Weak Fragmentability

by **V. Farmaki**

Abstract. We introduce the notion of weak-fragmentability which is identified with the weak-Radon Nikodym property and also with the Pettis property in weak*-compact, convex subsets of a dual Banach space. Also we prove that characteristic properties of a convex, weakly fragmented set K are that $\overline{convF}^{w^*} = \overline{convF}^{\| \ \|}$ for every weak*-compact subset F of K and that $L = \overline{convextL}^{\| \ \|}$ for every convex, weak*-compact subset L of K. Finally is proved that a convex, weak-fragmented set is affine homeomorphic to a weak*-compact subset of the dual space of a Banach space not containing l_1.

Introduction. The central theme of this paper is the notion of weak-fragmentability (Definition 1) which is inspired by the fragmentability defined in [8] and is identified with the scalar point of continuity property [9] in weak*-compact subsets of dual Banach spaces.

By Musial [7] and Janicka [6] it is proved that the dual Banach spaces with the weak-Radon-Nicodym property is characterized as the spaces with predual not containing l_1. Also it is known [11] that a dual Banach space has the w-R.N.P. if and only if its weak*-compact subsets are weakly-fragmented. On the other hand, every convex, weakly-fragmented subset of a dual Banach space is affine homeomorphic to a weak*-compact subset of a dual space with the w-R.N.P.(Corollary 13).

Every convex,weak*-compact subset K of a dual Banach space X^* is weakly fragmented if and only if it has the w-R.N.P. or equivalently if it is a Pettis set (Theorem 9).Also characteristic properties of a convex weakly fragmented set K are that $\overline{convF}^{w^*}=\overline{convF}^{\|\ \|}$ for every weak*-compact subset F of K and that $L=\overline{convextL}^{\|\ \|}$ for every convex, weak*-compact subset L of K (Theorem 9 and Corollary 10). Using a version of the Superlemma [1],[2], we prove that K is weakly fragmented if and only if for every $x^{**}\in X^{**}$ the set of extreme points of K which are also points of continuity of $x^{**}/K:(K,w^*) \to \mathbb{R}$ is a dense, G_δ set in extK (Proposition 4).

Finally, we prove that every bounnded linear operator $T:X \to Y^*$ from a Banach space X into a dual space Y^* such that the weak*-closure of the image of the unit ball of X is a weakly fragmented subset of Y^* factors through a dual space with predual not containing l_1(Corollary 14).This can also be proved by using a Theorem of E.Saab and P.Saab [11], we give a direct proof which uses the notion of weak-fragmentability.

Proposition 2. Let X be a Banach space and F a w^*-compact subset of the dual space X^*.The following statements are equivalent:

(i) F is w-fragmented.

(ii) For every non-empty, w^*-compact subset H of F, $\varepsilon>0$ and $x^{**}\in X^{**}$ there exists a non-empty relatively open subset U of (H,w^*) such that $O(x^{**},U)<\varepsilon$.

(iii) For every non-empty, w^*-compact subset H of F and $x^{**}\in X^{**}$ the restriction of x^{**} to (H,w^*) has a point of continuity.(Scalar point of continuity property, [9].)

(iv) For every w^*-compact subset H of F and $x^{**}\in X^{**}$ the set of points of continuity of the map $x^{**}/_H:(H,w^*)\to\mathbb{R}$ is a dence G_δ subset of (H,w^*).

Proof. The implications (i) $\Rightarrow$ (ii), (iv) $\Rightarrow$ (iii) and (iii) $\Rightarrow$ (ii) are obvious.

(ii) $\Rightarrow$ (iv). Let $n\in\mathbb{N}$ and Z_n be the set of all $x\in H$ for which there exists a relatively open subset U of (H,w^*) with $x\in U$ and $O(x^{**},U)<1/n$. Clearly the sets Z_n ,$n\in\mathbb{N}$ are relatively open subsets of (H,w^*) and from (ii) they are dence in (H,w^*).The set $Z=\bigcap_{n=1}^{\infty}Z_n$ is precisely the set of points of continuity of $x^{**}/_H:(H,w^*)\to\mathbb{R}$ and it is a dence,G_δ subset of (H,w^*), becauce (H,w^*) is a Baire space.

(iv) $\Rightarrow$ (i). For each $i=1,\dots,n$, let T_i be the set of points of continuity of $x_i^{**}/_H:(H,w^*)\to\mathbb{R}$.From (iv) the sets T_i are dence,G_δ in (H,w_δ^*).Hence $T=\bigcap_{i=1}^{n}T_i$ is also dence,G_δ in (H,w^*).Let $e\in T$.Then there exists a relatively open subset U of (H,w^*) such that $e\in U$ and $O(x_i^{**}.U)<\varepsilon$ for every $i=1,\dots,n$.

We will prove that the set of the extreme points of K which are also points of continuity of $x^{**}/_K:(K,w^*)\to\mathbb{R}$ for some $x^{**}\in X^{**}$ is dence, G_δ in $(\text{ext}K,w^*)$ if K is a w^*-compact, w-fragmented, convex subset of a dual space. This can be proved using analogous arguments as in the proof of Proposition 8

Notations. Let Z be a topological Hausdorff space and f a real valyed function on Z. If $A\subseteq Z$, the oscillation of f on A is $O(f,A)=\sup\{|f(y)-f(x)|: x,y\in A\}$ and the oscillation of f at a point $x\in Z$ is $O(f,x)=\inf\{O(f,U): U\subseteq Z \text{ is open and } x\in U\}$. Obviously, f is continuous at x if and only if $O(f,x)=0$. We denote by $f/_A$ the restriction of f to A.

Let X be a Banach space. We denote by X^* and X^{**} the dual and the second dual of X respectively. The closed unit ball of X is denoted by B_X and its surface by S_X. If A is a subset of the dual space X^* then we denote by $\overline{A}^{\|\ \|}$ the norm-closure of A, by $\overline{A}^{w}$ the weak-closure of A, by $\overline{A}^{w^*}$ the weak*-closure of A and by convA the convex hull of A. The symbol (A,w^*) means that A is endowed with relative w^*-topology. If K is a convex subset of X then the set of extreme points of K is denoted by extK.

If F is a bounded subset of a dual Banach space X^* then a w^*-slice (or w^*-open slice) of F is a set of the form $S=S(F,x,a) = \{ f\in F: f(x)>M(x,F)-a \}$ where $x\in X$, $a>0$ and $M(x,F)=\sup\{f(x):f\in F\}$. The corresponding w^*-closed slice of S is the set $S_1=S_1(F,x,a,) = \{ f\in F: f(x)\geq M(x,F)-a \}$.

Definition 1. Let X be a Banach space and F a non-empty subset of the dual space X^*.We say that F is **weakly fragmented** if for each non-empty subset A of F, $\varepsilon>0$ and $x_1^{**},\dots,x_n^{**}\in X^{**}$ there exists a non-empty relatively open subset U of (A,w^*) such that $O(x_i^{**},U)<\varepsilon$ for every $i=1,\dots,n$.

It is easy to check that F is w-fragmented if and only if for every non-empty relatively closed subset H of (F,w^*) there exists a non-empty relatively open subset U of (H,w^*) such that $O(x^{**},U)<\varepsilon$ for every $i=1,\dots,n$.

in [11].We will give another more elegant proof for this result using a version of the Superlemma [1],[2].

Lemma 3. Let X be a Banach space, K,K_0, and K_1 be w^*-compact, convex subsets of X^*, $\varepsilon>0$ and $x_1^{**},\dots,x_n^{**}\in S_{X^{**}}$ .Suppose that:

1. K_0 is a subset of K and $O(x_i^{**},K_0)<\varepsilon$ for every $i=1,\dots,n$.
2. K is not a subset of K_1.
3. K is a subset of $\mathrm{conv}(K_0\cup K_1)$.

Then there exists a w^*-slice S of K that intersects K_0 and $O(x_i^{**},S)<\varepsilon$ for every $i=1,\dots,n$.

The proof is virtually indentical with that of the Superlemma (weak* version).(See [3] Theorem 3.4.1.(w^*)).

Proposition 4. Let X be a Banach space, K a w^*-compact, w-fragmented, convex subset of X^* and $x^{**}\in X^{**}$. Then the set $T\cap \mathrm{ext}K$ is dense, G_δ in $(\mathrm{ext}K,w^*)$, where T is the set of the points of continuity of $x^{**}/_K(K,w^*)\to\mathbb{R}$. Consequently, $K=\overline{\mathrm{conv}(T\cap\mathrm{ext}K)}^{w^*}$.

Proof. Let $E=\mathrm{ext}K$. For each $\varepsilon>0$ let B_ε be the set of elements of E which have a w^*-open neighborhood V such that $O(x^{**},V\cap K)<\varepsilon$. Every B_ε is open in (E,w^*) and we will prove that B_ε is also dense. Let W be a w^*-open subset of X^* whith $W\cap E\neq\emptyset$. Since K is w-fragmented there exists a w^*-open subset U of X^* such that $U\subseteq W$, $U\cap E\neq\emptyset$ and $O(x^{**},\overline{\mathrm{conv}}^{w^*}(U\cap E))<\varepsilon$, ([11] Proposition 7). Define $K_0=\overline{\mathrm{conv}(U\cap E)}^{w^*}$ and $K_1=\overline{\mathrm{conv}(E\setminus U)}^{w^*}$.

The sets K_0, K_1, K satisfy the properties 1, 2, 3, of Lemma 3, hence we can find a w^* -slice S of K that intersects K_0, misses K_1 and $O(x^{**},S)<\varepsilon$.Let $u\in E\cap S$. Since $O(x^{**},S)<\varepsilon$, we have that $u\in B_\varepsilon$.Also $u\in E\cap U\subseteq W$ since $K_1\cap S=\emptyset$. Therefore B_ε is dense in (E,w^*).

Finally, $\text{ext}K \cap T = \bigcap_n B_{1/n}$ is dense, G_δ in $(\text{ext}K, w^*)$, since $(\text{ext}K, w^*)$ is a Baire space.

Corollary 5. Let A be a bounded subset of a dual space X^*. If $K = \overline{\text{conv}A}^{w^*}$ is w-fragmented then for every $\varepsilon > 0$ and $x_1^{**}, \ldots, x_n^{**} \in X^{**}$ there exists a w^*-slice S of K such that $S \cap A \neq \emptyset$ and $O(x_i^{**}, S) < \varepsilon$ for every $i = 1, \ldots, n$.

Proof. Let E_i $i=1,\ldots,n$ be the sets of extreme points of K which are points of continuity of $x_i^{**}/_K : (K, w^*) \to \mathbb{R}$ respectively. By Proposition 4 the sets E_i are dence, G_δ in $(\text{ext}K, w^*)$ for every $i=1,\ldots,n$. Since $(\text{ext}K, w^*)$ is a Baire space the set $E = \bigcap_{i=1}^n E_i$ is also dence, G_δ in $(\text{ext}K, w^*)$. Let $e \in E$, then there exists a w^*-slise S of K such that $e \in S$ and the corresponding closed slice S_1 of S has $O(x_i^{**}, S_1) < \varepsilon$ for every $i=1,\ldots,n$.

Of course $S \cap A \neq \emptyset$ and $O(x_i^{**}, \overline{\text{conv}(S \cap A)}^{w^*}) < \varepsilon$ for every $i=1,\ldots,n$.

Proposition 6. Let F be a w^*-compact subset of a dual Banach space X^*, such that $\overline{\text{conv}F}^{w^*}$ is w-fragmented. Then $\overline{\text{conv}F}^{w^*} = \overline{\text{conv}F}^{\|\ \|}$.

Proof. Let $f \in K = \overline{\text{conv}F}^{w^*}$. Then there is a regular Borel probability measure μ on (F, w^*) whose resualtant $\tau(\mu)$ is equal to f. Let $\varepsilon > 0$ and $x_1^{**}, \ldots, x_n^{**} \in X^{**}$. The support M of μ is a non-empty, w^*-compact subset of F and $M_1 = \overline{\text{conv}M}^{w^*}$ is w-fragmented subset of K. From Corollary 5 there exists a w^*-closed slice S_1 of M_1 such that $O(x_i^{**}, \overline{\text{conv}}^{w^*}(S_1 \cap M)) < \varepsilon$ for every $i=1,\ldots,n$ and $S_1 \cap M \neq \emptyset$. Therefore $\mu(S_1 \cap M) > 0$.

Let $\mathbf{A}$ be a maximal disjoint family of w^*-compact subsets A of F such that $\mu(A) > 0$ and $O(x_i^{**}, \overline{\text{conv}}^{w^*} A) < \varepsilon$ for every $i=1,\ldots,n$. Then $\mathbf{A}$ is countable and we will show that $\mu(\cup \mathbf{A}) = 1$. If $\mu(\cup \mathbf{A}) < 1$, then by the regularity of μ there exists a w^*-compact subset L of F such that $L \subseteq F \setminus \cup \mathbf{A}$ and $\mu(L) > 0$. Let μ_1 the Borel measure on F with $\mu_1(B) = \mu(L \cap B)$ for every w^*-Borel subset of F. If C is the support of μ_1, then C is a non-empty w^*-compact subset of L and $C_1=\overline{conv}^{w^*}C$ is w-fragmented subset of K. From Corollary 5 there is a w^*-closed slice S_1 of C_1 such that $O(x_i^{**},\overline{conv}^{w^*}(S_1\cap C))<\varepsilon$ and $S\cap C\neq\emptyset$. Hence $\mu(S_1\cap C)>0$. Therefore $\mathbf{A}\cup\{S\cap C\}$ is properly larger than $\mathbf{A}$, contradicting maximality. Hence $\mu(\cup\mathbf{A})=1$.

Let $A_1,\ldots,A_k\in\mathbf{A}$ so that $\sum_{i=1}^{k}\mu(A_i)>1-\varepsilon$. Then $\mu=t_1\mu_1+\ldots+t_k\mu_k+t_{k+1}\lambda$, where μ_i for $i=1,\ldots,k$ and λ are regular, Borel, probability measures of (F,w^*) with $\mu_i(A_i)=1$, $0\le t_i\le 1$ for $i=1,\ldots,k$, $t_{k=1}<\varepsilon$ and $\sum_{i=1}^{k+1}t_i=1$. Therefore $f=\tau(\mu)=t_1\tau(\mu_1)+\ldots+t_k\tau(\mu_k)+t_{k+1}\tau(\lambda)$. Choose $f_i\in A_i$ for $i=1,\ldots,k$ and $f_{k+1}\in F$. Since $\tau(\mu_i)\in\overline{convA_i}^{w^*}$, $i=1,\ldots,k$ we have that $x_j(f_i^{**}-\tau(\mu_i))<\varepsilon$ for every $j=1,\ldots,n$ and $i=1,\ldots,k$. Hence $x_j^{**}(f-\sum_{i=1}^{k+1}f_i)=\sum_{i=1}^{k}t_ix_j^{**}(\tau(\mu_i)-f_i)+t_{k+1}x_j^{**}(\tau(\lambda)-f_{k+1})<\varepsilon+diamF$ for every $j=1,\ldots n$.

Since $\varepsilon>0$ and $x_1^{**},\ldots,x_n^{**}$ are arbitrary we conclude that $f\in\overline{convF}^{w}=\overline{convF}^{\|\ \|}$.

Remark 7. For every w^*-compact, w-fragmented, convex subset K of a dual Banach space we have that $K=\overline{convextK}^{\|\ \|}$. This can be proved with the same as in the proof of Proposition 3.1 in [5]. Proposition 7 can be proved easily using the previus result. We gave a directly proof which has ideas of an analogous result of Namioka [8].

In the following we will prove that the property which is described in Proposition 6 is a characteristic property of convex, w-fragmented sets.

Definition 8. Let X be a Banach space and K a w^*-compact subset of the dual space X^*.

K is called a **Pettis** set if the identity map $I:(K,w^*)\to X^*$ is universally scalarly measurable, i.e. for each $x^{**}\in X^{**}$ the function $x^{**}/_K:(K,w^*)\to\mathbb{R}$ is μ-measurable for every Radon propability measure μ on (K,w^*).

K is called a **weak-Radon-Nikodym** set (w-R.N. set) if for every propability space (Ω,F,μ) and every bounded linear operator $T:L_1(\mu) \to X^*$ for which $T(\ \chi_E/\mu(E)\ )$ belongs to K for each non-null measurable set E, the operator T is represented by a Pettis integrable function with values in K.

Theorem 9. Let K be a w^*-compact,convex subset of a dual space X^*.The following are equivalent:

(i) K is w-fragmented.

(ii) K is a Pettis set.

(iii) K has the weak-Radon-Nikodym property.

(iv) For each w^*-compact subset F of K we have $\overline{convF}^{w^*}=\overline{convF}^{\|\ \|}$.

Proof. (i) $\Leftrightarrow$ (iii) K is w-fragmented if and only if K has the scalar point of continuity property, from Proposition 2.Hence we have the equivalence from the results in [9] and [10].

(ii) $\Leftrightarrow$ (iv) This equivalence is proved in [12].

(ii) $\Rightarrow$ (i) In [12] is proved that a w^*-compact subset F of X^* is a Pettis set if and only if $\overline{conv}^{w^*}(\ K\cup(-K)\ )$ is a Pettis set. Since every w^*-compact absolutely convex subset of X^* is w-fragmented ([9]) we have that every Pettis set is w-fragmented.

(i) $\Rightarrow$ (iv) It is obvious from Proposition 6.

Corollary 10. A convex,w^*-compact subset K of a dual Banach space X^* is w-fragmented if and only if $L=\overline{convextL}^{\|\ \|}$ for every convex subset L of K.

The following Theorem is unfluenced by the methods of Davis-Figiel-Johnson-Pelczynski [4] and Namioka [8]

Lemma 11 Let F be a w^*-compact, w-fragmented subset of a dual space X^*, B the unit ball of X^*, $\varepsilon>0$, $\delta>0$ and $x_1^{**},\dots,x_n^{**}\in S_{X^{**}}$. If A is a non-empty subset of $K+\delta B^*$ then there is a w^*-open subset V of X^* such that $V\cap A\neq\emptyset$ and $O(x_i^{**},V\cap A)<2\delta+\varepsilon$ for every $i=1,\dots,n$.

Proof. We can assume that A is w^*-compact.We define the function $f:F\times\delta B \to X^*$ by $f((x^*,y^*)=x^*+y^*$. If $F\times\delta B$ has the product of the weak*-topologies and X^* the weak*-topology the function f is continuous and of course $A\subseteq f(F\times\delta B)$.

Let M be a minimal compact subset of $F\times\delta B$ such that $f(M)=A$.Then $\pi_1(M)$ (where π_1 is the projection map) is a w-fragmented subset of F, hence there exists a relatively w^*-open subset U of F such that $U\cap\pi_1(M)\neq\emptyset$ and $O(x_i^{**},V\cap\pi_1(M))<\varepsilon$ for every $i=1,..,n$.By the minimality of M, $f(M\setminus\pi_1^{-1}(V))$ is a proper subset of A, hence $W=A\setminus f(M\setminus\pi_1^{-1}(V))\subseteq f(M\cap\pi_1^{-1}(V))$ is a non-empty, relatively w^*-open subset of A. Let $x^*,y^*\in W$, then $x=x_1^*+x_2^*$ and $y^*=y_1^*+y_2^*$ with $(x_1^*,x_2^*),(y_1^*,y_2^*)\in(V\cap\pi_1^{-1}(M))\times\delta B$.Then

$$x_i^{**}(x^*-y^*) = x_i^{**}(x_1^*-y_1^*) + x_i^{**}(x_2^*-y_2^*) < \varepsilon+2\delta.$$

and $O(x_i^{**},W)<2\delta+\varepsilon$ for every $i=1,\dots,n$ as was to be shown.

Theorem 12. Let X be a Banach space and K a w^*-compact, w-fragmented, absolutely convex subset of the dual space X^*.Then there exists a bounded linear operator T from X onto a dence subspace of a Banach space E not containing l_1 such that $K\subseteq T^*(B)$, where B is the unit ball of E^*.

Proof. Let V be the unit ball of X^*, $U_n=2^nK+(1/2^n)V$ for every $n=1,2,\dots$ and $\|\ \|_n$ a norm for X^* whose unit ball is U_n. The norms $\|\ \|_n$ are dual and equivalent to the original one on X^*. Let $p(x^*)=\sum_{n=1}^{\infty}\|x^*\|_n$ for every $x^*\in X^*$, $H=\{x^*\in X^*:p(x^*)<\infty\}$ and $U=\{x^*\in X^*:p(x^*)\leq 1\}$.

As proved in [8], (H,p) is a dual Banach space and U is a w^*-compact, absolutely convex subset of X^* with $K \subseteq U$. We will prove that U is also w-fragmented. Let F be a non-empty, w^*-compact subset of U, $\varepsilon>0$ and $x^{**} \in X^{**}$. Fix $n \in \mathbb{N}$ such that $1/2^n<(\varepsilon/3)\|x^{**}\|$. Then $F \subseteq U \subseteq U_n = 2^n K+(1/2^n)V \subseteq 2^n K+(\varepsilon/2\|x^{**}\|)V$. From Lemma 11 there is a relatively w^*-open subset W of F with $O(x^{**},W)<\varepsilon$. Hence U is a w-fragmented subset of X^*.

We will show that (H,p) has the w-R.N.P..It is sufficient to prove that U is a w-fragmented subset of (H,p).Let F be a non-empty, w^*-compact subset of U, $\varepsilon>0$ and $x^{**} \in S_{H^*}$. If $m=\sup\{\ p(f):f \in F\ \}$, then we choose an $f_0 \in F$ such that $p(f_0)>m-\varepsilon/3$ and consequently $n_0 \in \mathbb{N}$ such that $\sum_{n=1}^{n_0} \|f_0\|_n>m-\varepsilon/3$.Write $q(g)=\sum_{n=1}^{n_0} \|g\|_n$ for every $g \in X^*$.Then q is an equivalent norm in E^* and also q is w^*-lower semicontinuous.Let $W=\{\ f \in F:q(f)>m-\varepsilon/3\ \}$.Then $f_0 \in W$ and W is a relatively w^*-open subset of F.We define $Z=(\sum_n X_n)_1$ where $X_n=(X^*,\|\ \|_n)$ and $\varphi:H \to Z$ by $\varphi(F)=(jf,jf,\dots)$ where $j:H \to X^*$ is the inclusion map.Then $\varphi(H)$ is a closed linear subspace of Z and (H,p) is isometric to $\varphi(H)$.Hence $\varphi^*:(\sum_n X_n^*)_\infty \to H^*$ is onto H^* and therefore there exists $y^*=(x_1^*,x_2^*,\dots) \in \sum_{n=1}^{\infty} X_n^*$ with $\|y^*\|_\infty \leq 1$ such that $y^*(f)=\sum_n x_n^*(f)$ for every $f \in H$. Let $z^*:X^* \to \mathbb{R}$ with $z^*(g)=\sum_{n=1}^{n_0} x_n^*(g)$ for every $g \in X^*$.Then $z^* \in X^{**}$, because the norm q is equivalent to the original norm of X^*. Since U is a w-fragmented subset of X^*, there exists a non-empty relatively w^*-open subset G of W such that $O(z^*,G)<\varepsilon/3$. The set G is relatively w^*-open in F and $O(x^*,G)<\varepsilon$. Indeed, let $g_1,g_2 \in G$, then $x^*(g_1-g_2)=$ $\sum_{n=1}^{\infty} x_n^*(g_1-g_2)=\sum_{n=1}^{n_0} x_n^*(g_1-g_2)+\sum_{n=n_0+1}^{\infty} x_n^*(g_1-g_2)<\varepsilon/3+\sum_{n=n_0+1}^{\infty} \|g_1\|_n+\sum_{n=n_0+1}^{\infty} \|g_2\|_n<\varepsilon$ because $g_1,g_2 \in W$.This shows that U is w-fragmented in (H,p).

Let C(U) be the Banach space of all continuous scalar-valued functions on (U,w^*) with the supremum norm and let $R:X \to C(U)$ be the bounded linear operator defined by $R(x)(u)=u(x)$ for every $x \in X$ and $u \in U$. Let $E=\overline{R(X)}$ and

$T:X \to E$ be the map which is obtained from R by restricting the range. Then $T^*:E^* \to X^*$ is an isometry onto (H,p) and $K \subseteq T^*(B)$ where B is the unit ball of E^*.

Corollary 13. Every Pettis set (F,w^*) of a dual Banach space X^* is (affine) homeomorphic to a w^*-compact subset of the dual E^* of a Banach space E not containing l_1.

Proof. If F is a Pettis set then $K=\overline{\text{conv}(F \cup -F)}^{w^*}$ is also a Pettis set [1]. According to Theorems 9 and 12 there exists a space E not containing l_1 and a bounded linear map $T:X \to E$ with dence range such that $K \subseteq T^*(B_{E^*})$. Since the restriction of $T^*/B_{E^*} :(B_{E^*},w^*) \to (T^*(B_{E^*}),w^*)$ is a homeomorphism we have that (F,w^*) is (affine) homeomorphic to a w^*-compact subset of E^*.

Corollary 14. Let X,Y be Banach spaces and $T:X \to Y^*$ a bounded linear operator such that $\overline{T(B_X)}^{w^*}$ is a w-fragmented subset of Y^*. Then T factors through a dual Banach space E^* with E not containing l_1.

Proof. Apply Theorem 12 to $K=\overline{T(B_X)}^{w^*}$.Then there exists a dual Banach space (H,p) with predual not containing l_1 such that $K \subseteq B_H$, $H \subseteq Y^*$ and the inclusion map $J:H \to Y^*$ be continuous. Hence $T=J \circ S$ where $S=J^{-1} \circ T:X^* \to H$.

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Vassiliki Farmaki
University of Athens
Department of Mathematics
Panepistemiopolis-Ilissia
15784 ATHENS
GREECE