New matrix perturbation bounds via contour analysis

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ABSTRACT

Matrix perturbation bounds (such as Weyl and Davis-Kahan) are among the most frequently used tools in many branches of mathematics, statistics, and theoretical computer science.

Most of the classical results in this area are optimal, in the worst case analysis. However, in modern applications, both the ground and the noise matrices frequently have extra structural properties. For instance, it is often assumed that the ground matrix is essentially low rank, and the noise matrix is random or pseudo-random. It is of fundamental interest to see if one can improve upon the classical theory under these modern and popular assumptions.

We aim to systematically rebuild a part of perturbation theory, adapting to these modern assumptions. We will do this using a method called "contour expansion" which may be of independent interest.

With this method, we are able to exploit the skewness among the leading eigenvectors of the ground matrix and the noise matrix

(which is significant when the two matrices are uncorrelated) to our advantage. This has led to a number of quantitative improvements over the classical results in well-known problems, which, in turn, has direct applications in data science and theoretical computer science.

Joined work with Phuc Tran (Yale).