Shadows of Finite Groups

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Many properties of $\mathrm{GL}_n(q)$ - the general linear group over a finite field with q elements - behave generically with respect to q. Among them: the order of the group, its Sylow theory and to some extent its representation theory, all of which are governed by the symmetric group $\mathfrak{S}_n$ acting on the lattice $\mathbb{Z}^n$. This holds more generally for a finite reductive group $G(q)$ such as $\mathrm{SO}_n(q), \mathrm{Sp}_{2n}(q), \ldots, E_8(q)$ where $\mathfrak{S}_n$ is replaced by a reflection group over $\mathbb{Z}$.

During a conference in 1993 on the Greek island of Spetses, Broué, Malle and Michel observed that the reflection group on $\mathbb{Z}$ may be replaced by a *complex* reflection group so that the numerical data (such as the order of $G(q)$ and the dimensions of its representations) seem to still make sense, without being attached to a group. Since then, that mysterious object is called *Spets*, referring to the eponymous island.

In this talk I will explain the story behind that discovery, or: how to construct representations of non-existing groups by counting points of non-existing algebraic varieties.