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Properties of a troposphere with zero EPV gradients on isentropes.

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It has recently been noted that baroclinic instability can act in a direction to neutralize the atmosphere (with respect to baroclinic instability) without eliminating surface temperature gradients (the condition for neutrality using the Charney-Stern Theorem) by wiping out gradients of potential vorticity up to some finite level (Lindzen, 1993). Such a process leads to a concentration of potential vorticity gradient at this level, which, it was suggested, could be identified with the tropopause. Sun and Lindzen (1994) have examined the data, and have found that the troposphere is indeed associated with reduced PV gradients; however, the data does not permit us to distinguish between $q_y=0$ and $q_y=\beta$. This is illustrated in Figure 1 which shows the distribution of q, the potential vorticity, on four isentropes. The 300°K isentrope originates in the tropics and intersects the tropopause; the 290°K isentrope originates at the ground and intersects the tropopause; the 275°K isentrope originates at the ground and reaches the pole before intersecting the tropopause. There are evident concentrations of q_y at the tropopause. The difficulty of determining q_y in the troposphere is illustrated in Figure 2. Here we show zonally averaged U and T v. height for two idealized cases: in one, U and T are associated with constant U_z and N^2 up to 10 km; in the other, U and T have been adjusted to have $q_y=0$ below 10 km. Above 10 km, the profiles have identical gradients and have been smoothly matched to the profiles below 10 km. Clearly, U and T are almost within observational uncertainty of each other. Perhaps more interestingly, associated with the constant U_z, N^2 profile is $q_y=2.6\ \beta$. This is illustrated in Figure 3 where we also show q_y for states intermediate between those shown in Figure 2. Quite clearly from the study of Sun and Lindzen, the observed q_y is much smaller than 2.6β

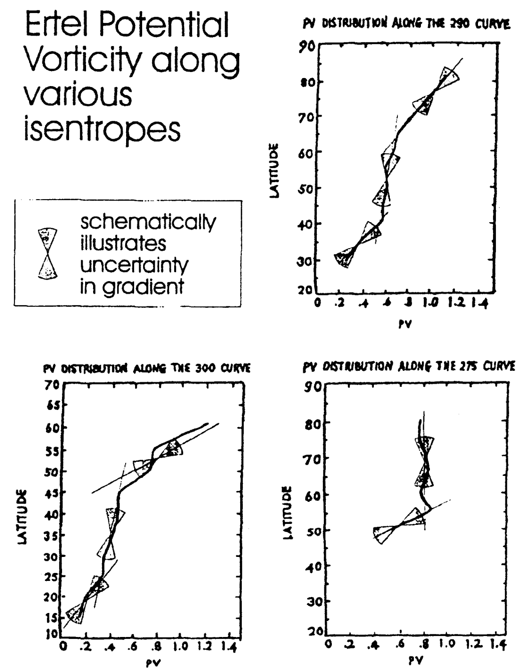

Figure 1

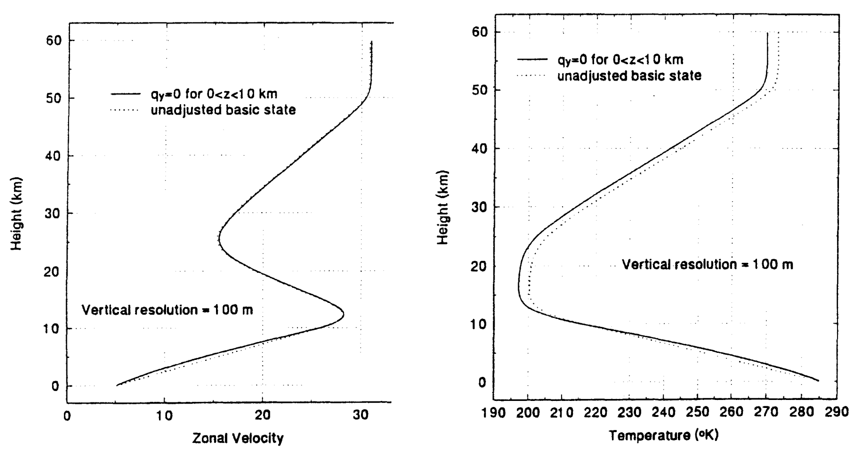

Figure 2. Vertical profiles of basic zonal velocity, U (left panel), and temperature, T (right panel), for constant U_z, N^2 state and for state adjusted to $q_y=0$ for z<10 km.

though some of the intermediate states can't be ruled out. Thus, we may reasonably conclude that, not surprisingly, baroclinic wave activity acts to diminish q_y along isentropes within the troposphere whether or not actual neutrality is formally achieved. This has been suggested earlier in the literature (Pfeffer, 1981, Hoskins, et al, 1985). It has also been noted in the ocean (Marshall and Nurser, 1991).

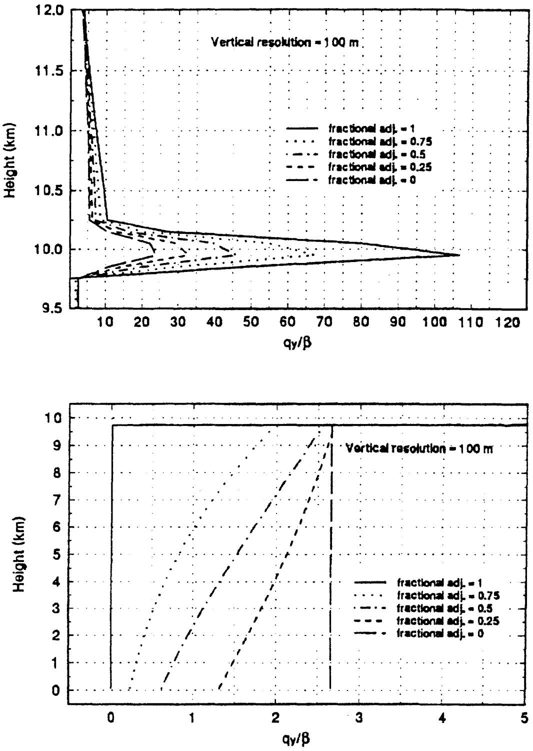

Figure 3. q_y/β for fractional adjustments to q_y=0 for z<10 km. Above 12 km, q_y is always O(1).

The main purpose of the present paper is to examine the implications of the above view for our understanding of atmospheric waves. Despite the fact that q_y provides the basic restoring force for synoptic and planetary scale waves, little attention has been given to the details of q_y when studying either baroclinic instability or stationary waves. In the classic papers in the literature, one used either constant U_z and N^2 on a β-plane (Charney, 1947, Charney and Drazin, 1961) or considered a bousinnesq fluid on an f-plane where q_y was indeed zero in the interior despite the fact that U_z and N^2 were constant (Eady, 1949). For the former case, we have seen that q_y is greatly exaggerated in the troposphere. In the latter case, it is not evident that q_y=0 in the presence of β is equivalent to setting β=0. In this paper we will reexamine the Eady and Charney-Drazin problems, paying special attention to the distribution of PV gradient. In both these problems, the mathematics and equations are well known, and won't be repeated here.

We will first consider the Eady problem on a β-plane where U and T have been adjusted to provide q_y=0 despite the presence of β. Figure 4 shows such distributions of U and T compared with constant U_z, N^2 profiles. We will compare stability properties for a conventional (β=0) Eady problem using the constant U_z, N^2 profiles with those calculated for a modified Eady problem where $\beta \neq 0$, but where U and T have been adjusted to produce q_y=0. The results for the classic Eady problem are easily summarized (and can be found in many texts). There is a short wave cut-off for instability associated with the fact that waves at the upper and lower boundary can no longer effectively interact. There is no long wave cut-off, and, equivalently, there are no neutral long waves. The steering level for all unstable waves is the middle level, and the unstable waves are symmetric about this level. The modified problem is mathematically almost as simple as the classic Eady problem (though hyperbolic trig functions are replaced with Bessel Functions); however, as we see in Figure 5, the stability properties

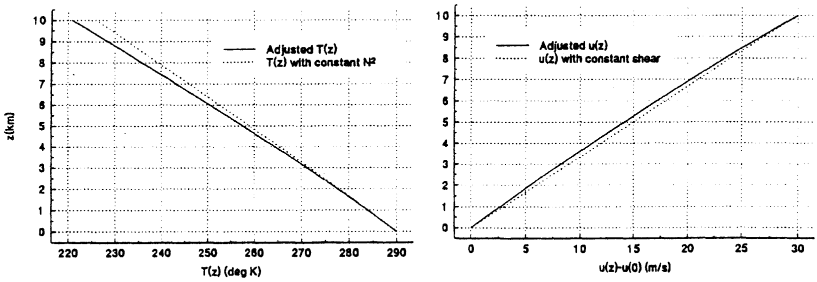

Figure 4. Basic T (left panel) and U (right panel) for classic Eady Problem and for $\beta \neq 0$ with basic state adjusted to q_y=0.

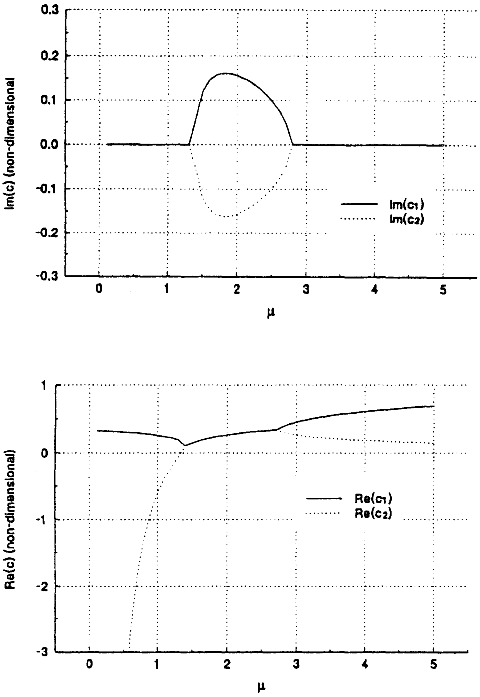

Figure 5. Imaginary and real parts of complex phase speed as functions of total horizontal wavenumber, μ, for modified Eady Problem.

are markedly modified. There remains the short wave cut-off essential to the argument of Lindzen (1993); however, there is now a long wave cut-off. In addition, the steering level for unstable waves is now depressed below the middle level, consistent with observed synoptic waves. Although we will not present the details here, it is possible to show that when the Eady problem, as modified here, with the additional consideration of exponential decrease of basic density, now allows some absolute instability (Farrell, 1982). What this means is that there are now unstable modes whose group velocity is somewhat easterly relative to the basic surface flow. This, in turn, implies that in the presence of surface westerlies there will be instabilities that are either stationary or slowly moving with respect to the surface itself. Such disturbances can effectively tap localized sources of instability and are important for the existence of storm paths. Previous studies have failed to demonstrate absolute instability for traditional Eady, Charney and 2-Level Problems. Only the barotropic instability associated with cyclogenesis over the Bay of Bengal seemed to be clearly absolutely unstable (Lindzen, et al, 1983).

Below the long wave cut-off, we have a pair of neutral waves, one of which is associated with phase speeds that approach $-\infty$ as $k \to 0$. This implies the presence of stationary neutral (unforced) waves which can resonate with stationary forcing at realistic flow speeds. Such resonances are not properties of the classic Eady configuration, and would be crucial to the response to stationary forcing which also plays a dominant role in the determination of storm paths.

Of course, the presence of a rigid upper boundary in the Eady Problem calls into question any prediction of stationary wave resonance. We have, therefore, examined the traditional Charney-Drazin Problem with surface orographic forcing using the two basic states illustrated in Figure 2, as well as intermediate states whose q_y is shown in Figure 3. The vertical structure equation in Charney and Drazin (1961) is solved numerically with 100 m vertical resolution and a radiation condition at z=60 km. Coarser resolutions down to 2 km were also examined. The results indicate some measure of resonance. More importantly, the responses are sensitive to the precise degree of smoothing of PV gradients in the troposphere and the associated concentration of PV gradients in the neighborhood of the tropopause. An example of this is illustrated in Figure 6 where the amplitude of the response function at z=10 km is shown as a function of zonal wavenumber, s. Concentrating PV gradients at the tropopause greatly increases the magnitude of the response. Using the basic state corresponding to $q_y=0$ in the troposphere, we also calculated the response using coarser vertical resolution. We find that even

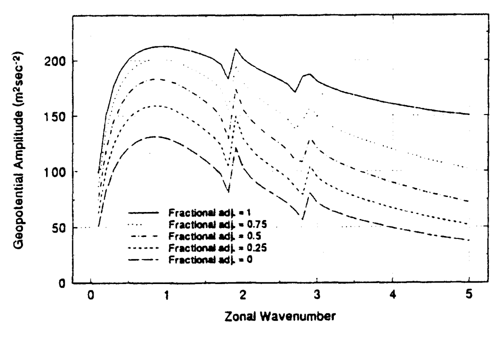

Figure 6. Response amplitude (scaled for 100 m forcing amplitude) at 10 km as a function of zonal wavenumber for varying degrees of adjustment to $q_y=0$ for z<10 km.

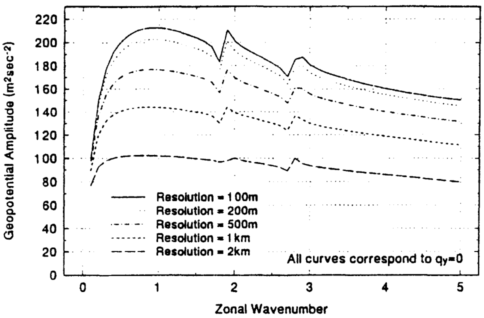

Figure 7. Response amplitude at 10 km as a function of zonal wavenumber for varying vertical resolutions and the basic state adjusted to q_y=0 for z<10 km.

resolutions as coarse as 2 km adequately describe the overall vertical structure of the solutions. However, as one sees in Figure 7, coarser resolution also leads to much smaller overall amplitudes. Coarser resolution, it should be noted, leads to numerical smoothing of the PV gradients at the tropopause, and, as it turns out, the amplitudes obtained are comparable to those found in the high resolution integrations with similarly smoothed PV gradients. These results are of potentially great importance for numerical models. They suggest that the intensity of stationary waves and associated storm paths depends critically on the effectiveness of baroclinic eddies in smoothing PV gradients in the troposphere and concentrating them at the tropopause. Similarly, the ability of a numerical scheme to resolve the concentrated PV gradients at the tropopause determines its ability to correctly produce stationary wave amplitudes. It should be noted that coarse observational sampling also potentially smooths the magnitude of q_y.

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