The algebraic-geometric approach to the Kakeya conjecture

Keith Rogers
Institute of Mathematical Sciences, Madrid
keith.rogers@icmat.es

ABSTRACT

We will consider compact sets that contain a unit line segment in every direction and how small such sets can be. To this end, we will consider thin tubes which point in different directions and how much they can be made to overlap by positioning them strategically. On the one hand, we will see that the tubes cannot be compressed too much if they are positioned in an algebraic way. The proof employs tools from semialgebraic geometry including the Tarski-Seidenberg projection theorem and the Gromov-Yomdin algebraic lemma. On the other hand, a technique known as polynomial partitioning can be used to show that the expected bound holds in the absence of any algebraic structure. Balancing between the two extremes yields improved bounds for the Kakeya maximal conjecture in higher dimensions.