# An introduction to quantitative ergodic theory

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### ABSTRACT

Ergodic theory is the study of measure preserving actions of groups on a probability space. These may be studied from two different angles: up to isomorphism, or up to "orbit equivalence". For the latter we merely require an isomorphism between the probability spaces, which preserves the orbits of the group actions, but the groups themselves may no longer be isomorphic. Orbit equivalence has been intensively studied since the eighties, and one of the most impressive results, due to Ornstein and Weiss, says that any two free ergodic actions of infinite amenable groups (such as Z^d for instance) are orbit equivalent. In other words, all information on the (amenable) groups is lost under orbit equivalence. We shall present a new theory, which emerged from the need to nuance Orstein-Weiss' theorem. Roughly, one defines a way to measure how ``good" an orbit equivalence map is in order to restore some information on the group.