Magnetohydrodynamics of relativistic jets

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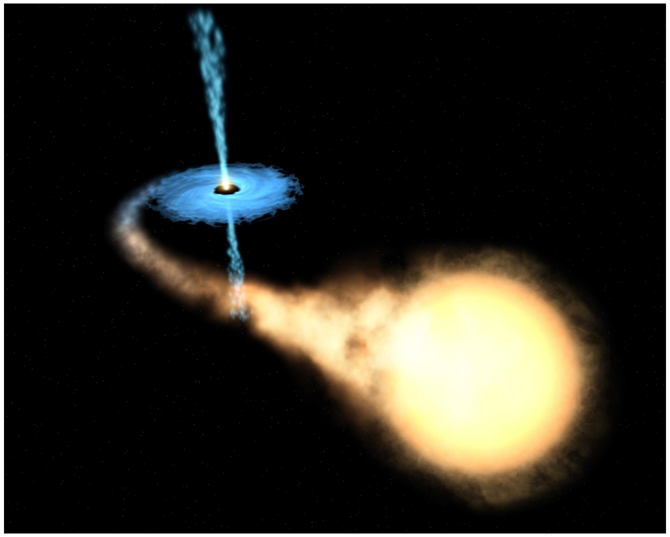
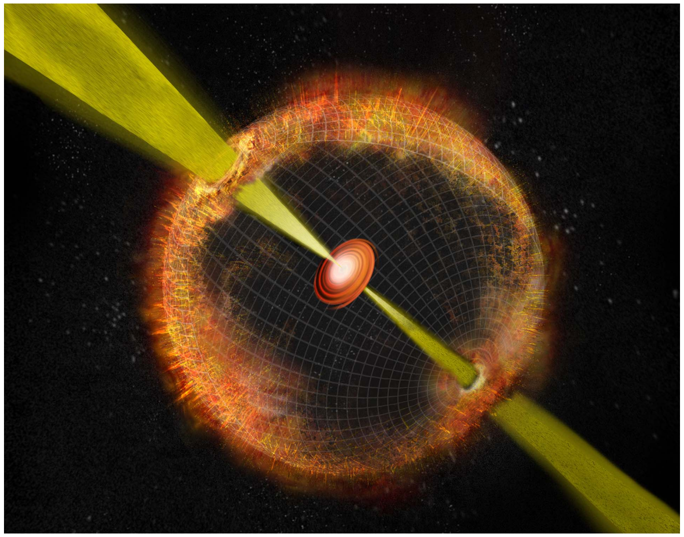
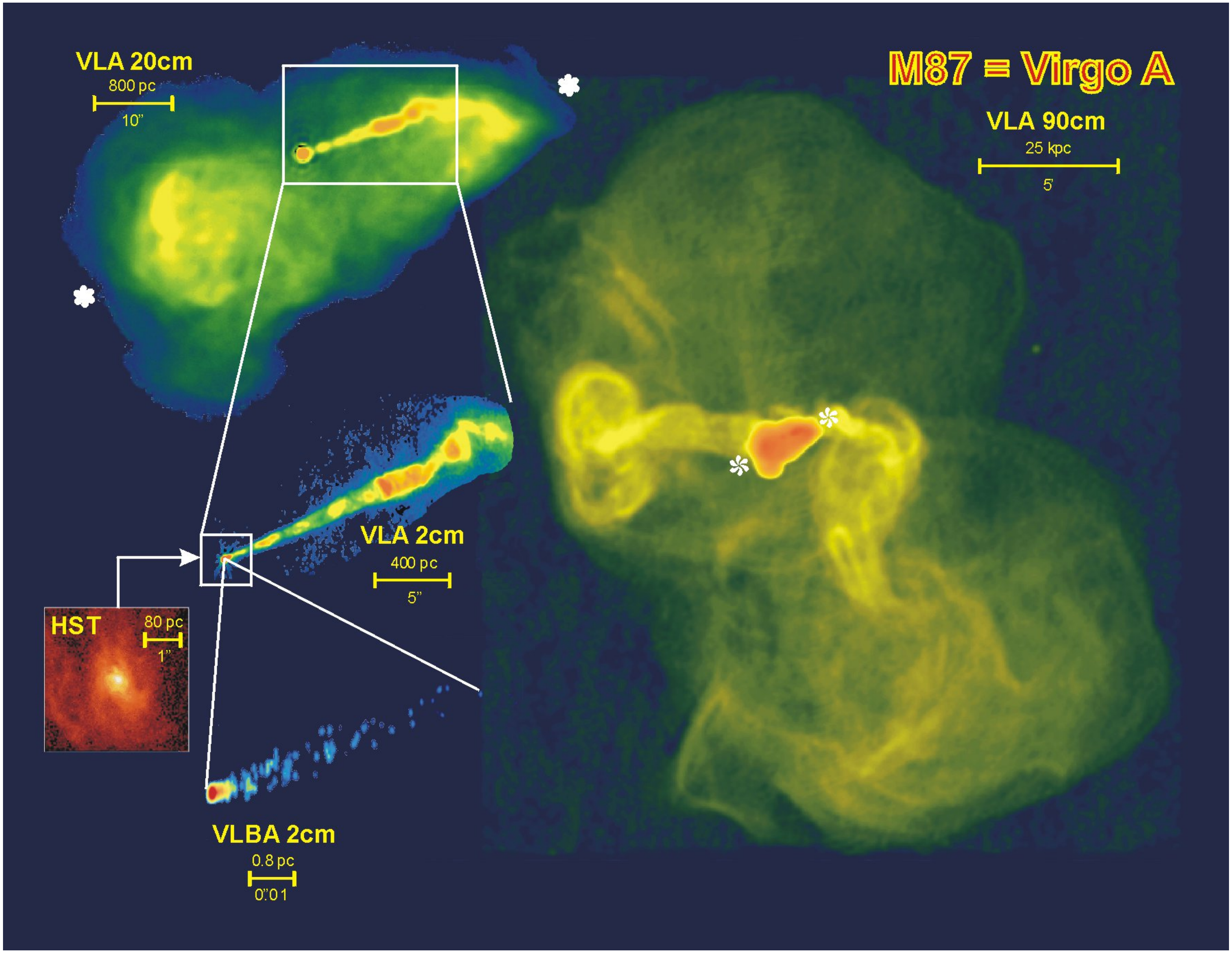
Outline

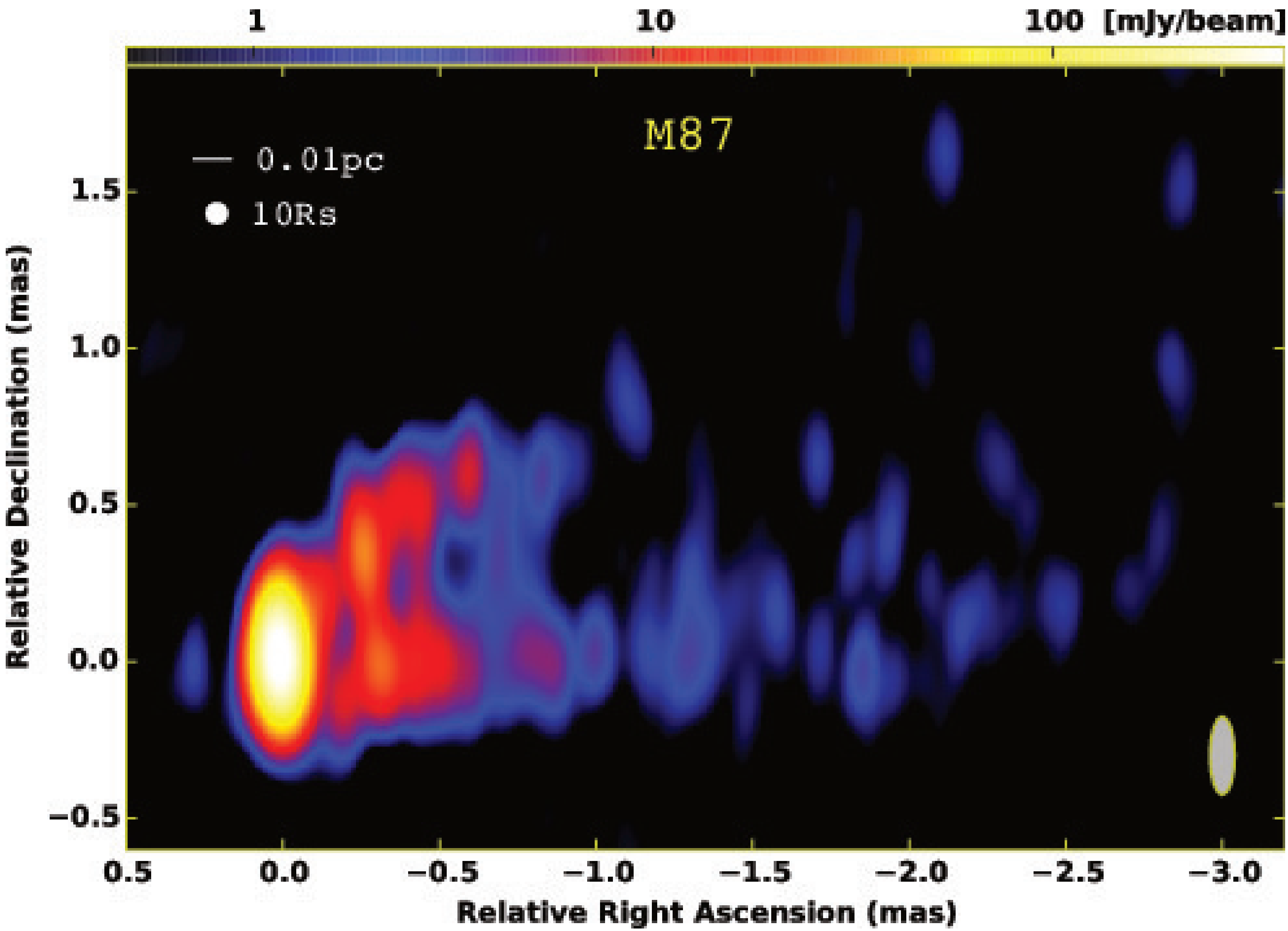
- introduction (jet characteristics)
- collimation-acceleration paradigm
- rarefaction acceleration

Examples of astrophysical jets

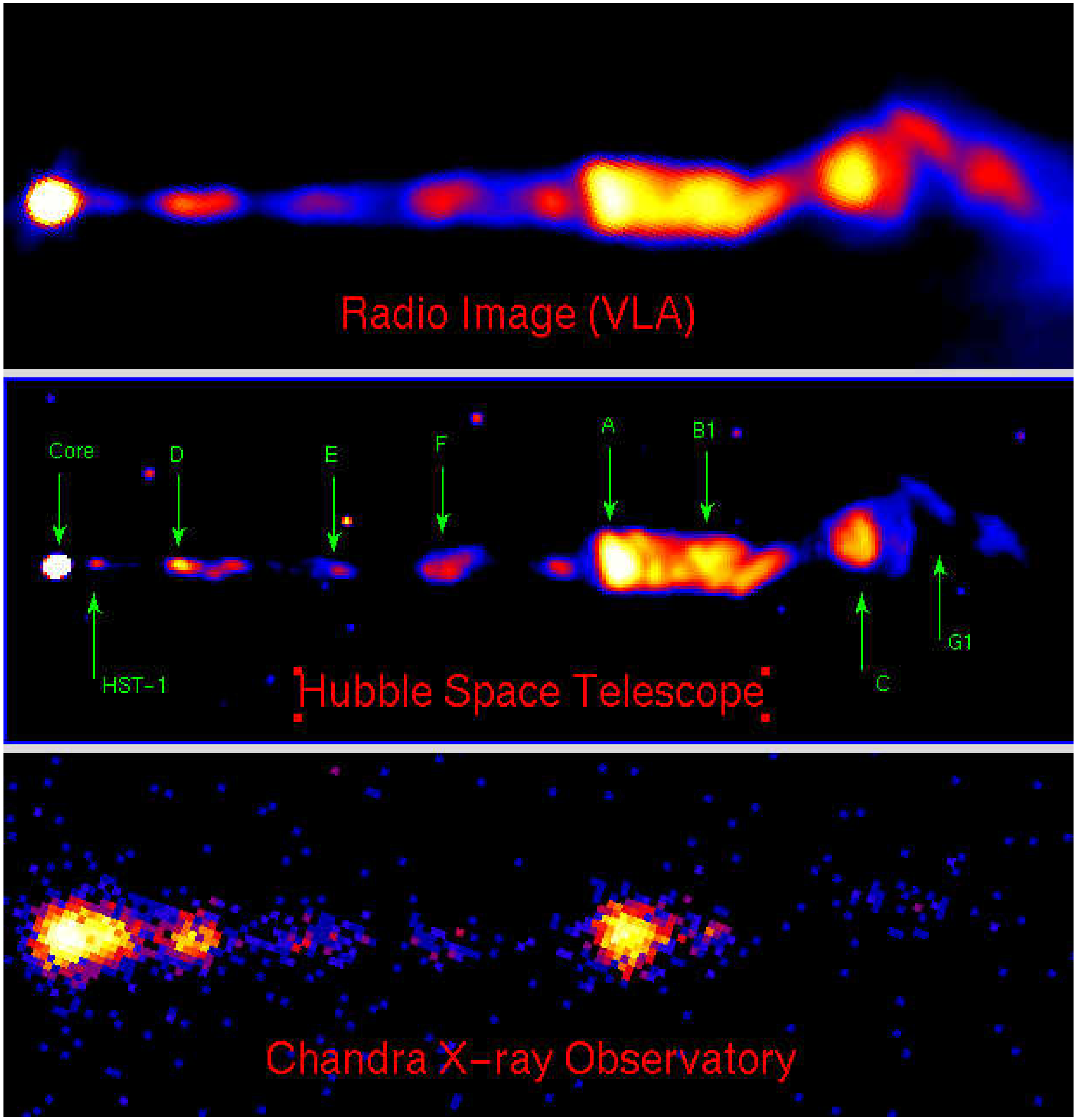




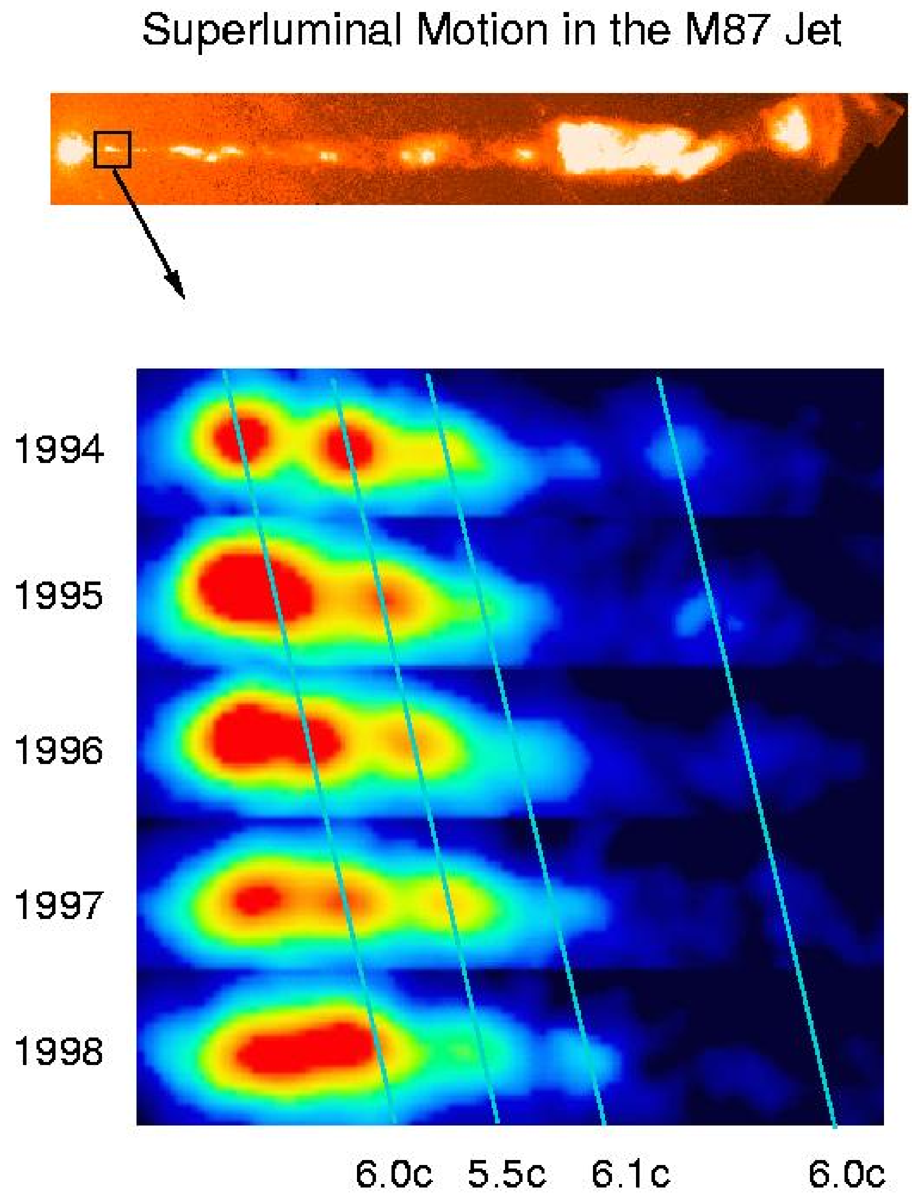

M87 = Virgo A
VLA 20cm
800 pc
10"
VLA 90cm
25 kpc
5'
VLA 2cm
400 pc
5"
HST
80 pc
1"
VLBA 2cm
0.8 pc
0."01



(Hada et al 2016)


Radio Image (VLA)
Core
D
E
F
A
B1
HST-1
Hubble Space Telescope
C
G1
Chandra X-ray Observatory

Jet speed


Superluminal Motion in the M87 Jet
1994
1995
1996
1997
1998
6.0c
5.5c
6.1c
6.0c

On the bulk acceleration

- More distant components have higher apparent speeds
- Brightness temperature increases with distance (Lee, Lobanov, et al)
- A more general argument on the acceleration (Sikora et al):
  - ⋆ lack of bulk-Compton features $\rightarrow$ small ($\gamma < 5$) bulk Lorentz factor at $\lesssim 10^3 r_g$
  - ⋆ the γ saturates at values $\sim$ a few 10 around the blazar zone $(10^3 - 10^4 r_g)$

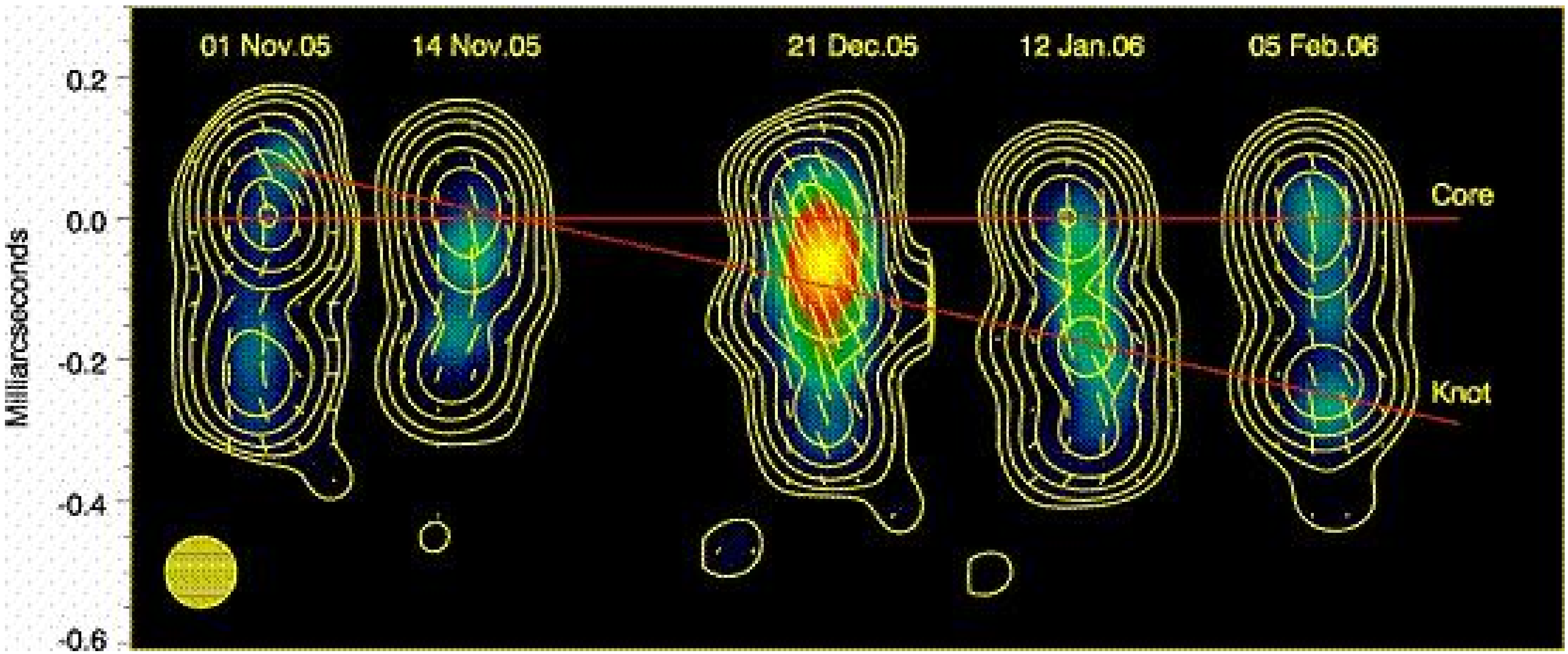
So, relativistic AGN jets undergo the bulk of their acceleration on parsec scales ($\gg$ size of the central black hole)

Hydro-Dynamics

- In case $n_e \sim n_p$, $\gamma_{\max} \sim kT_i/m_pc^2 \sim 1$ even with $T_i \sim 10^{12}K$
- If $n_e \neq n_p$, $\gamma_{\max} \sim (n_e/n_p) \times (kT_i/m_pc^2)$ could be $\gg 1$
- With some heating source, $\gamma_{\max} \gg 1$ is in principle possible

However, even in the last two cases, HD is unlikely to work because the HD acceleration saturates at distances comparable to the sonic surface where gravity is still important, i.e., very close to the disk surface (certainly at $\ll 10^3 r_g$)

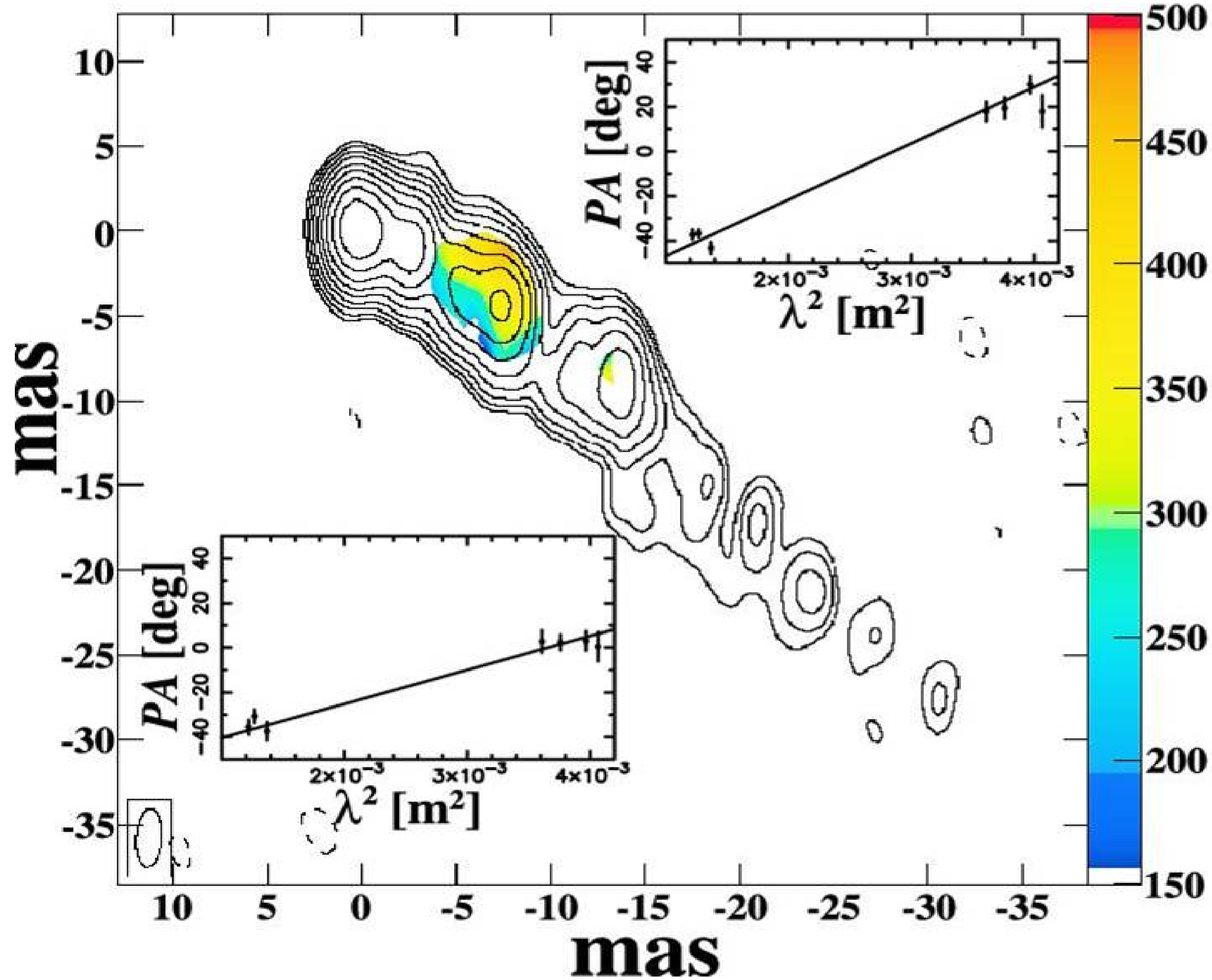
Polarization

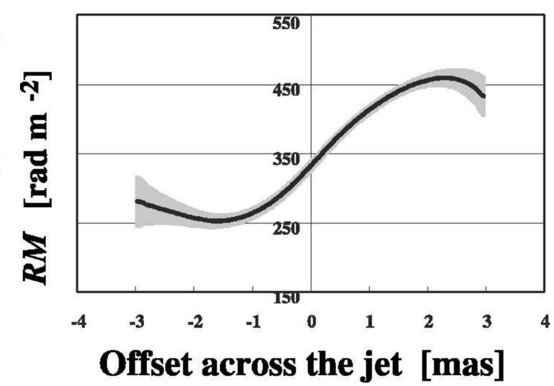


(Marscher et al 2008, Nature)

observed $\boldsymbol{E}_{\rm rad} \perp \boldsymbol{B}_{\perp{\rm los}}$
(modified by Faraday rotation and relativistic effects)

Faraday RM gradients across the jet



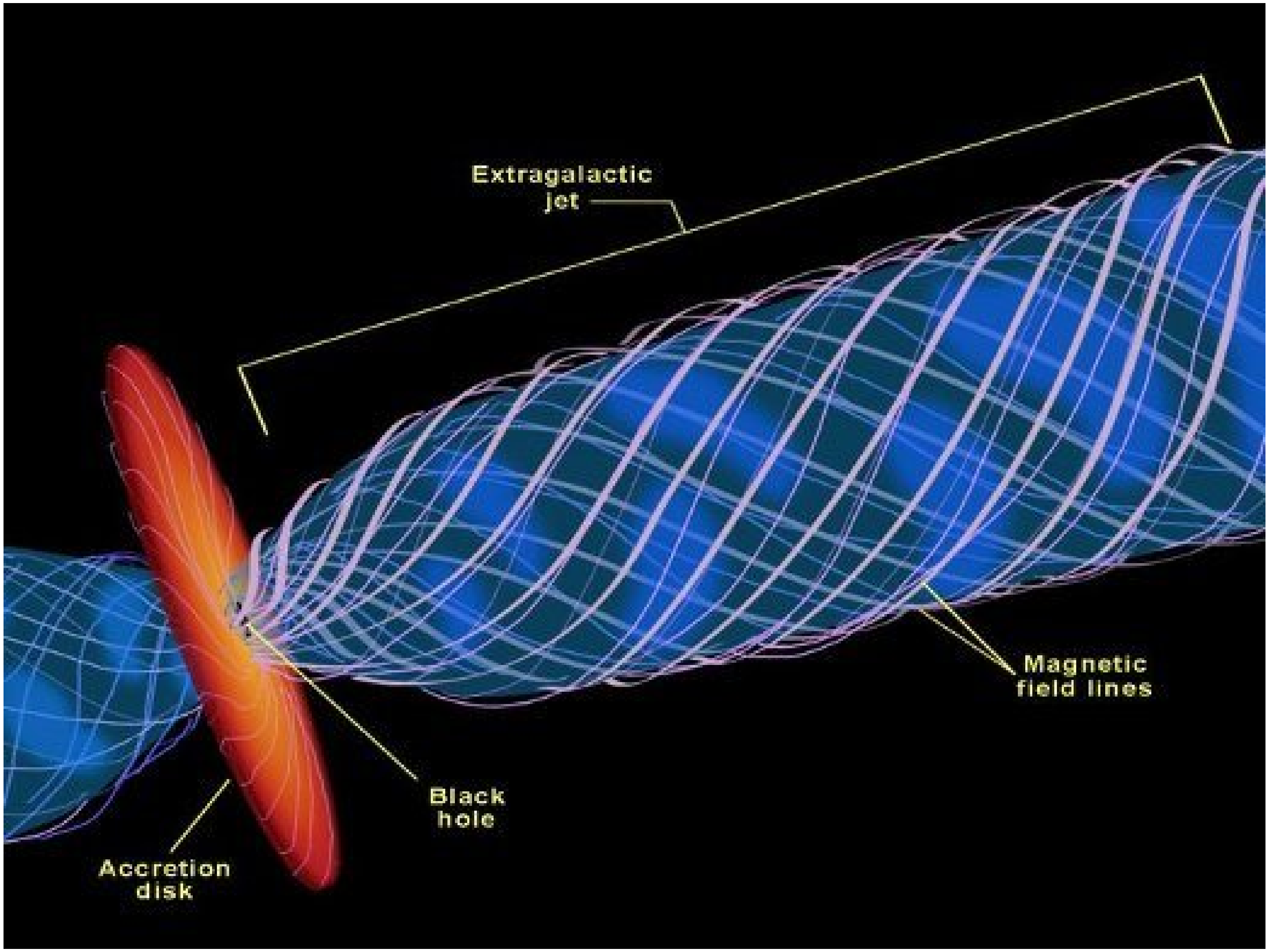


(Asada et al)

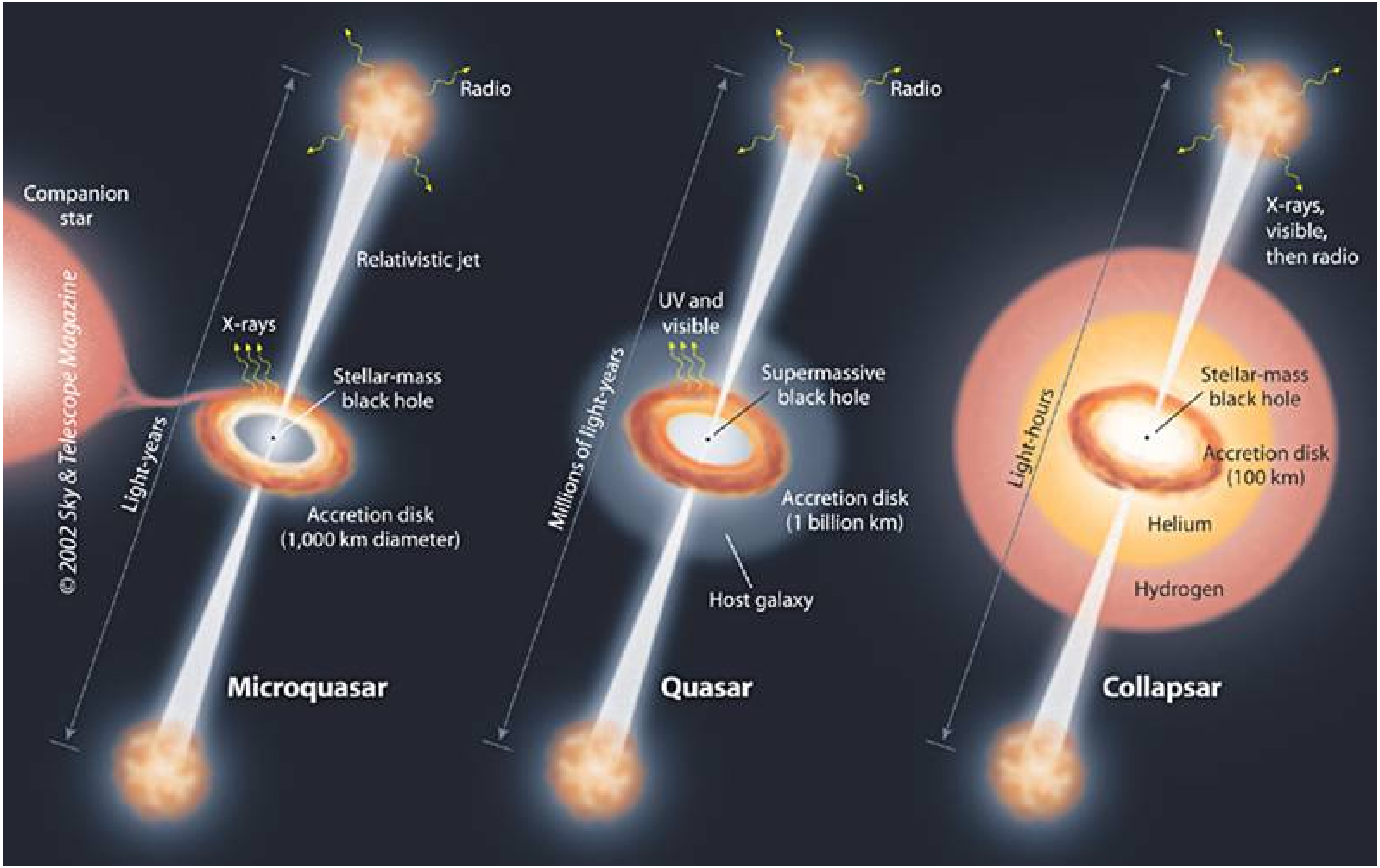
helical field surrounding the emitting region (Gabuzda)

What magnetic fields can do

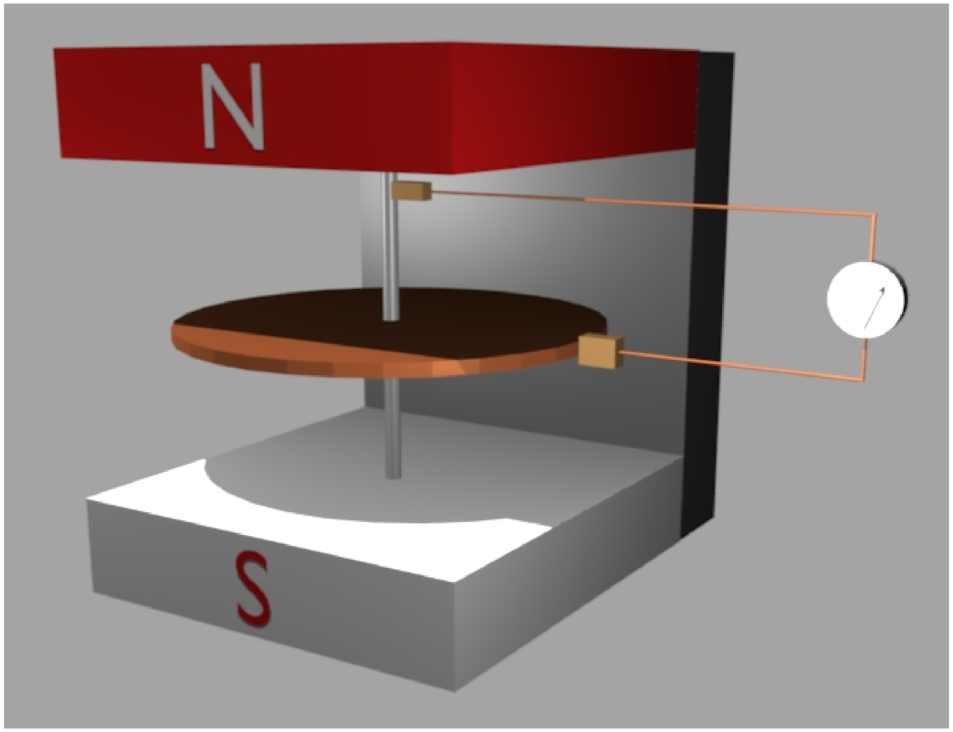
- ⋆ extract energy (Poynting flux)
- ⋆ extract angular momentum
- ⋆ transfer energy and angular momentum to matter
- ⋆ explain relatively large-scale acceleration
- ⋆ self-collimation
- ⋆ synchrotron emission
- ⋆ polarization and Faraday RM maps



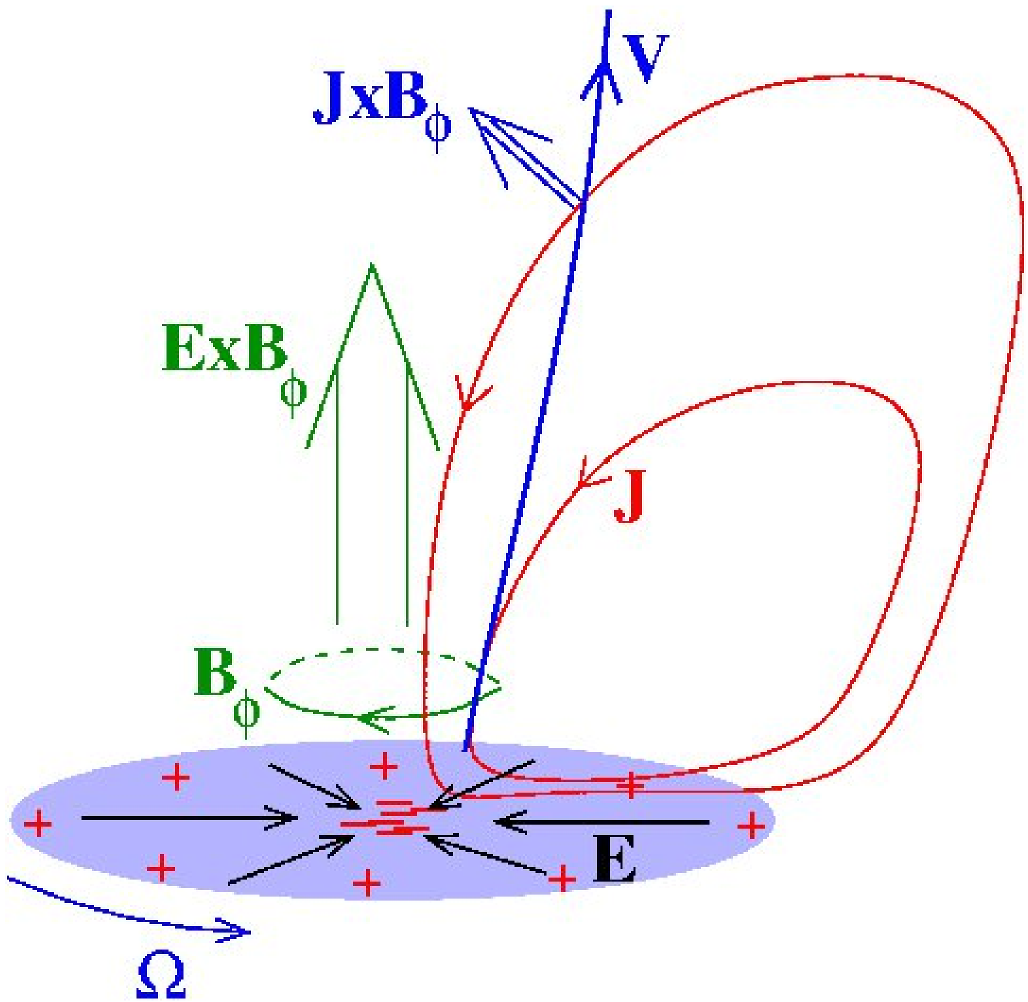
B field from advection, or dynamo, or cosmic battery (Poynting-Robertson drag).


Companion star
Radio
Relativistic jet
X-rays
Stellar-mass black hole
Accretion disk (1,000 km diameter)
Light-years
© 2002 Sky & Telescope Magazine
Microquasar
Radio
UV and visible
Supermassive black hole
Accretion disk (1 billion km)
Host galaxy
Millions of light-years
Quasar
X-rays, visible, then radio
Stellar-mass black hole
Accretion disk (100 km)
Helium
Hydrogen
Light-hours
Collapsar

A unipolar inductor



magnetic field + rotation



current $\leftrightarrow B_\phi$
Poynting flux $\frac{c}{4\pi}EB_\phi$ is extracted (angular momentum as well)

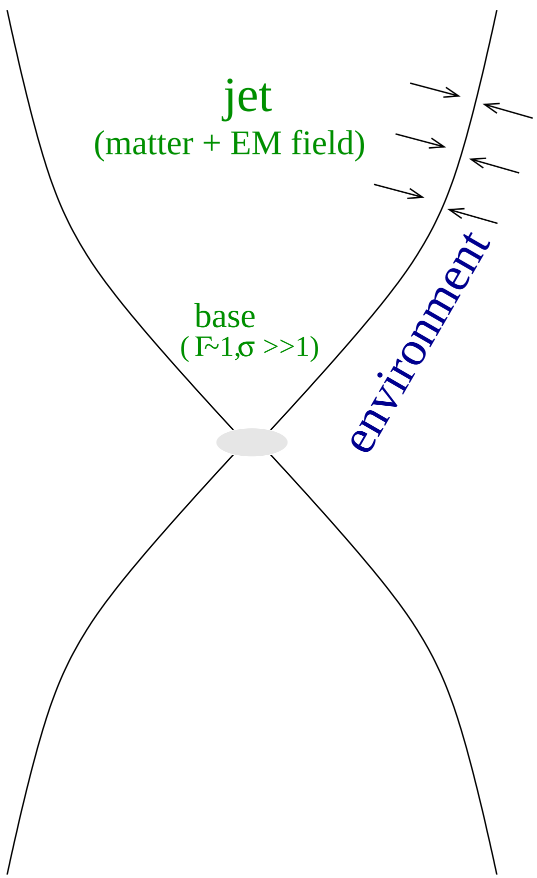
The Faraday disk could be the rotating accretion disk, or the frame dragging if energy is extracted from the ergosphere of a rotating black hole (Blandford & Znajek mechanism)

How to model magnetized outflows?

⋆ as pure electromagnetic energy (force-free, magnetodynamics, electromagnetic outflows):
– ignore matter inertia (reasonable near the origin)
– this by assumption does not allow to study the transfer of energy form Poynting to kinetic
– wave speed $= c \rightarrow$ no shocks
– there may be some dissipation (e.g. reconnection) $\rightarrow$ radiation

⋆ as magneto-hydro-dynamic flow
– the force-free case is included as the low inertia limit
– the back reaction from the matter to the field is included

Magnetized outflows



- Extracted energy per time $\dot{\mathcal{E}}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)

$$\dot{\mathcal{E}} = \frac{c}{4\pi} \underbrace{\frac{r}{r_{\rm lc}} B_p}_{E} \, B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{\mathcal{E}}/\dot{M}c^2$ gives the maximum possible bulk Lorentz factor of the flow

- Magnetohydrodynamics:
matter (velocity, density, pressure)
+ large scale electromagnetic field

Basic questions

☞ bulk acceleration

- thermal (due to ∇P) $\rightarrow$ velocities up to C_s
- magnetocentrifugal $\rightarrow$ velocities up to $V_{\phi i}$
- relativistic thermal (thermal fireball) gives $\gamma \sim \left(\dfrac{\text{enthalpy}}{\text{mass} \times c^2}\right)_{\mathrm{i}}$.
- magnetic ($\boldsymbol{J} \times \boldsymbol{B}$ force)
  acceleration efficiency $\gamma_\infty/\mu (= \gamma_\infty \dot{M} c^2/\dot{\mathcal{E}}) =$?
  terminal γ_∞ ?

☞ collimation

hoop-stress + electric force

degree of collimation ?

jet opening angle ?

some key steps on relativistic MHD modeling

- Michel 1969: assuming monopole flow (crucial) $\rightarrow$ inefficient acceleration with $\gamma_\infty \approx \mu^{1/3} \ll \mu$

- Li, Chiueh & Begelman 1992; Contopoulos 1994: cold self-similar model $\rightarrow \gamma_\infty \approx \mu/2$ (50% efficiency)

- Vlahakis & Königl 2003: generalization of the self-similar model (including thermal and radiation effects) $\rightarrow \gamma_\infty \approx \mu/2$ (50% efficiency)

- Vlahakis 2004: complete asymptotic transfield force-balance connect the flow-shape (collimation) with acceleration explain why Michel's model is inefficient

- Beskin & Nokhrina 2006: parabolic jet with $\gamma_\infty \approx \mu/2$

some key steps (cont'd)

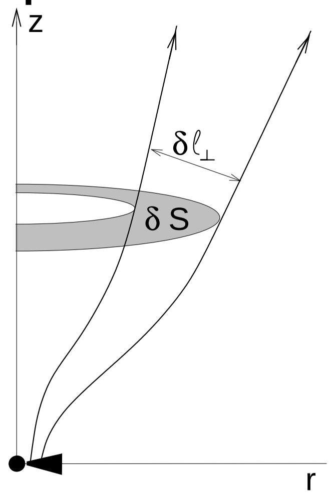
- Komissarov, Barkov, Vlahakis & Königl 2007 and
  Komissarov, Vlahakis, Königl & Barkov 2009:
  possible for the first time to simulate high γ MHD flows and follow the acceleration up to the end
  + analytical scalings
  + role of causality, role of external pressure

- Tchekhovskoy, McKinney & Narayan, 2009: simulations of nearly monopolar flow (more detailed than in Komissarov et al 2009)
  Even for nearly monopolar flow the acceleration is efficient near the rotation axis

- Lyubarsky 2009:
  generalization of the analytical results of Vlahakis 2004 and Komissarov et al 2009

"Standard" model for magnetic acceleration

☞ component of the momentum equation



$$\gamma n(\boldsymbol{V}\cdot\nabla)\,(\gamma w\boldsymbol{V}) = -\nabla p + J^0\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}$$

along the flow (wind equation): $\gamma \approx \mu - \mathcal{F}$

where $\mathcal{F} \propto r^2 n\gamma V_p = r^2\times$ mass flux

since mass flux $\times\delta S =$ const,

$\mathcal{F} \propto r^2/\delta S \propto r/\delta\ell_\perp$

acceleration requires the separation between streamlines (or fieldlines) to increase faster than the cylindrical radius

the collimation-acceleration paradigm:
$\mathcal{F}\downarrow$ through stronger collimation of the inner streamlines relative to the outer ones (differential collimation)

the resulting core with strong poloidal magnetic field may affect jet stability

☞ transfield component of the momentum equation

$$\frac{\gamma^2 r}{\mathcal{R}} \approx \frac{\left(\frac{2I}{\Omega B_p r^2}\right)^2 r \nabla_\perp \ln \left|\frac{I}{\gamma}\right|}{1 + \frac{w}{\rho c^2} \frac{4\pi\rho u_p^2}{B_p^2} \frac{r_{\rm lc}^2}{r^2}} - \gamma^2 \frac{r_{\rm lc}^2}{r^2} \nabla_\perp r, \text{ with } \nabla_\perp \sim 1/r,$$

for ($\gamma \gg 1$) simplifies to $\underbrace{\frac{\gamma^2 r}{\mathcal{R}}}_{inertia} \approx \underbrace{1}_{EM} - \underbrace{\gamma^2 \frac{r_{\rm lc}^2}{r^2}}_{centrifugal}$

• if centrifugal negligible then $\gamma \approx z/r$ (since $\mathcal{R}^{-1} \approx -\frac{d^2 r}{dz^2} \approx \frac{r}{z^2}$)
power-law acceleration regime
(for parabolic shapes $z \propto r^a$, γ is a power of r)

• if inetria negligible then $\gamma \approx r/r_{\rm lc}$ linear acceleration regime

• if electromagnetic negligible then ballistic regime

☞ role of external pressure

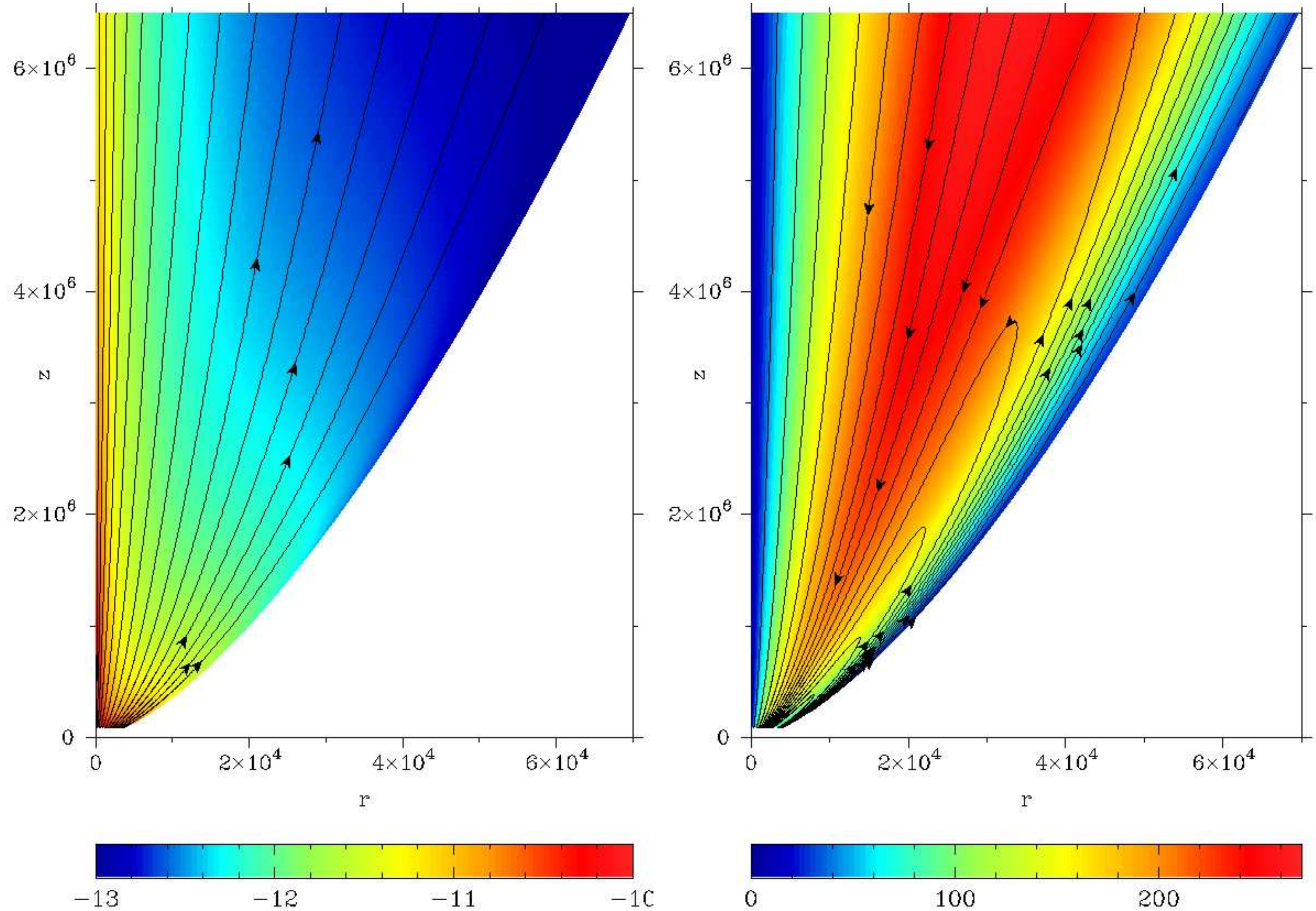
$p_{\rm ext} = B_{\rm co}^2/8\pi \simeq (B^{\hat\phi})^2/8\pi\gamma^2 \propto 1/r^2\gamma^2$

Assuming $p_{\rm ext} \propto z^{-\alpha_p}$ we find $\gamma^2 \propto z^{\alpha_p}/r^2$.

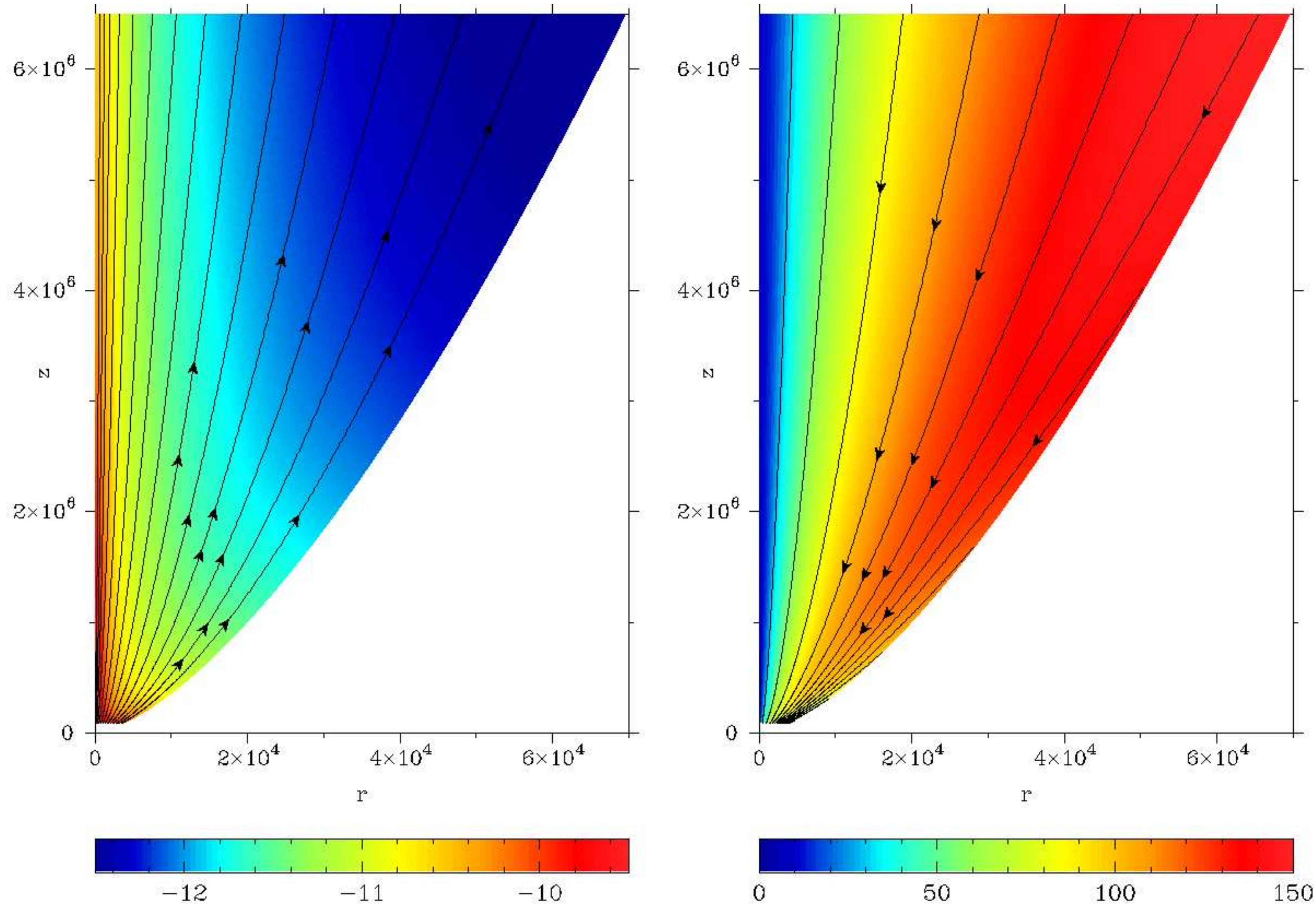
Combining with the transfield $\frac{\gamma^2 r}{\mathcal{R}} \approx 1 - \gamma^2 \frac{r_{\rm lc}^2}{r^2}$ we find the jet shape (we find the exponent a in $z \propto r^a$).

- if the pressure drops slower than z^{-2} then
  - ⋆ shape more collimated than $z \propto r^2$
  - ⋆ linear acceleration $\gamma \propto r$
- if the pressure drops as z^{-2} then
  - ⋆ parabolic shape $z \propto r^a$ with $1 < a \leq 2$
  - ⋆ first $\gamma \propto r$ and then power-law acceleration $\gamma \sim z/r \propto r^{a-1}$
- if pressure drops faster than z^{-2} then
  - ⋆ conical shape
  - ⋆ linear acceleration $\gamma \propto r$ (small efficiency)

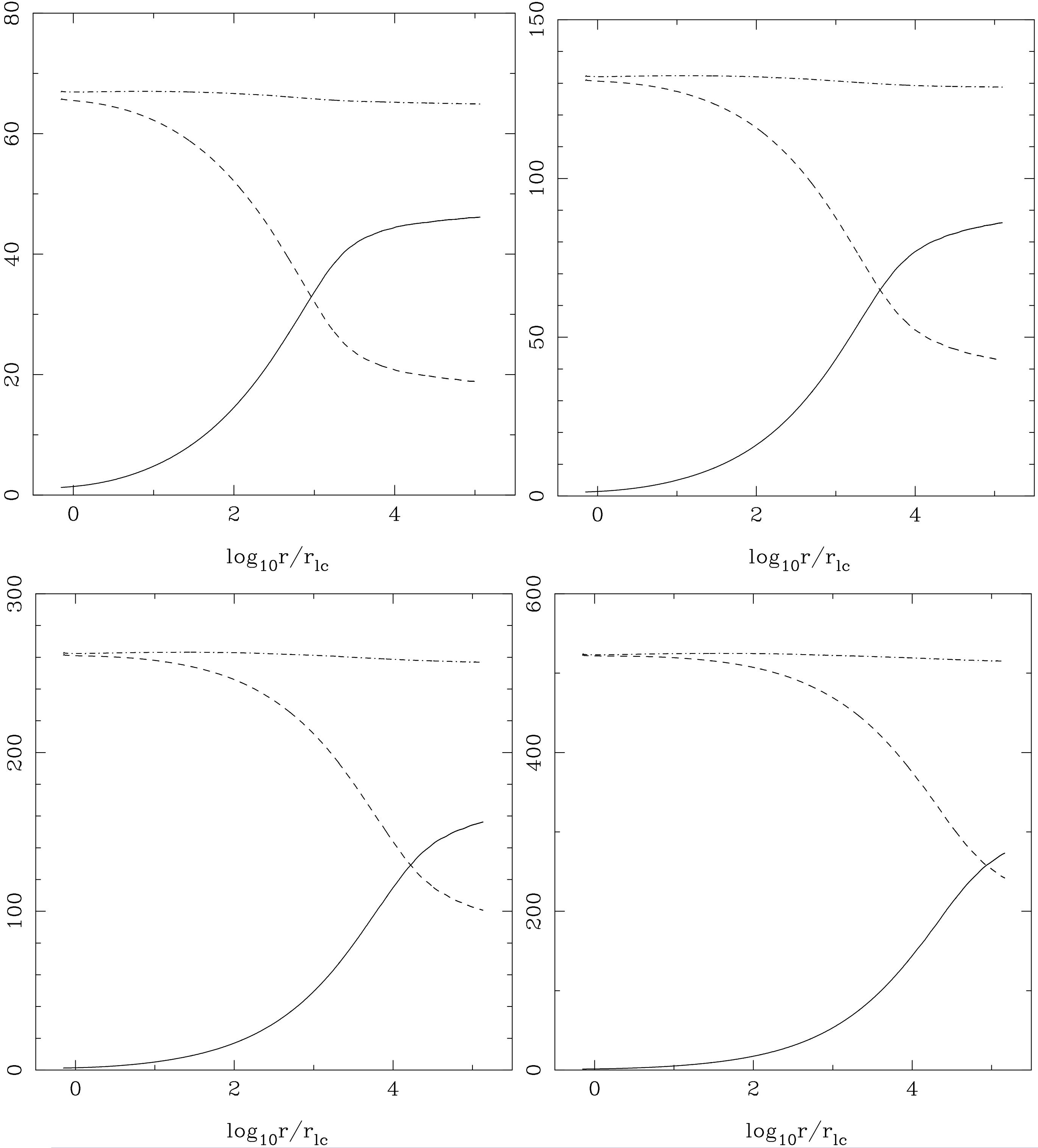
Komissarov, Vlahakis, Königl, & Barkov 2009



left: density/field lines, right: Lorentz factor/current lines (jet shape $z \propto r^{1.5}$)
Differential rotation $\rightarrow$ slow envelope



Uniform rotation $\rightarrow \gamma$ increases with r



energy flux ratios:

$$\gamma = \frac{\text{kinetic}}{\text{rest mass}}$$

$$\gamma\sigma = \frac{\text{Poynting}}{\text{rest mass}}$$

$$(\sigma = \frac{\text{Poynting}}{\text{kinetic}})$$

$$\mu = \gamma + \gamma\sigma$$

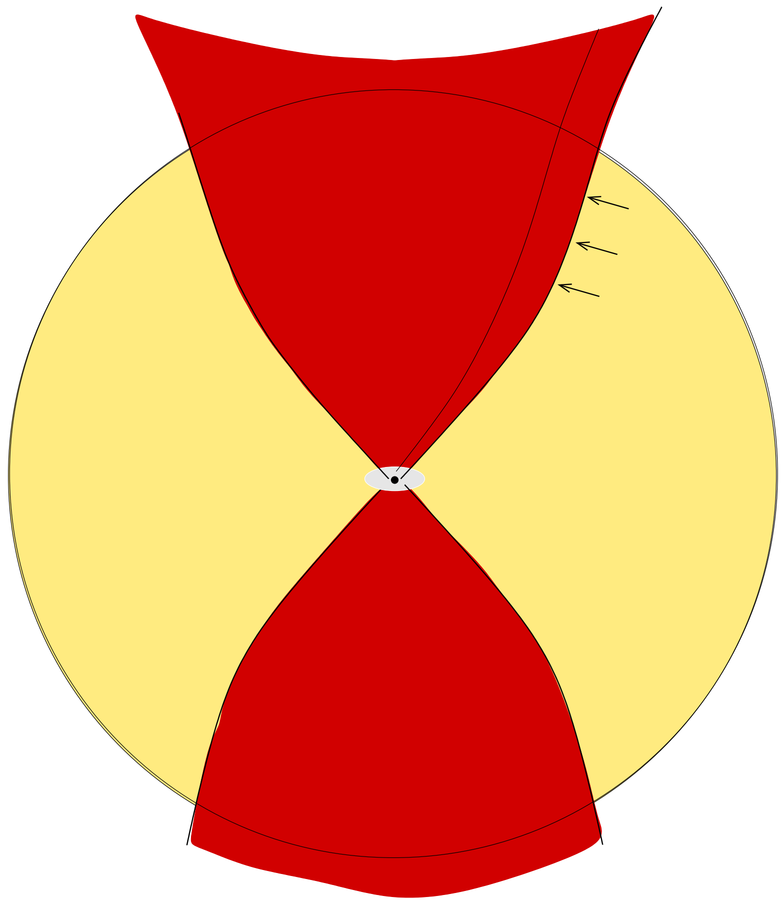
γ (increasing),

$\gamma\sigma$ (decreasing),

and μ (constant)

efficiency $> 50\%$

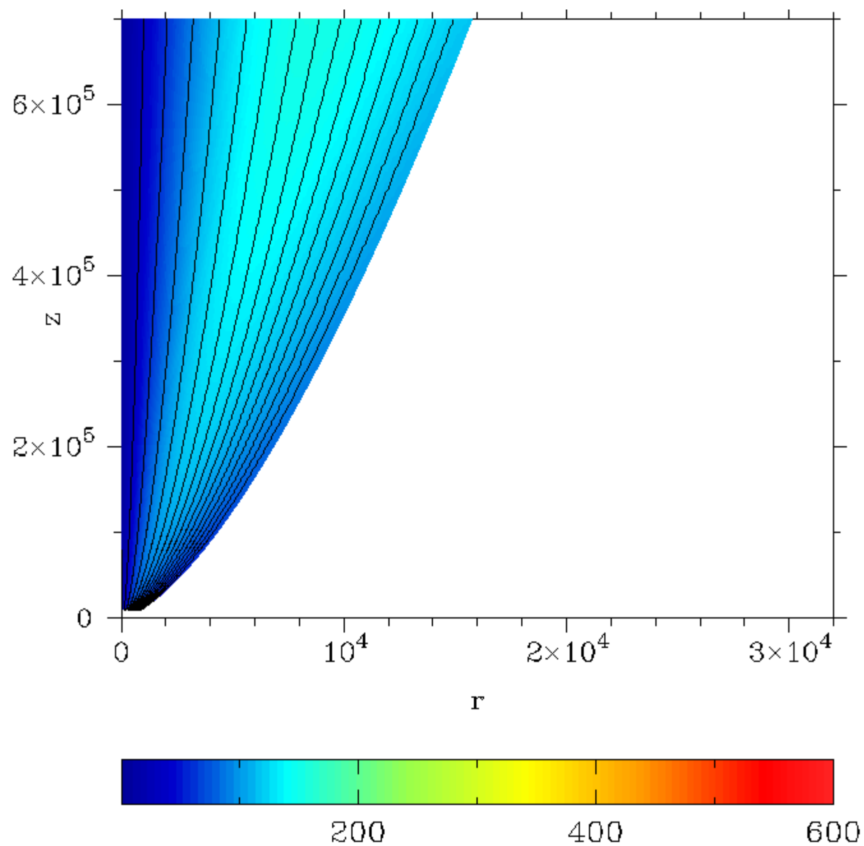
Rarefaction acceleration

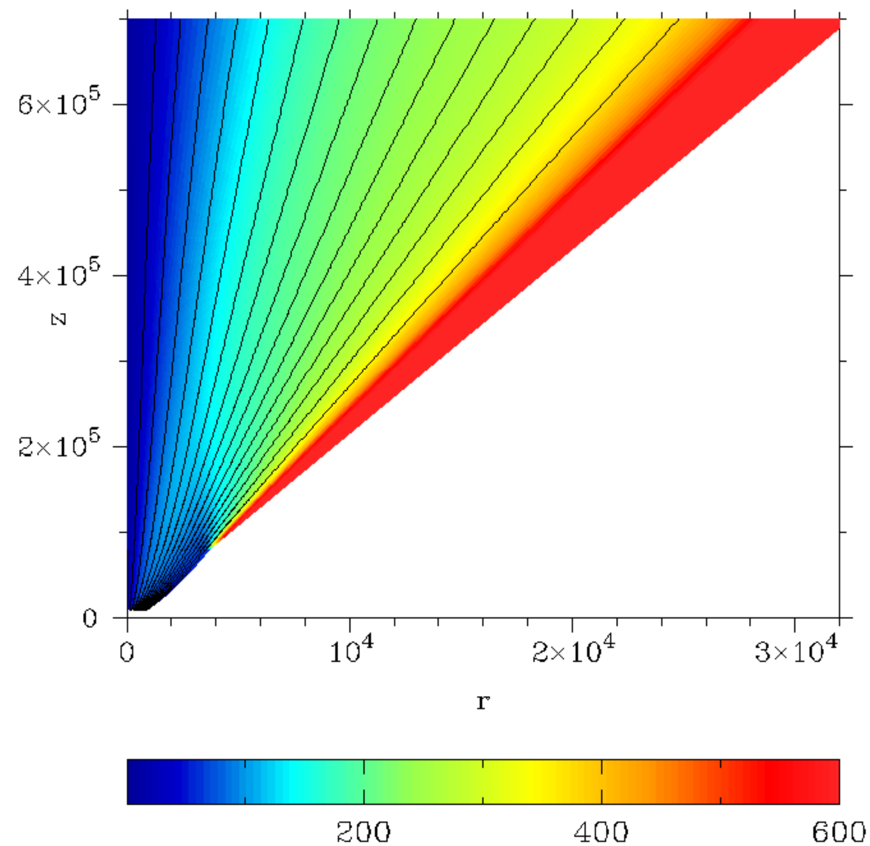


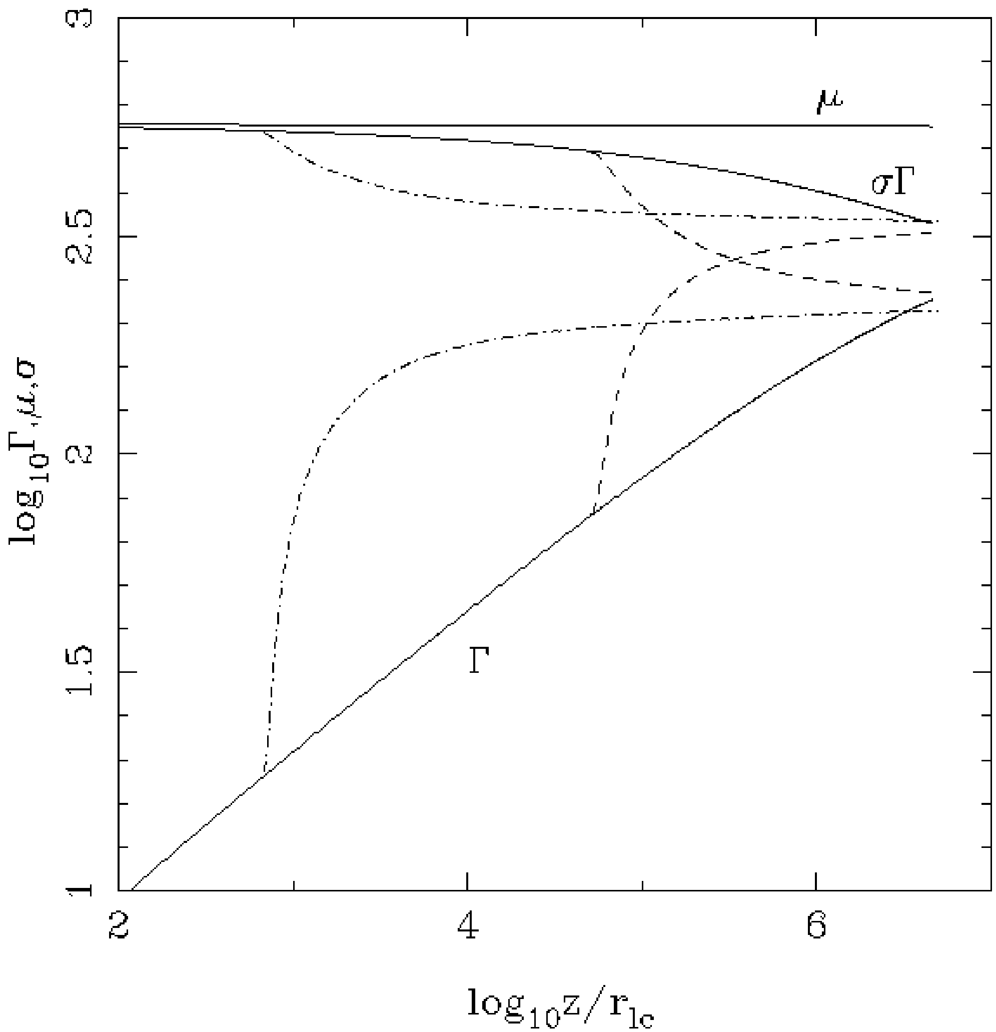
Simulation results

Komissarov, Vlahakis & Königl 2010

(see also Tchekhovskoy, Narayan & McKinney 2010)

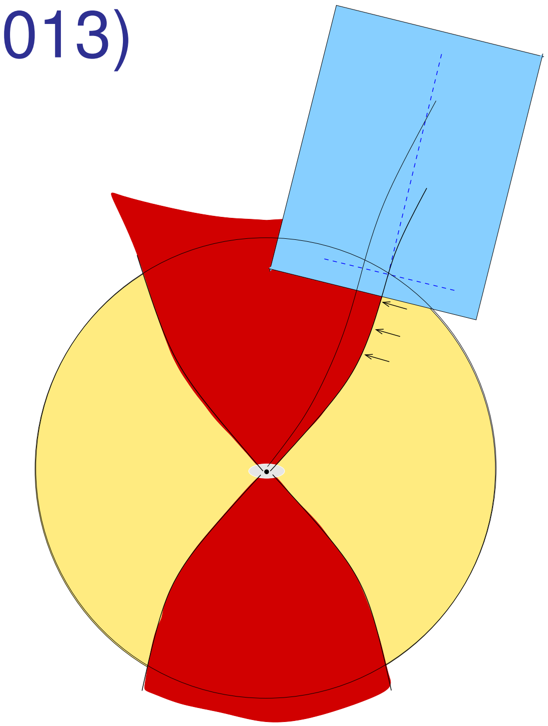




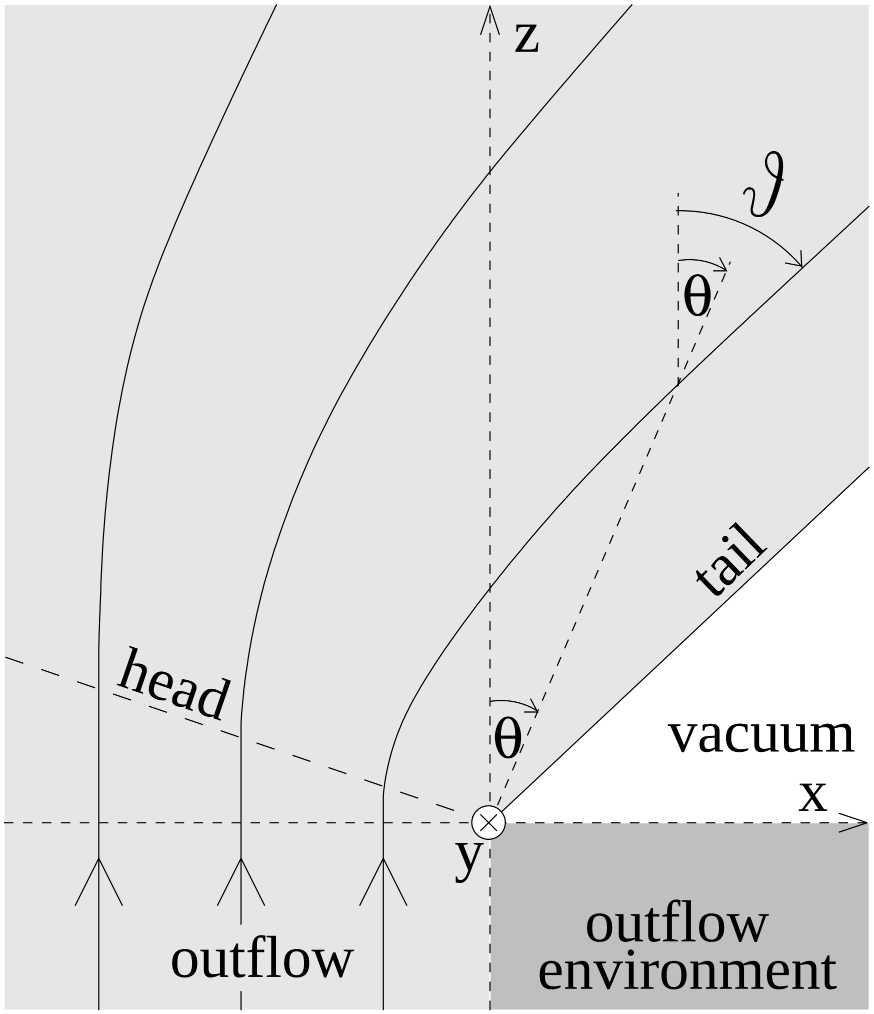

3
2.5
2
1.5
1
2
4
6
μ
$\sigma\Gamma$
Γ
$\log_{10}\Gamma, \mu, \sigma$
$\log_{10} z/r_{lc}$

Steady-state rarefaction wave

Sapountzis & Vlahakis (2013)



- “flow around a corner”
- planar geometry
- ignoring B_p (nonzero B_y)
- similarity variable x/z (angle θ)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)

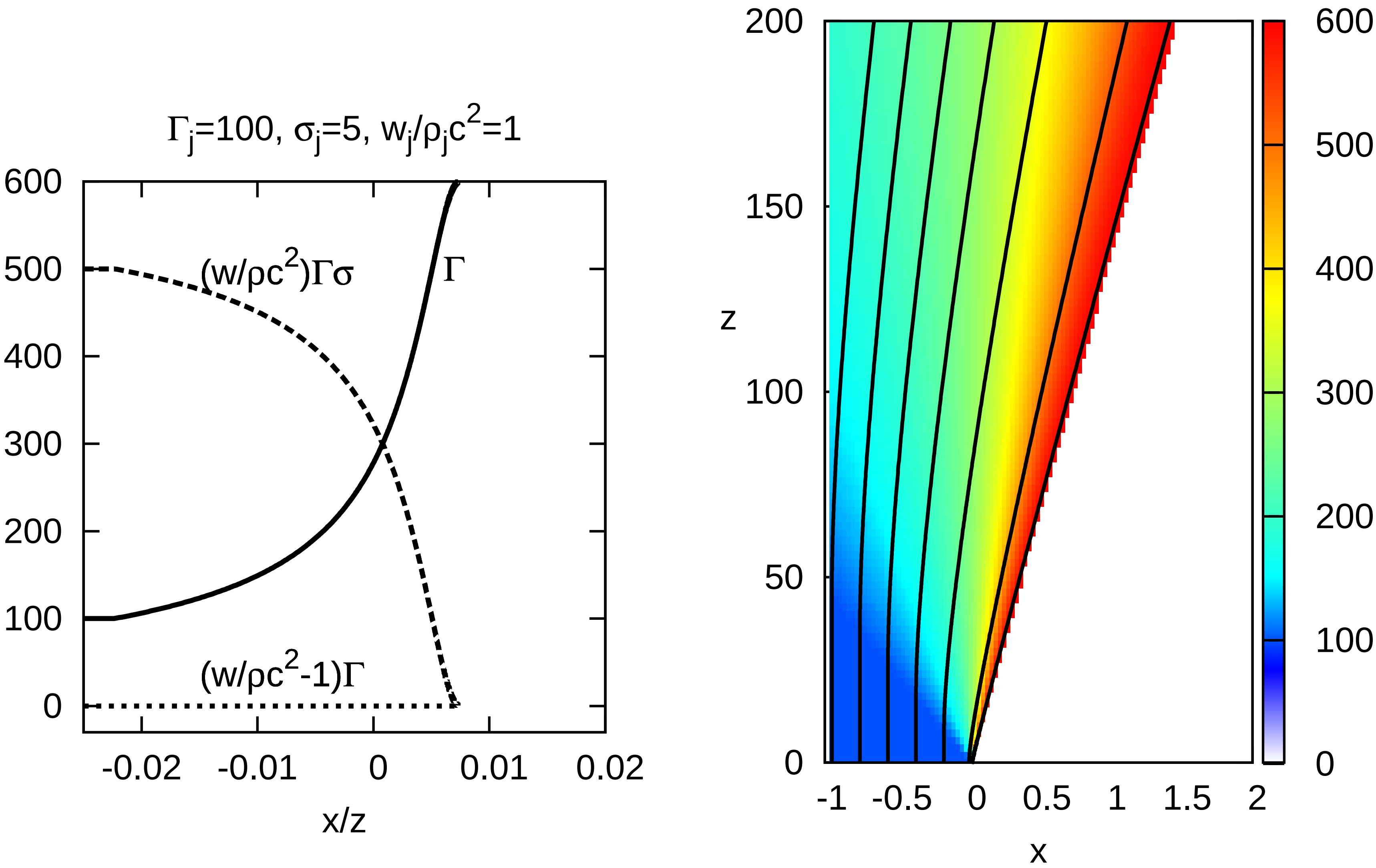


$$\theta_{\rm head} = -\frac{\sigma_j^{1/2}}{\gamma_j}$$

$$\theta_{\rm tail} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)}$$

$$\sigma = (\sigma_j \gamma_j x_i / z)^{2/3}$$

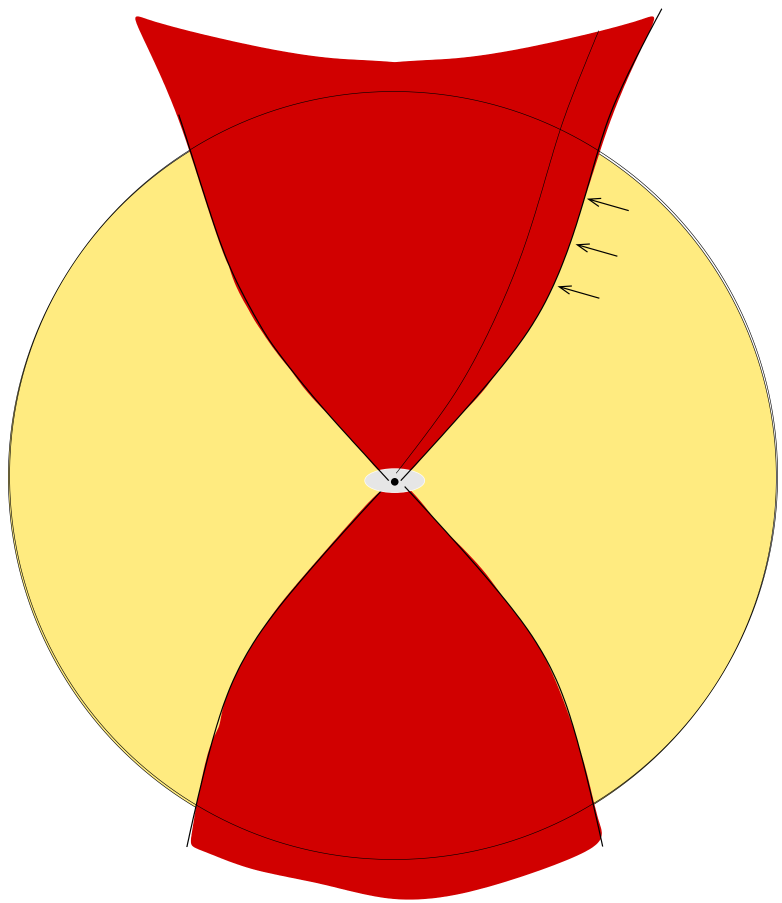
$\sigma = 1$ at $r = \sigma_j \gamma_j |x_i| = 7\times 10^{11} \sigma_j \left(\frac{|x_i|}{R_\star/\gamma_j}\right) \left(\frac{R_\star}{10 R_\odot}\right)$ cm



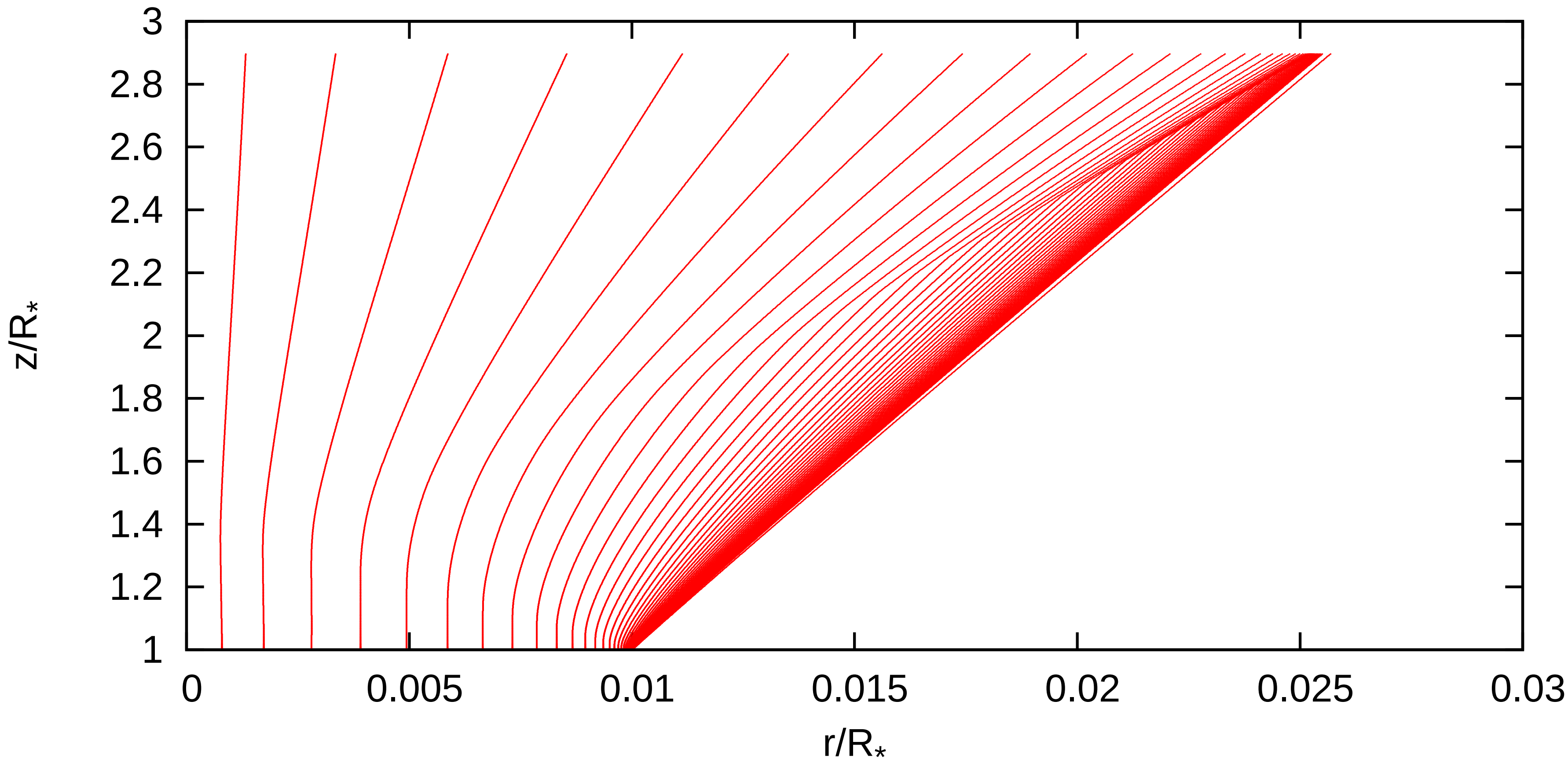
(distance unit $= R_\star/\gamma_j \sim 10^{10}$ cm)

Axisymmetric model

Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)

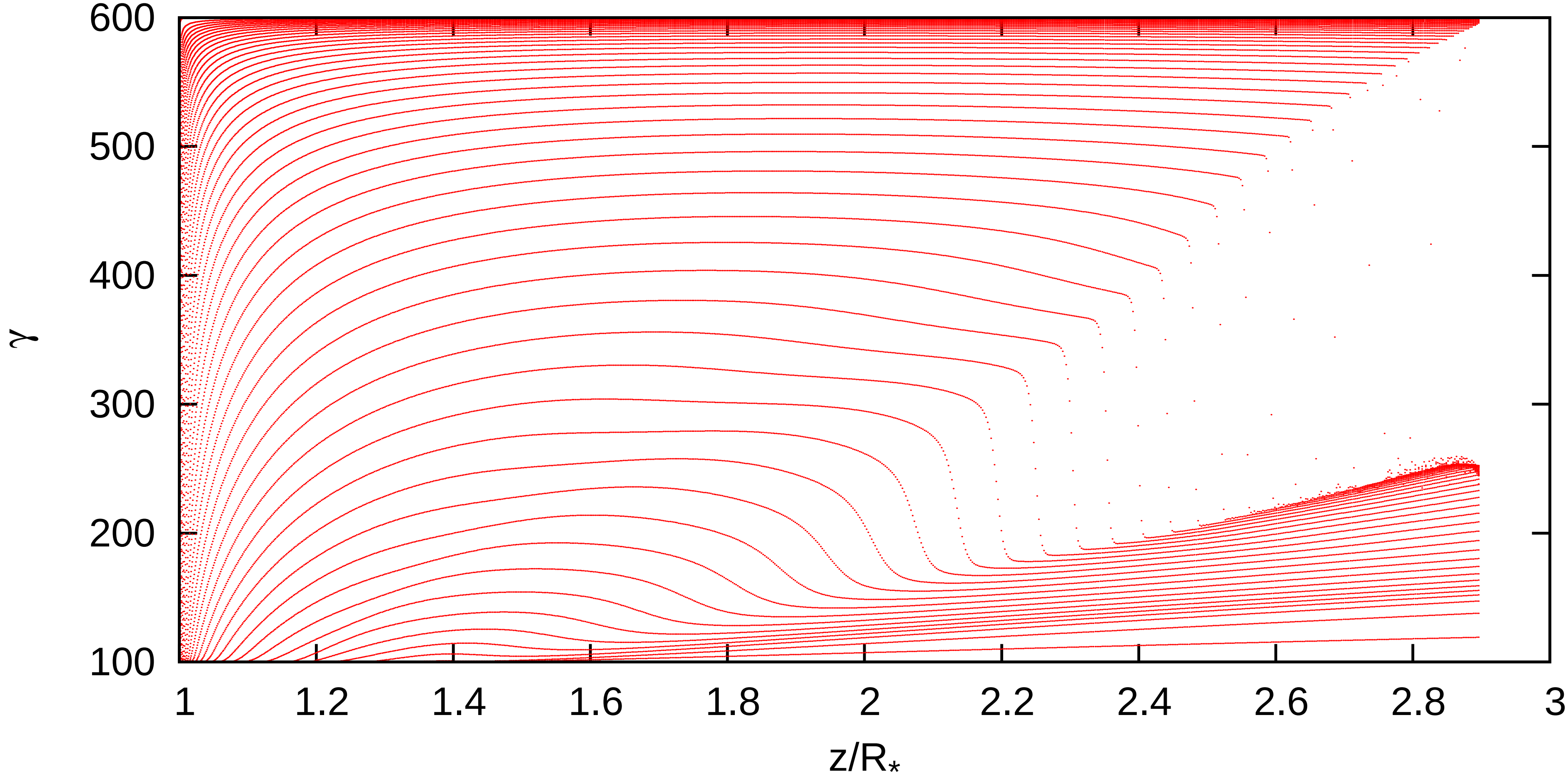


$\gamma_j = 100$, $\sigma_j = 5$



(not in scale!)

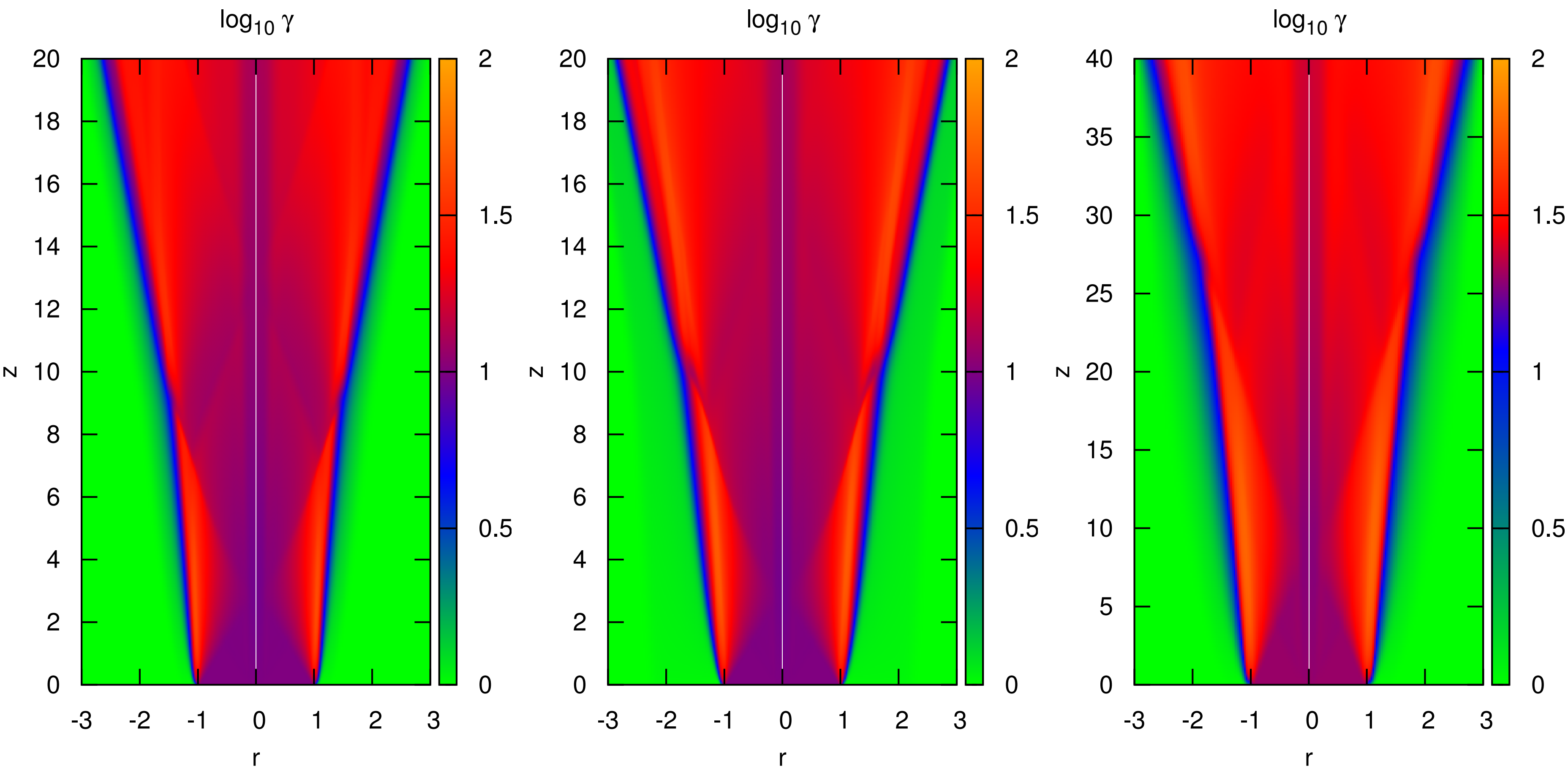
Reflection of the wave from the axis



Reflection causes sudden deceleration – standing shock

Numerical simulations

(preliminary results using PLUTO code)

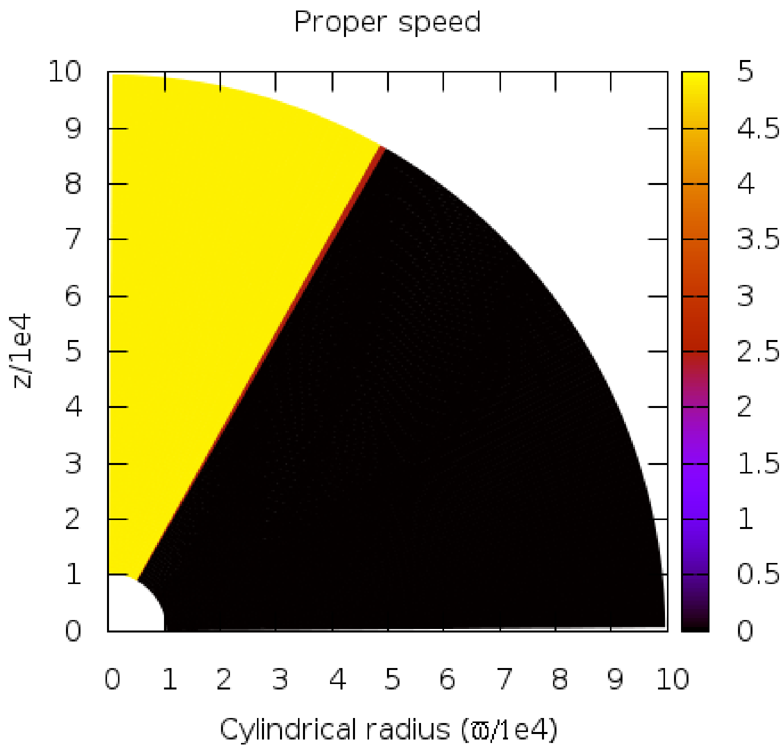


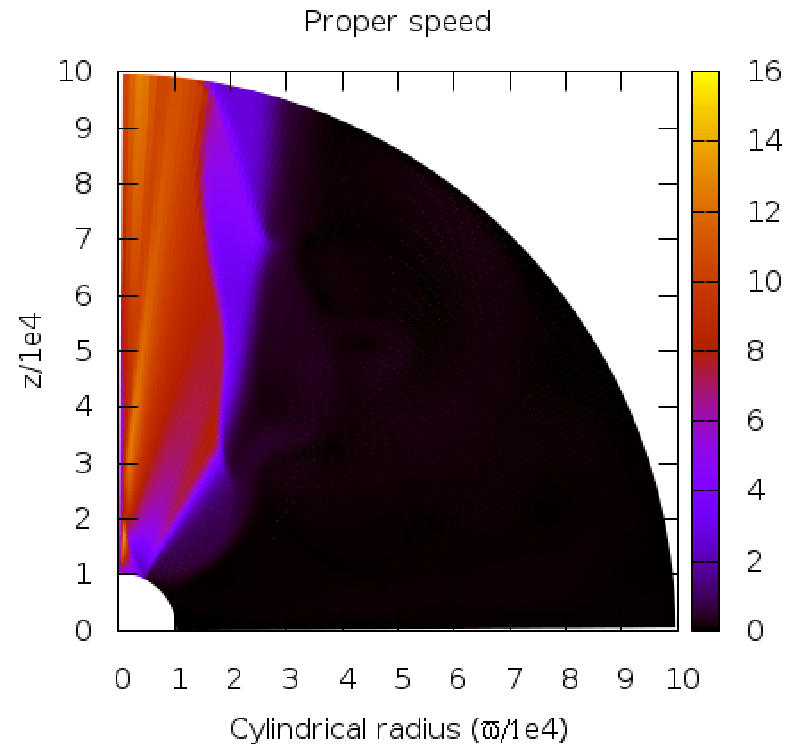
Left: $\gamma_j = 10$, $\sigma_j = 10$ (RW hit the axis at $z/r_j \sim \gamma_j/\sqrt{\sigma_j} \approx 3$)
Middle: 10 times lower density and pressure outside
Right: $\gamma_j = 20$, $\sigma_j = 5$ (note the different z-scale; $z/r_j \approx 9$)

Simulations of AGN jets

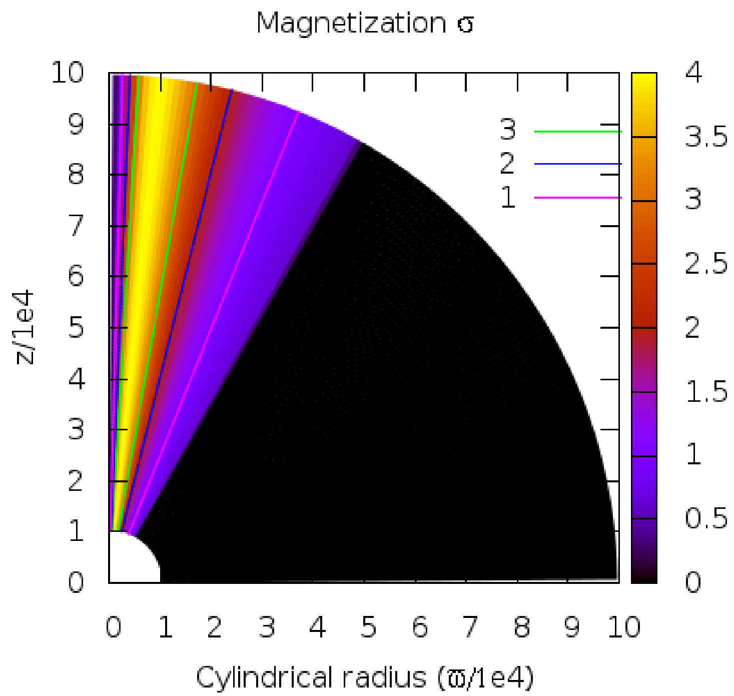
(Millas & Vlahakis)

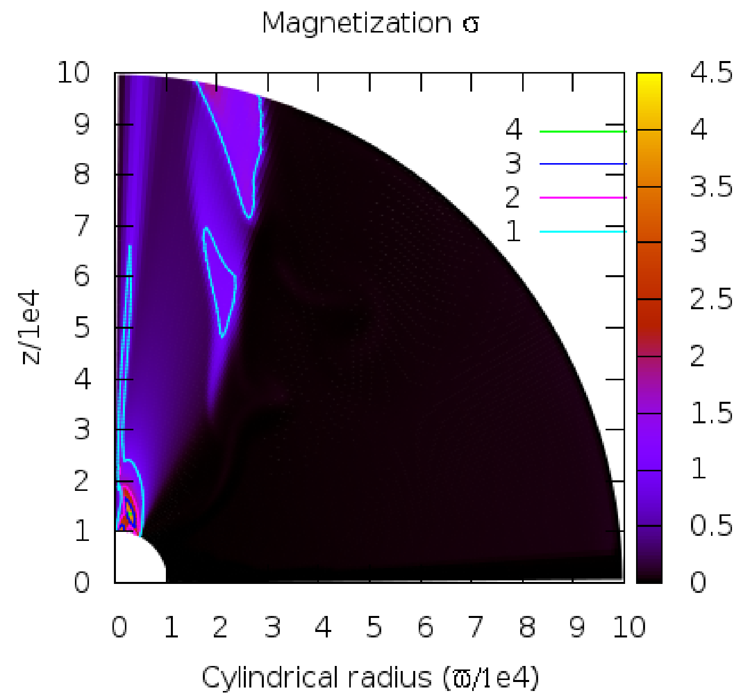
- ⋆ simulate both, jet+environment (static atmosphere or Bondi accretion)
- ⋆ use realistic values for M87 jet around HST-1 (external temperature from Bondi radius, density ratio from pressure equilibrium at base $\rightarrow$ sound crossing time $\gg$ light crossing time)
- ⋆ bell-shaped B_ϕ, ignore B_p since we are at $r \gg$ light cylinder radius
- ⋆ include gravity
- ⋆ using PLUTO code





$u = \dfrac{\Gamma V}{c}$ for $t = 0$ (left) and $t = 10$ external sound crossing times (right)

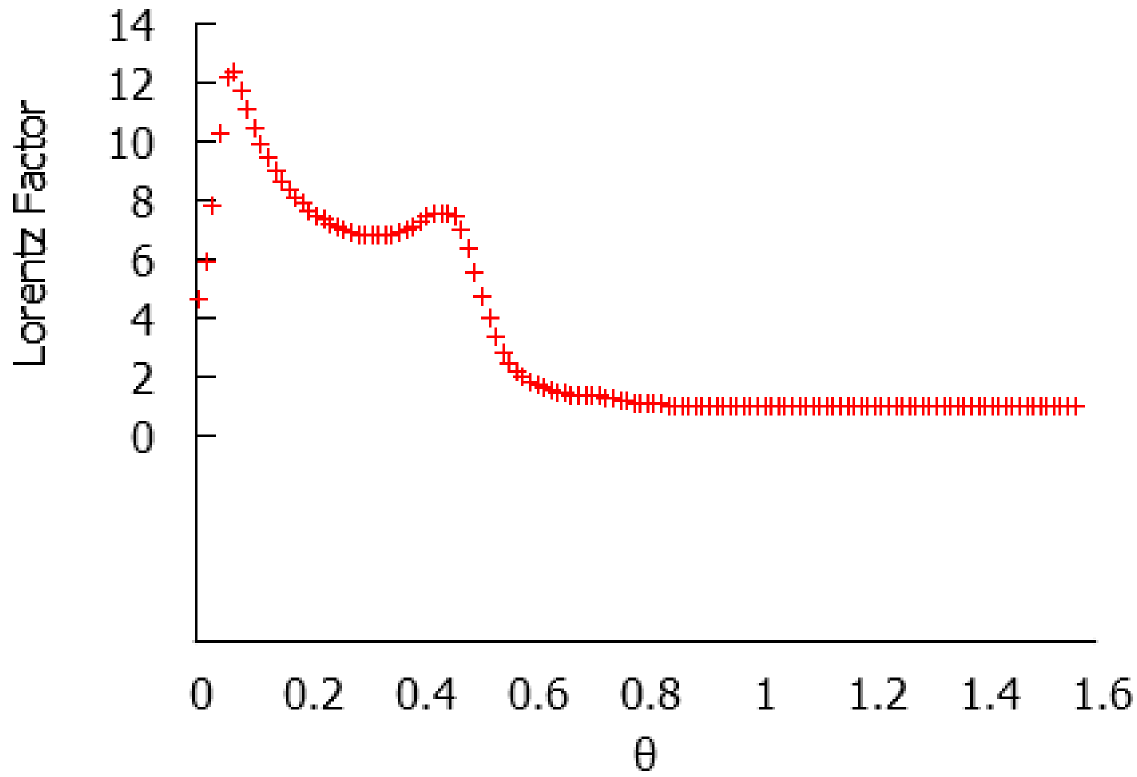




σ for $t = 0$ (left) and $t = 10$ external sound crossing times (right)

A quasi-steady jet is found (with well defined characteristics near the axis and at $R < R_{Bondi}$ where the environment is shocked)

Shocks AND rarefaction in the volume of the jet



Lorentz factor at distance $R = 3 \times 10^5$. The first peak corresponds to the magnetic acceleration/collimation mechanism and the second to the rarefaction acceleration.

Summary

- $\star$ magnetic field + rotation $\rightarrow$ Poynting flux extraction
- $\star$ the collimation-acceleration mechanism is very efficient – provides a viable explanation for the bulk acceleration in relativistic jets
- $\star$ terminal Lorentz factors $\gamma_\infty \gtrsim 0.5 \frac{\mathcal{E}}{Mc^2}$
- $\star$ environment significantly affects jet dynamics (jet-shape, re-confinement shocks, rarefaction waves, shocks)
- $\star$ rarefaction acceleration
  - important in GRB jets
  - weaker but still present in AGN jets