Profinite rigidity of Affine Coxeter groups

Rizos Sklinos

Chinese Academy of Sciences rizos.sklinos@amss.ac.cn

ABSTRACT

In this talk I will explore two seemingly disconnected notions of rigidity in group theory: profinite rigidity and first-order rigidity. Roughly speaking, profinite rigidity asks to what extent a group is determined by its family of finite images, while first-order rigidity poses the analogous question with respect to its first-order theory. Both concepts express in different languages the idea that a group's abstract structure may be recoverable from limited information. The study of profinite rigidity has a long and rich history, reaching back to Remeslennikov's question whether nonabelian free groups are profinitely rigid - a problem that remains open to this day and has inspired deep work at the intersection of algebra, topology, and number theory. Only recently have "full-sized" examples of profinitely rigid groups, that is, groups containing nonabelian free groups, been discovered, marking a turning point in the field. Parallel to this, first-order rigidity, originating from classical model theory, has witnessed major progress and rekindled interest with some works on higher-rank arithmetic lattices. Although the two rigidity notions arise from distinct traditions, they meet in a remarkable way. It turns out that for the class of Abelian-by-finite groups, the two coincide. Building on this connection, in joint work with G. Paolini, we have shown that every affine Coxeter group is first-order rigid and, consequently, profinitely rigid. Along the way, I will also survey the landscape of known rigidity phenomena, contrasting the tame behaviour of finite extensions of abelian groups with the wild side exhibited by non elementary hyperbolic groups.