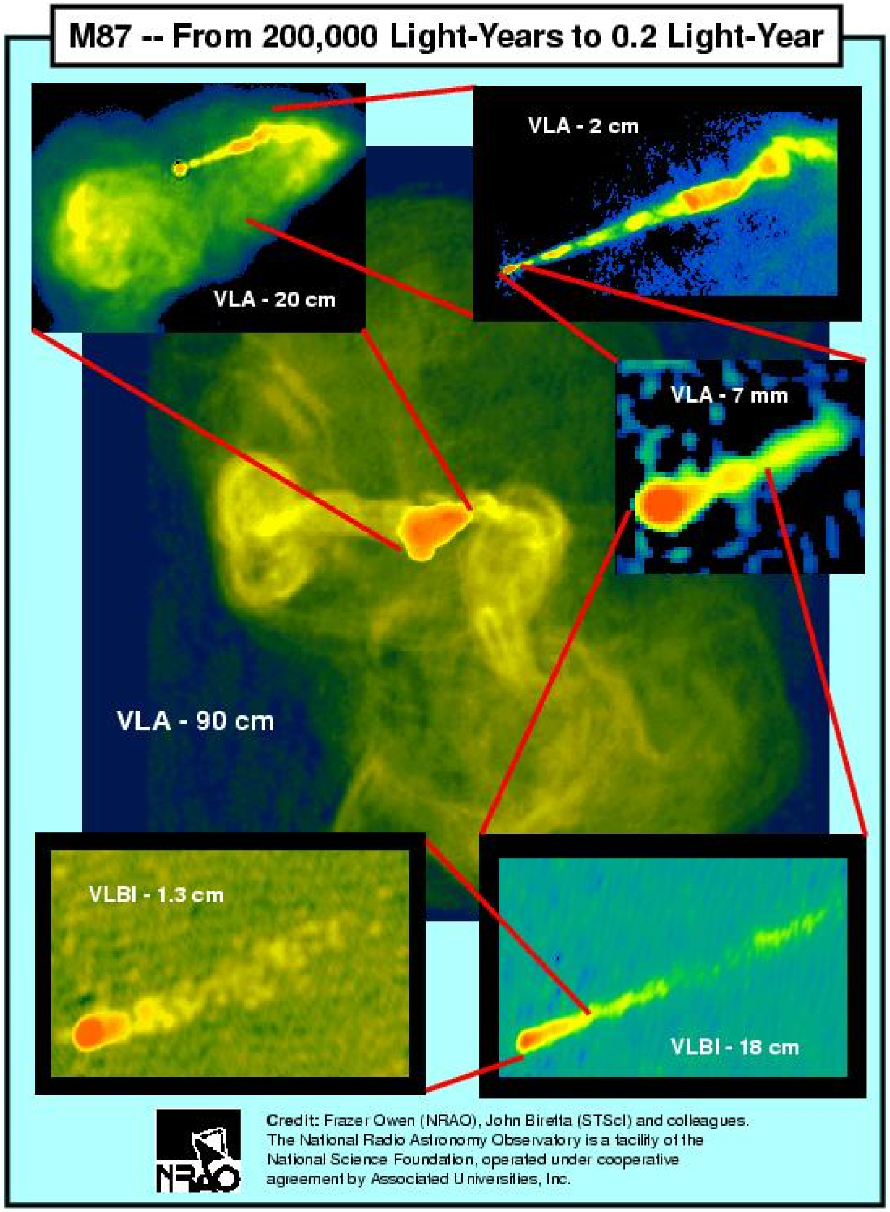
- introduction: astrophysical jets
- the MHD description
  - ⋆ acceleration – collimation
  - ⋆ models (semi-analytical – simulations)

Jets from Young Stars

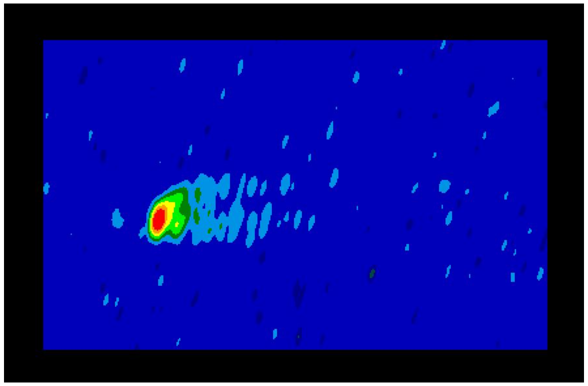


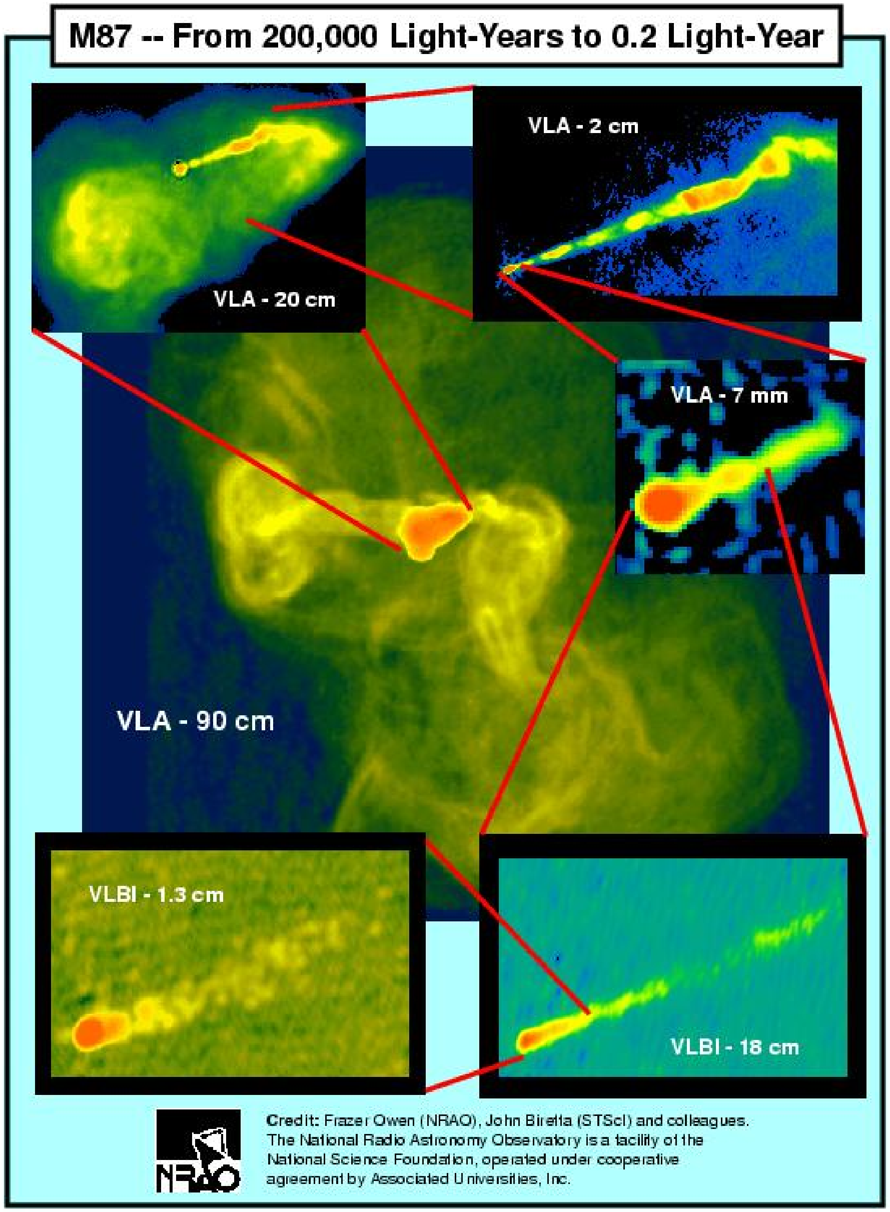
(scale =1000 AU, $V_\infty = a few 100$km/s)

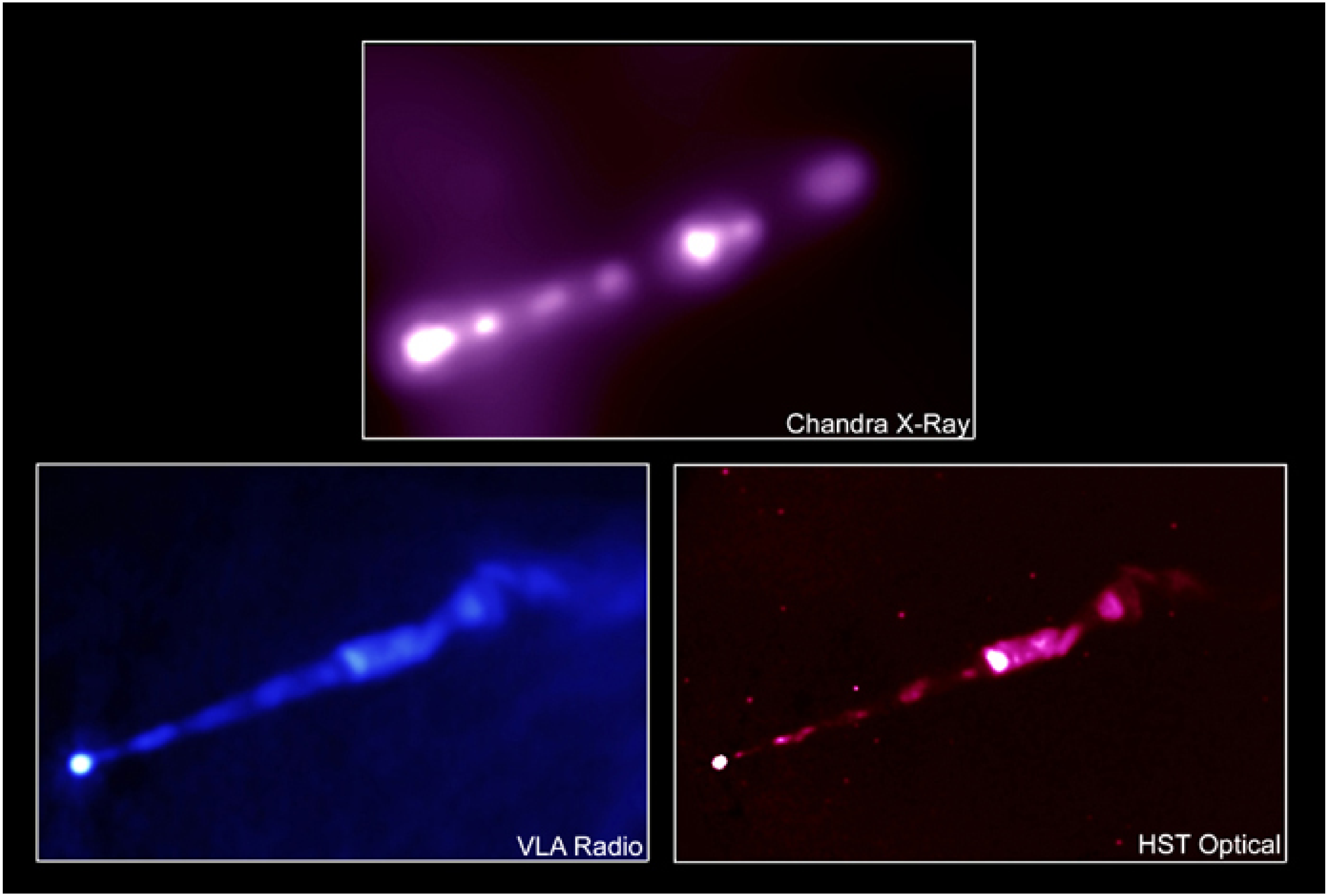
Jets from Active Galactic Nuclei

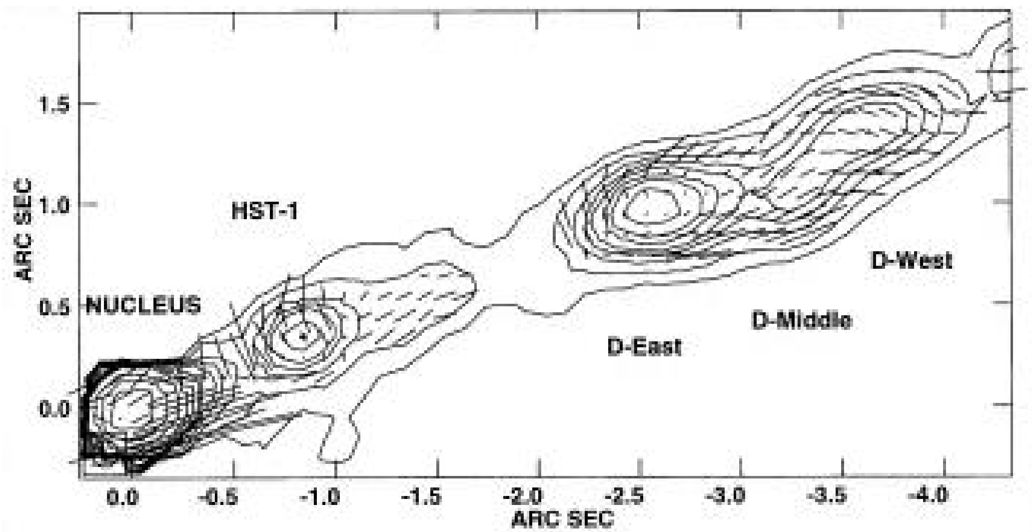


Jets from Active Galactic Nuclei

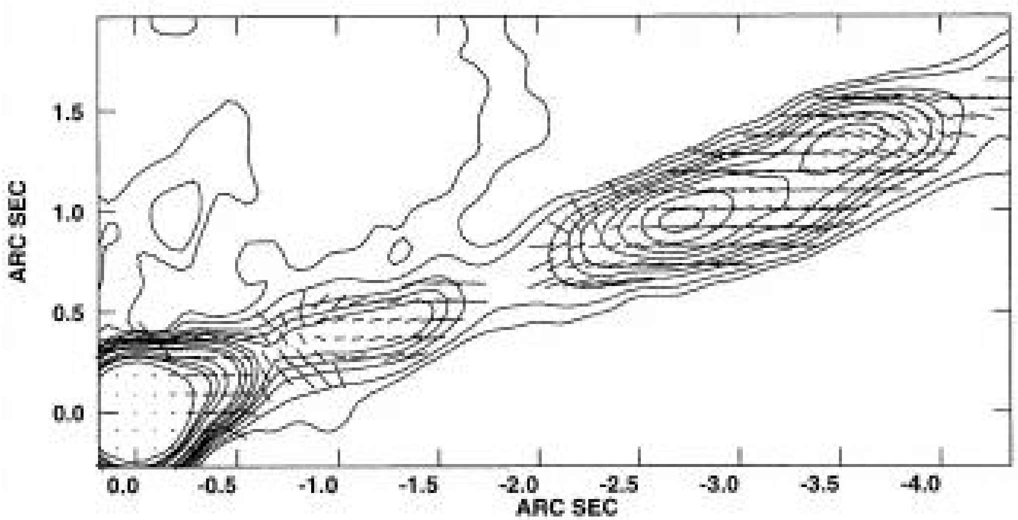





Chandra X-Ray
VLA Radio
HST Optical

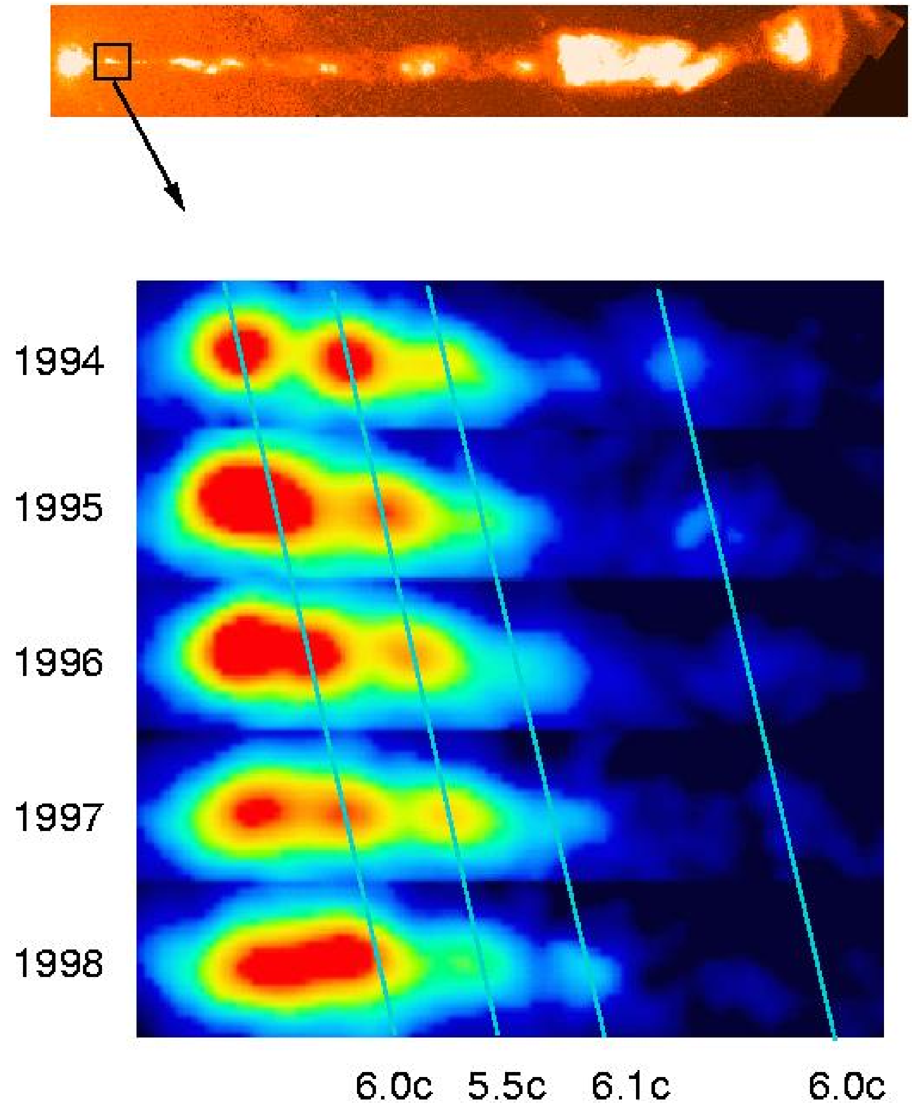


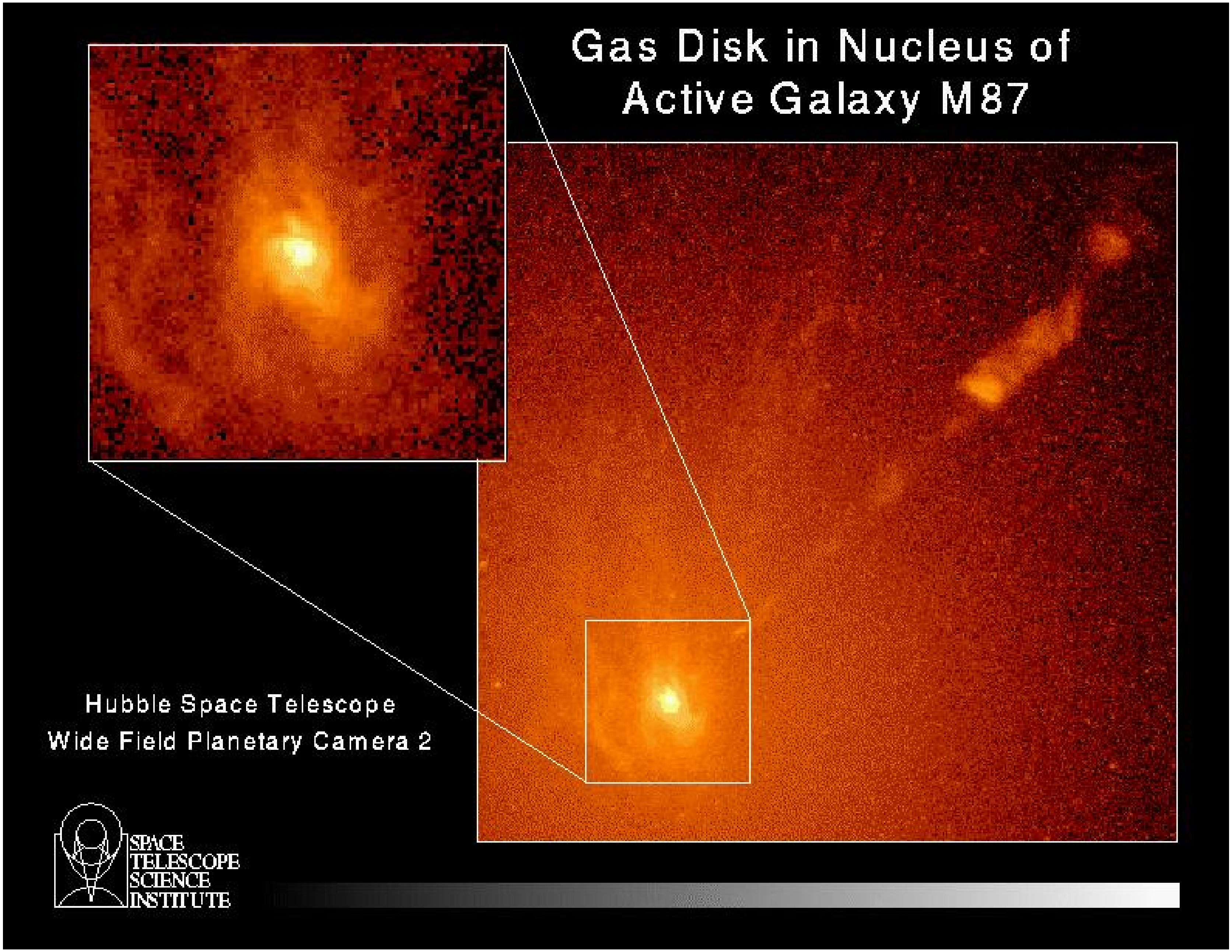
M87 - OPTICAL POLARIZATION

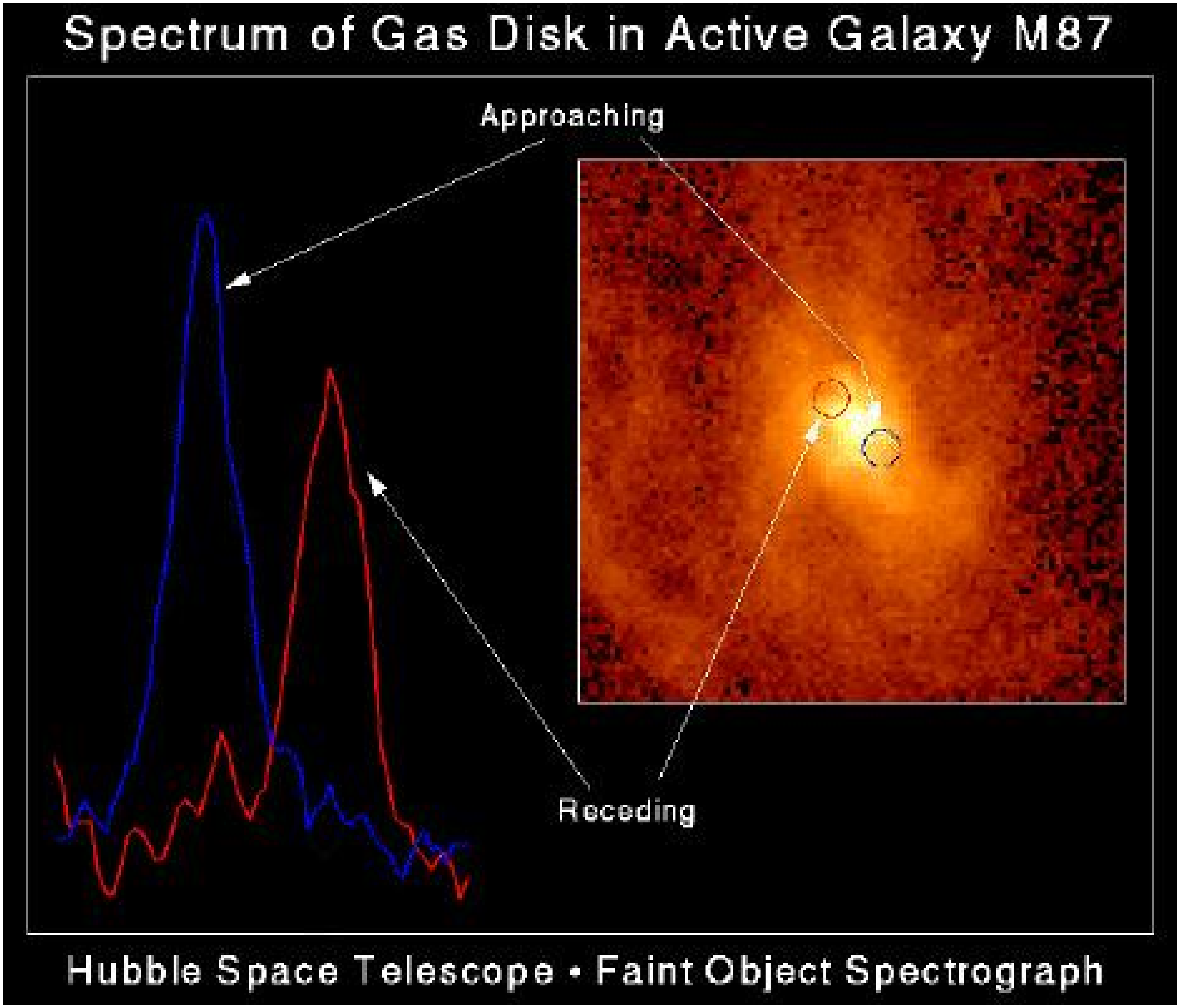


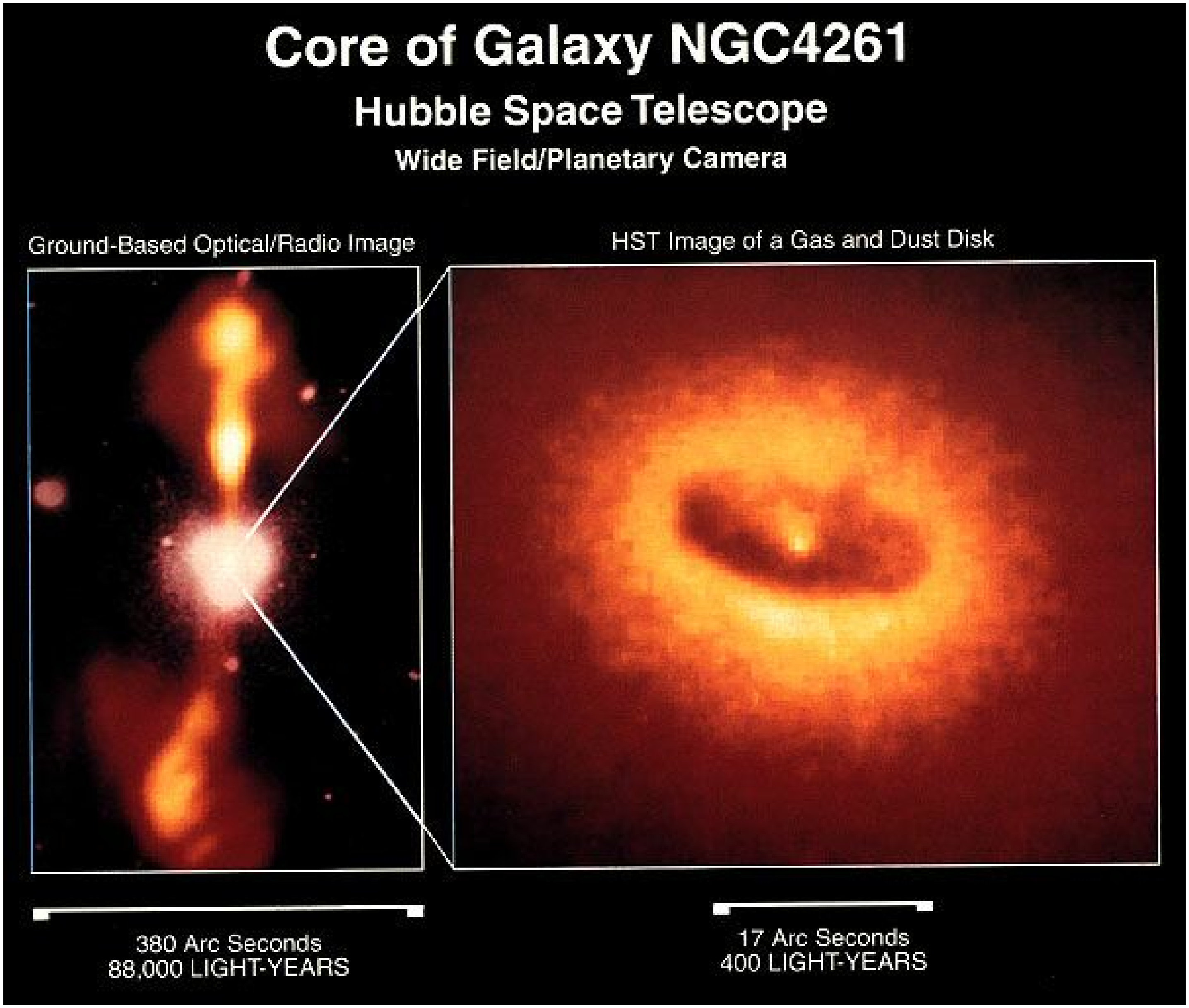
M87 - RADIO POLARIZATION

Superluminal Motion in the M87 Jet

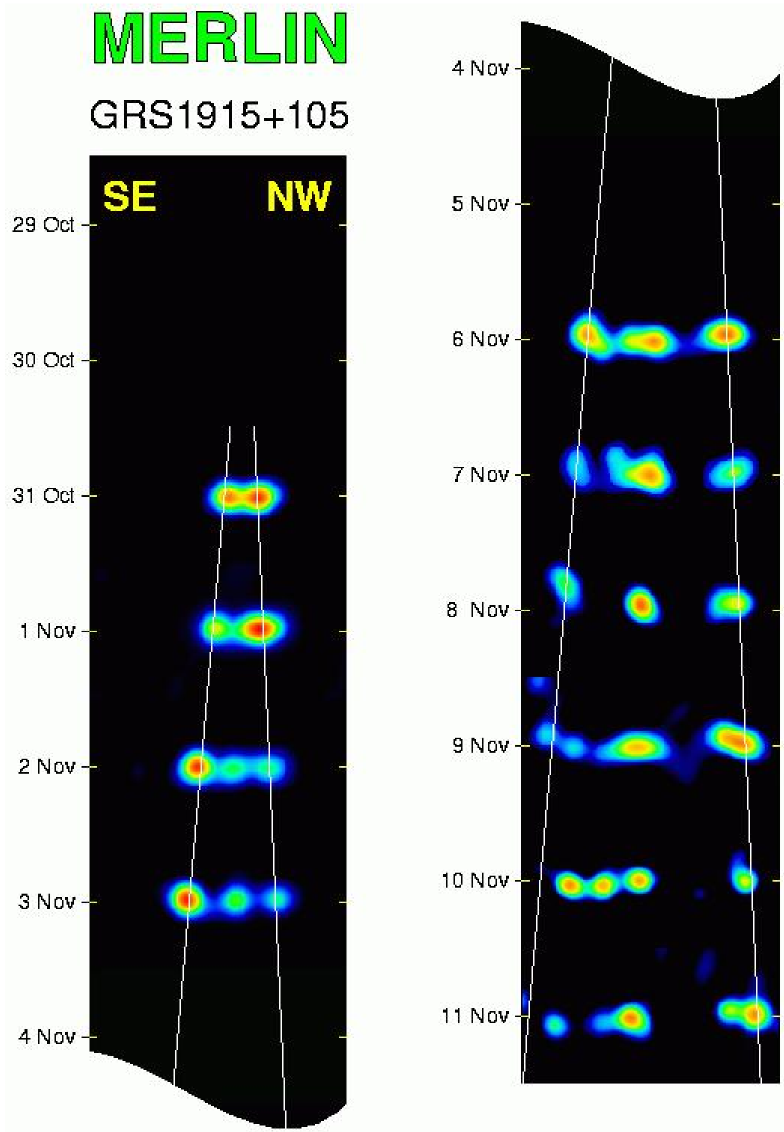



Gas Disk in Nucleus of
Active Galaxy M87
Hubble Space Telescope
Wide Field Planetary Camera 2
SPACE
TELESCOPE
SCIENCE
INSTITUTE


Spectrum of Gas Disk in Active Galaxy M87
Approaching
Receding
Hubble Space Telescope • Faint Object Spectrograph


Core of Galaxy NGC4261
Hubble Space Telescope
Wide Field/Planetary Camera
Ground-Based Optical/Radio Image
HST Image of a Gas and Dust Disk
380 Arc Seconds
88,000 LIGHT-YEARS
17 Arc Seconds
400 LIGHT-YEARS

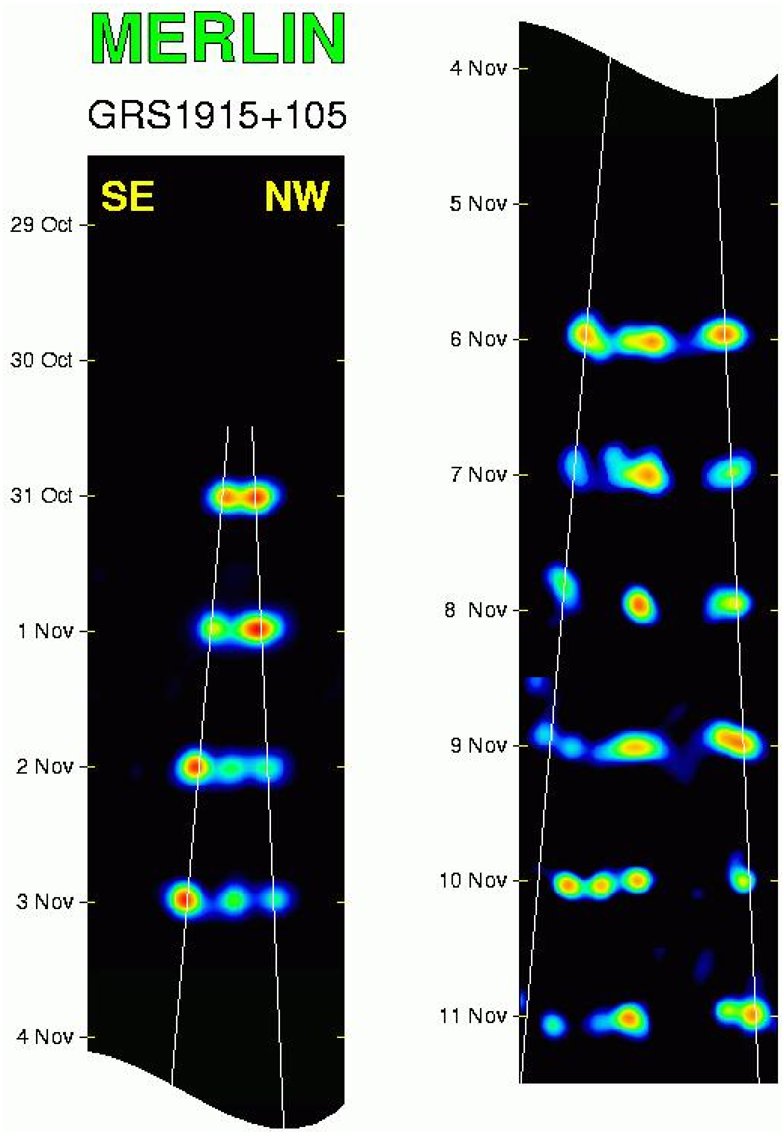
microquasars



scale-down of quasars

speed $\sim 0.9 - 0.99c$

microquasars



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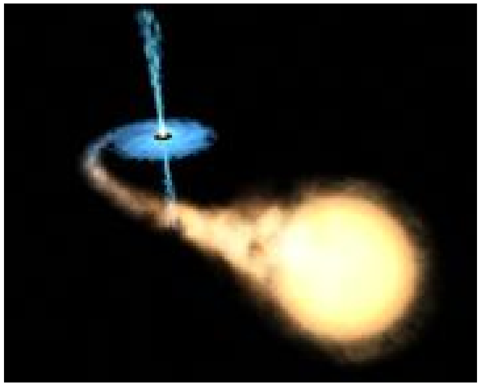


GRBs

- ⋆ high Lorentz factors (compactness problem)
- ⋆ collimated outflows (energy reservoir, achromatic afterglow breaks)

GRBs

⋆ high Lorentz factors (compactness problem)

⋆ collimated outflows (energy reservoir, achromatic afterglow breaks)

☞ similar characteristics

☞ MHD offers a unified picture

We need magnetic fields

- ⋆ to extract energy (Poynting flux)
- ⋆ to extract angular momentum
- ⋆ to transfer energy and angular momentum to matter
- ⋆ to explain relatively large-scale acceleration
- ⋆ to collimate outflows and produce jets
- ⋆ for synchrotron emission
- ⋆ to explain polarization maps

MHD (Magneto-Hydro-Dynamic) description

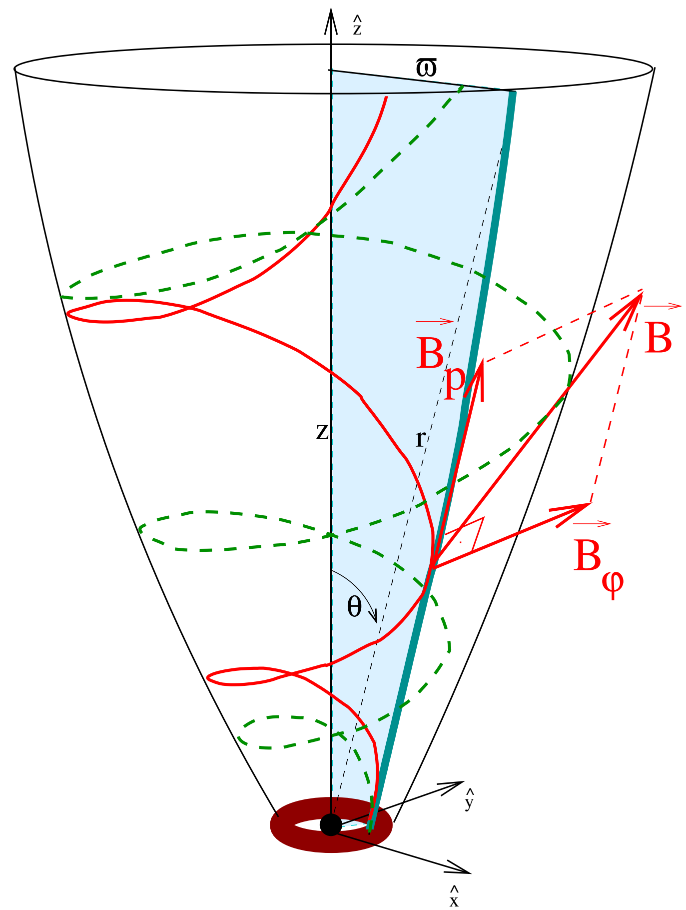
- How the jet is collimated and accelerated? Need to examine outflows taking into account
  - matter: velocity $\boldsymbol{V}$, rest density ρ_0, pressure P, specific enthalpy ξc^2
  - electromagnetic field: $\boldsymbol{E}$, $\boldsymbol{B}$
- ideal MHD equations in special relativity:
  - Maxwell:
    $$\nabla \cdot \boldsymbol{B} = 0 = \nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{c \partial t} , \ \nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{c \partial t} + \frac{4\pi}{c} \boldsymbol{J} , \ \nabla \cdot \boldsymbol{E} = \frac{4\pi}{c} J^0$$
  - Ohm: $\boldsymbol{E} + \frac{\boldsymbol{V}}{c} \times \boldsymbol{B} = 0$
  - mass conservation: $\frac{\partial (\gamma \rho_0)}{\partial t} + \nabla \cdot (\gamma \rho_0 \boldsymbol{V}) = 0$
  - specific entropy conservation: $\left(\frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla \right) \left(\frac{P}{\rho_0^{\Gamma}} \right) = 0$
  - momentum: $\gamma \rho_0 \left(\frac{\partial}{\partial t} + \boldsymbol{V} \cdot \nabla \right) (\xi \gamma \boldsymbol{V}) = -\nabla P + \frac{J^0 \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}}{c}$
- The system gives $\boldsymbol{B}$, $\boldsymbol{V}$, ρ_0, P.

Integrals of motion
under the assumption of steady-state and axisymmetry

From $\nabla \cdot \boldsymbol{B} = 0$

$$\boldsymbol{B}_p = \frac{\nabla A \times \hat{\boldsymbol{\phi}}}{\varpi}, \text{ or, } \boldsymbol{B}_p = \nabla \times \left(\frac{A\,\hat{\boldsymbol{\phi}}}{\varpi} \right)$$

$$A = \frac{1}{2\pi} \iint \boldsymbol{B}_p \cdot d\boldsymbol{S}$$



From $\nabla \times \boldsymbol{E} = 0$, $\boldsymbol{E} = -\nabla \Phi$
Because of axisymmetry $E_\phi = 0$.
Combining with Ohm's law ($\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B}/c$) we find $\boldsymbol{V}_p \parallel \boldsymbol{B}_p$.

Because $\boldsymbol{V}_p \parallel \boldsymbol{B}_p$ we can write

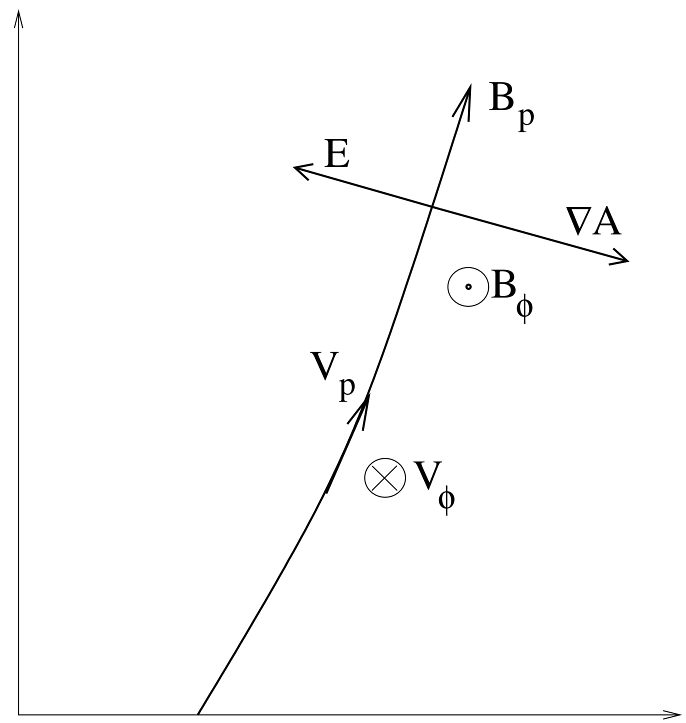
$$\boldsymbol{V} = \frac{\Psi_A}{4\pi\gamma\rho_0}\boldsymbol{B} + \varpi\Omega\hat{\boldsymbol{\phi}}\,, \qquad \frac{\Psi_A}{4\pi\gamma\rho_0} = \frac{V_p}{B_p}\,,$$

$$V_\phi = \frac{\Psi_A}{4\pi\gamma\rho_0}B_\phi + \varpi\Omega = \frac{V_p}{B_p}B_\phi + \varpi\Omega\,.$$

The Ω and Ψ_A are constants of motion, $\Omega = \Omega(A)$, $\Psi_A = \Psi_A(A)$.

- $\Omega =$ angular velocity at the base
- $\Psi_A =$ mass-to-magnetic flux ratio

The electric field $\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B}/c = -(\varpi\Omega/c)\hat{\boldsymbol{\phi}} \times \boldsymbol{B}_p$ is a poloidal vector, normal to $\boldsymbol{B}_p$. Its magnitude is $E = \frac{\varpi\Omega}{c}B_p$.



So far, we've used Maxwell's eqs, Ohm's law and the continuity.

The entropy eq gives $P/\rho_0^\Gamma =$ constant of motion (entropy).

We are left with the momentum equation

$$\gamma\rho_0 \left(\boldsymbol{V}\cdot\nabla\right)\left(\xi\gamma\boldsymbol{V}\right) = -\nabla P + \frac{J^0\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}}{c}, \text{ or,}$$

$$\gamma\rho_0 \left(\boldsymbol{V}\cdot\nabla\right)\left(\xi\gamma\boldsymbol{V}\right) = -\nabla P + \frac{(\nabla\cdot\boldsymbol{E})\boldsymbol{E} + (\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi}$$

Due to axisymmetry, the toroidal component can be integrated to give the total angular momentum-to-mass flux ratio:

$$\xi\gamma\varpi V_\phi - \frac{\varpi B_\phi}{\Psi_A} = L(A)$$

Poloidal components of the momentum eq

$$\gamma\rho_0 \left(\boldsymbol{V}\cdot\nabla\right)\left(\xi\gamma\boldsymbol{V}\right) = -\nabla P + \frac{(\nabla\cdot\boldsymbol{E})\boldsymbol{E} + (\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi} \Leftrightarrow$$

$$\boldsymbol{f}_G + \boldsymbol{f}_T + \boldsymbol{f}_C + \boldsymbol{f}_I + \boldsymbol{f}_P + \boldsymbol{f}_E + \boldsymbol{f}_B = 0$$

$$\left.\begin{array}{lll} \boldsymbol{f}_G = -\gamma\rho_0\xi\left(\boldsymbol{V}\cdot\nabla\gamma\right)\boldsymbol{V} & & \\ \boldsymbol{f}_T = -\gamma^2\rho_0\left(\boldsymbol{V}\cdot\nabla\xi\right)\boldsymbol{V} & : & \text{“temperature” force} \\ \boldsymbol{f}_C = \hat{\boldsymbol{\varpi}}\gamma^2\rho_0\xi V_\phi^2/\varpi & : & \text{centrifugal force} \\ \boldsymbol{f}_I = -\gamma^2\rho_0\xi\left(\boldsymbol{V}\cdot\nabla\right)\boldsymbol{V} - \boldsymbol{f}_C & & \end{array}\right\} \begin{array}{c}\text{inertial}\\ \text{force}\end{array}$$

$$\begin{array}{lll} \boldsymbol{f}_P = -\nabla P & : & \text{pressure force} \\ \boldsymbol{f}_E = \left(\nabla\cdot\boldsymbol{E}\right)\boldsymbol{E}/4\pi & : & \text{electric force} \\ \boldsymbol{f}_B = \left(\nabla\times\boldsymbol{B}\right)\times\boldsymbol{B}/4\pi & : & \text{magnetic force} \end{array}$$

Acceleration mechanisms

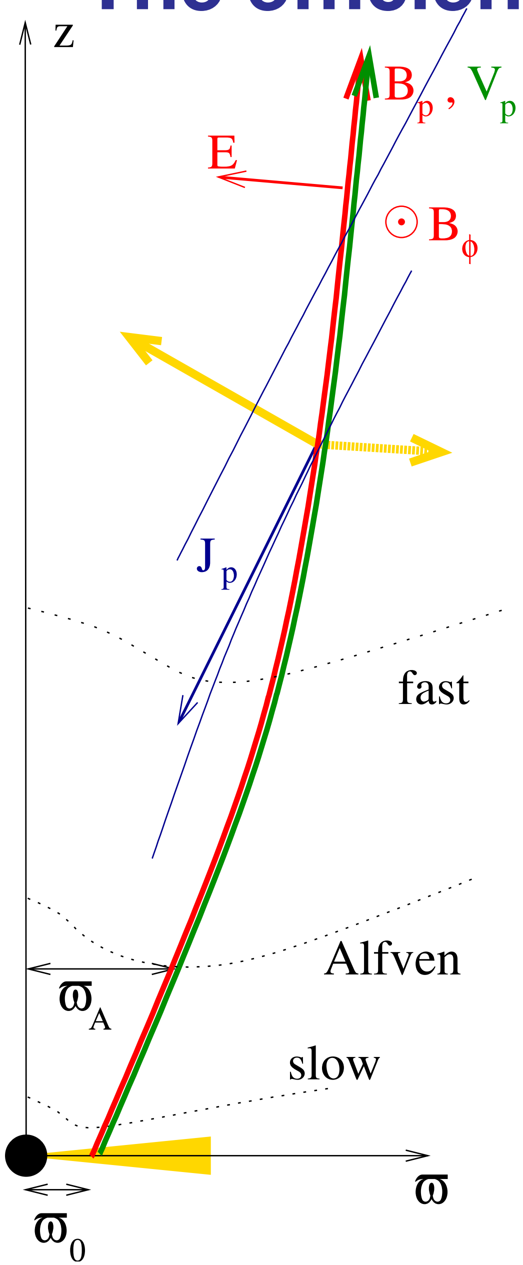
- thermal (due to ∇P) $\longrightarrow$ velocities up to C_s
- magnetocentrifugal (beads on wire - Blandford & Payne)
  - in reality due to magnetic pressure
  - initial half-opening angle $\vartheta > 30^o$
  - the $\vartheta > 30^o$ not necessary for nonnegligible P
  - velocities up to $\varpi_0 \Omega$
- relativistic thermal (thermal fireball) gives $\gamma \sim \xi_i$, where $\xi = \dfrac{\text{enthalpy}}{\text{mass} \times c^2}$.
- magnetic

All acceleration mechanisms can be seen in the energy conservation equation

$$\mu = \xi\gamma + \frac{\Omega}{\Psi_A c^2}\varpi|B_\phi| \left(\text{where } \mu = \frac{\dfrac{dE}{dSdt}}{\dfrac{dM}{dSdt}c^2} \right)$$

So $\gamma \uparrow$ when $\xi \downarrow$ (thermal, relativistic thermal), or,
$\varpi|B_\phi| \downarrow \Leftrightarrow I_p \downarrow$ (magnetocentrifugal, magnetic).

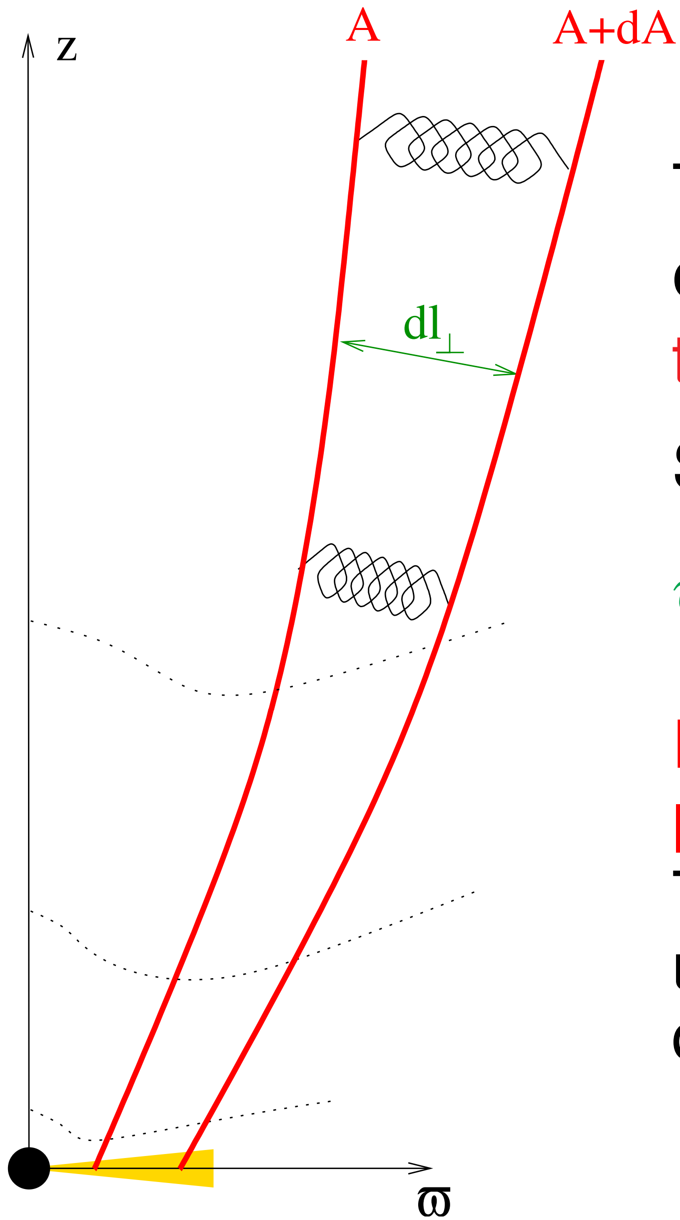
The efficiency of the magnetic acceleration



The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force strongly depends on the angle between field-lines and current-lines.

Are we free to choose these two lines? NO! All MHD quantities are related to each other and should be found by solving the full system of equations.

From Ferraro's law, $V_\phi = \frac{V_p}{B_p} B_\phi + \varpi\Omega \rightarrow \varpi|B_\phi| \approx \varpi^2 B_p \Omega / V_p$. So, the transfield force-balance determines the acceleration.



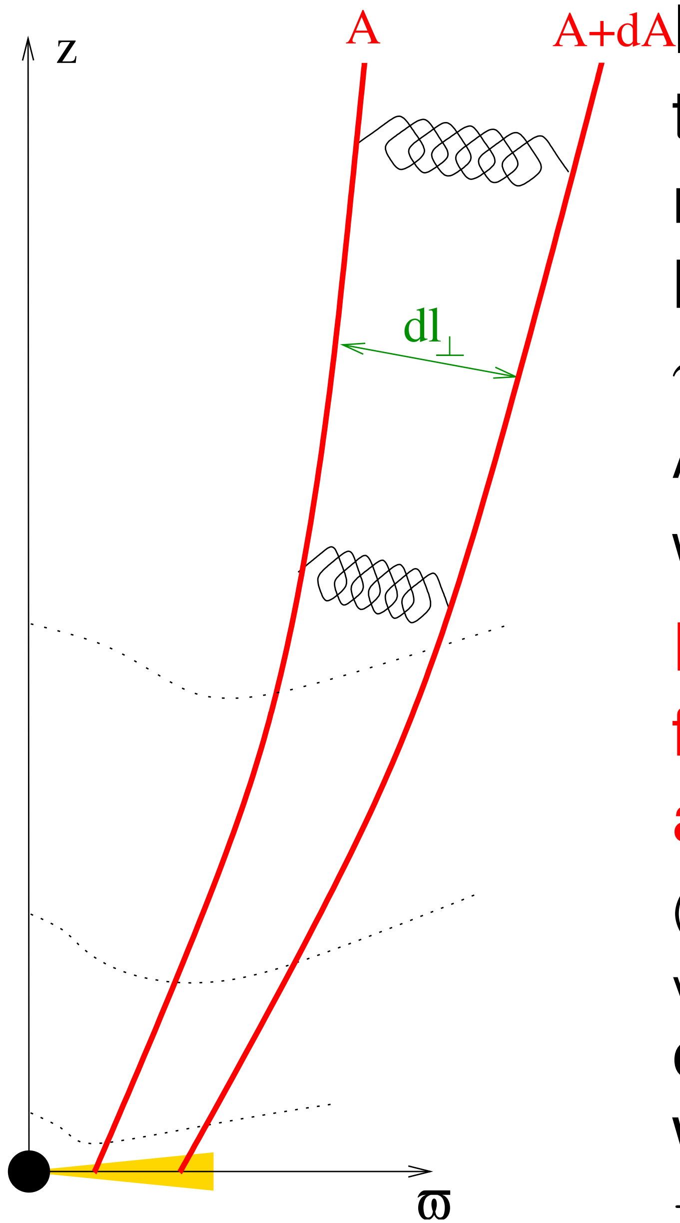
The magnetic field minimizes its energy under the condition of keeping the magnetic flux constant.

So, $\varpi|B_\phi| \downarrow$ for decreasing

$$\varpi^2 B_p = \frac{\varpi^2}{2\pi\varpi dl_\perp}\underbrace{(B_p dS)}_{dA} \propto \frac{\varpi}{dl_\perp}.$$

Expansion with increasing $dl_\perp/\varpi$ leads to acceleration (Vlahakis 2004).
The expansion ends in a more-or-less uniform distribution $\varpi^2 B_p \approx A$ (in a quasi-monopolar shape).

Conclusions on the magnetic acceleration



If we start with a uniform distribution the magnetic energy is already minimum → no acceleration. Example: Michel's (1969) solution which gives $\gamma_\infty \approx \mu^{1/3} \ll \mu$.

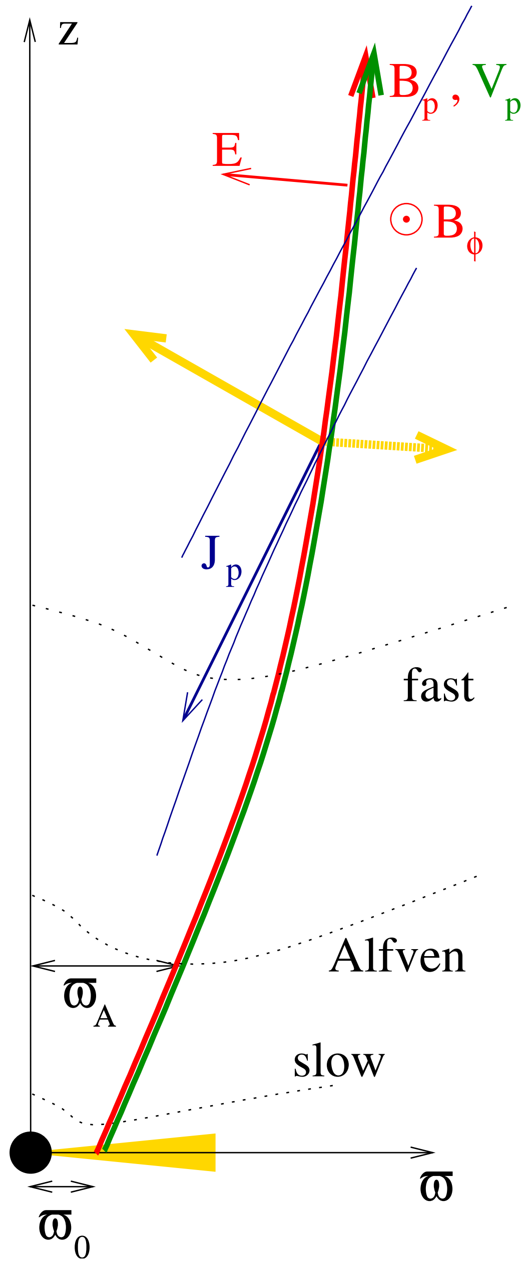
Also Beskin et al (1998); Bogovalov (2001) who found quasi-monopolar solutions.

For any other (more realistic) initial field distribution we have efficient acceleration!

(details and an analytical estimation of the efficiency in Vlahakis 2004, ApSS 293, 67).

example: if we start with $\varpi^2 B_p / A = 2$ we have asymptotically $\varpi^2 B_p / A = 1$ $\rightarrow 50\%$ efficiency

On the collimation



The $\boldsymbol{J}_p \times \boldsymbol{B}_\phi$ force contributes to the collimation (hoop-stress paradigm).
In relativistic flows the electric force plays an opposite role (a manifestation of the high inertia of the flow).

- collimation by an external wind (Bogovalov & Tsinganos 2005, for AGN jets)
- surrounding medium may play a role (in the collapsar model)
- self-collimation mainly works at small distances where the velocities are mildly relativistic (Vlahakis & Königl 2003)

For $\gamma \gg 1$, the transfield force-balance gives

$$\gamma^2 \frac{\varpi}{\mathcal{R}} \approx \underbrace{\left(1 - \frac{\gamma}{\mu}\right) \varpi \nabla \ln \left| \frac{\Psi_A}{\Omega} \left(\frac{\mu}{\gamma} - 1\right) \right| \cdot \frac{\nabla A}{|\nabla A|}}_{\mathcal{O}(1)} - \left(\frac{\gamma}{\varpi \Omega / c}\right)^2 \underbrace{\frac{\hat{\varpi} \cdot \nabla A}{|\nabla A|}}_{\mathcal{O}(1)}$$

- If the last term is negligible then the curvature radius $\mathcal{R} \sim \gamma^2 \varpi \ (\gg \varpi)$.

Collimation more difficult, but not impossible!

$$\frac{\varpi}{\mathcal{R}} = -\varpi \frac{\partial^2 \varpi}{\partial z^2} \left(\frac{B_z}{B_p}\right)^3 \sim \left(\frac{\varpi}{z}\right)^2$$

Combining the above, we get

$$\gamma \sim \frac{z}{\varpi}$$

- If the first term is negligible (quasi-radial flow) then

$$\gamma \approx \varpi \Omega / c$$

(linear accelerator, Contopoulos & Kazanas 2002)

r self-similarity

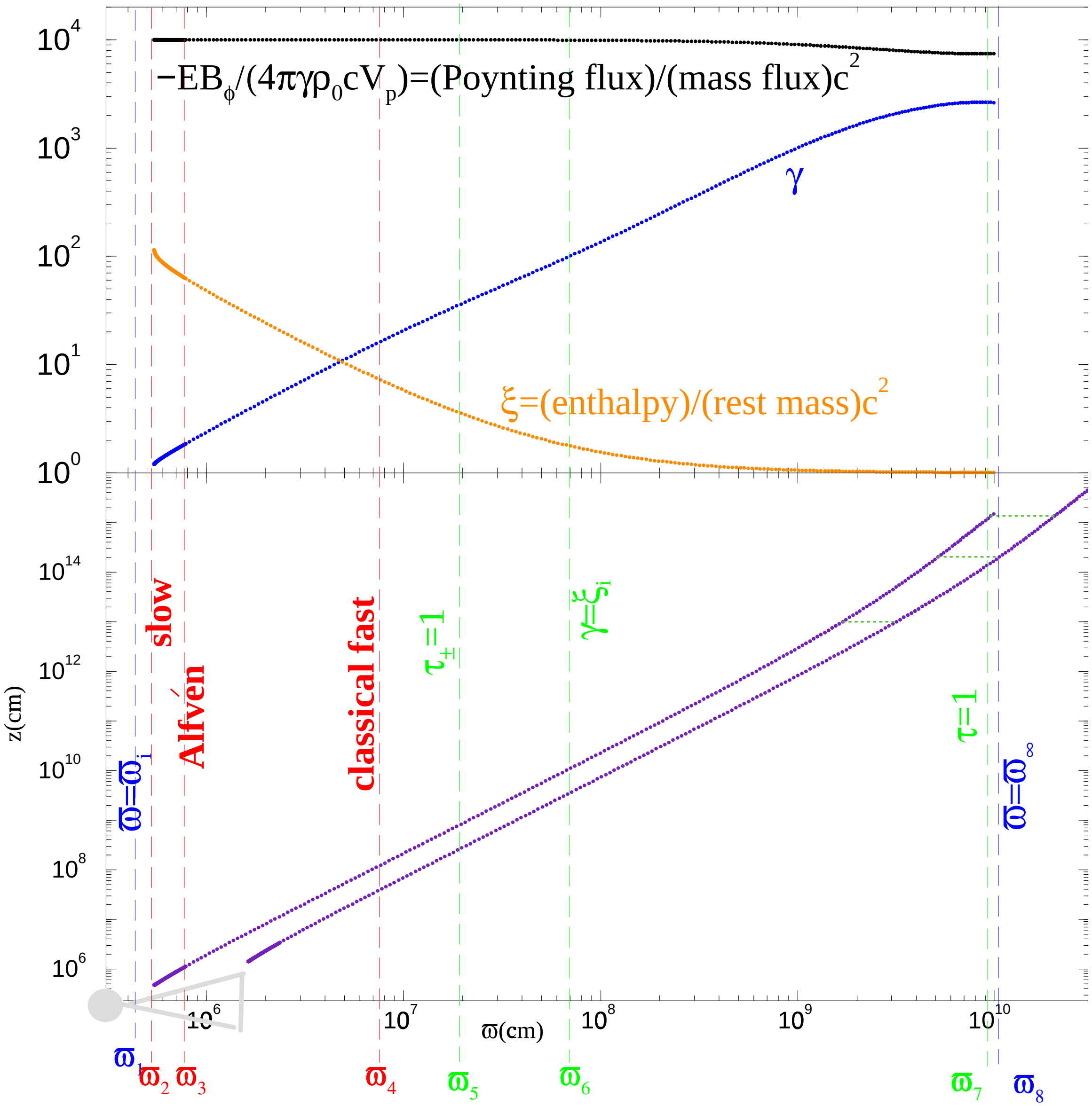
Assume that all physical quantities (velocity and magnetic field components, pressure, density) scale as a power of r times a function of θ (in spherical coordinates).

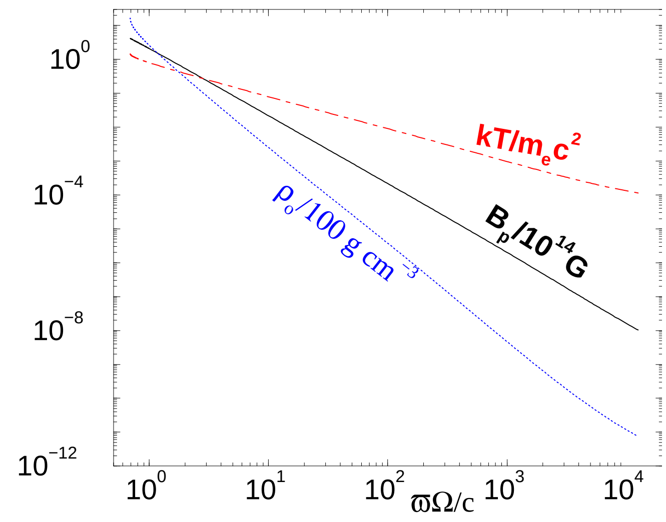
$$B_r = r^{F-2}\mathcal{C}_1(\theta)\,,\; B_\phi = r^{F-2}\mathcal{C}_2(\theta)\,,$$
$$V_r/c = \mathcal{C}_3(\theta)\,,\; V_\theta/c = -\mathcal{C}_4(\theta)\,,\; V_\phi/c = \mathcal{C}_5(\theta)\,,$$
$$\rho_0 = r^{2(F-2)}\mathcal{C}_6(\theta)\,,\; P = r^{2(F-2)}\mathcal{C}_7(\theta)\,.$$

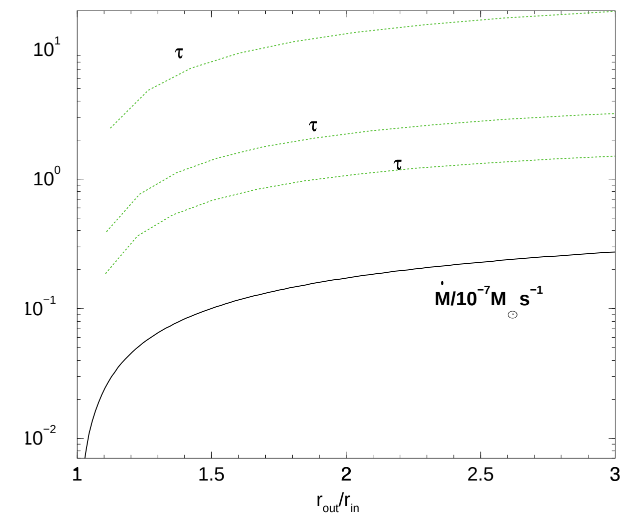
The variables $r\,,\theta$ are separable and the system reduces to ODEs.

- Blandford & Payne – (nonrelativistic)
- Li, Chiueh, & Begelman (1992) and Contopoulos (1994) – (cold)
- Vlahakis & Königl (2003, 2004) – (including thermal/radiation effects)

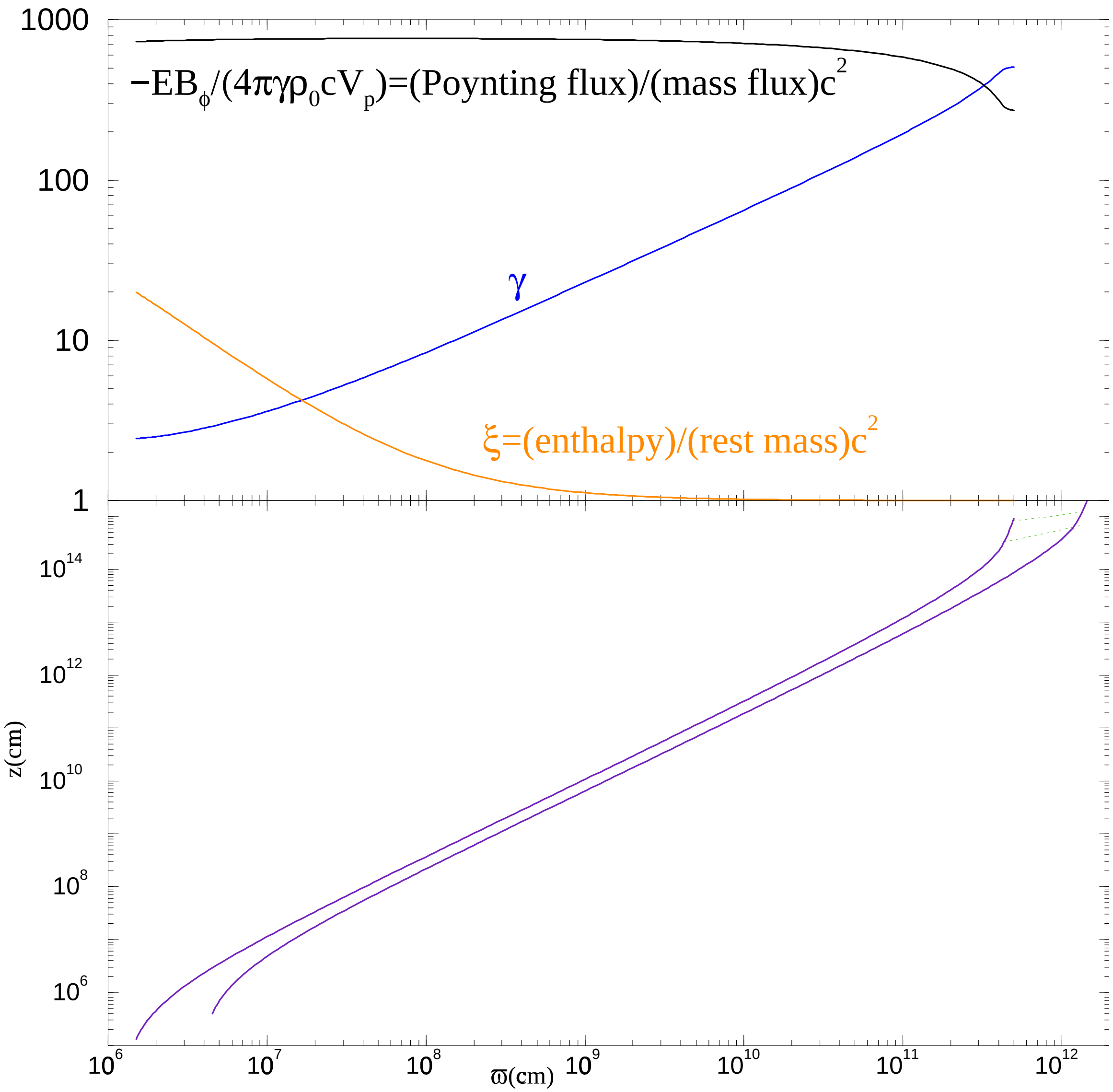
Semi-analytic solutions for GRB Jets (NV & Königl 2001, 2003a,b)

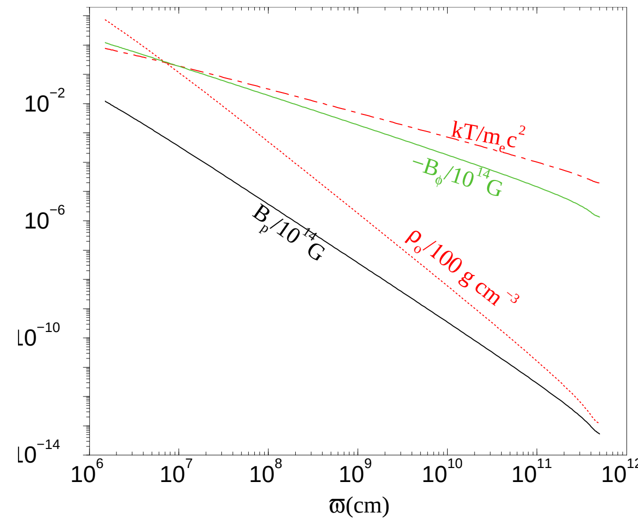


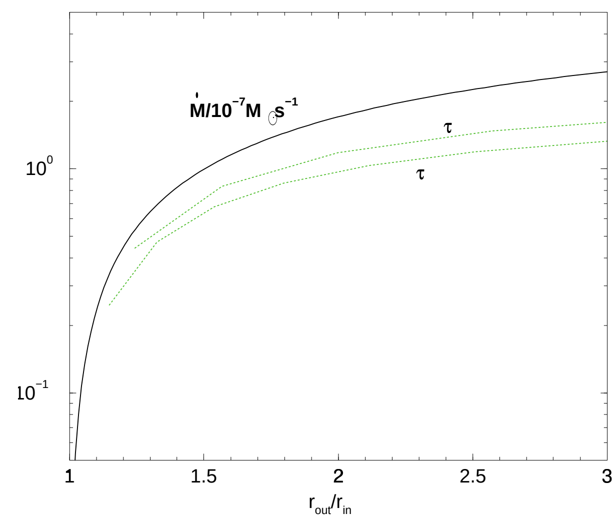




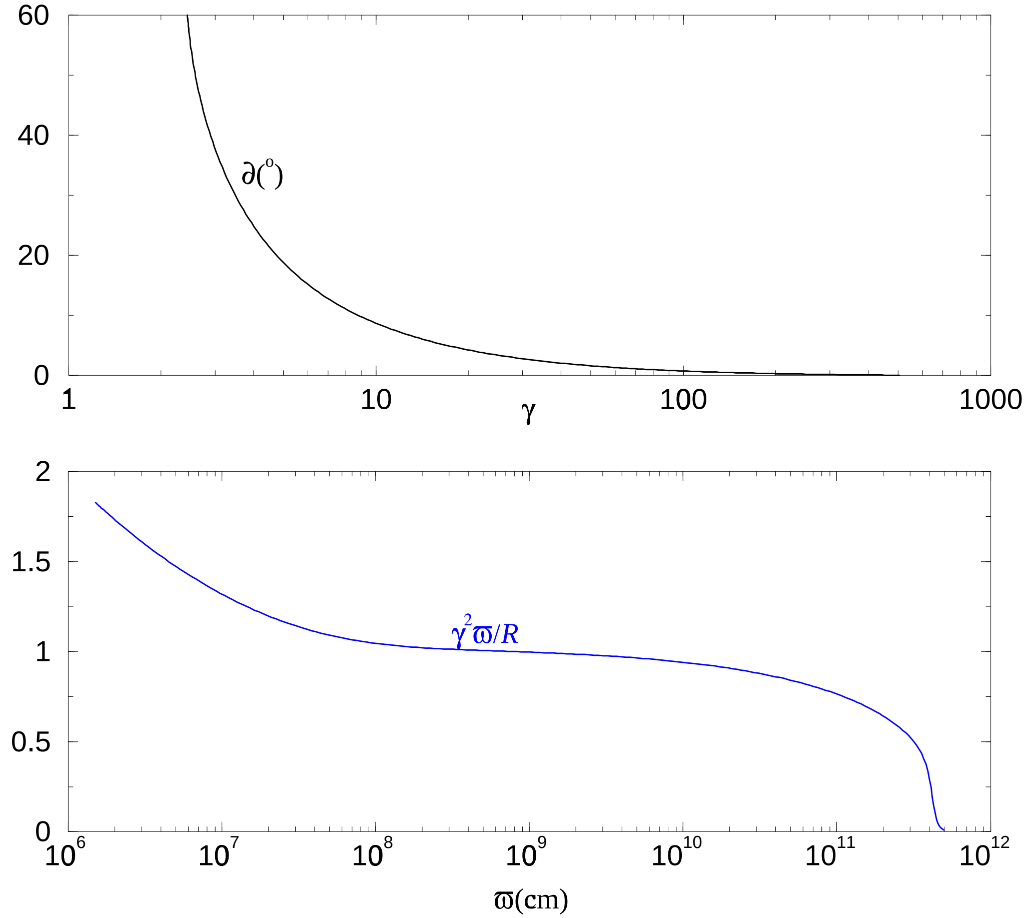
- $\varpi_1 < \varpi < \varpi_6$: Thermal acceleration - force free magnetic field ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$, $T \propto \varpi^{-1}$, $\varpi B_\phi = const$, parabolic shape of fieldlines: $z \propto \varpi^2$)
- $\varpi_6 < \varpi < \varpi_8$: Magnetic acceleration ($\gamma \propto \varpi$, $\rho_0 \propto \varpi^{-3}$)
- $\varpi = \varpi_8$: cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$







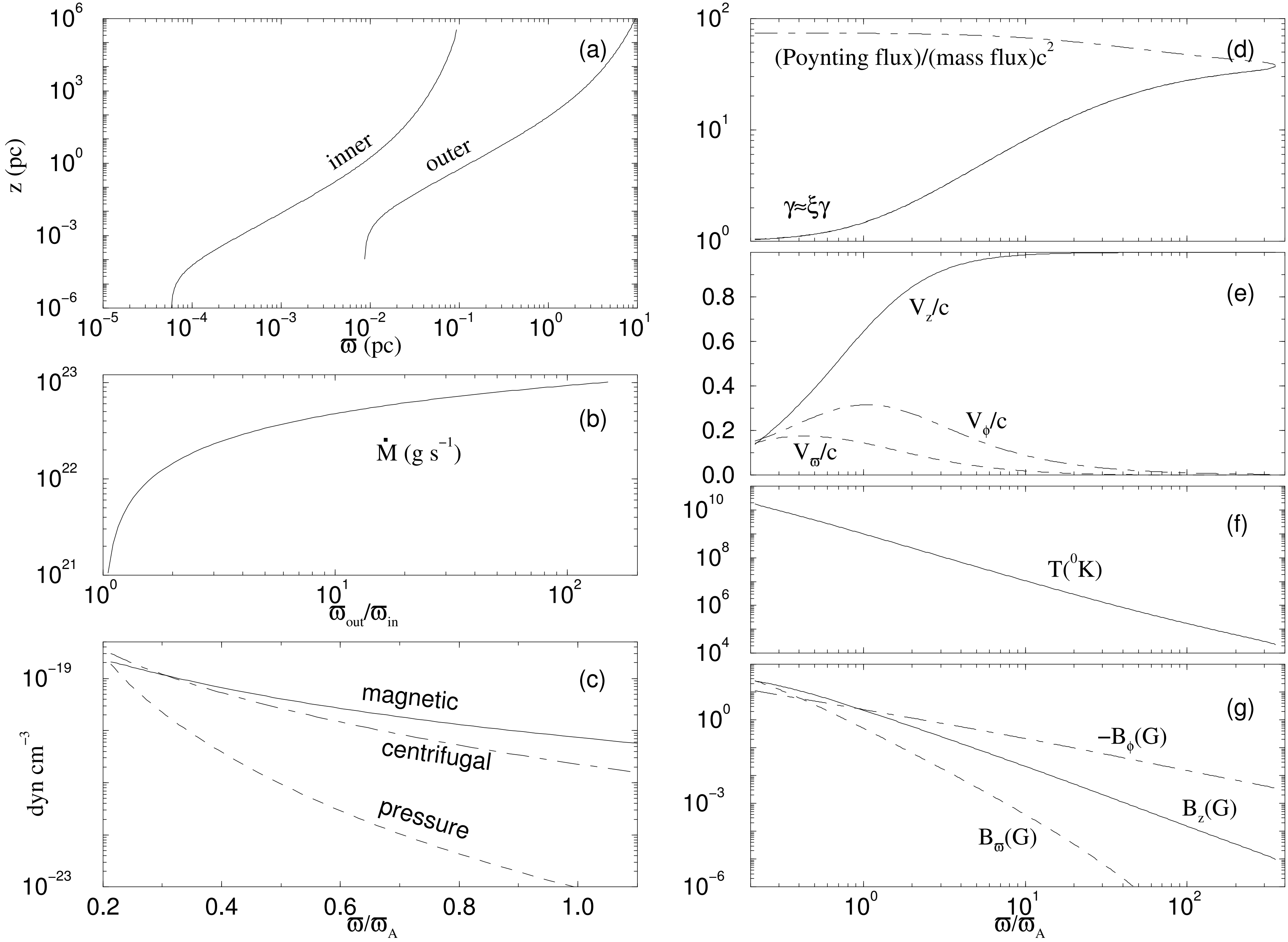
- Thermal acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$, $T \propto \varpi^{-0.8}$, $B_\phi \propto \varpi^{-1}$, $z \propto \varpi^{1.5}$)
- Magnetic acceleration ($\gamma \propto \varpi^{0.44}$, $\rho_0 \propto \varpi^{-2.4}$)
- cylindrical regime - equipartition $\gamma_\infty \approx (-EB_\phi/4\pi\gamma\rho_0 V_p)_\infty$



$\star$ At $\varpi = 10^8$cm – where $\gamma = 10$ – the opening half-angle is already $\vartheta = 10^o$

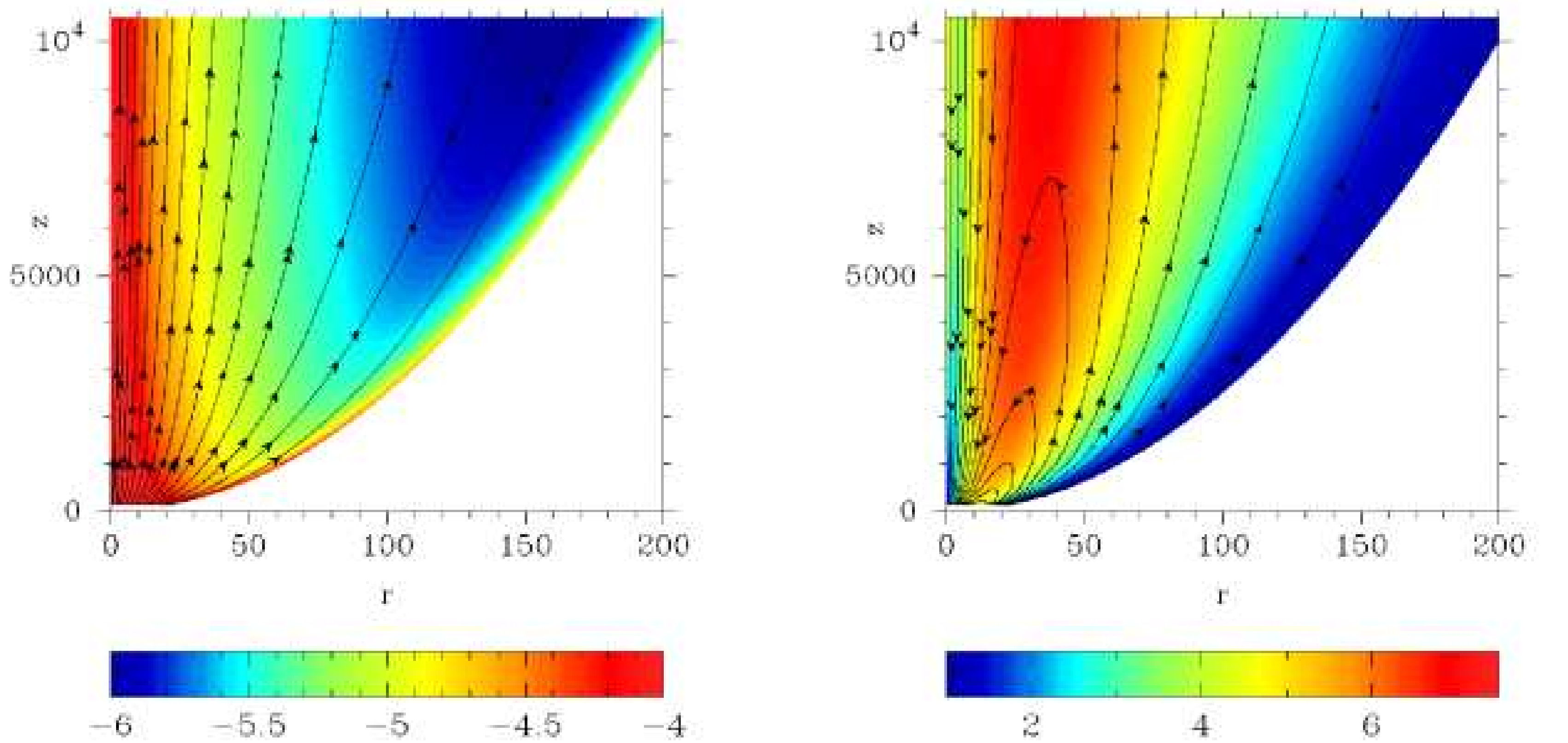
$\star$ For $\varpi > 10^8$cm, collimation continues slowly ($\mathcal{R} \sim \gamma^2\varpi$)

Semi-analytic solutions for AGN jets (Vlahakis & Königl 2004)

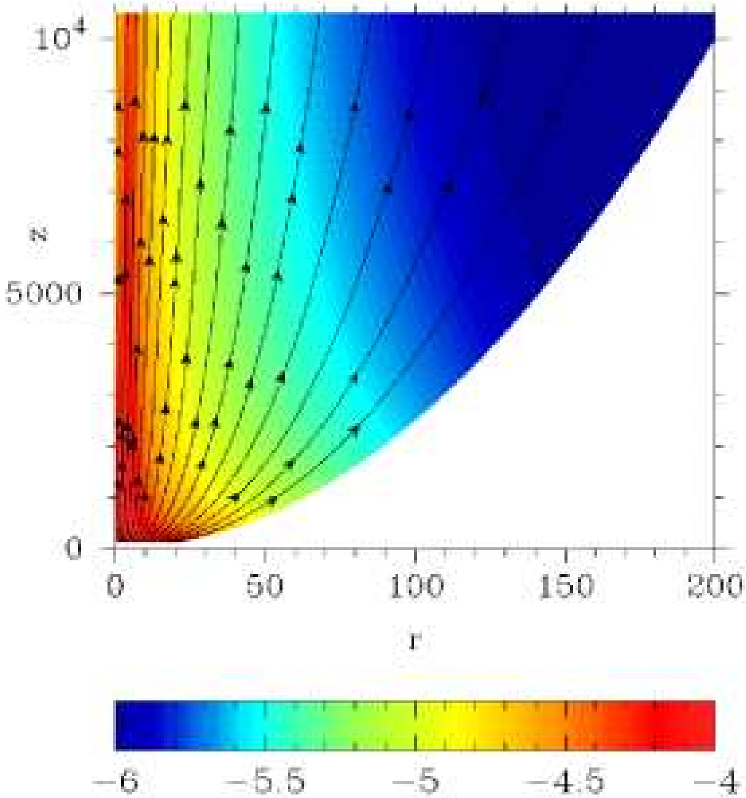


Simulations of relativistic AGN jets

Komissarov, Barkov, Vlahakis, & Königl (2007)



Left panel shows density (colour) and magnetic field lines.
Right panel shows the Lorentz factor (colour) and the current lines.



10^4
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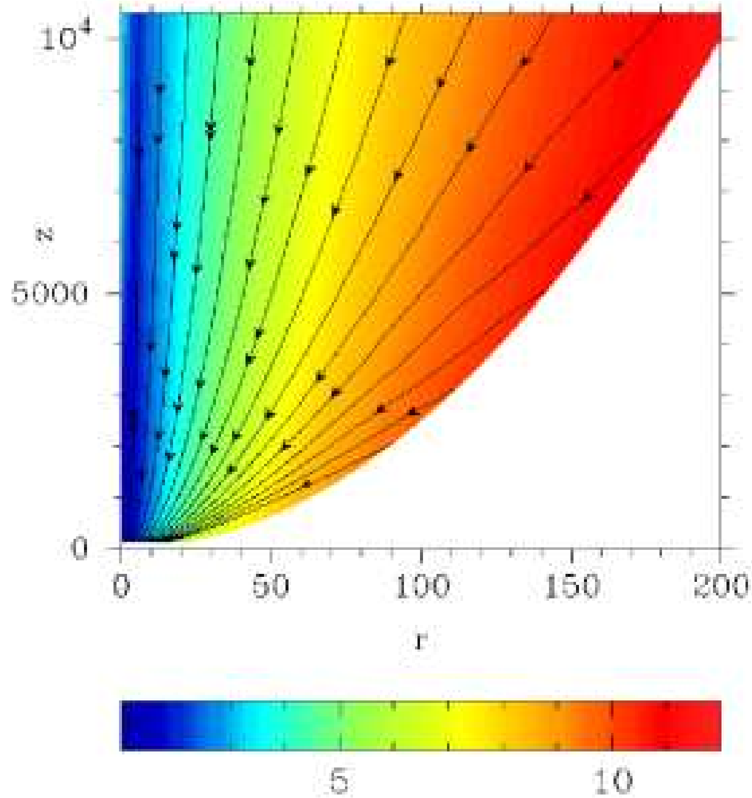
0
0
50
100
150
200
r
−6
−5.5
−5
−4.5
−4



10^4
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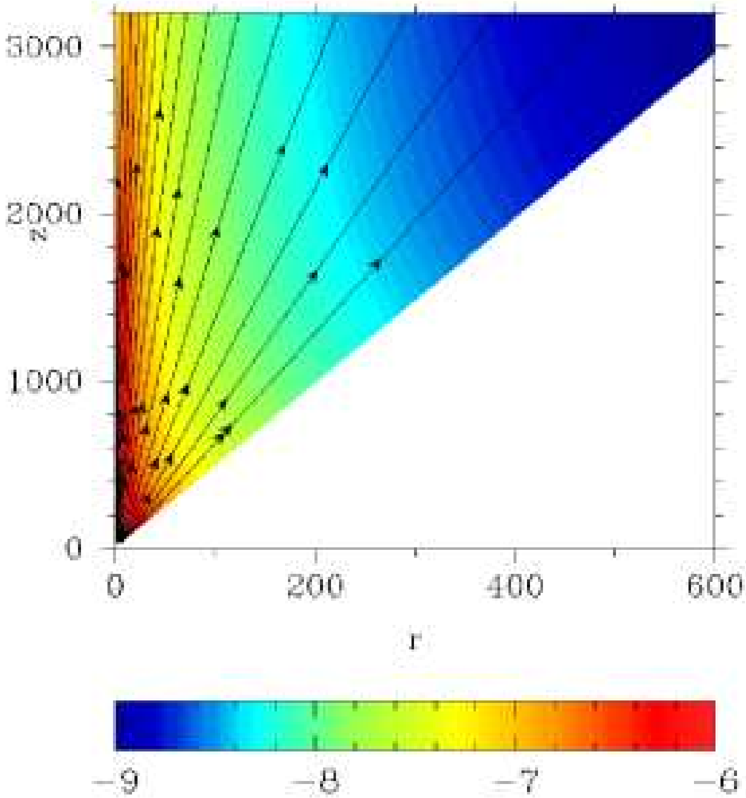
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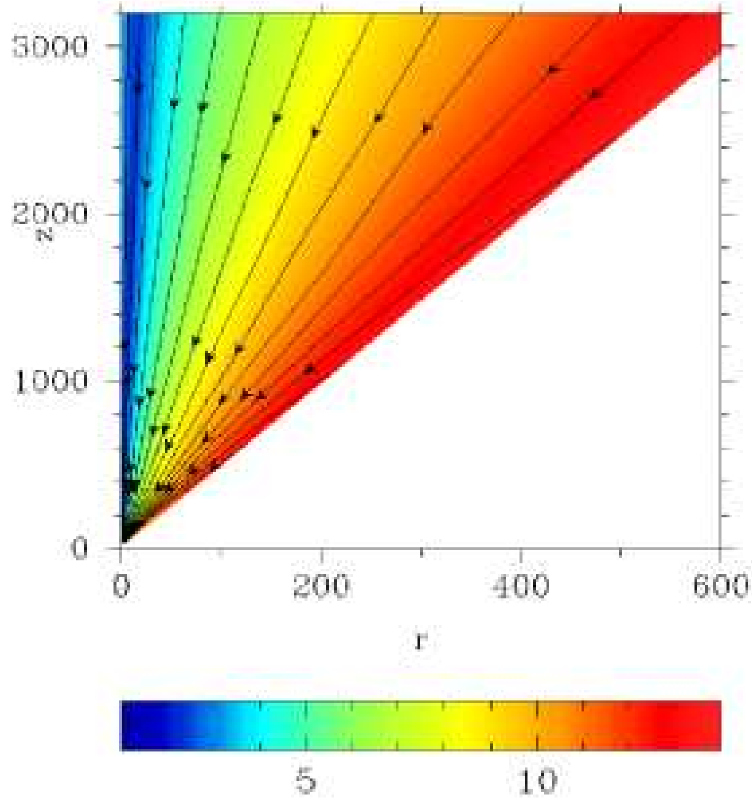
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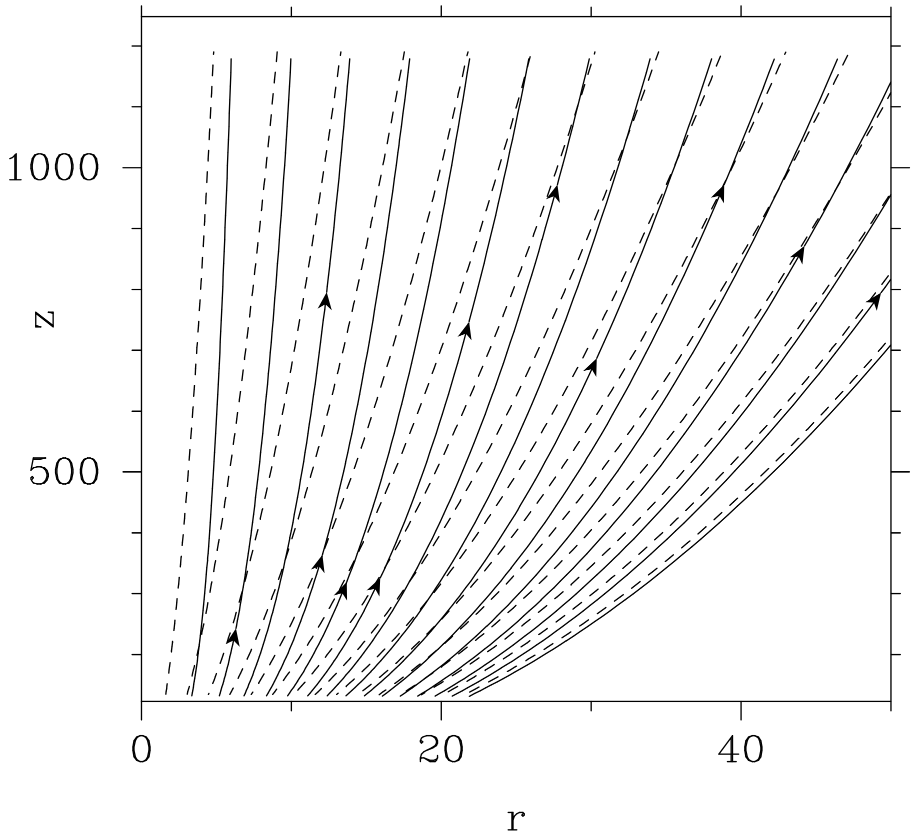
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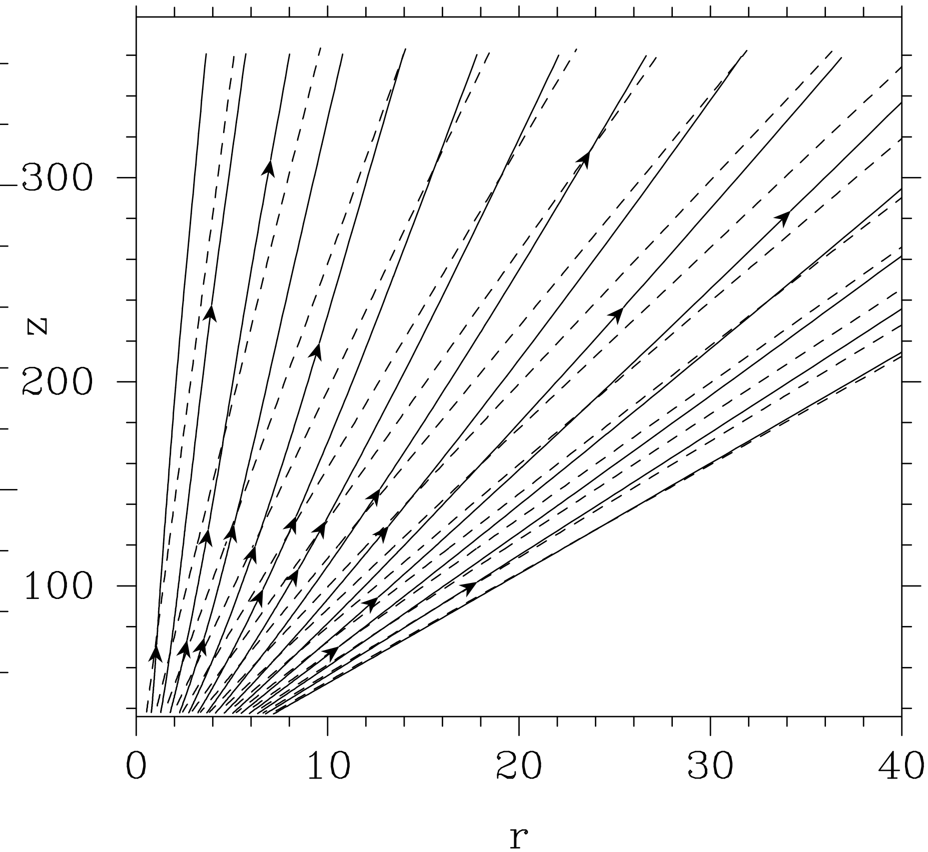
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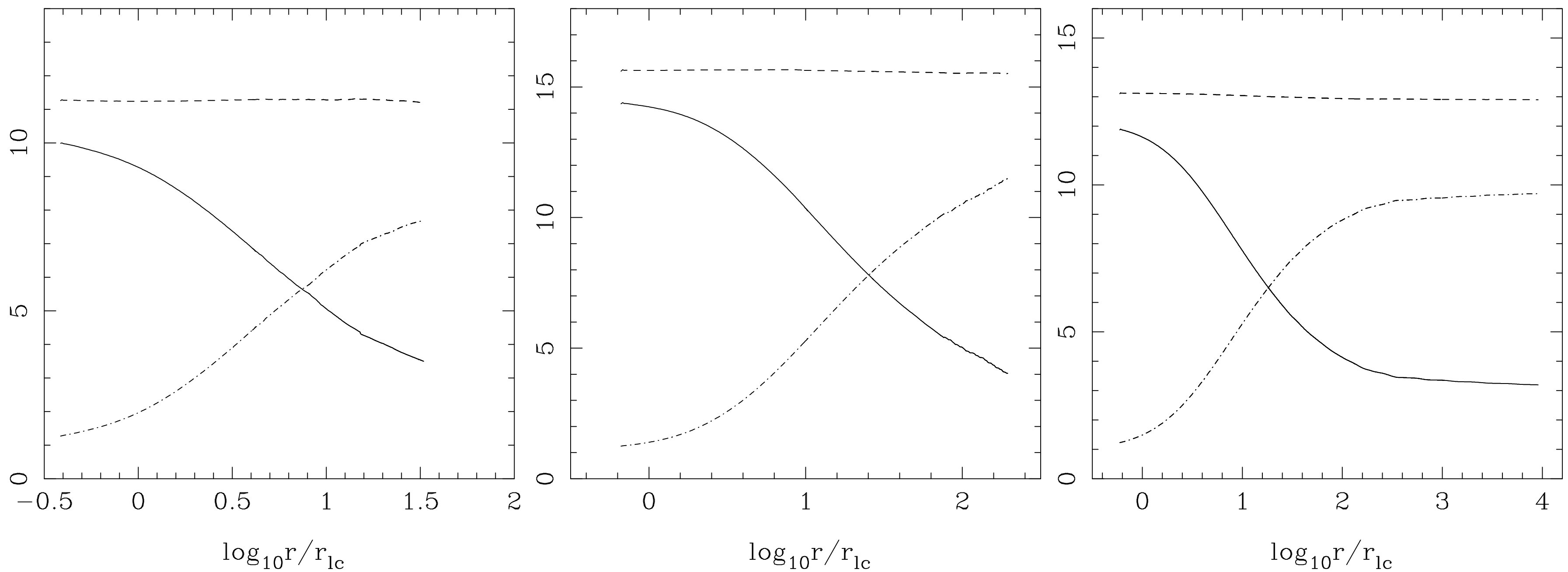
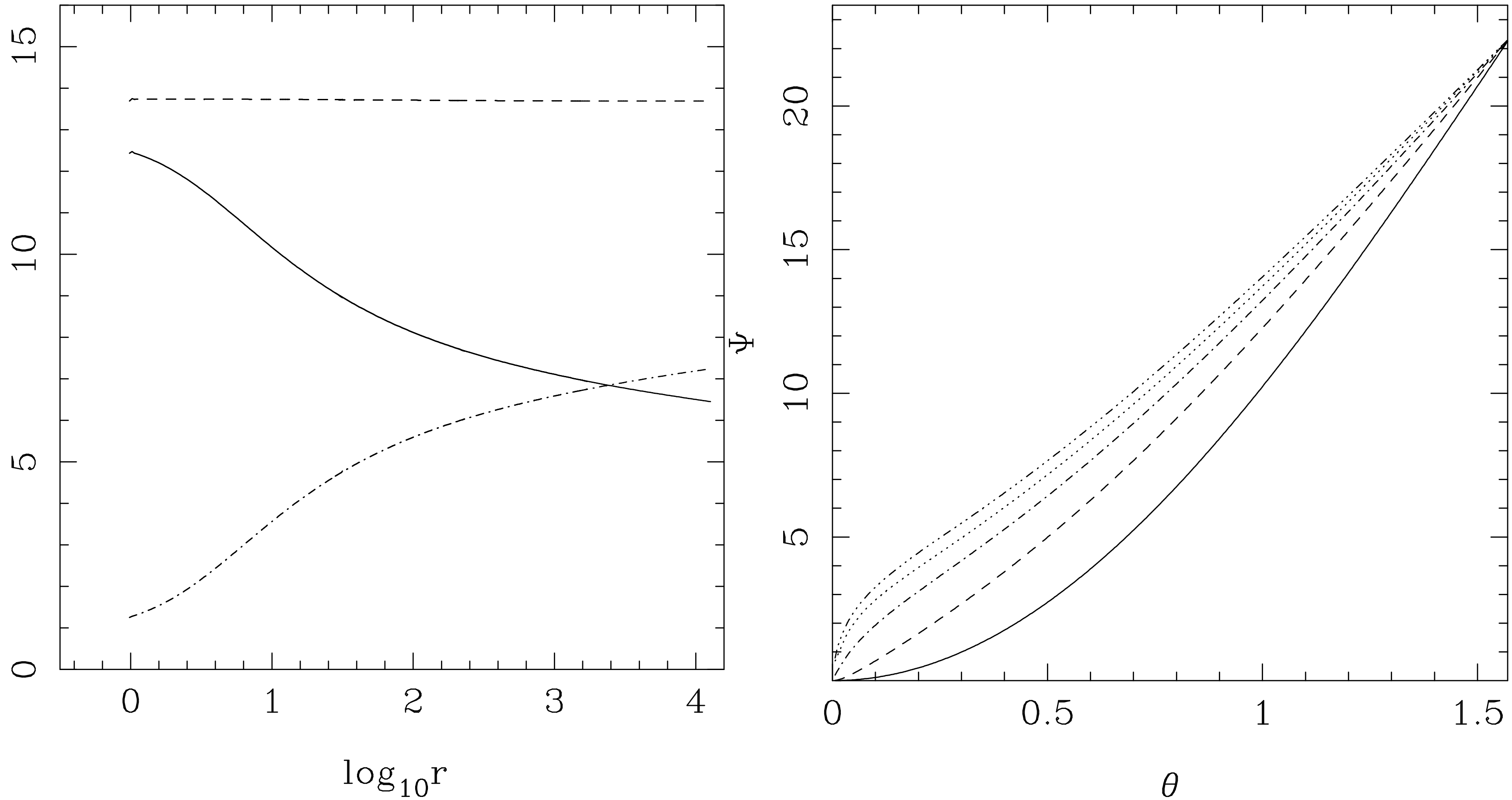


$\gamma\sigma$ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius for models C1 (left panel), C2 (middle panel) and A2 (right panel).

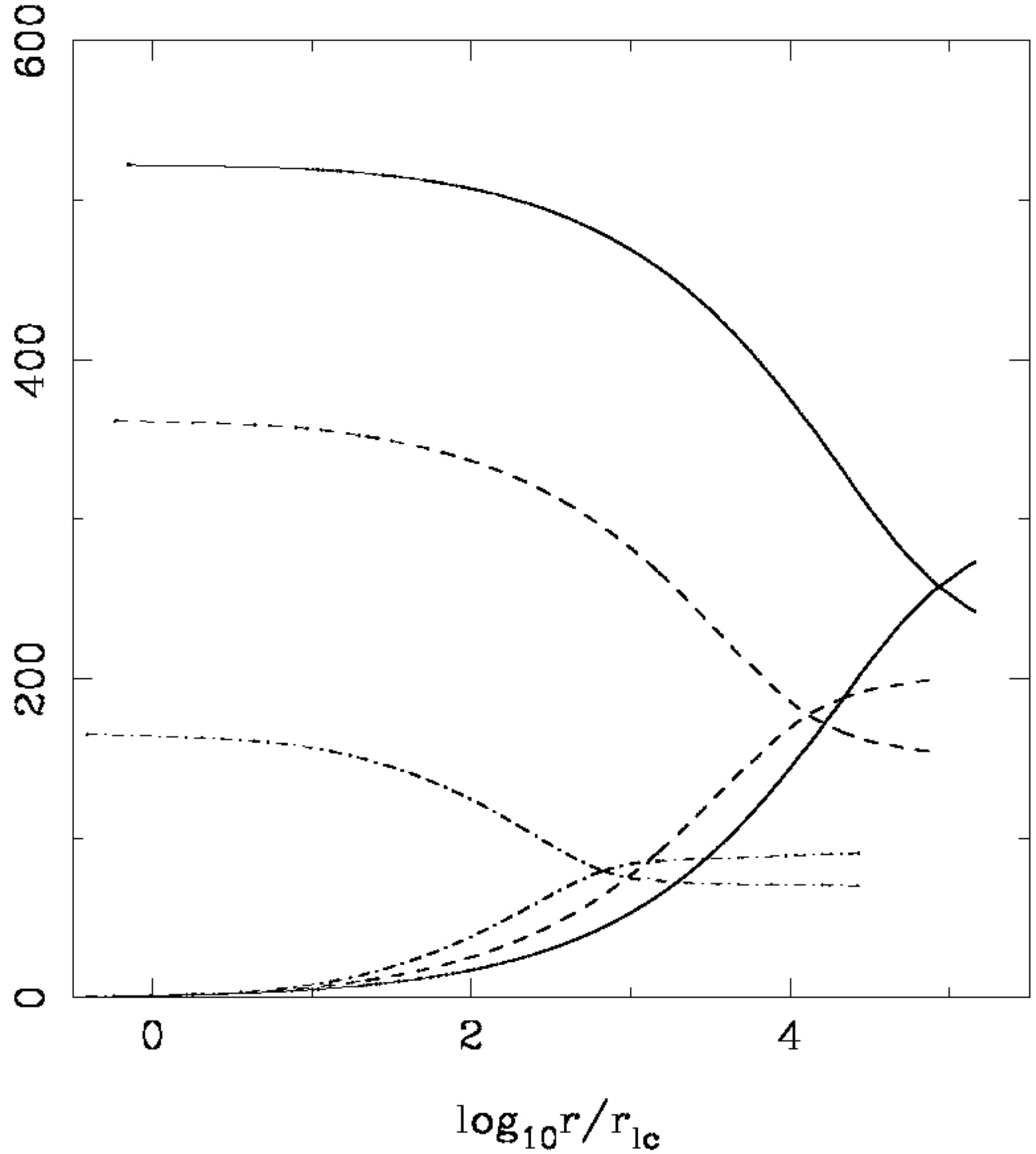
(without a wall)

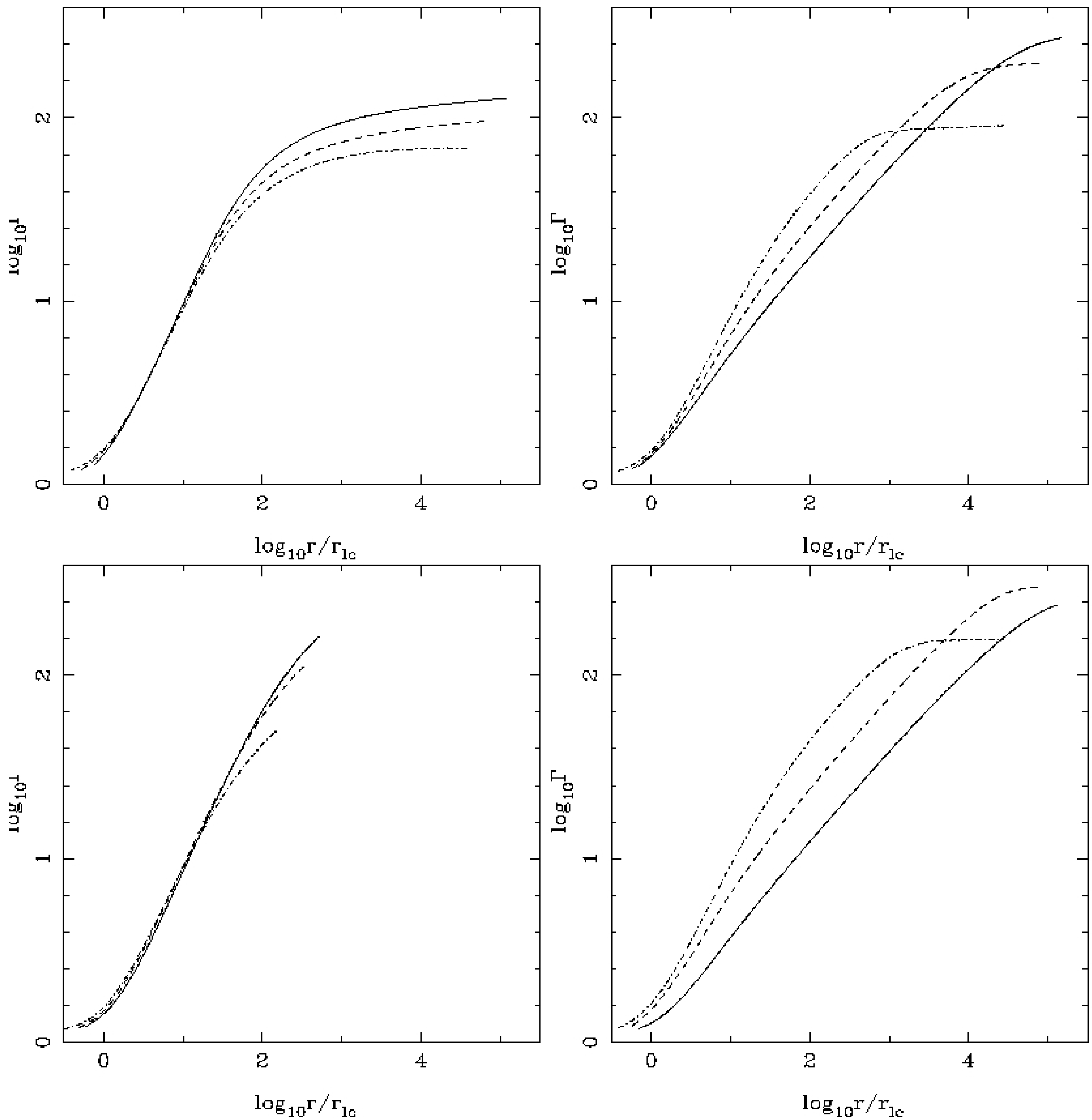


e.g. for $\Psi = 10$, $\vartheta = 57^{\rm o} \to 40^{\rm o}$
while for $\Psi = 5$, $\vartheta = 40^{\rm o} \to 15^{\rm o}$

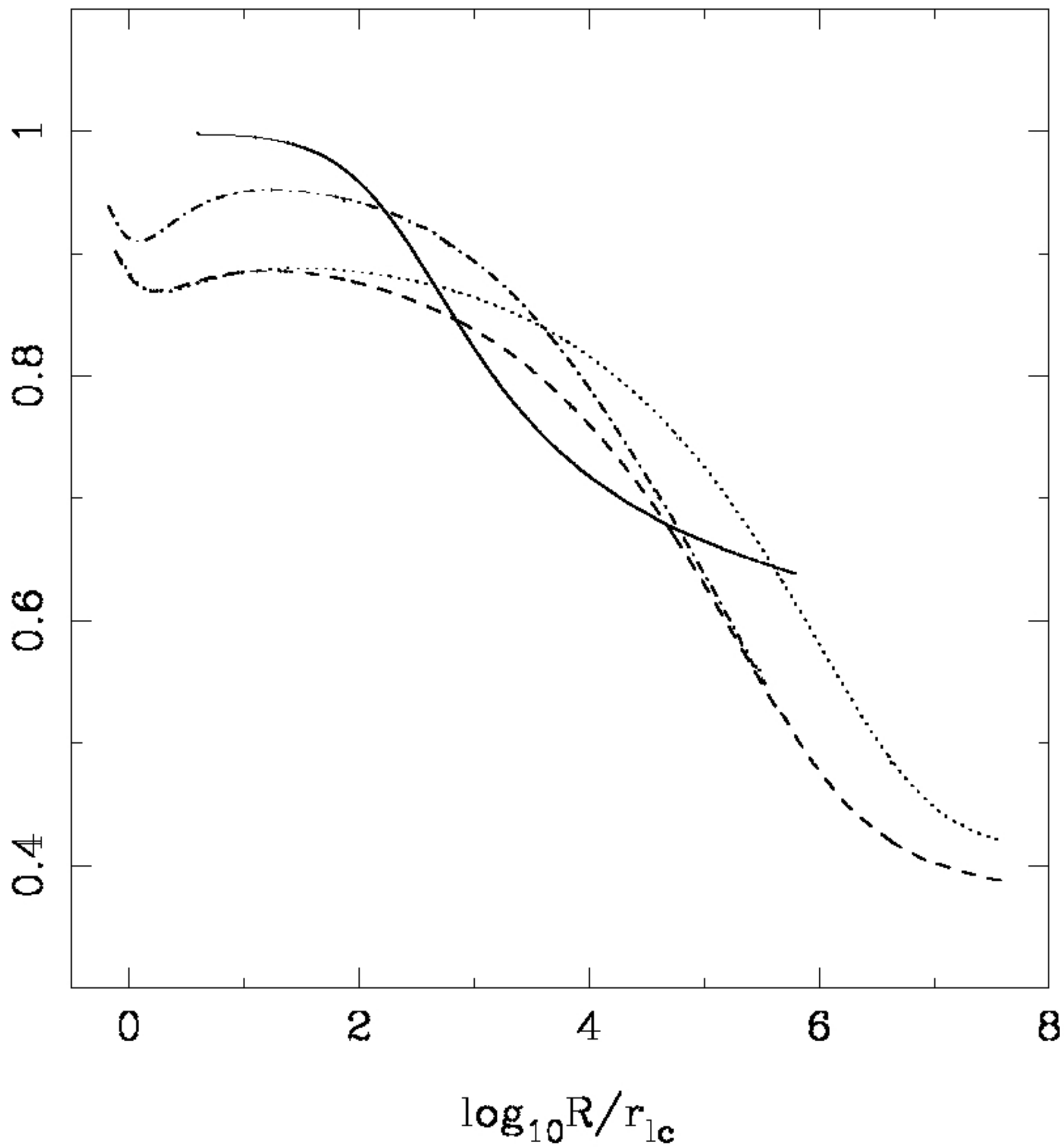
Simulations of relativistic GRB jets

Komissarov, Barkov, Vlahakis, & Königl, in preparation




log10Γ
log10r/rlc
0
1
2
4

$B_p \varpi^2/(2A)$



Summary

⋆ MHD could explain the dynamics of relativistic jets:

- acceleration (the flow is initially thermally, and subsequently magnetically accelerated up to Lorentz factors corresponding to rough equipartition between kinetic and Poynting fluxes)
  $\gamma_\infty \approx 0.5 \frac{\mathcal{E}}{Mc^2}$
- collimation
  parabolic shape $z \propto \varpi^{\beta+1}$ consistent with $\gamma \propto \varpi^\beta$

⋆ The paradigm of MHD jets works in a similar way in all astrophysical jets!