Emergence of non-zonal coherent structures in barotropic turbulence

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Atmospheric turbulence is observed to self-organize into large scale structures such as zonal jets and coherent vortices. One of the simplest models that retains the relevant dynamics is a barotropic flow in a beta-plane channel with turbulence sustained by random stirring. Non-linear integrations of this model show that as the energy input rate of the forcing is increased, the homogeneity of the flow is first broken by the emergence of non-zonal, coherent, westward propagating structures and at larger energy input rates by the emergence of zonal jets. We study the emergence of non-zonal coherent structures using a statistical theory, Stochastic Structural Stability Theory (S3T). S3T directly models a second order approximation to the statistical mean turbulent state and allows identification of statistical turbulent equilibria and study of their stability. We find that when the homogeneous turbulent state becomes S3T unstable, non-zonal large scale structures emerge and we obtain analytic expressions for their characteristics (scale, amplitude and phase speed). Numerical simulations of the non-linear equations are found to reproduce the characteristics of the structures predicted by S3T.

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1 Introduction

Atmospheric turbulence is commonly observed to be organized into large scale zonal jets with long-lasting coherent vortices embedded in them. The jets control the transports of heat and chemical species in the atmosphere, while the coherent vortices sequester chemical species and heat and produce significant spatiotemporal variability. It is therefore important to understand the mechanisms for the emergence, equilibration, and maintenance of these coherent structures.

The simplest model that retains the relevant dynamics is a turbulent barotropic flow on a β-plane. Numerical simulations of this model have shown that robust, large scale zonal jets emerge in the flow and are sustained at finite amplitude. In addition, large scale westward propagating coherent waves were found to coexist with the zonal jets (Galperin et al. 2010). These large scale waves either obey a Rossby wave dispersion, or propagate with different phase speeds and appear to be sustained by non-linear interactions between Rossby waves. However the mechanism for their excitation and maintenance remains elusive. In this work, we present a theory that predicts the formation and nonlinear equilibration of large scale coherent structures in barotropic β-plane turbulence and then test this theory against nonlinear simulations.

Since turbulent dynamics involve complex interactions among many degrees of freedom, an attractive approach for understanding the emergence of coherent structures is to study the evolution of the flow statistics rather than the evolution of the complex flow itself, investigate the dynamics of the corresponding equations and the stability of the statistical equilibria that emerge. This approach is followed in Stochastic Structural Stability Theory (S3T) (Farrell and Ioannou 2003), which is a non-equilibrium statistical theory applied to atmospheric turbulence (Farrell and Ioannou 2008). While recent studies have demonstrated that S3T can predict the structure of zonal flows in turbulent fluids (Constantinou et al. 2013), the results presented in this work demonstrate that an extended version of S3T can predict the emergence of both zonal and non-zonal coherent structures and can capture their finite amplitude manifestations.

2 Formulation of Stochastic Structural Stability Theory

Consider a non-divergent barotropic flow on a β-plane with cartesian coordinates $\mathbf{x}=(x,y)$. The velocity field, $\mathbf{u}=(u,v)$, is given by $(u,v)=(-\partial_y\psi,\partial_x\psi)$, where ψ is the streamfunction. Relative vorticity $\zeta(x, y, t) = \Delta\psi$, evolves according to the non-linear (NL) equation:

$$(\partial_t+\mathbf{u}\cdot\nabla)\zeta+\beta v=-r\zeta-\nu\Delta^2\zeta+f_e \qquad (1)$$

where $\Delta=\partial_{xx}^2+\partial_{yy}^2$ is the horizontal Laplacian, β is the gradient of planetary vorticity, r is the coefficient of linear dissipation that typically parameterizes Ekman drag and ν is the coefficient of hyper-diffusion that dissipates enstrophy flowing into unresolved scales. The exogenous forcing term f_e, parameterizes processes such as small scale convection or baroclinic instability, that are missing from the barotropic dynamics and is necessary to sustain turbulence. We assume that f_e is a temporally delta correlated and spatially homogeneous random stirring. We also assume that the forcing is isotropic, injecting energy at a rate ε in a narrow ring of wavenumbers with radius K_f=10. The calculations in this work are for β=10, r=0.01 and $\nu=1.9\times10^{-6}$.

S3T describes the statistical dynamics of the first two same time moments of (1). The first moment is the ensemble mean of the vorticity $Z(\mathbf{x}, t) \equiv \langle\zeta\rangle$, where the brackets denote an ensemble average over forcing realizations. The second moment $C(\mathbf{x}_1, \mathbf{x}_2, t) \equiv\langle\zeta'_1\zeta'_2\rangle$, is the two point correlation function of the vorticity deviation from the mean $\zeta'_i\equiv \zeta_i - Z_i$, where the subscript i = 1, 2 refers to the value of the relative vorticity at $\mathbf{x}_i = (x_i, y_i)$. We adopt the general interpretation that the ensemble average is a Reynolds average over the fast turbulent motions (Bernstein and Farrell 2010). With this definition of the ensemble mean, we seek to obtain the statistical dynamics of the interaction of the coarse-grained ensemble average field, which can be zonal or non-zonal coherent structures represented by their vorticity Z, with the fine-grained incoherent field represented by the vorticity covariance C. The equations governing the evolution of the first two moments are obtained as follows. Under the decomposition of vorticity into an ensemble mean and a deviation from the mean, (1) is split into two equations governing the evolution of the vorticity of the coherent structures Z and of the eddy (deviation from the mean) vorticity ζ'. The mean vorticity Z evolves according to:

$$(\partial_t + \mathbf{U}\cdot\nabla)Z + \beta V + rZ + \nu\Delta^2 Z = -\nabla\cdot\langle\mathbf{u}'\zeta'\rangle = G(C) \quad (2)$$

where $\mathbf{U}$, $\mathbf{u'}$ are the ensemble mean and the eddy velocity fields respectively. The mean vorticity Z is therefore forced by the divergence of the ensemble mean vorticity fluxes that can be expressed as a function of the vorticity covariance $\nabla\cdot\langle\mathbf{u}'\zeta'\rangle = G(C)$. The covariance evolves as:

$$\partial_t C + (A_1 + A_2)C = \Xi \quad (3)$$

where

$$A = -\mathbf{U}\cdot\nabla - \left(\beta + \partial_y Z\right)\partial_x\Delta^{-1} + \partial_x Z\partial_y\Delta^{-1} - r + \nu\Delta^2$$

governs the dynamics of linear perturbations about the instantaneous mean flow $\mathbf{U}$ and Ξ is the spatial correlation function of the external forcing. In obtaining (3), we have ignored the f_{nl} term describing the eddy-eddy interactions, so that (2)-(3) form a closed deterministic system that governs the joint evolution of the coherent flow field and of the eddy statistics. The S3T system has bounded solutions and the fixed points Z^E and C^E, if they exist, define statistical equilibria of the coherent structures with vorticity Z^E, in the presence of an eddy field with covariance C^E.

3 Results

The S3T system (2), (3) has for ν=0 the equilibrium ZE=0, CE=Ξ/2r, that has zero large scale flow and a homogeneous eddy field with the spatial covariance of the forcing. We now investigate the stability of this equilibrium as a function of the energy input rate ε and the characteristics of the equilibrated unstable structures and relate the outcome of the analysis to the results in the nonlinear simulations of (1). The stability of the homogeneous equilibrium is assessed by introducing perturbations δZ=einx+imy+σt, δC, linearizing (2), (3) about the equilibrium and calculating the eigenvalues σ. The resulting stability equation for σ(n, m) can be solved explicitly (Bakas and Ioannou 2013). For small values of the energy input rate of the forcing ε, the homogeneous state is stable. When ε exceeds a critical εc, the homogeneous flow becomes S3T unstable and coherent structures emerge.

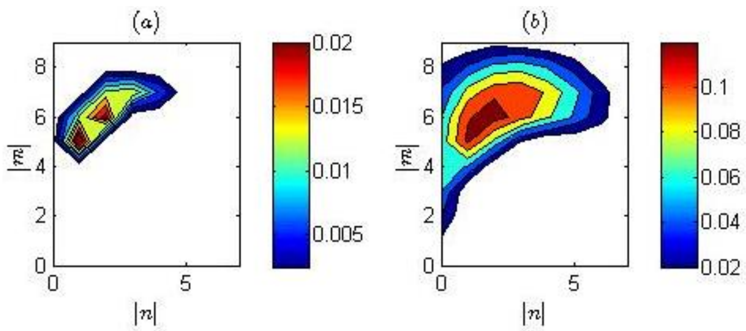

Fig. 1. Growth rate Re(σ) as a function of the integer valued wavenumbers ($|n|$,$|m|$) of the emerging structure for (a) $\varepsilon=4\varepsilon_c$ and (b) $\varepsilon=30\varepsilon_c$.

The growth rates as a function of the integer valued wave numbers (n, m) of the structure are shown in Fig. 1. For $\varepsilon=4\varepsilon_c$, the structure with the largest growth rate is non-zonal with $(|n|,|m|)=(1,5)$ and has $\mathrm{Im}(\sigma)>0$, implying westward propagation of the eigenstructure. Note also that for this energy input rate, zonal flows ($n=0$) are stable. For $\varepsilon=30\varepsilon_c$, both stationary zonal jets (having $\mathrm{Im}(\sigma)=0$) and westward propagating non-zonal structures are unstable, but the zonal jets have smaller growth rates compared to the non-zonal structures. Numerical integration of the S3T system (2)-(3) shows that for $\varepsilon > \varepsilon_c$ the unstable structures equilibrate at finite amplitude after an initial period of exponential growth. Fig. 2 shows the equilibrated structure arising from random initial conditions when $\varepsilon=4\varepsilon_c$. This structure coincides with the finite amplitude equilibrium of the fastest growing eigenfunction and propagates as shown in Fig. 2 westwards with a speed approximately equal to the phase speed of this eigenfunction. For $\varepsilon=30\varepsilon_c$, a mixed structure that consists of a zonal jet with $(|n|, |m|)=(0,5)$ and lower amplitude $(|n|, |m|)=(1,5)$ westward propagating waves embedded in it, is the finite amplitude equilibrium of the S3T system (not shown).

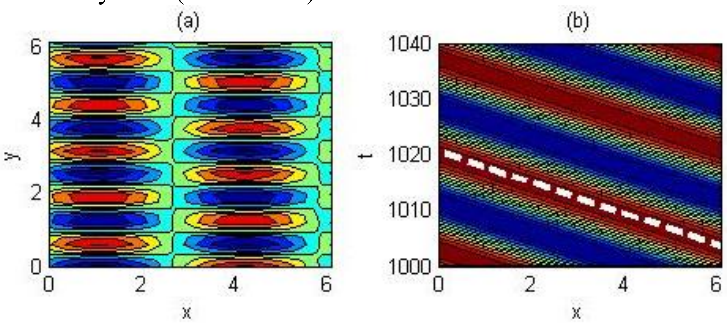

Fig. 2. (a) Streamfunction of the equilibrated structure for $\varepsilon=4\varepsilon_c$. (b) Hovmoller diagram of $\psi(x,y=\pi/4,t)$. The phase speed of the most unstable eigenfunction is also shown (dashed line).

A proxy for the amplitude of these equilibrated structures are the zmf and nzmf indices defined as the ratio of the energy of zonal jets and non-zonal structures respectively with scales lower than the scale of the forcing over the total energy:

$$zmf = \frac{\sum_{l<K_f} \hat{E}(k=0,l)}{\sum_{k,l} \hat{E}(k,l)}, \quad nzmf = \frac{\sum_{k,l<K_f} \hat{E}(k,l)}{\sum_{k,l} \hat{E}(k,l)} - zmf, \quad (4)$$

where $\hat{E}(k,l)$ is the time averaged energy power spectrum of the flow and k, l are the zonal and meridional wave numbers, respectively. These indices that are calculated for the S3T integrations, are shown in Fig. 3.

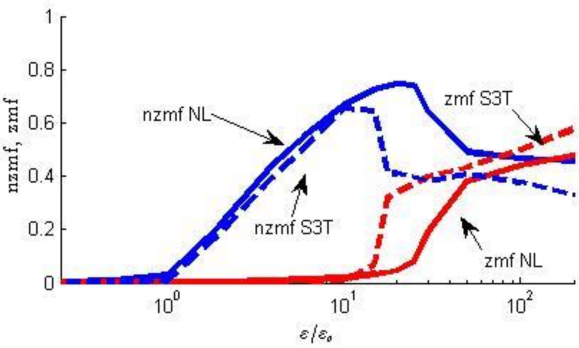

Fig. 3. zmf and nzmf indices defined in (4) as a function of the energy input rate, for the NL and S3T integrations.

As the energy input rate increases, the non-zonal structures equilibrate at larger amplitudes and nzmf increases. However, for $\varepsilon/\varepsilon_c>15$ the finite amplitude non-zonal equilibria are S3T unstable to zonal jet perturbations. As a result, the structures with the largest domain of attraction are zonal jets and the flow is dominated by these structures resulting in an increase of zmf and a concomitant decrease of nzmf.

The results of the S3T analysis are now compared to nonlinear simulations for which the zmf and nzmf indices are calculated as well. The stability analysis accurately predicts the critical ε_c for emergence of non-zonal structures in the nonlinear simulations as shown in Fig. 3. The finite amplitude equilibria obtained when $\varepsilon>\varepsilon_c$ also correspond to the dominant structures in the nonlinear simulations. For $\varepsilon=4\varepsilon_c$, the time averaged energy spectra shown in Fig. 4 exhibit significant power at $(|k|,|l|)=(1,5)$, corresponding to the equilibrated S3T structure shown in Fig. 2. Remarkably, the phase speed of these waves observed in the nonlinear simulations and the amplitude of these structures as illustrated by the nzmf index are approximately equal to the phase speed and amplitude of the corresponding S3T translating equilibrium structure (cf. Figs. 2, 4). In addition, the spectra for $\varepsilon=30\varepsilon_c$ exhibit a peak at $(|k|,|l|)=(0,5)$, as in the S3T integrations (not shown).

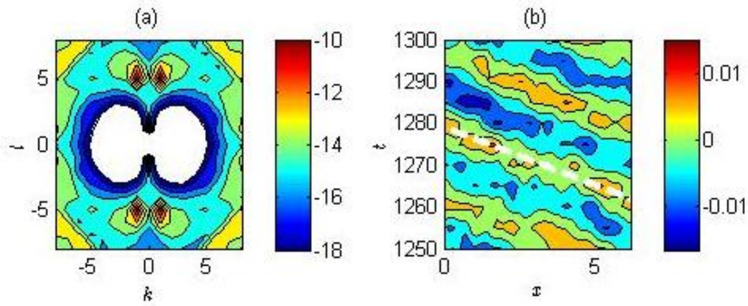

Fig. 4. (a) Time averaged energy power spectra obtained from the NL simulations for $\varepsilon=4\varepsilon_c$. (b) Hovmoller diagram of $\psi(x,y=\pi/4,t)$. The phase speed of the most unstable eigenfunction is also shown (dashed line).

4 Conclusions

In summary, we presented a theory (S3T) that shows that large scale structure in barotropic turbulence arises through systematic self-organization of the turbulent Reynolds stresses, a process that is captured by a second order closure of the flow statistics. The theory allowed the determination of conditions for the emergence of westward propagating, coherent non-zonal structures in homogeneously forced flows and we have demonstrated, through comparison with nonlinear simulations, that it predicts both the emergence and the finite amplitude equilibration of these structures. The relation of these states to westward propagating vortex rings in the ocean and the atmosphere will be the subject of future research.

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References

Bakas NA, and Ioannou PJ (2013) Emergence of large scale structure in barotropic beta-plane turbulence. Phys. Rev. Lett. 110, 224501

Bernstein J and Farrell BF (2009) Low frequency variability in a turbulent baroclinic jet: eddy-mean flow interactions in a two-level model. J. Atmos. Sci. 67:452–467

Constantinou NC, Farrell BF and Ioannou PJ (2014) Emergence and equilibration of jets in barotropic beta-plane turbulence: applications of Stochastic Structural Stability Theory. J. Atmos. Sci. 71, 1818-1842

Farrell BF and Ioannou PJ (2003) Structural stability of turbulent jets. J. Atmos. Sci. 60, 2101-2118

Farrell BF and Ioannou PJ (2008) Formation of jets in baroclinic turbulence. J. Atmos. Sci. 65, 3352-3355

Galperin BH, Sukoriansky S and Dikovskaya N (2010) Geophysical flows with anisotropic turbulence and dispersive waves: flows with a β-effect. Ocean Dyn 60, 427–441