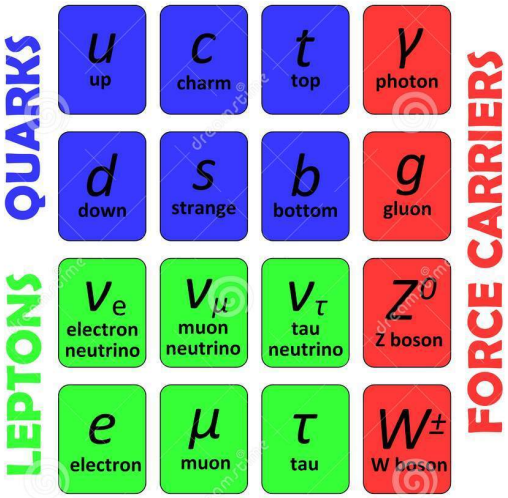
Exact results in gauge theories

Elli Pomoni

ΗΜΕΡΙΔΑ ΠΡΟΣ ΤΙΜΗΝ ΤΟΥ ΦΩΚΙΩΝΑ ΧΑΤΖΗΙΩΑΝΝΟΥ

Athens, 22 December 2017

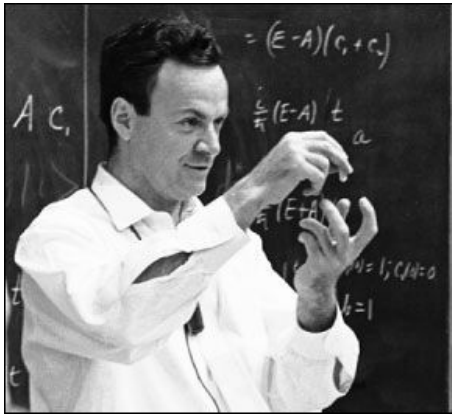
Gauge theories

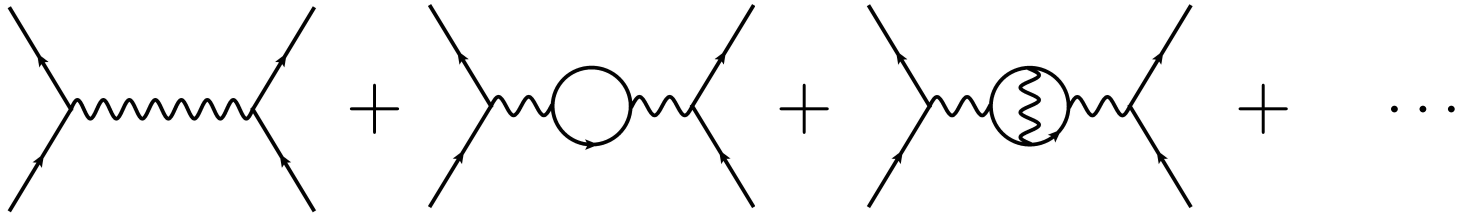


U(N) or SU(N) gauge group
gauge bosons force carriers: A_μ

Matter fields (quarks and leptons): ψ

Feynman: how to calculate a process **perturbatively**:

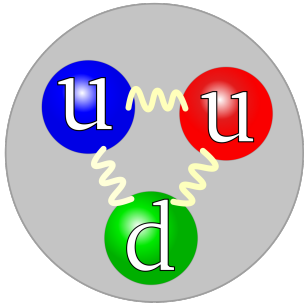




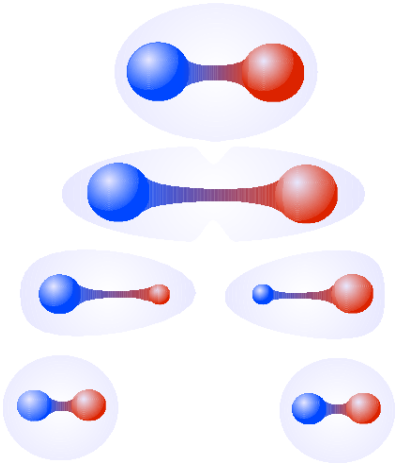
$$= \quad c_1 \lambda \quad + \quad c_2 \lambda^2 \quad + \quad c_3 \lambda^3 \quad + \quad \cdots$$

$$\lambda << 1$$

A big open problem



Confinement:
Quarks cannot be isolated in Nature, and cannot be directly observed.



Find an analytic proof that
quantum chromodynamics (QCD) should be confining.

Cannot be done pertubatively

Emmy Noether

Symmetry ➡ *Conservation law*

Use the Symmetry to solve the problem.

The more symmetry the easier it is to solve the problem.

Gauge theories are very hard to understand:
Let's add **Supersymmetry.**

boson ↔ **fermion**

Supersymmetric Gauge theories in 4D

$\mathcal{N} = 4$ Super-Yang-Mills (SYM)

$A_\mu\ ,\ \lambda^A\ ,\ \phi^{AB} \qquad A, B = 1, \ldots, 4$

No matter fields are allowed! Only "gluons"!

Conformal: $\lambda \neq \lambda(E)$!

More susy easier to calculate

$\mathcal{N} = 2$ $\left\{ \begin{array}{ll} \text{Vector multiplet} & A_\mu\ ,\ \lambda^{\mathcal{I}}\ ,\ \phi \\ \text{Matter multiplet} & Q_{\mathcal{I}}\ ,\ \psi\ ,\ \tilde{\psi} \end{array} \right. \qquad \mathcal{I} = 1, 2$

$\mathcal{N} = 1$ $\left\{ \begin{array}{ll} \text{Vector} & A_\mu\ ,\ \lambda \\ \text{Matter} & q\ ,\ \psi \end{array} \right.$

Less susy more realistic

$\mathcal{N} = 0$ $\left\{ \begin{array}{ll} \text{Vector} & A_\mu \\ \text{Matter} & \psi \end{array} \right.$

The real world

The success story

Exact results for many observables

$$c_1\,\lambda + c_2\,\lambda^2 + c_3\,\lambda^3 + \cdots = F(\lambda)$$

due to:

- *AdS/CFT correspondence*
- *Integrability*
- *Localization*
- *4D/2D relations*

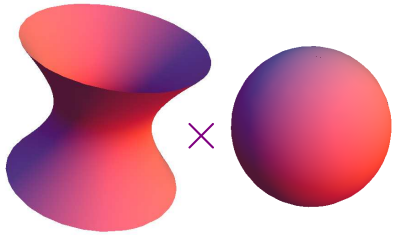
AdS/CFT correspondence

['t Hooft 1993 Susskind 1995]

A relation: Gravity theories in d+1 and gauge theories d-dimensions.

[Maldacena 1998]

The 4D $\mathcal{N} = 4$ SYM: *10D critical string theory on*



$AdS_5 \times S^5$

A *duality*:

Use weakly coupled gravity: strongly coupled gauge theory.

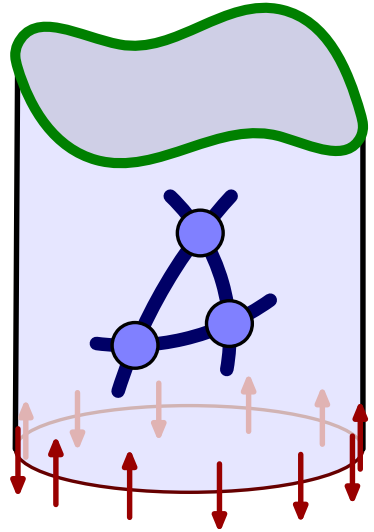
Holographic models: strong coupling regime
Geometric description of:

- Confinement
- Chiral symmetry breaking
- Ultrahot QuarkQluonPlasma/ Ultracold atoms
- Applications of Condensed matter systems

Integrability

$\mathcal{N} = 4$ SYM is integrable in the planar limit for **any coupling**

- *Perturbation theory*: mapped to an integrable spin chain
- *Strong coupling*: integrable 2D theory on the string world-sheet



Powerful integrability toolkit

- The spectral problem is solved **exactly**: for **any coupling**

Integrability now is applied to **other observables.**

Localization

[Pestun 2007]

$$Z_{S^4} = \int [D\Phi]\, e^{-S[\Phi]} = \int da\, |\mathcal{Z}(a)|^2$$

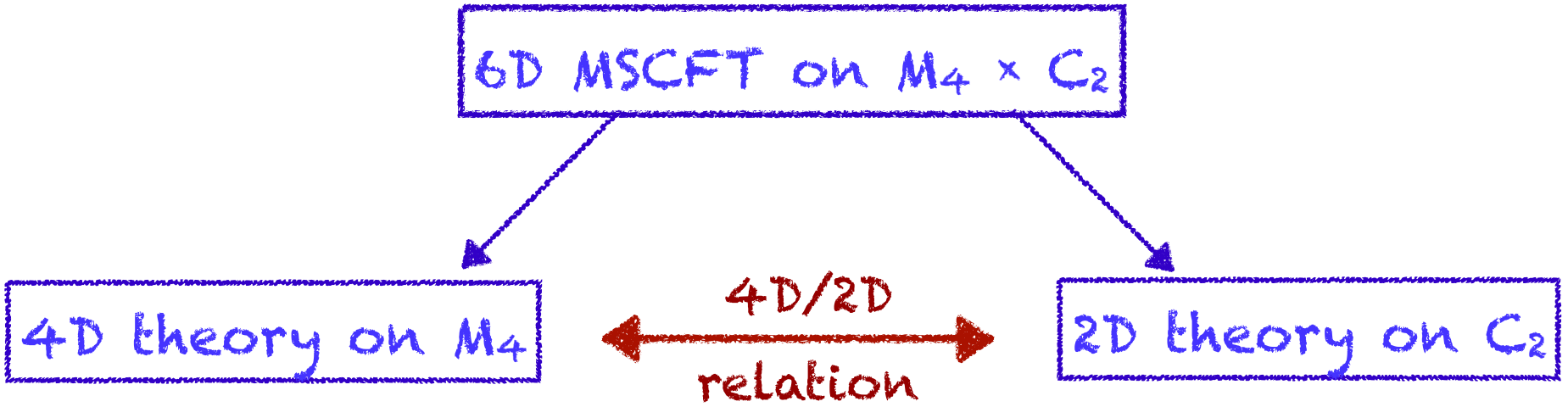
The **path integral** localizes to an **ordinary integral**
(*Cancelations due to supersymmetry*)

We can do an ordinary integral.
Compute the path integral exactly.
For **any value of the coupling constant.**

Example of exact observable:

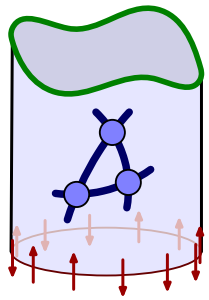
$$W(\lambda) = 2\,\frac{I_1(\sqrt{\lambda})}{\sqrt{\lambda}} = \begin{cases} 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \frac{\lambda^3}{9216} + \cdots, & \lambda << 1 \\ \sqrt{\frac{2}{\pi}}\lambda^{-\frac{3}{4}} e^{\sqrt{\lambda}} + \cdots, & \lambda >> 1 \end{cases}$$

4D/2D relations



- *4D N=4 SYM: 6D MSCFT on* T^2 *(EM duality = modular transformations)*
- *4D N=2 class: 6D MSCFT on Riemann surface* $C_{g,n}$ [Gaiotto 2009]
- *4D partition functions = 2D CFT correlators* [Alday,Gaiotto,Tachikawa 2009]
- *4D SC Index = 2D TFT correlators* [Gadde,EP,Rastelli,Razamat 2009]

AdS/CFT integrability beyond N=4 SYM



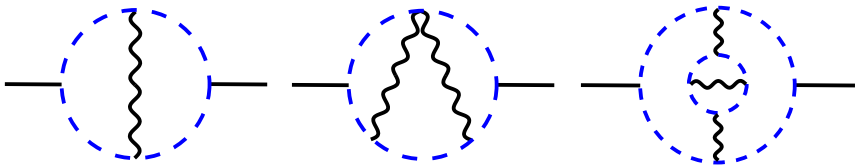
Any $\mathcal{N}=2$ superconformal gauge theory
has a closed **purely gluonic sector**
that is **integrable** in the planar limit.

[EP 2013]
[Mitev, EP 2014 + 2015]

Observable in this sector is obtained
from its $\mathcal{N}=4$ counterpart by replacing: $\lambda \longrightarrow f(\lambda)$

Relative finite renormalization: Can calculate it with Feynman diagrams

$$f(\lambda) = \lambda + \lambda\left(Z_{\mathcal{N}=2} - Z_{\mathcal{N}=4}\right)$$



Compute it exactly using localization.

And check with AdS/CFT.

Lesson: *Gauge symmetry is important: not supersymmetry!*

Similar sector for N=1 theories

[Carstensen,EP work in progress]

Beyond Localization

We can do even better than usual localization:
The partition function for a theory with no Lagrangian description.
Using powerful **string theory** tools.

[Bao, Mitev, EP, Taki,Yagi 2013]

Solving a **long standing open problem in Mathematical physics**:
3-point functions of 2D Toda CFT

[Mitev, EP 2014]

[Isachenkov, Mitev, EP 2014]

[Coman, EP, Teschner to appear]

Carving out a 2D CFT

Can we have 4D/2D relations for N=1 theories?

4D partition functions
=
2D CFT correlators

Developing a method for finding such relations.

A large class of N=1 theories [Coman,EP,Taki,Yagi 2015]

- *The 2D symmetry algebra and representations* [Mitev,EP 2017]
- *Exact results for N=1 instantons* [Bourton, EP 2017]
- *Partition functions for N=1 SCFTs* [work in progress]

Summary (past and future)

- *AdS/CFT and integrability beyond N=4 SYM*
- *4D/2D relations*
- *Enlarging the list of Exact N=2 observables*
- *Exact results for N=1 theories!*
- *N=0 ??? (QCD conformal window)*


N=4 SYM
Thanks for your attention!
Real world QCD