Simple waves in relativistic magnetized outflows

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Outline

- Introduction to simple waves – Riemann problem
- Applications to GRB outflows:
  - impulsive acceleration
  - rarefaction acceleration
  - axisymmetric model – simulations

Simple waves

- Finite amplitude waves caused by pressure imbalances

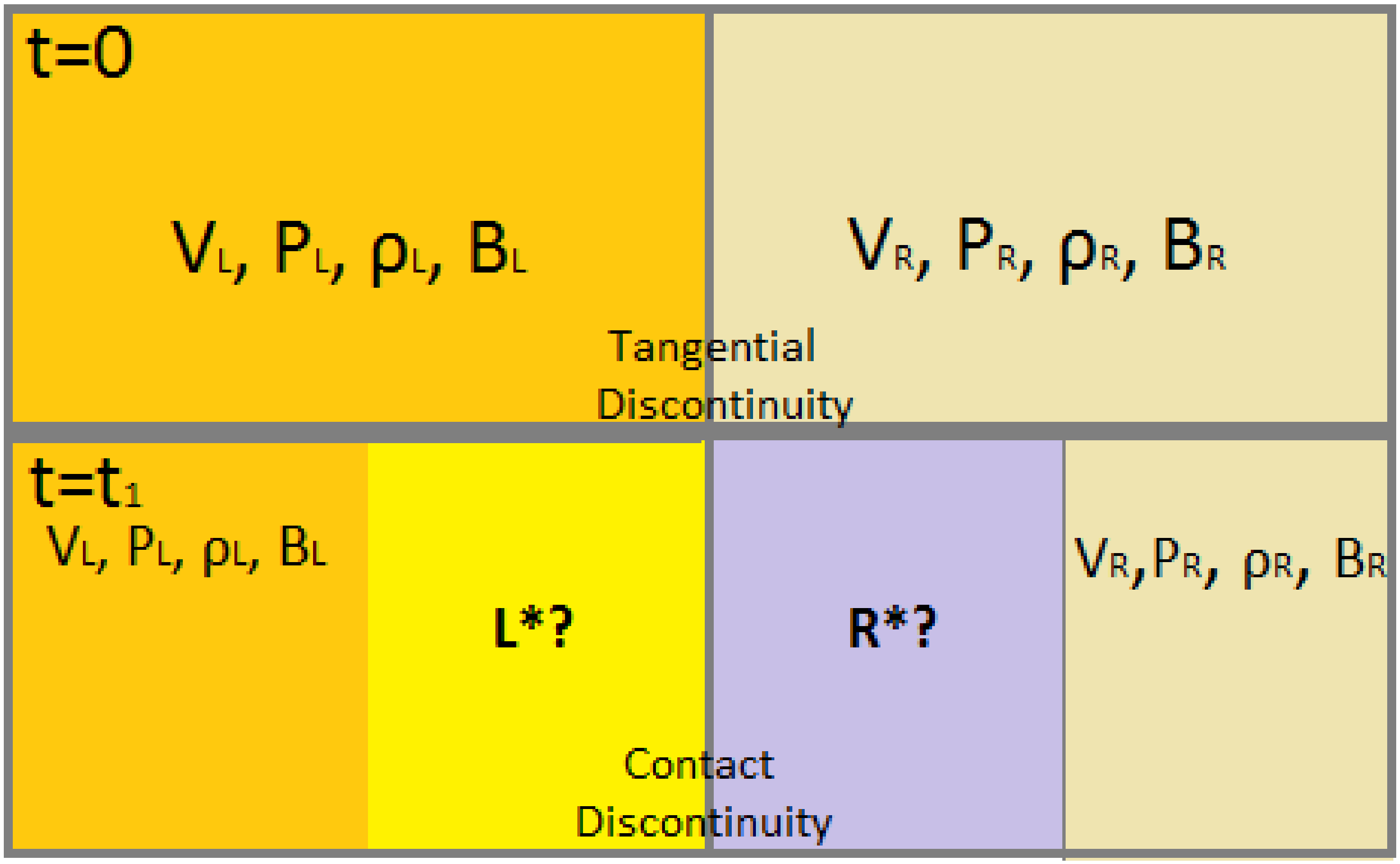
- Form of the solution in the textbook case (1D nonrelativistic HD):
  $\rho = \rho(\xi), \quad P = P(\xi), \quad \boldsymbol{V} = V_x(\xi)\hat{x},$
  where $\xi = \xi(x, t)$

- In the absence of scale $\xi = x/t$ (self-similarity)

- Relativistic MHD generalization:
  $\rho = \rho(\xi), \quad P = P(\xi), \quad \boldsymbol{V} = V_x(\xi)\hat{x} + V_z(\xi)\hat{z}, \quad \boldsymbol{B} = B_y(\xi)\hat{y}$
  where $\xi = x/t$

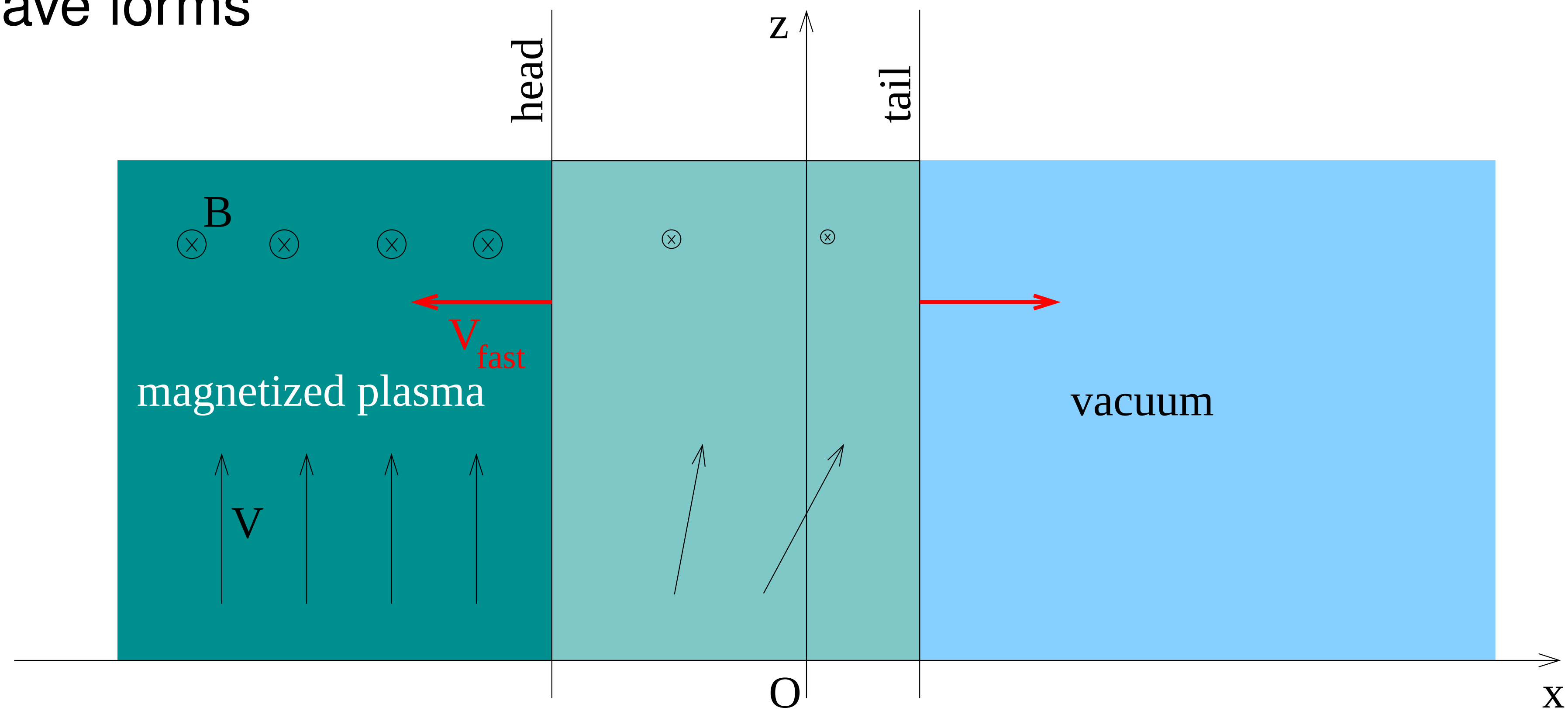
The Riemann problem

Initially two uniform states are in contact.
If the total pressure is not the same and (or) the x-velocities are different, two travelling waves are formed (shock or rarefaction).

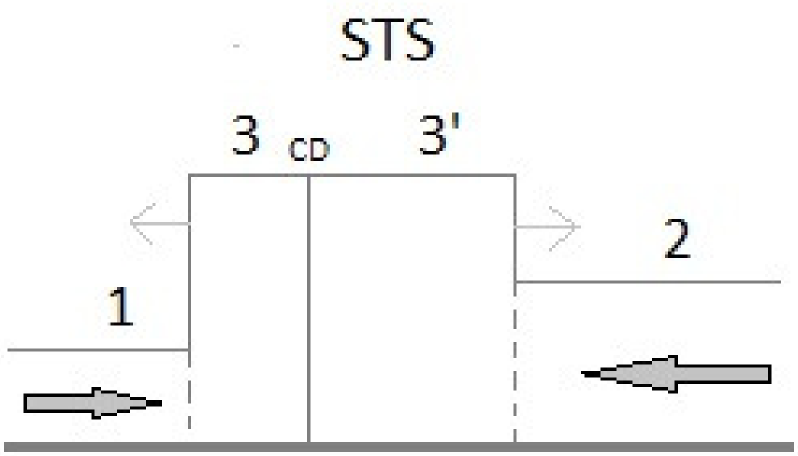


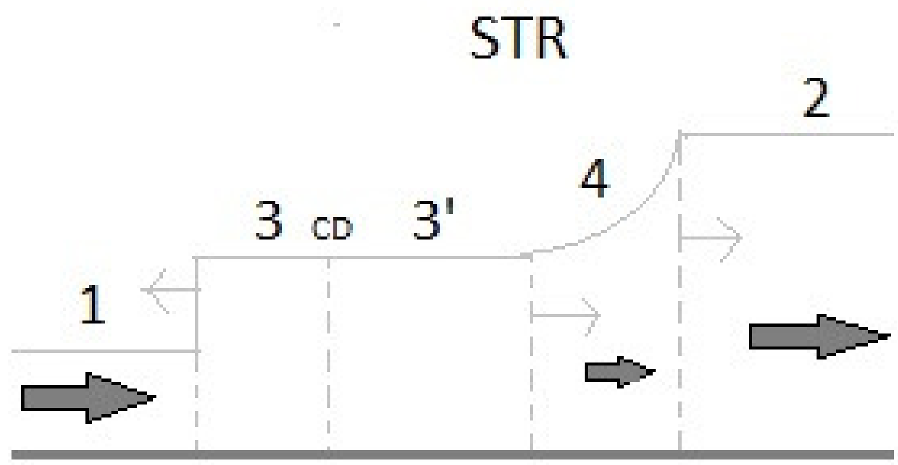
Rarefaction simple waves

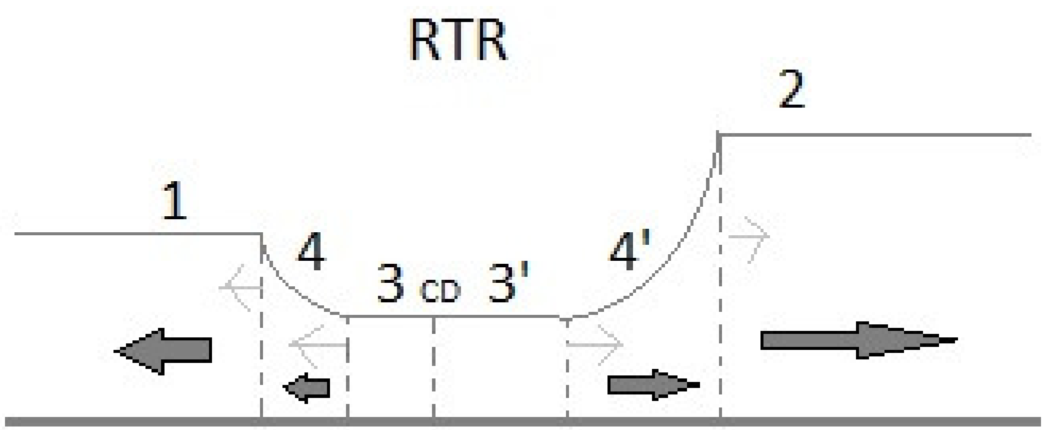
- when $\rho_R/\rho_L = 0$ (vacuum on the right) a simple rarefaction wave forms



Possible cases:







Solving the problem

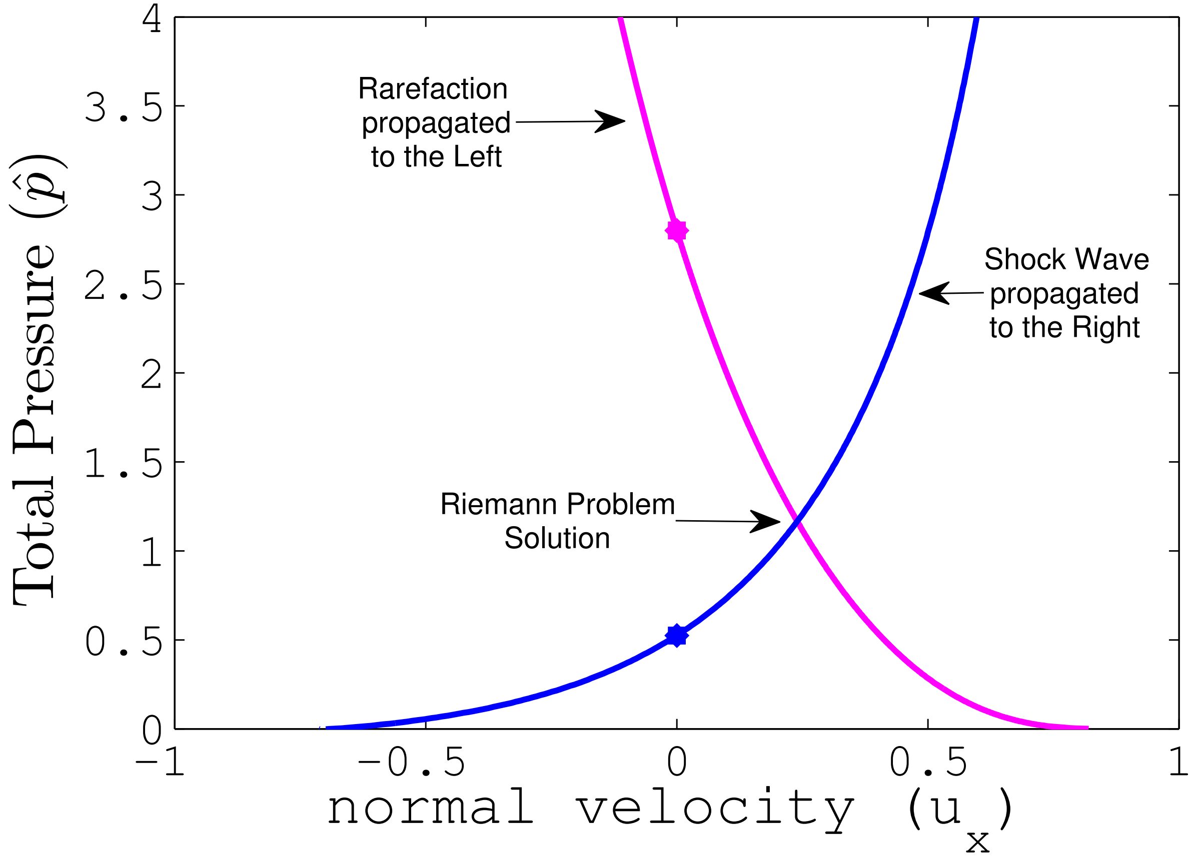
Katsoulakos & NV in preparation (see also Marti+1994, Lyutikov 2010)

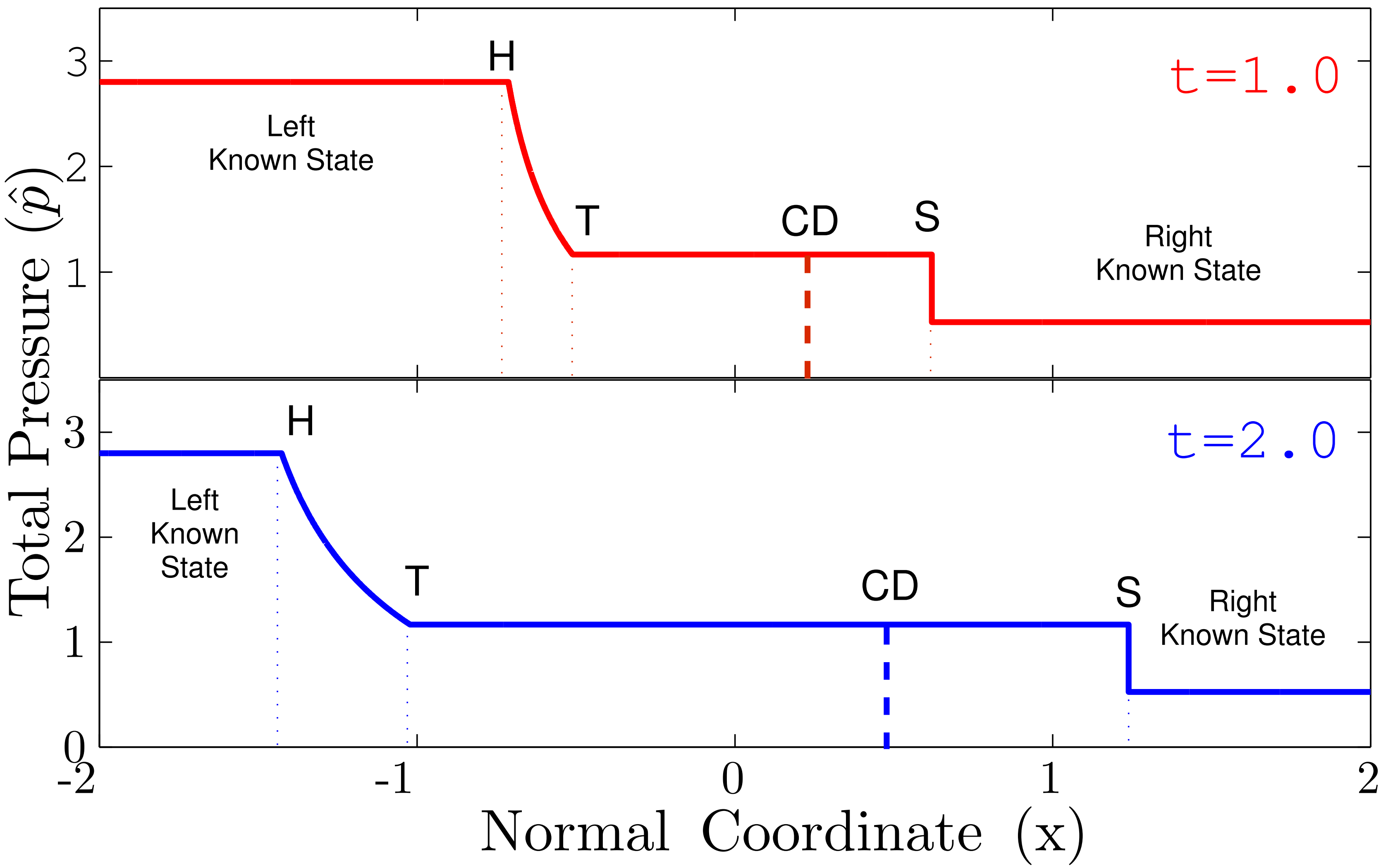
For the rarefaction, solve the MHD equations (all quantities functions of x/t).
For the shock, solve the jump conditions for various shock speeds.
The solution is found requiring same total pressure and x-velocity at CD.

Example:

Left state:
$P = 0.80$,
$V_x = 0.0$,
$\rho = 1.0$,
$V_z = 0.6$,
$B_y = 2.0$.

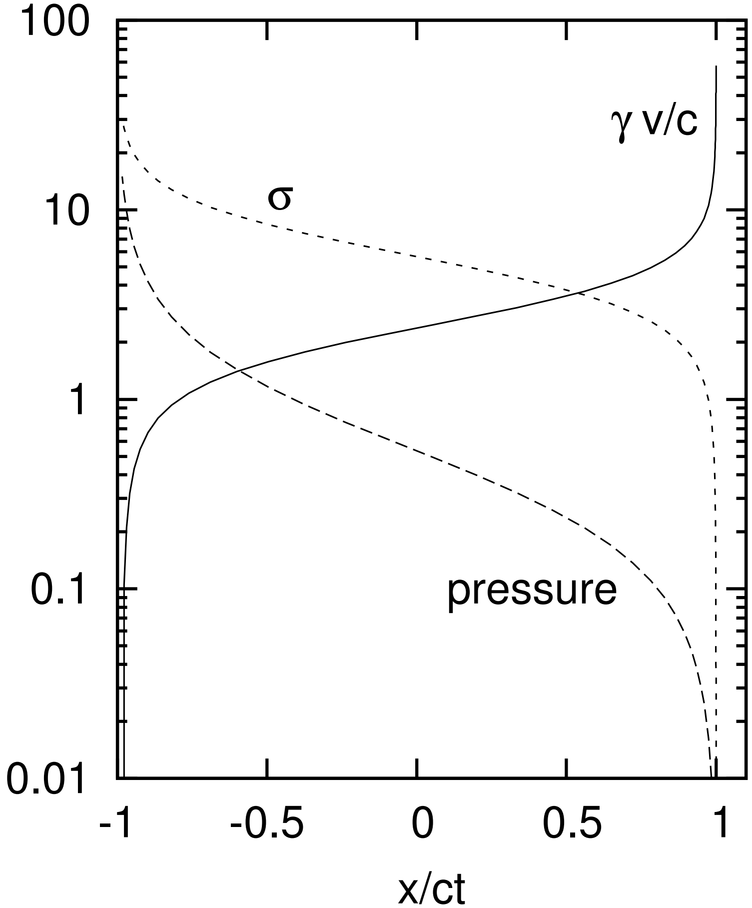
Right state:
$P = 0.40$,
$V_x = 0.0$,
$\rho = 0.5$,
$V_z = 0.6$,
$B_y = 0.5$.

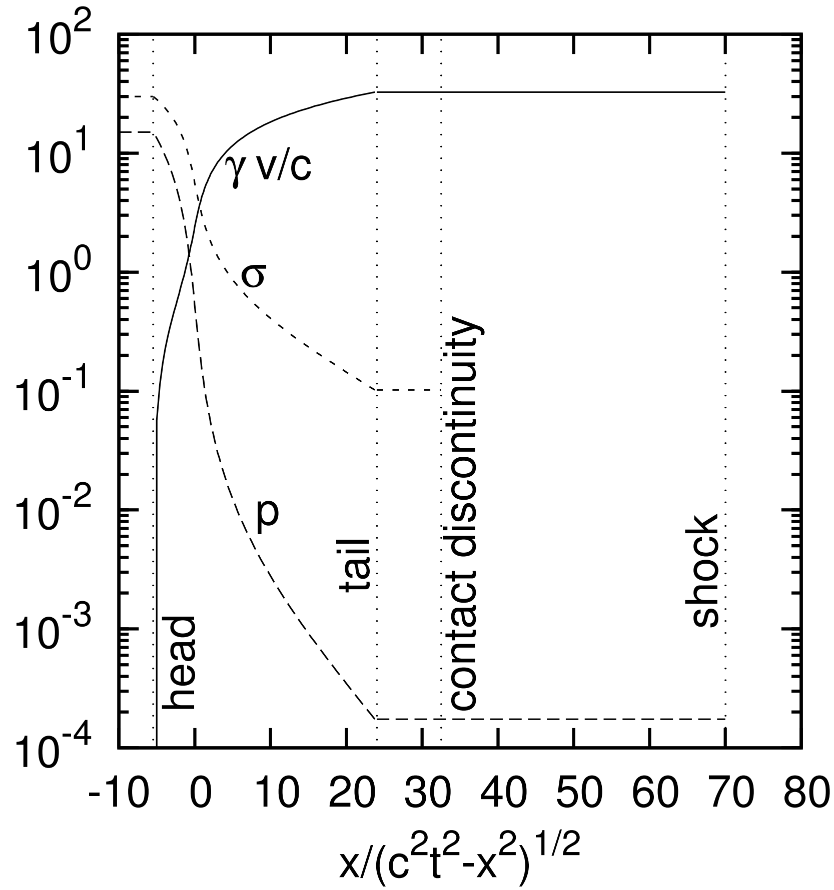



t=1.0
H
Left
Known State
T
CD
S
Right
Known State
t=2.0
H
Left
Known
State
T
CD
S
Right
Known State
Total Pressure ($\hat{p}$)
Normal Coordinate (x)
0
1
2
3
-2
-1
0
1
2

GRB application 1: impulsive acceleration

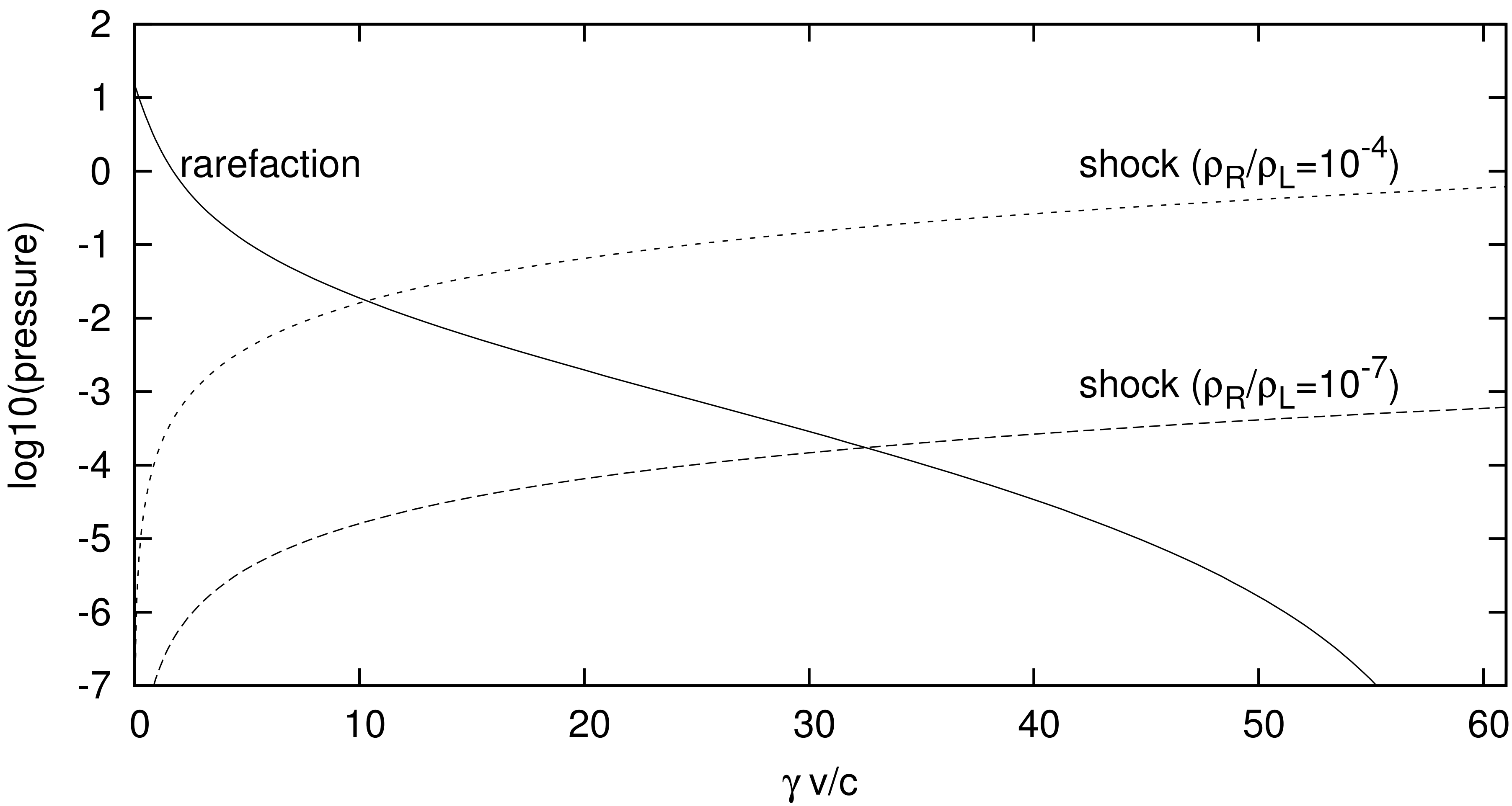
(Granot, Komissarov & Spitkovsky 2011)





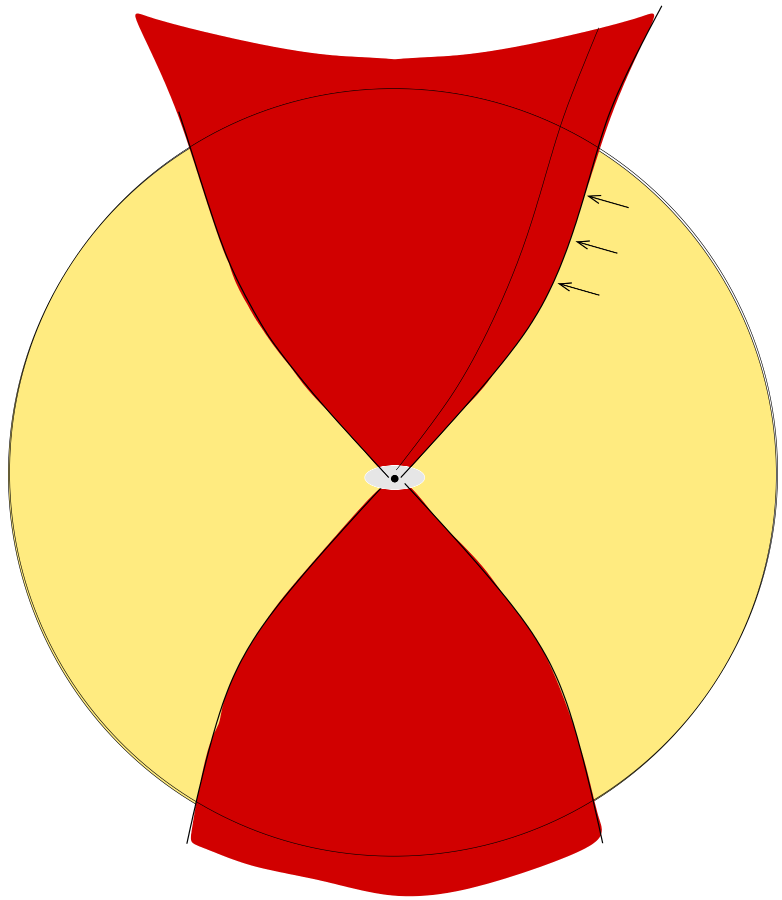
for $\rho_R/\rho_L = 0$

for $\rho_R/\rho_L = 10^{-7}$, $P_R = 0$

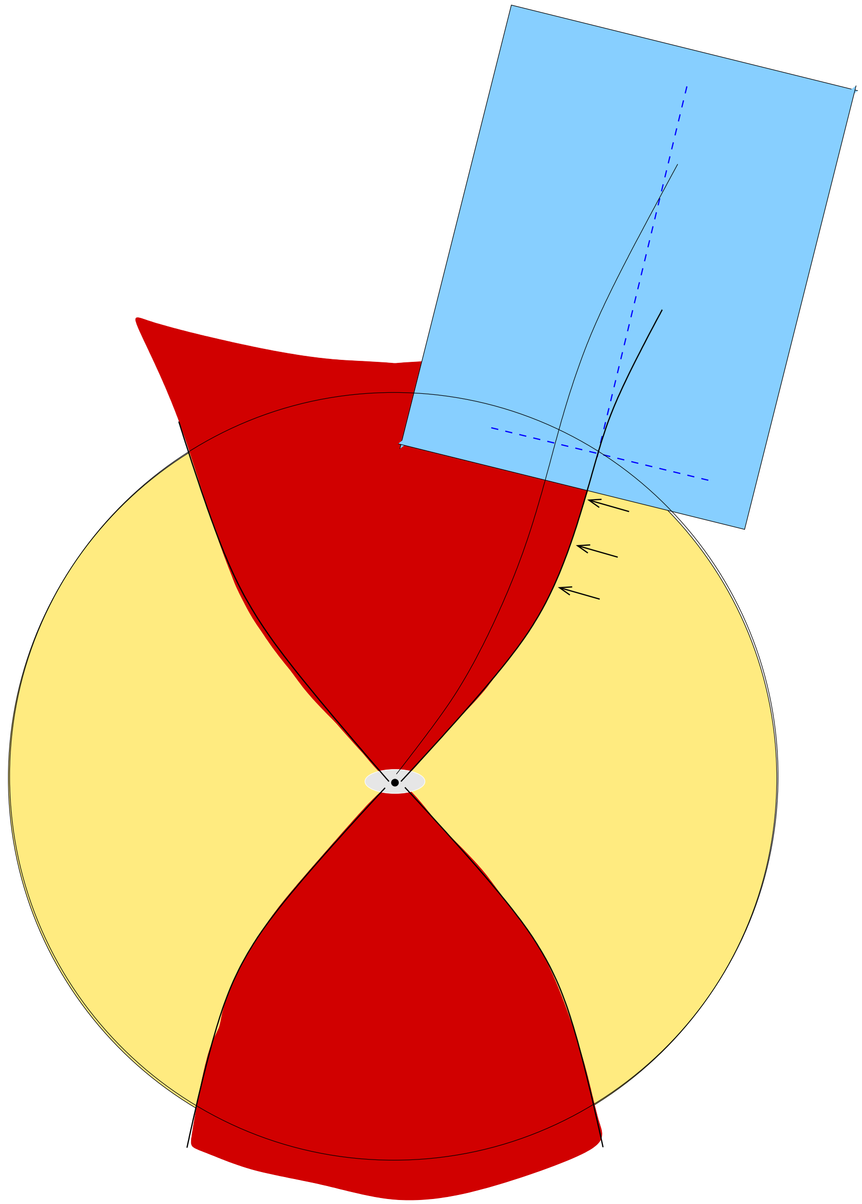


- the crossing of the two curves gives the maximum γ
- even tiny ρ_R/ρ_L affect γ_{max}

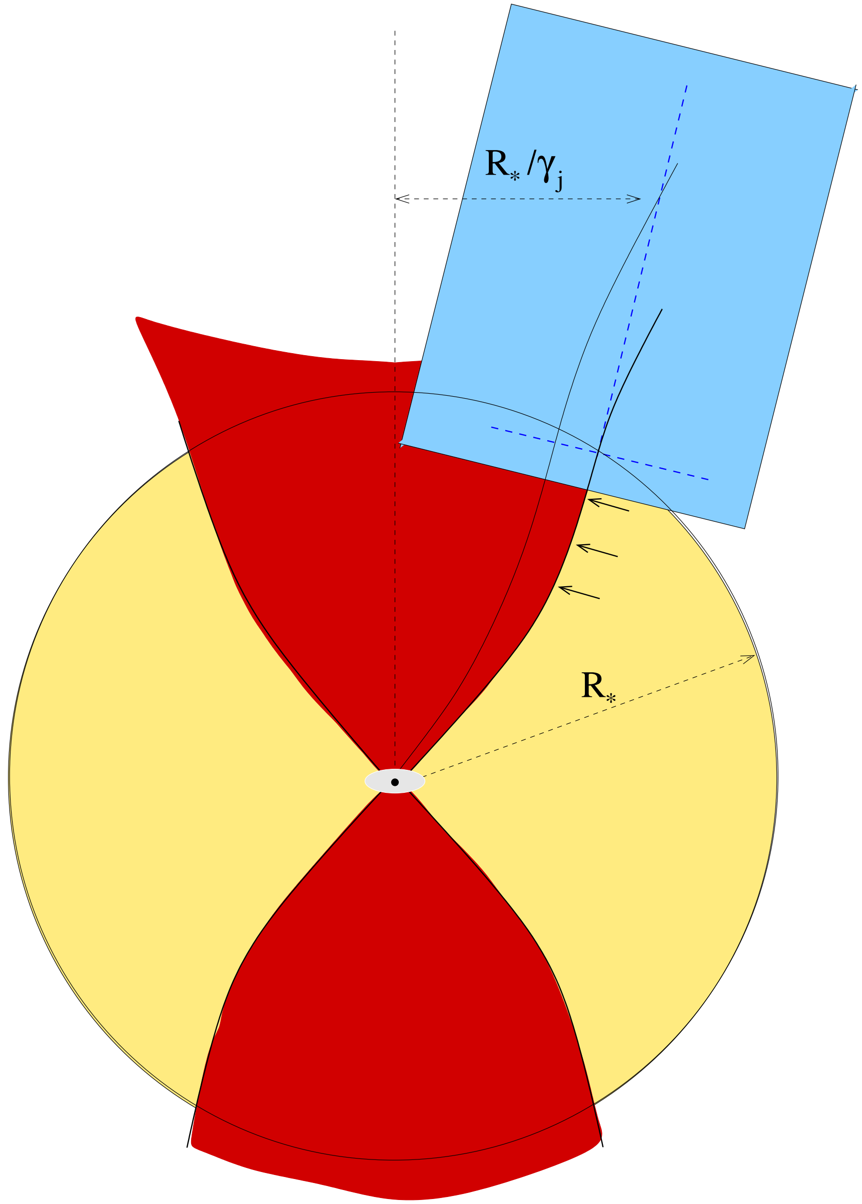
GRB application 2: rarefaction acceleration



GRB application 2: rarefaction acceleration



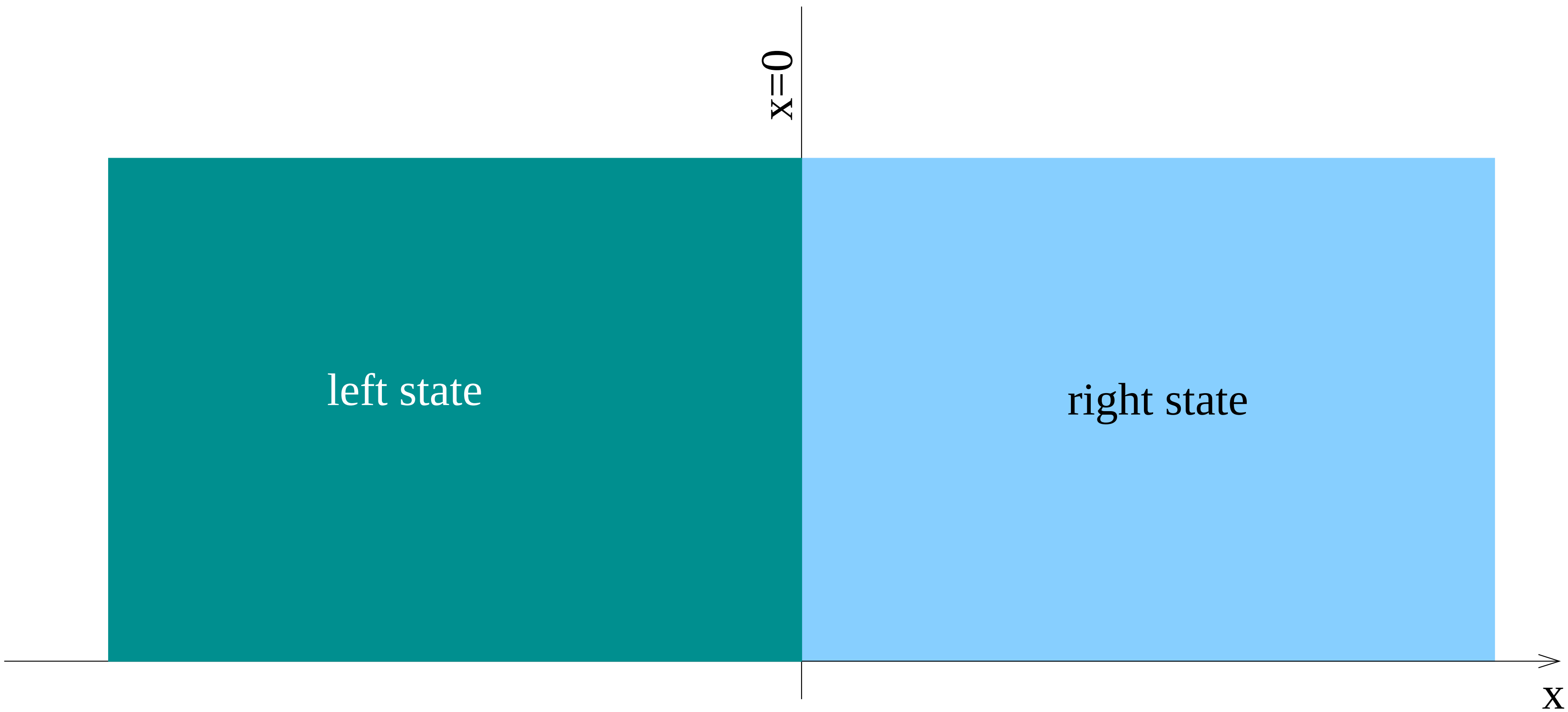
GRB application 2: rarefaction acceleration



Rarefaction simple waves with $V_z \neq 0$

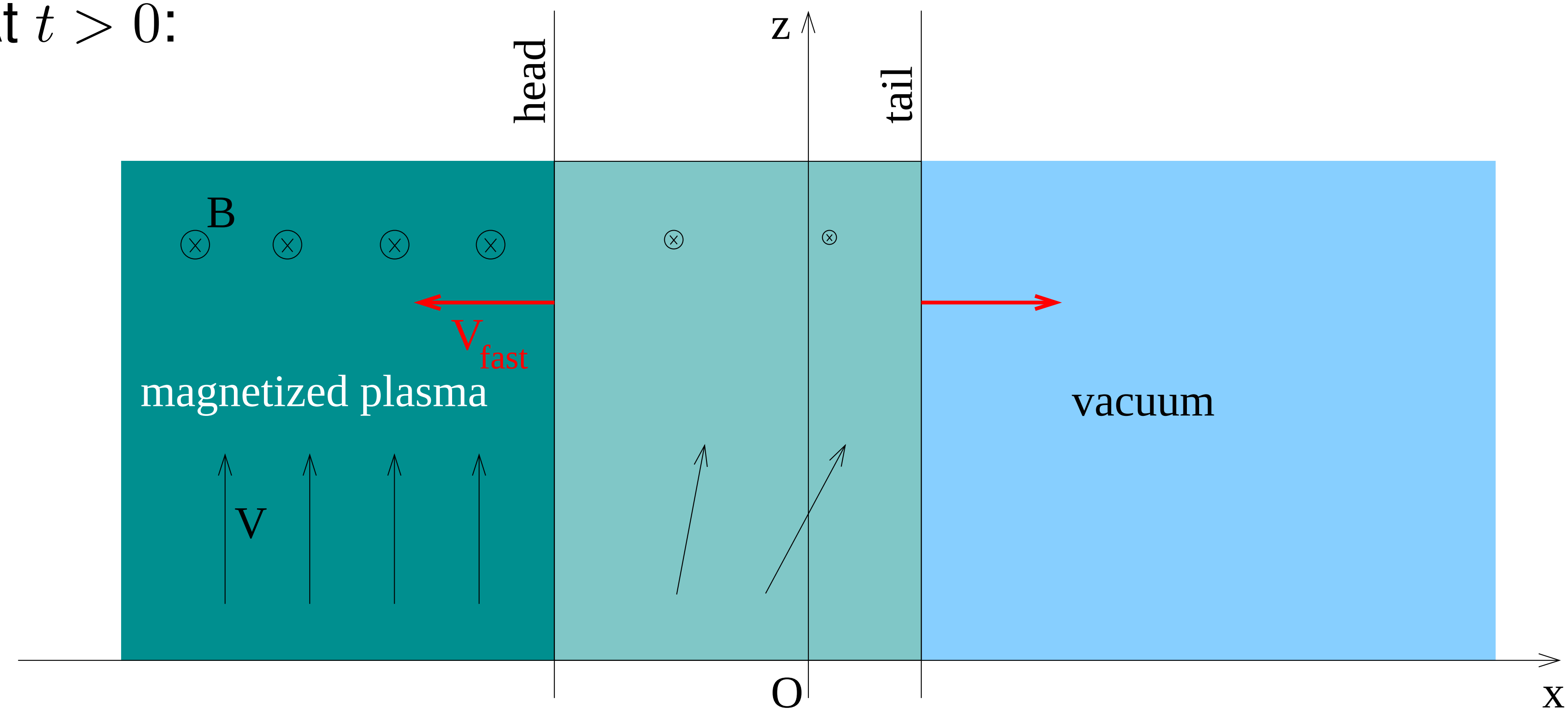
Komissarov, Vlahakis & Königl 2010

At $t = 0$ two uniform states are in contact:



- when $\rho_R/\rho_L = 0$ simple rarefaction wave

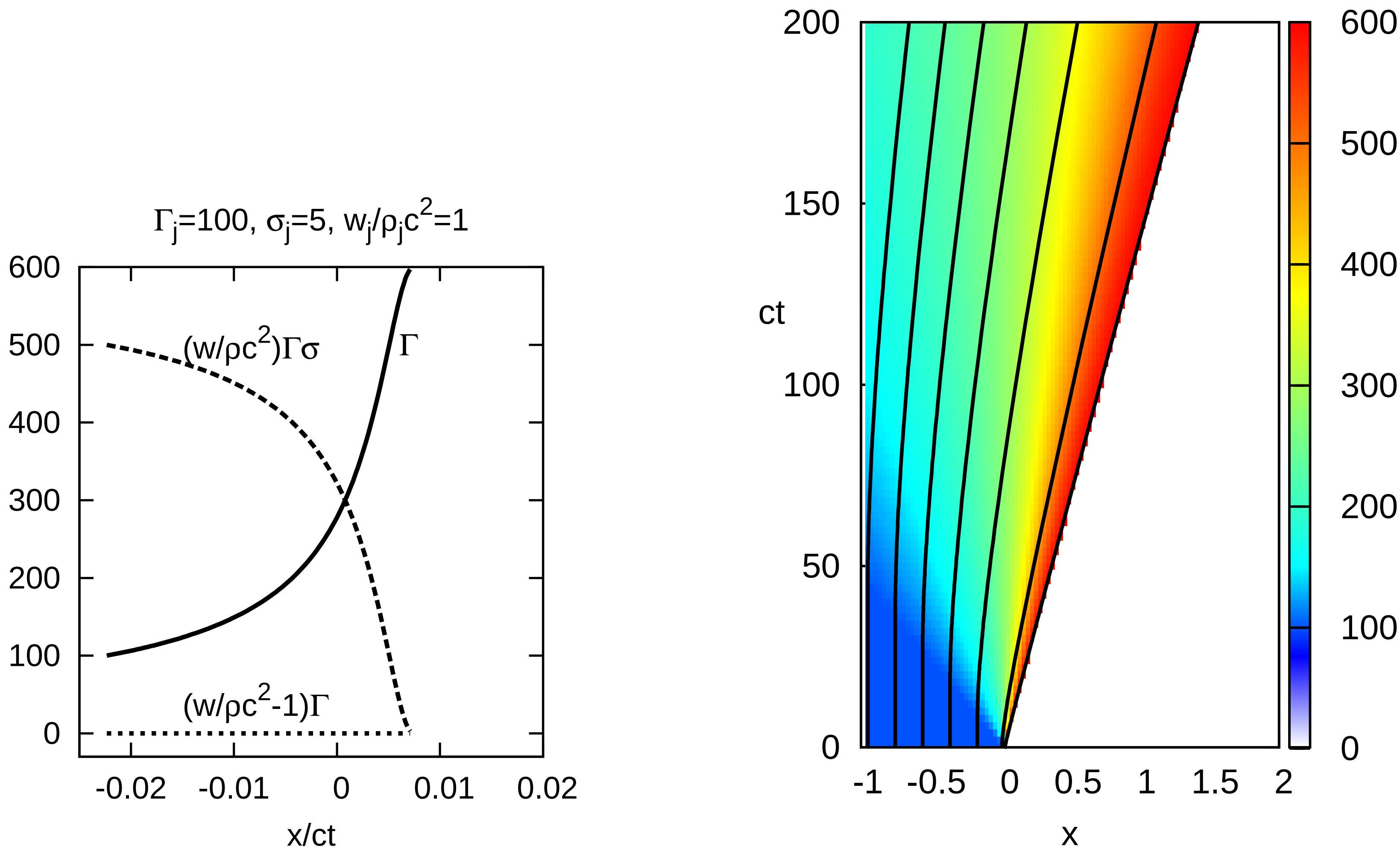
At $t > 0$:



for the cold ultrarelativistic case the MHD equations (through the Riemann invariants) imply

$$V_x = \frac{c}{\gamma_j} \frac{2\sigma_j^{1/2}}{1+\sigma_j} \left[1 - \left(\frac{\rho}{\rho_j} \right)^{1/2} \right], \ \gamma = \frac{\gamma_j (1+\sigma_j)}{1+\sigma_j \rho / \rho_j}, \ \rho = \frac{4\rho_j}{\sigma_j} \sinh^2 \left[\frac{1}{3} \operatorname{arcsinh} \left(\sigma_j^{1/2} - \frac{\mu_j}{2} \frac{x}{t} \right) \right]$$

$$V_{head} = -c \frac{\sigma_j^{1/2}}{\gamma_j}, \qquad V_{tail} = c \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1+\sigma_j}, \qquad \Delta\vartheta = \frac{V_{tail}}{c} < \frac{1}{\gamma_i}$$

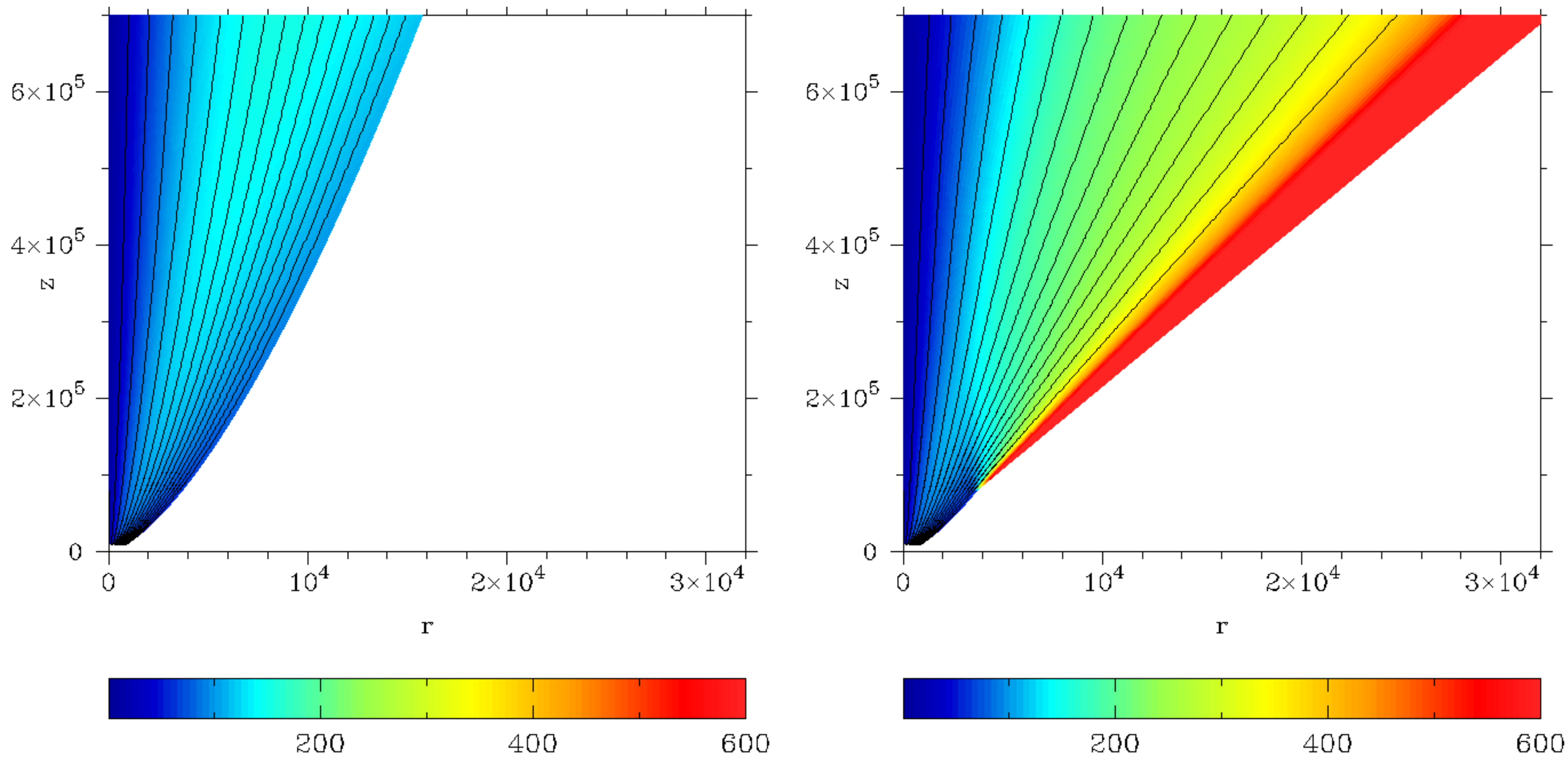


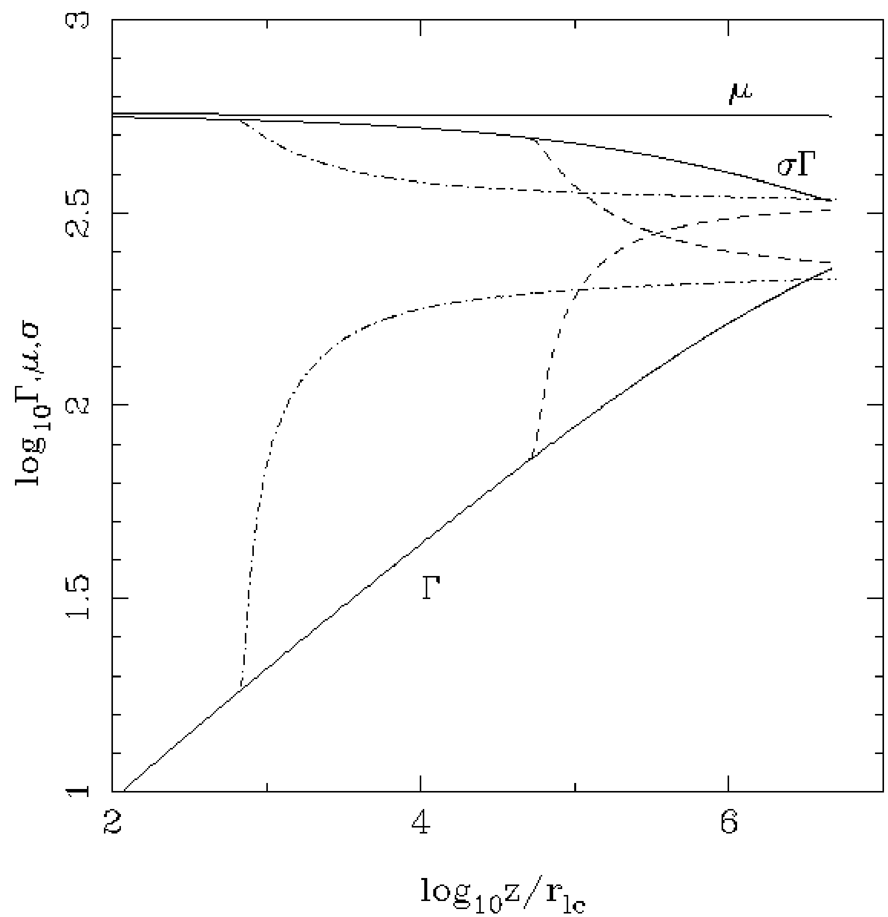
The colour image in the Minkowski diagram represents the distribution of the Lorentz factor and the contours show the worldlines of various fluid parcels (see also Aloy & Rezzolla 2006 for HD, Mizuno+2008 for MHD simulations)

Simulation results

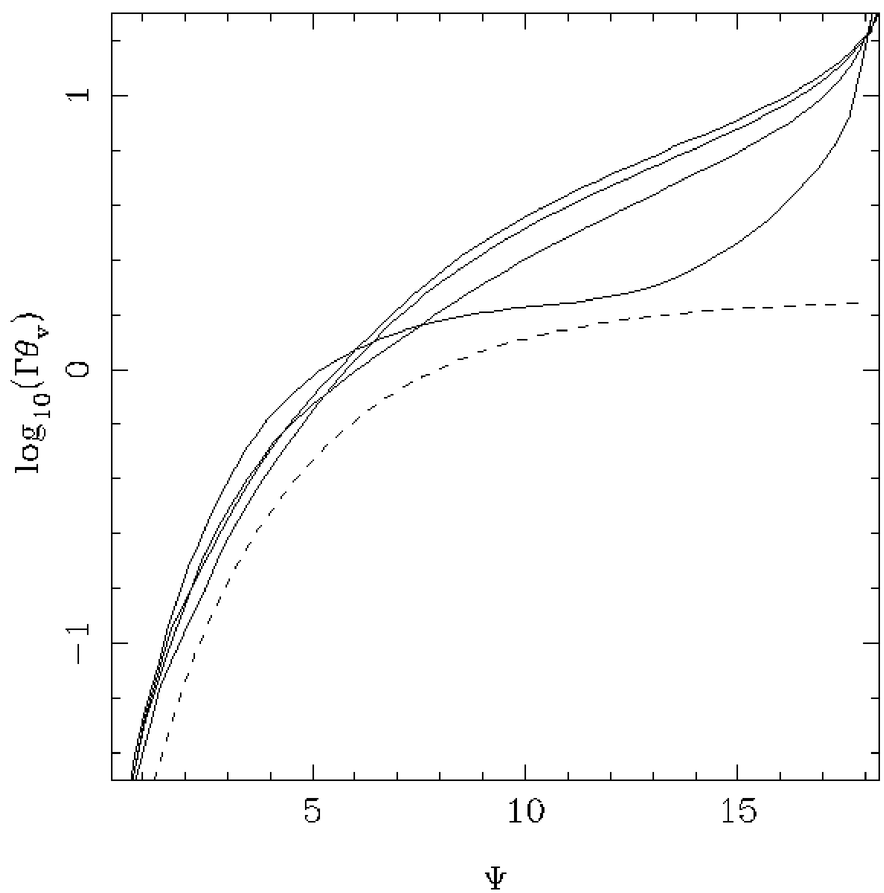
Komissarov, Vlahakis & Königl 2010

(see also Tchekhovskoy, Narayan & McKinney 2010)





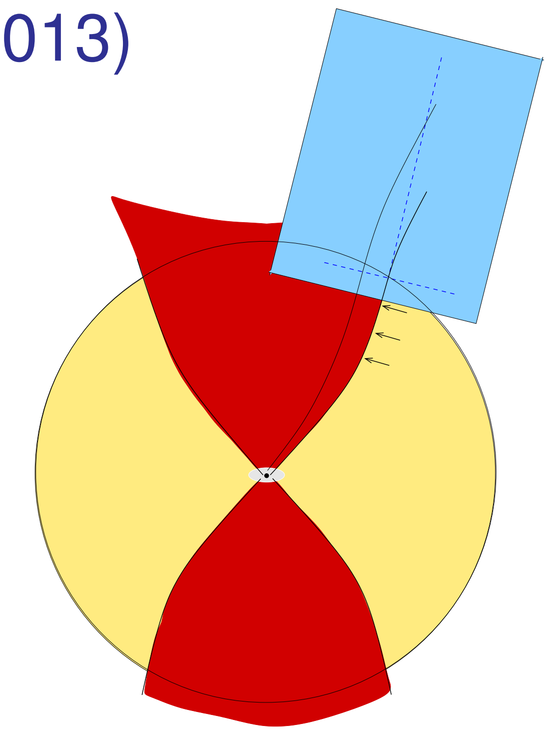
3
2.5
2
1.5
1
$\log_{10}\Gamma,\mu,\sigma$
μ
$\sigma\Gamma$
Γ
2
4
6
$\log_{10}z/r_{lc}$



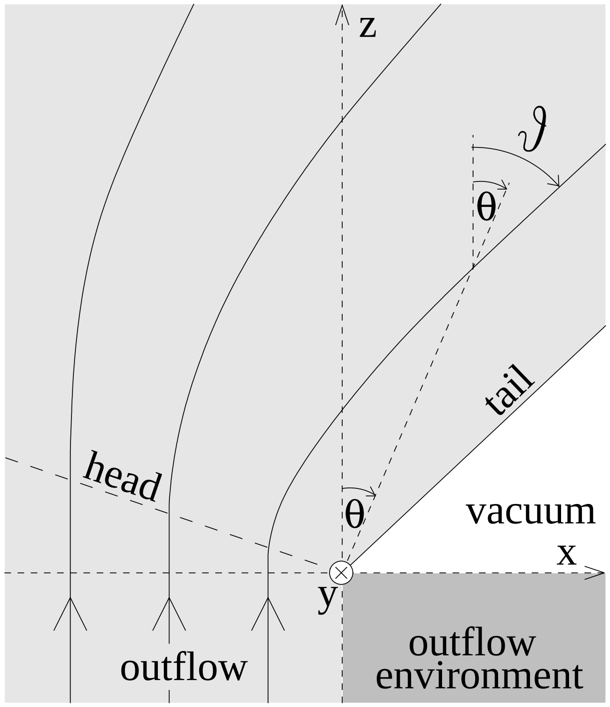
1
0
−1
$\log_{10}(\Gamma\theta_v)$
5
10
15
Ψ

Steady-state rarefaction wave

Sapountzis & Vlahakis (2013)



- “flow around a corner”
- planar geometry
- ignoring B_p (nonzero B_y)
- similarity variable x/z (angle θ)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)

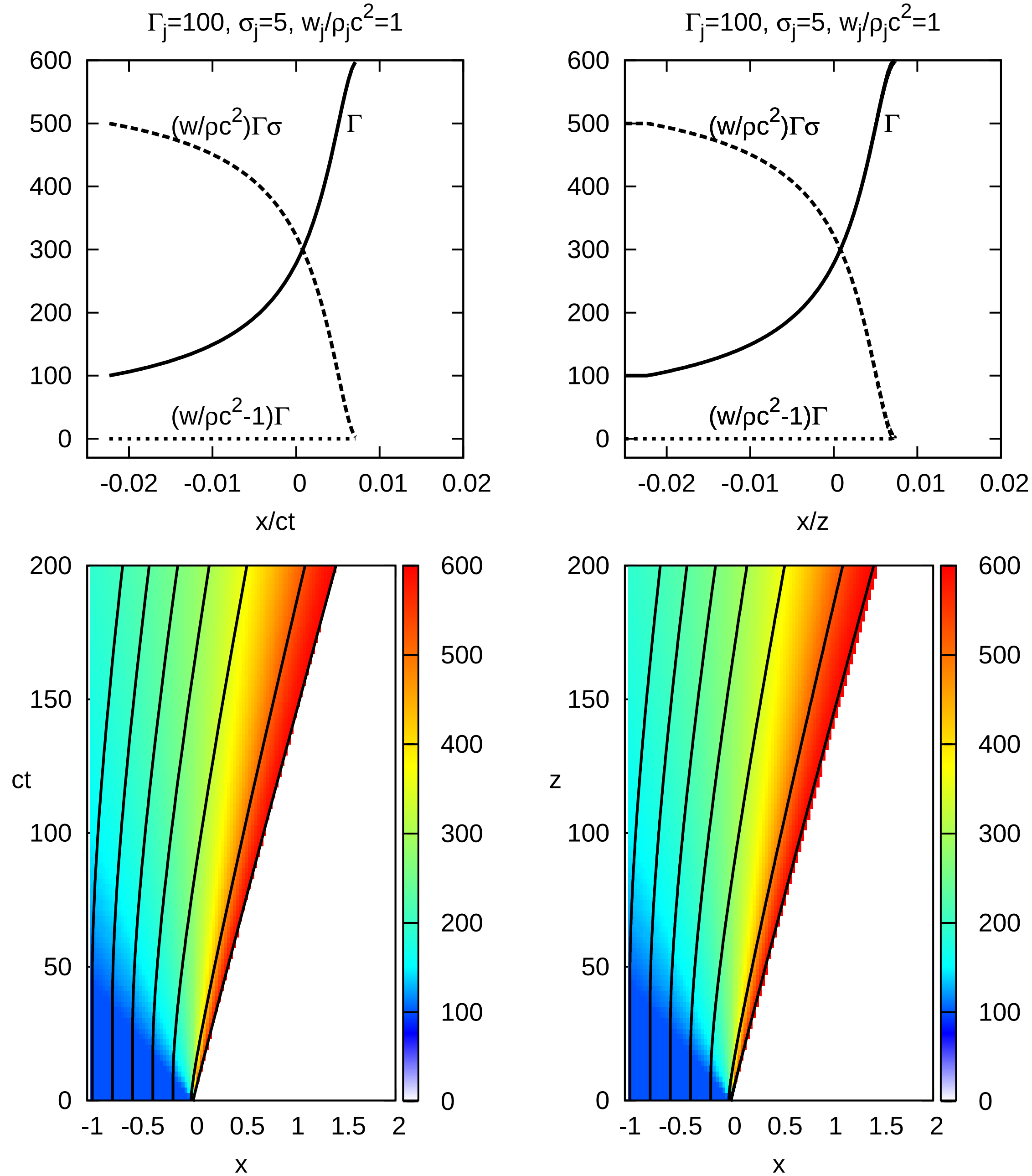


$$\theta_{\rm head} = -\frac{\sigma_j^{1/2}}{\gamma_j}$$

$$\theta_{\rm tail} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)}$$

$$\sigma = (\sigma_j \gamma_j x_i / z)^{2/3}$$

$\sigma = 1$ at $r = \sigma_j \gamma_j |x_i| = 7\times 10^{11} \sigma_j \left(\frac{|x_i|}{R_\star/\gamma_j}\right)\left(\frac{R_\star}{10 R_\odot}\right)$ cm



time-dependent (left) and steady-state (right) rarefaction (similar; $ct \to z$)
(distance unit $= R_\star/\gamma_j \sim 10^{10}$ cm)

The frozen pulse approximation

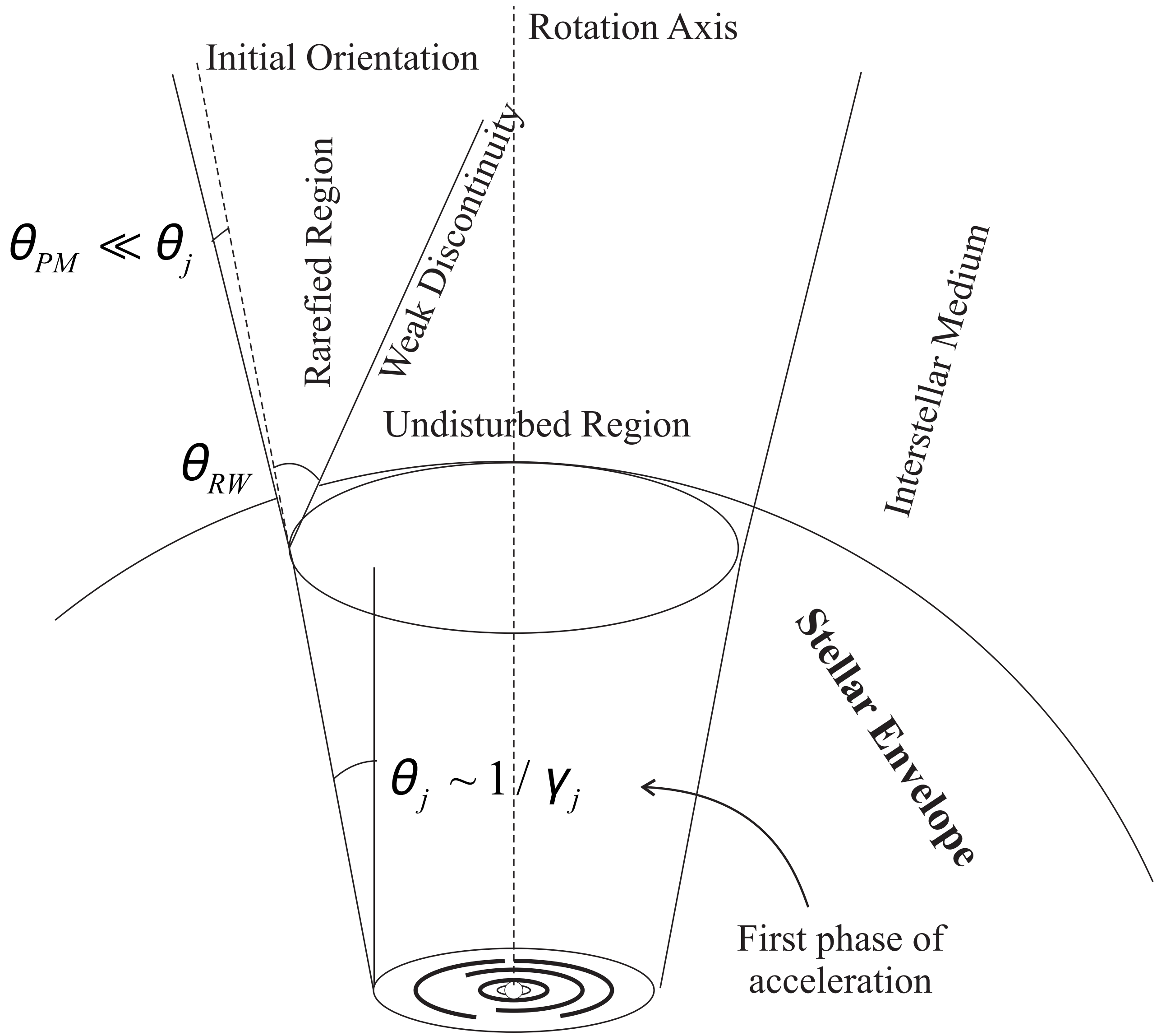
Introduced by Piran+1993 for HD flows and extended by NV & Königl (2003) in the MHD case.

In a superfast magnetosonic, ultrarelativistic flow any possible disturbance is travelling with it and cannot affect the neighbouring parts. As a result the evolution of each fluid parcel is essentially steady-state.

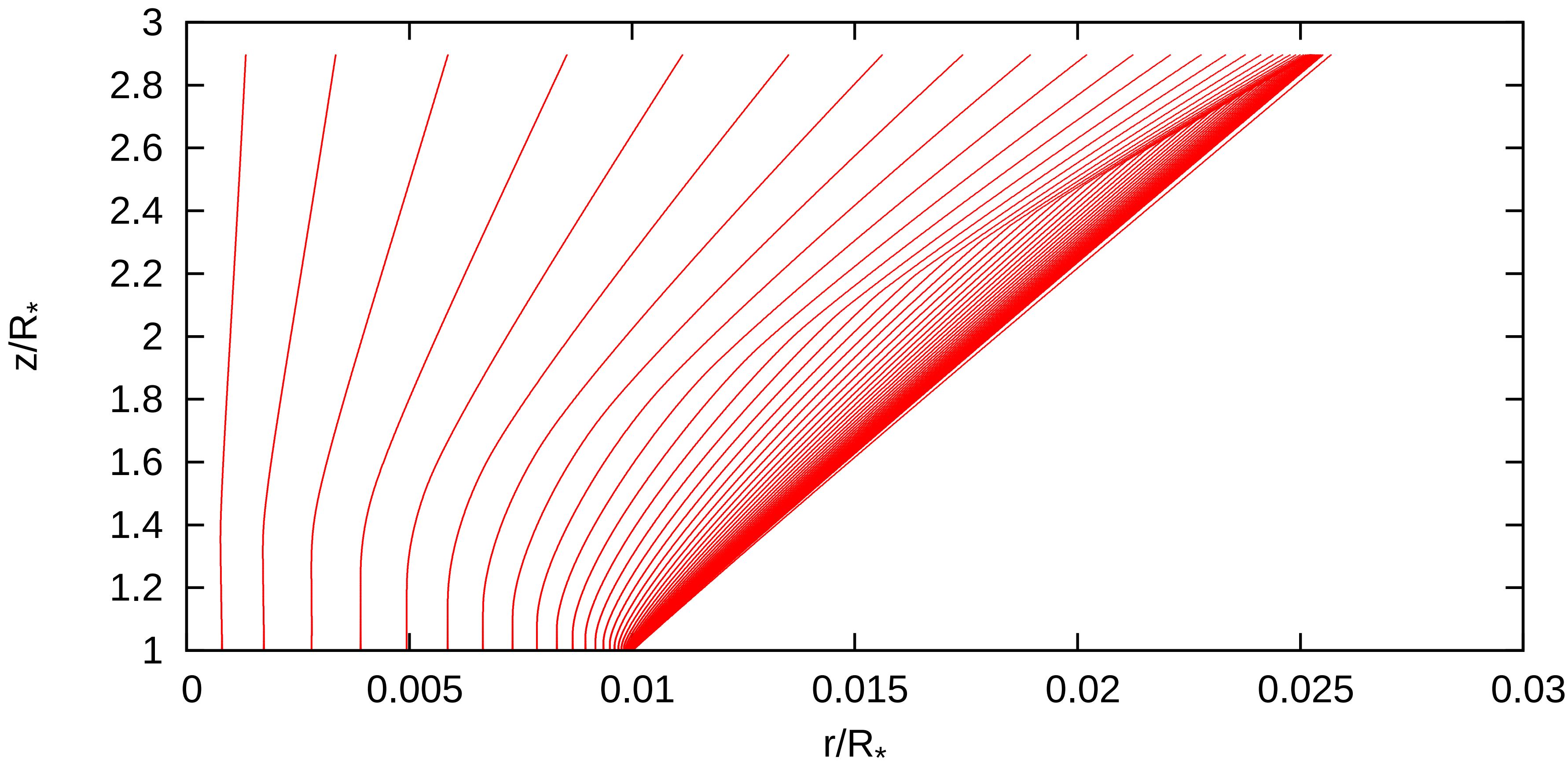
In mathematical terms, if we change coordinates from (x, z, t) to $(x, z, s = ct - z)$ the approximate equations do not contain derivatives $\dfrac{\partial}{\partial s}$.

Axisymmetric model

Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)

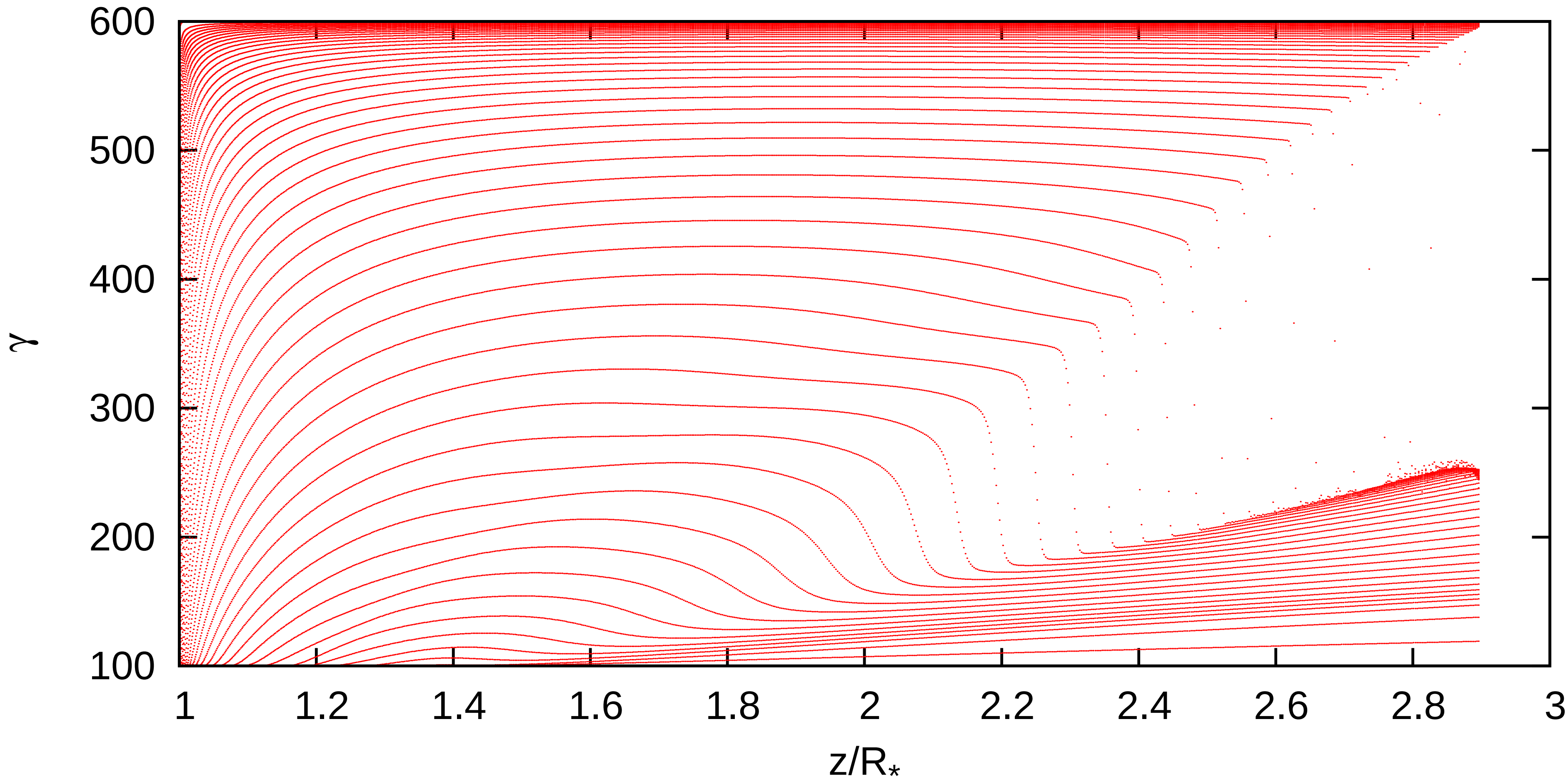


$\gamma_j = 100$, $\sigma_j = 5$, $\rho_{ext} = 0$



(not in scale!)

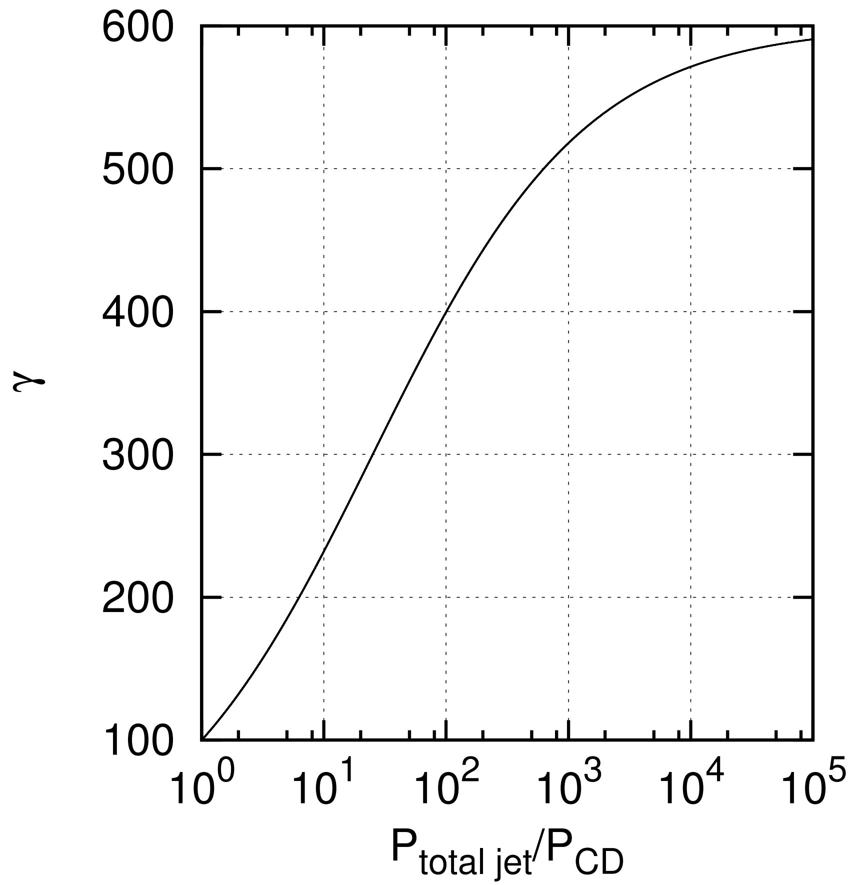
Reflection of the wave from the axis



Reflection causes sudden deceleration – standing shock

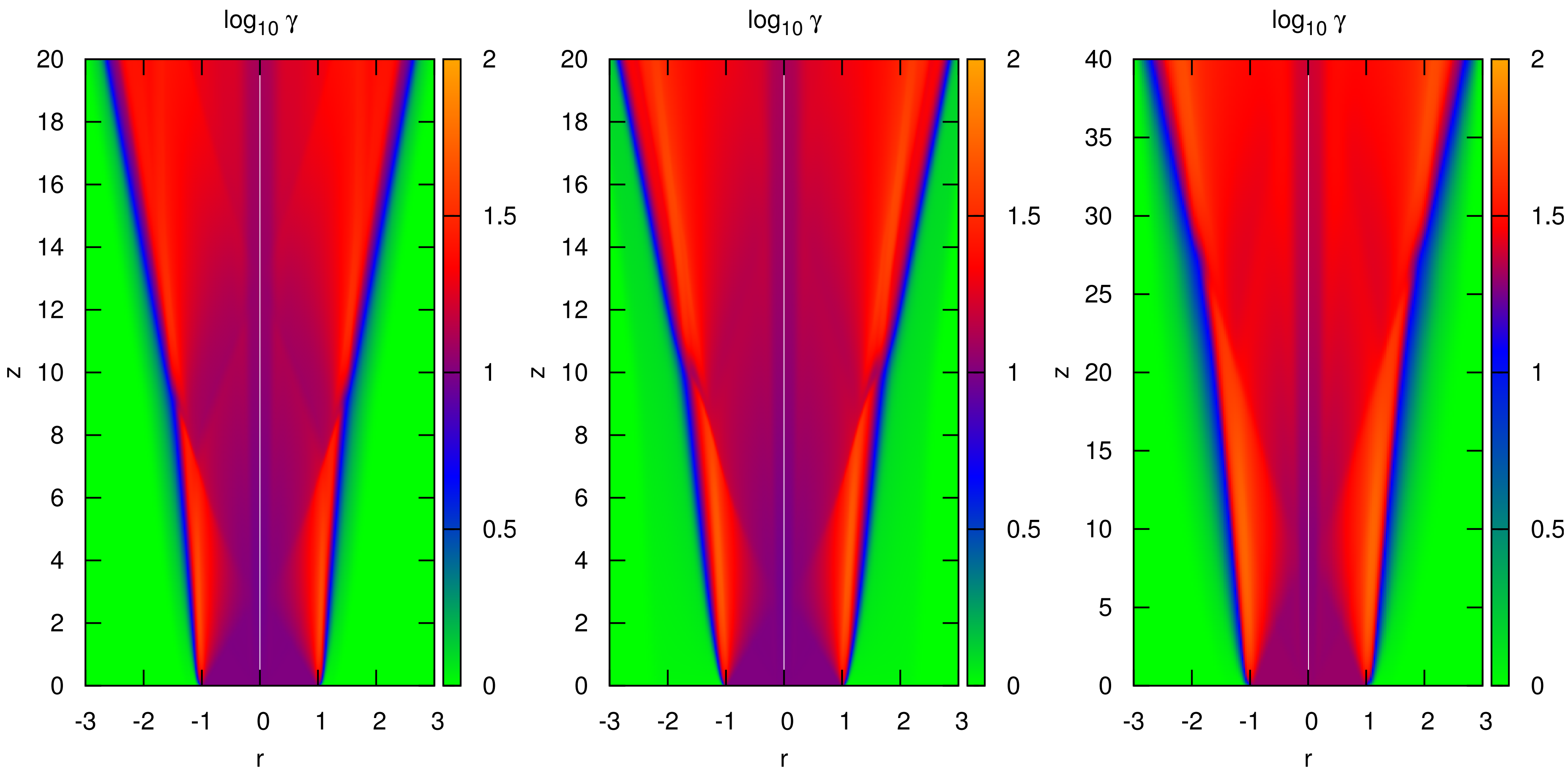
The role of the environment

- for nonzero ρ_{ext} Riemann problem: rarefaction on the left state / contact discontinuity / shock on the right



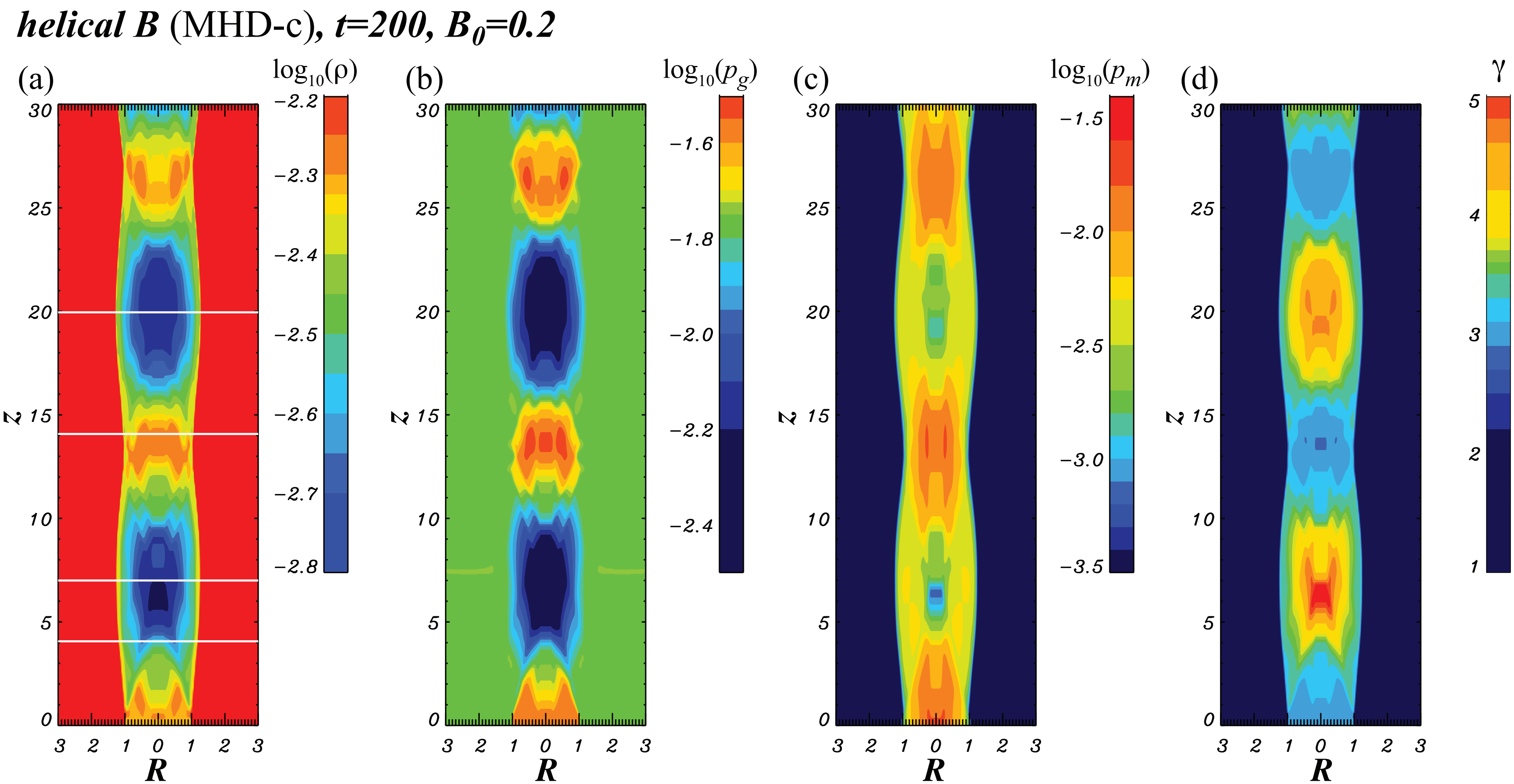
(for $\gamma_j = 100$, $\sigma_j = 5$)

Numerical simulations with PLUTO (preliminary results)



Left: $\gamma_j = 10$, $\sigma_j = 10$ (RW hit the axis at $z/r_j \sim \gamma_j/\sqrt{\sigma_j} \approx 3$)
Middle: 10 times lower density and pressure outside
Right: $\gamma_j = 20$, $\sigma_j = 5$ (note the different z-scale; $z/r_j \sim \gamma_j/\sqrt{\sigma_j} \approx 9$)

Similar to recollimation shock structure, e.g. Mizuno+2015



(Their σ is < 0.36 and the jet is not cold. Also $\rho_j \ll \rho_{out}$ and $P_j = 1.5P_{out}$.)

Summary

Simple (rarefaction) waves could significantly affect the dynamics of GRB outflows

⋆ contribute to the jet bulk acceleration

⋆ make magnetically accelerated GRB jets with $\gamma\vartheta \gg 1$

⋆ create series of shocks (that are standing and do not depend on the engine activity)