PLANCK SCALE ARITHMETIC GEOMETRY

AND

FINITE QUANTUM MECHANICS

ON THE HORIZON OF BLACK HOLES

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CONFERENCE IN HONOUR OF
PROF EMERITUS FOKION HADJIOANNOU
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RECENT WORK

- Modular discretization of the AdS2/CFT1 Holography
  M.Axenides,E.Floratos,S.Nicolis
  JHEP 1402(2014)109 arXiv:1306.5670

- Chaotic Information Processing by Extremal Black Holes
  M.Axenides,E.G.Floratos,S.Nicolis
  Int. J. Mod. Phys. D24 (2015) 1542.0122
  arXiv:1504.00483

- Quantum cat map dynamics on AdS2

  Minos Axenides, Emmanuel Floratos,

  Stam Nicolis
- arXiv:1608.07845

- Arithmetic Circuits for Multilevel Qudits Based on Quantum Fourier Transform
- Archimedes Pavlidis, Emmanuel Floratos
- . arXiv:1707.08834

A SHORT SUMMARY
OF BH INFORMATION PARADOX STATE OF THE ART

- BLACK HOLE THERMODYNAMICS
  BEKENSTEIN –CHRISTODOULOU BH ENTROPY 1972-3
- HAWKING RADIATION-BH INFORMATION PARADOX 1976
- T'HOOFT CHAOTIC SCATTERING ON BH HORIZONS HOLOGRAPHY 1986
- T'HOOFT -SUSSKIND-PAGE QM UNITARITY ANYWAY 1990-94
  BH HORIZON HOLOGRAPHY -COMPLEMENTARITY PRINCIPLE
- EIGENSTATE THERMALIZATION HYPOTHESIS
  BH –PROBE CLOSED QUANTUM SYSTEM SREDNICKI 1994
- PROBLEM OF HORIZON MICROSCOPIC DOF = PLANCK SCALE PHYSICS-UKNOWN
  BUT GENERAL CONSTARAINTS FROM QUANTUM INFORMATION THEORY
- NO-QUANTUM CLONING HAYDEN-PRESKILL 2006
  HORIZON CHAOTIC EXPONENTIALY FAST DIFUSION OF INFORMATION-
- SUSSKIND FAST SCRAMBLING-> NONLOCAL QM DYNAMICS 2008
  POLCHINSKI TRIPARTITE ENTANGLEMENT –OUTSIDE-HORIZON-INSIDE BH REGIONS
  FIREWALLS –HARD HAWKING QUANTA 2012
- SHOCK WAVE CHAOTIC HORIZON GEOMETRIES SHENKER-STANFORD-MALDACENA 2014
- HOLOGRPHIC BH INTERIORS
- MALDACENA-SUSSKIND QUANTUM WORMHOLES CONNECTING BH EXTERIOR-INTERIOR -> ER=EPR ,COMPLEXITY=GEOMETRY 2015
- BOUND ON BH CHAOS MALDACENA 2015

ARITHMETIC GEOMETRY A PROPER FRAMEWORK FOR THE BH QUANTUM INFORMATION PARADOX

- ASSUMPTIONS
- FINITENES OF BH ENTROPY S ->

  FINITE DIMENSIONAL HILBERT SPACE OF BH MICROSCOPIC STATES Dim[H]=Exp[S]
- FINITE DIMENSIONAL HILBERT SPACE -> DISCRETE AND FINITE SPACE-TIME
- THE SIMPLEST MODEL FOR SINGLE PARTICLE DYNAMICS A DISCRETE AND FINITE PHASE SPACE Z[N]XZ[N]

  Z[N]={0,1,2,3,..,N-1} ALL INTEGERS MODULO N

  FOR N=PRIME INTEGER=p GALOIS FIELD F[p]

ARITHMETIC GEOMETRY FOR HORIZON BH DYNAMICS A MODEL FOR PLANCK SCALE SPACE-TIME

- CONSEQUENCES
- MODULAR ARITHMETIC -> RANDOM POINT GEOMETRIES
- MODULAR DYNAMICS A= 2X2 MODULAR MATRICES IN SL[2,Z[N]] -> CHAOS AUTOMATIC
- FINITE DIMENSIONAL HILBERT SPACES
- FINITE QUANTUM MECHANICS AND QUANTUM FIELD THEORIES
- N=p^n, QUDITS OF INFORMATION,
- FINITE DIMENSIONAL UNITARY RANDOM EVOLUTION OPERATORS U[A]
- QUANTUM COMPLEXITY -> COUNTING THE GATES OF
- QUANTUM CIRCUITS FOR CONSTRUCTIN U[A]

- MOTIVATION

1)THE EIGENSTATE THERMALIZATION HYPOTHESIS

GAUSSIAN PDF OF EIGENSTATE' S PROB VALUES

FLAT PDF OF EIGENSTATE'S PHASES

2) SATURATION OF THE SCRAMBLING TIME BOUND

3) RELATION OF QUANTUM COMPLEXITY WITH

BH ENTROPY

SIMPLEST EXAMPLE
DIFUSION OF SINGLE PARTICLE WAVE PACKETS ON ADS2
TIME-RADIAL GEOMETRY OF EXTREMAL BH'S

- OLD AND LARGE NEAR EXTREMAL BH'S

- GEOMETRY =AdS2 XΣ , Σ=COMPACT ANGULAR DIRECTIONS

- ADS2 RADIAL MOTION
- AdS2[R]=SL[2,R]/SO[1,1,R]

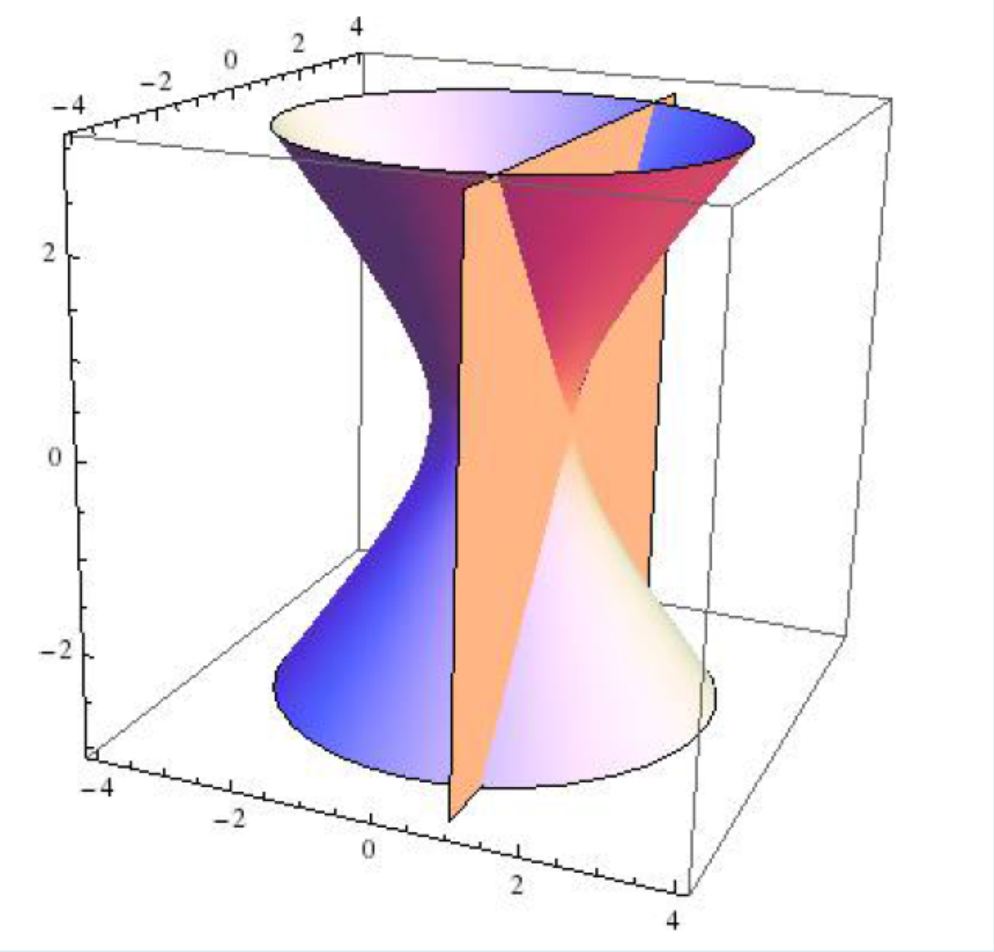
- DISCRETIZE

- ℗ AdS2[N]=SL[2,ZN]/SO[1,1,ZN]

- CONSTRUCT THE AdS2 UNITARY EVOLUTION MATRIX
  OF PROBE STRING BITS
  -> USE SUPERDCONFORMAL QM
  OF SL[2,ZN] ISOMETRY QUANTUM MAPS
  (TOWNSEND-STROMINGER-KALOSH 1998)

AdS2
NEAR HORIZON DEOMETRY OF
EXTREMAL BH s

$$x_0^2 + x_1^2 - x_2^2 = 1$$



WEYL ACTION OF SL[2,R] ON AdS2

To every point $x_\mu \in \mathrm{AdS}_2$, $\mu = 0, 1, 2$, we assign the traceless and real, 2×2 matrix

$$\mathsf{M}(x) \equiv \begin{pmatrix} x_0 & x_1 + x_2 \\ x_1 - x_2 & -x_0 \end{pmatrix} \tag{2.3}$$

Its determinant is $\det \mathsf{M}(x) = -x_0^2 - x_1^2 + x_2^2 = -1$.

The action of any $\mathsf{A} \in SL(2, \mathbb{R})$ on AdS_2 is defined through the non-linear mapping

$$\mathsf{M}(x') = \mathsf{A}\mathsf{M}(x)\mathsf{A}^{-1} \tag{2.4}$$

This induces an $SO(1,2)$ transformation on $(x_\mu)_{\mu=0,1,2}$,

$$x' \equiv \mathsf{L}(\mathsf{A})x \tag{2.5}$$

Choosing as the origin of coordinates the base point $\boldsymbol{p} \equiv (1, 0, 0)$, its stability group $SO(1,1)$ is the group of Lorentz transformations in the $x_0 = 0$ plane of $\mathscr{M}^{1,2}$ or equivalently, the "scaling" subgroup D of $SL(2, \mathbb{R})$

$$\mathsf{D} \ni \mathsf{S}(\lambda) \equiv \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix} \tag{2.6}$$

for $\lambda \in \mathbb{R}^\bullet$.

For this choice of the stability point, we define the coset h_A by decomposing A as

$$\mathsf{A} = h_\mathsf{A}\mathsf{S}(\lambda_\mathsf{A}) \tag{2.7}$$

Thus, we associate uniquely to every point $x \in \mathrm{AdS}_2$ the corresponding coset representative $h_\mathsf{A}(x)$.

ARITHMETIC DISCRETIZATION OF AdS2=SL[2,R]/SO[1,1] =>AdS2[N]=SL[2,Z[N]]/SO[1,1,[Z[N]]

$X0^2+X1^2-X2^2=1$ MOD[N]

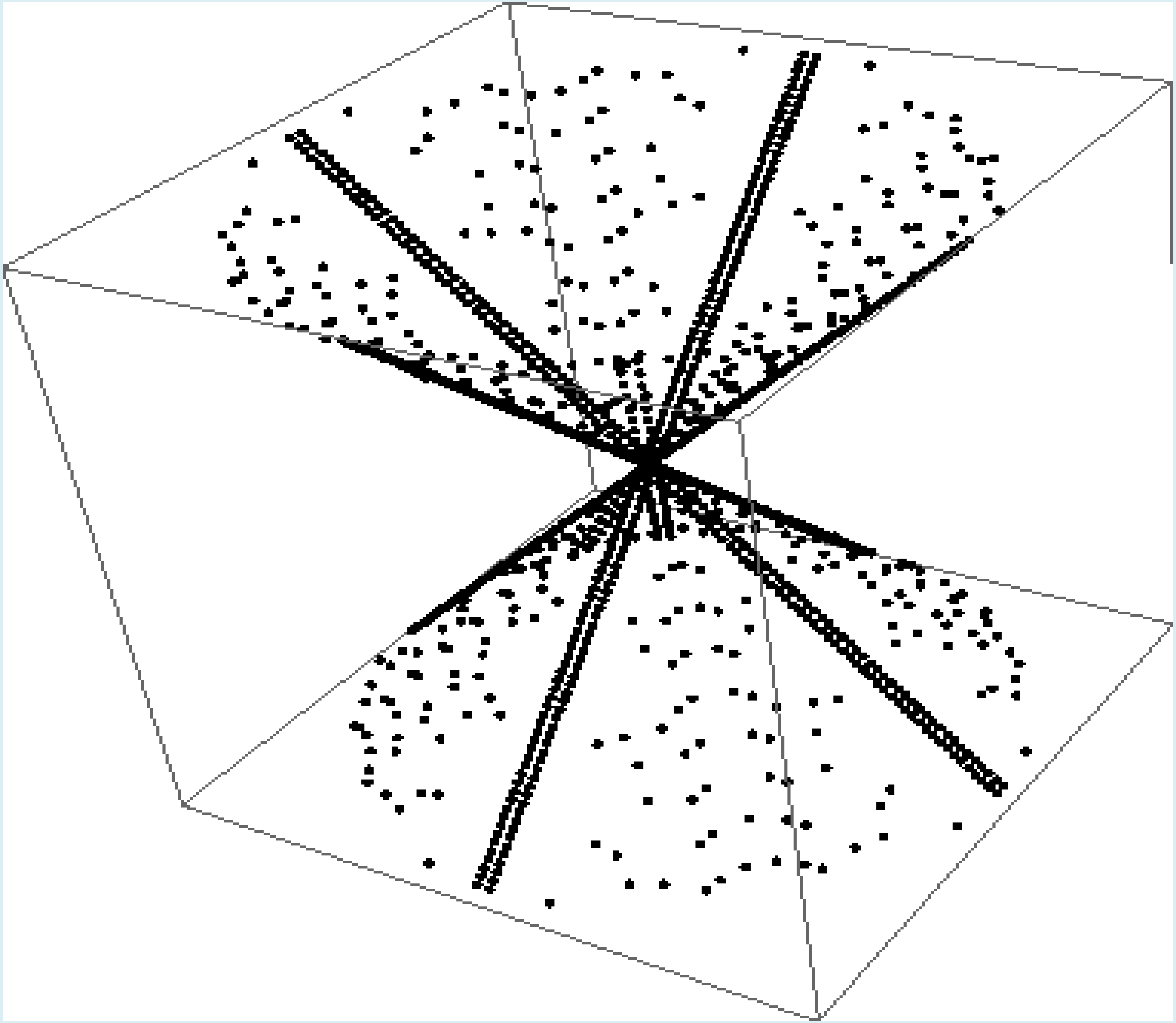
ALL INTEGER SOLUTIONS MOD[N]=> DISCRETE SET OF POINTS=ADS2[N]

X0=A- M B,
X1=B+M A
X2=M A,B,M=0,1,..,N-1

$A^2+B^2=1$ MOD[N], DISCRETE CIRCLE S1[N]

ROTATING THE ‘LINE’, { X0=0,X1=M,X2=M M=0,1,2,..,N-1} AROUND S1[N]

SO[1,1,Z[N]] STABILITY GROUP OF P: X0=1,X1=X2=0



DISCRETE SUPERCONFORMAL CONFORMAL DYNAMICS

ARNOLD CAT MAP A={{1,1},{1,2}},

WEYL ACTION ON AdS2[N] X -> A.X.A^(-1)

PROPERTIES

a)STRONG ARITHMETIC CHAOS

(ARNOLD,FORD,BERRY VOROS,VIVALDI,DI VIZENZO)

b)HOLOGRAPHY=>NON LOCAL REDUNDANT STORAGE OF INFORMATION

c)MIXING TIME -LIAPUNOV EXPONENT

d)GENERATION OF KOLMOGOROV –SINAI ENTROPY

BASIC PROPERTY OF ARNOLD CAT MAP =FIBONACCI CHAOS

A^n={{f(2n-1),f(2n)},{f(2n),f(2n+1)}},

f(n) FIBONACCI INTEGERS

f[n+1]=f[n]+f[n-1];f[0]=0,f[1]=1

FOR ANY INTEGER N

Periods of A MOD[N] A^(T(N))=IdentityMatrix MOD[N]

DYSON:IF N=f[m] → T(N)=2 m

BUT SINCE FOR m->INFINITY f[m]->Exp[c m],c=log[Goldenratio]

WE OBTAIN T[N]->Log[N]

LOGARITHMIC TIME CHAOTIC MIXING(SCRAMBLING)
For FIBONACCI SEQUENCE OF INTEGER HILBERT SPACE DIMENSIONS

FINITE AND DISCRETE QUANTUM MECHANICS

HEISENBERG-WEYL MAGNETIC TRANSLATION GROUP

HEISENBERG CCR' S $QP=\omega PQ$ $\omega=Exp[2 \pi i/N]$

$Q(kl)=\omega^{(k.l)}$ POSITION OPERATOR ON THE UNIT CIRCLE

P(kl)=Delta[k-1,l],, MOMENTUM OPERATOR ON THE DUAL UNIT CIRCLE

$P=F^{(-1)} .Q. F$

$F(k,l)=1/Sqrt[N]\ \omega^{(k l)}$, FINITE FOURIER TRANSFORM $k,l=0,1,2,\ldots N-1$

MAGNETIC TRANSLATIONS

- $J[r,s]=\omega^{(rs/2)}P^r Q^s$, $J[r,s].J[r',s']=\omega^{(r's-rs')} J[r',s']J[r,s]$ SU[N] TORUS BASIS $r,s=0,1,2,..N-1$

QUANTUM HALL EFFECT ON A TOROIDAL LATTICE, MAGNETIC FLUX $\Phi= 2 \pi/N$

PLANCK'S CONSTANT $h=\Phi$

QUANTIZATION OF CAT MAPS A=>U[A]

- CONSTRUCTION OF SPECIAL UNITARY N DIM IRREP OF SL[2,Z[N]]
  CALLED METAPLECTIC (A.WEIL 1962)

- U[A]J[r,s]U[A]^(-1)=J[(r,s)A]

- U[AB]=U[A]U[B] -> U[A^k]=U[A]^k !

- FOR N=PRIME INTEGER->EXACT- NOT PROJECTIVE!

  ONE-TIME STEP QUANTUM MECHANICAL EVOLUTION OF STATES

- |n+1>=U[A]|n> , n=0,1,2,..,T[A] , T[A]= period of A, A^T[A] = I

  |0> =STATE at ITERATION TIME n=0

ANY INTEGER (COARSE GRAINING)FACTORIZATION,

- N=N1xN2 ,N=Exp[R^2], R^2=>R1^2 + R2^2
- SL[2,Z[N]]=SL[2,Z[N1]]xSL[2,Z[N2]]
- A[N]=A[N1]A[N2]
- H[N]=H[N1]+H[N2], SCHWINGER HEIS-WEYL FACTORIZATION
- U[A[N]]=U[A[N1]]U[A[N2]]

FAST QUANTUM MAPS N^2->N LogN

- S[N]=S[N1]+S[N2] ADDITIVITY OF COARSE GRAINED ENTROPIES

EIGENSTATE THERMALIZATION SENARIO
PAGE,DEUTSCH,BERRY,SREDNICKI

IF THE EIGENSTATES OF A CLOSED QM SYSTEM ARE RANDOM

(RANDOM PHASES AND GAUSSIAN DISTRIBUTED AMPLITUDES)

THEN ANY INITIAL PURE STATE OF A SUBSYSTEM THERMALIZES TO THE THERMAL DENSITY MATRIX OF THE SUBSYSTEM

RELATION TO THE INFORMATION PARADOX

2013 SREDNICKI TALK TO KITP

ARNOLD QUANTUM CAT MAP A={{1,1},{2,1}}

EXACT CONSTRUCTION OF THE SPECTRUM AND EIGENSTATES FOR N=p,prime,

LINEAR SPECTRUM

RANDOM EIGENSTATES ->LINEAR COMBINATIONS OF MULTIPLICATIVE CHARACTERS OF GF[P]

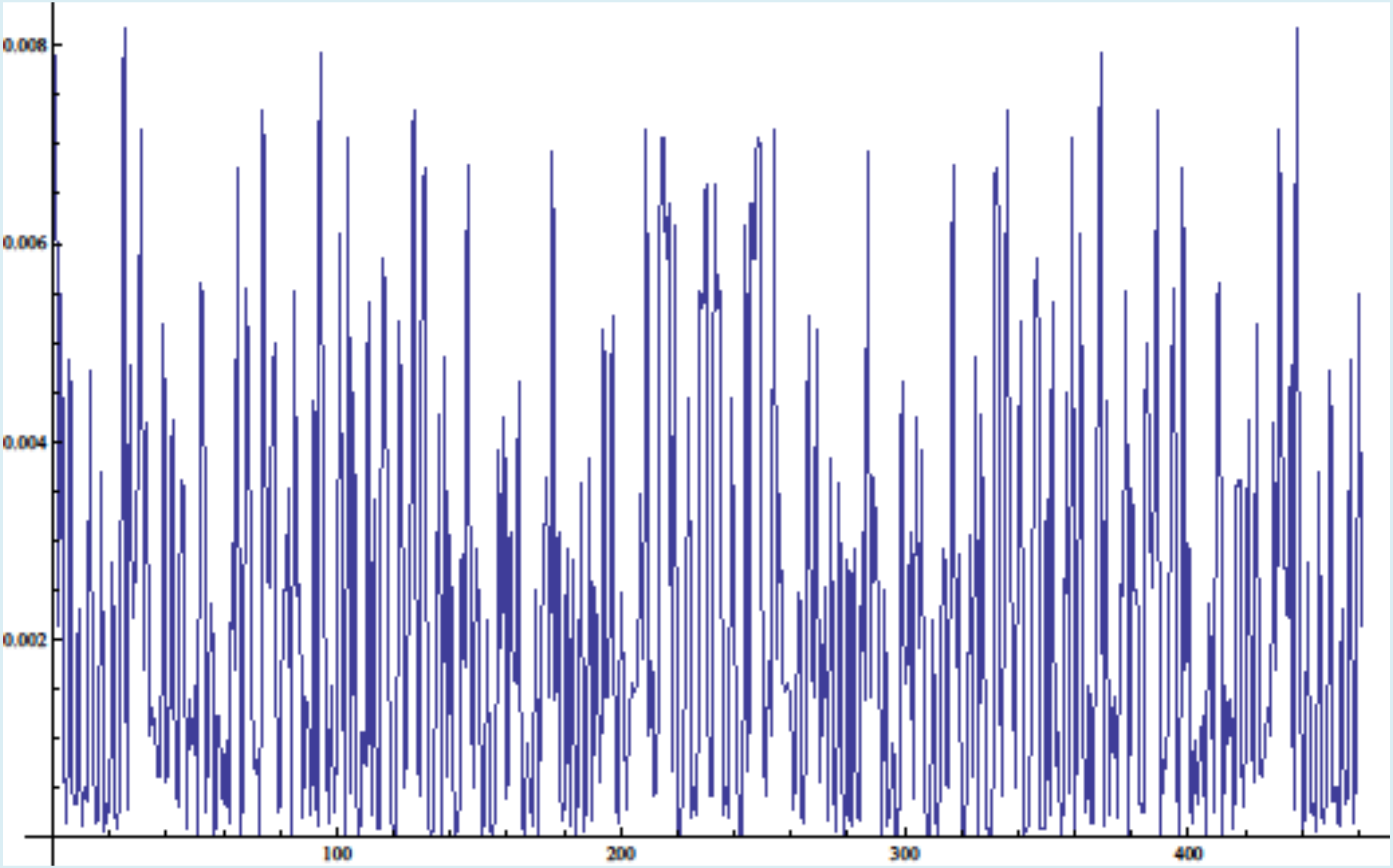
1)RANDOM PHASES
2)GAUSSIAN RANDOM AMPLITUDES

BUT SCARS(VOROS.,NONEMACHER)F
FOR SEQUENCES OF N's WITH SHORT PERIODS

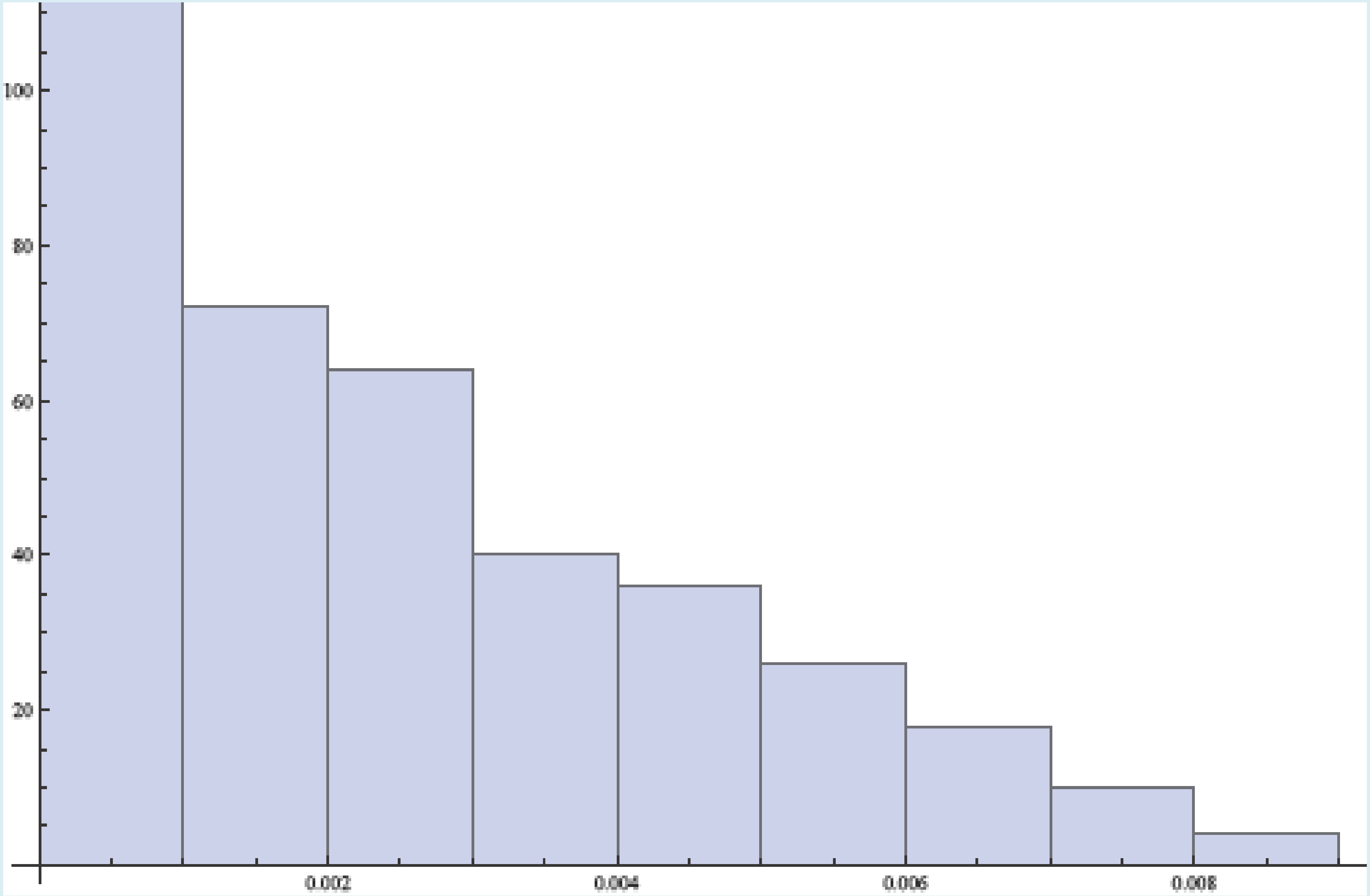
QUANTUM CHAOS, STRONG MIXING

- **FACTORIZATION FOR ARNOLD CAT MAPS IMPLIES LOGARITHMIC IMPROVEMENT FROM N^2->N LOGN**
- **USING QUANTUM CIRCUITS FOR THE IMPLEMENTATION OF THE QUANTUM MAP AND COUNTING THE NUMBER OF GATES NLOGN->(LOGN)^2**
- **EXACTLY AS FOR THE QUANTUM FOURIER FACTORIZATION ALGORITHM OF SHOR.**

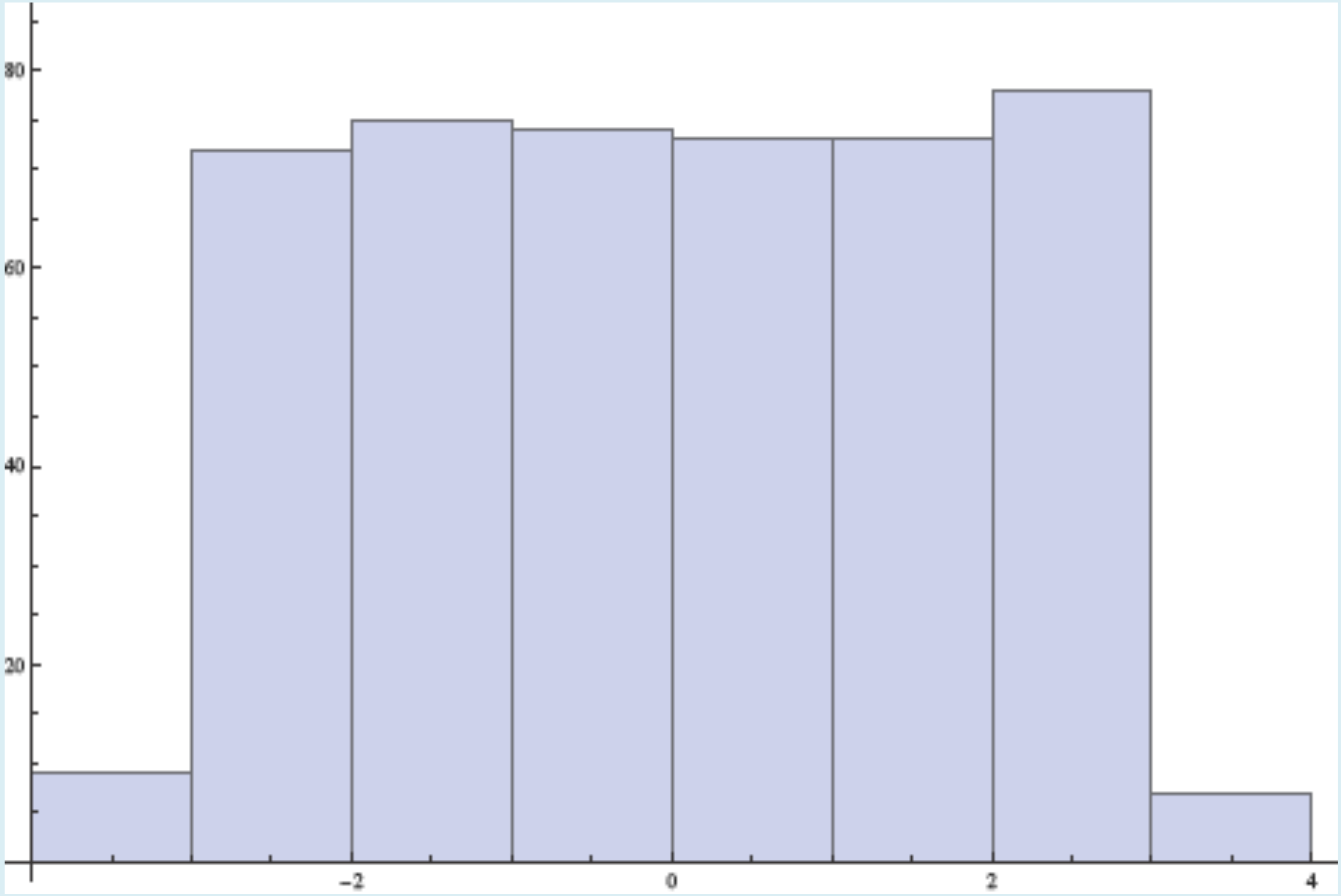
- N=461,T[461]=23 ,GROUND STATE,PHI=0



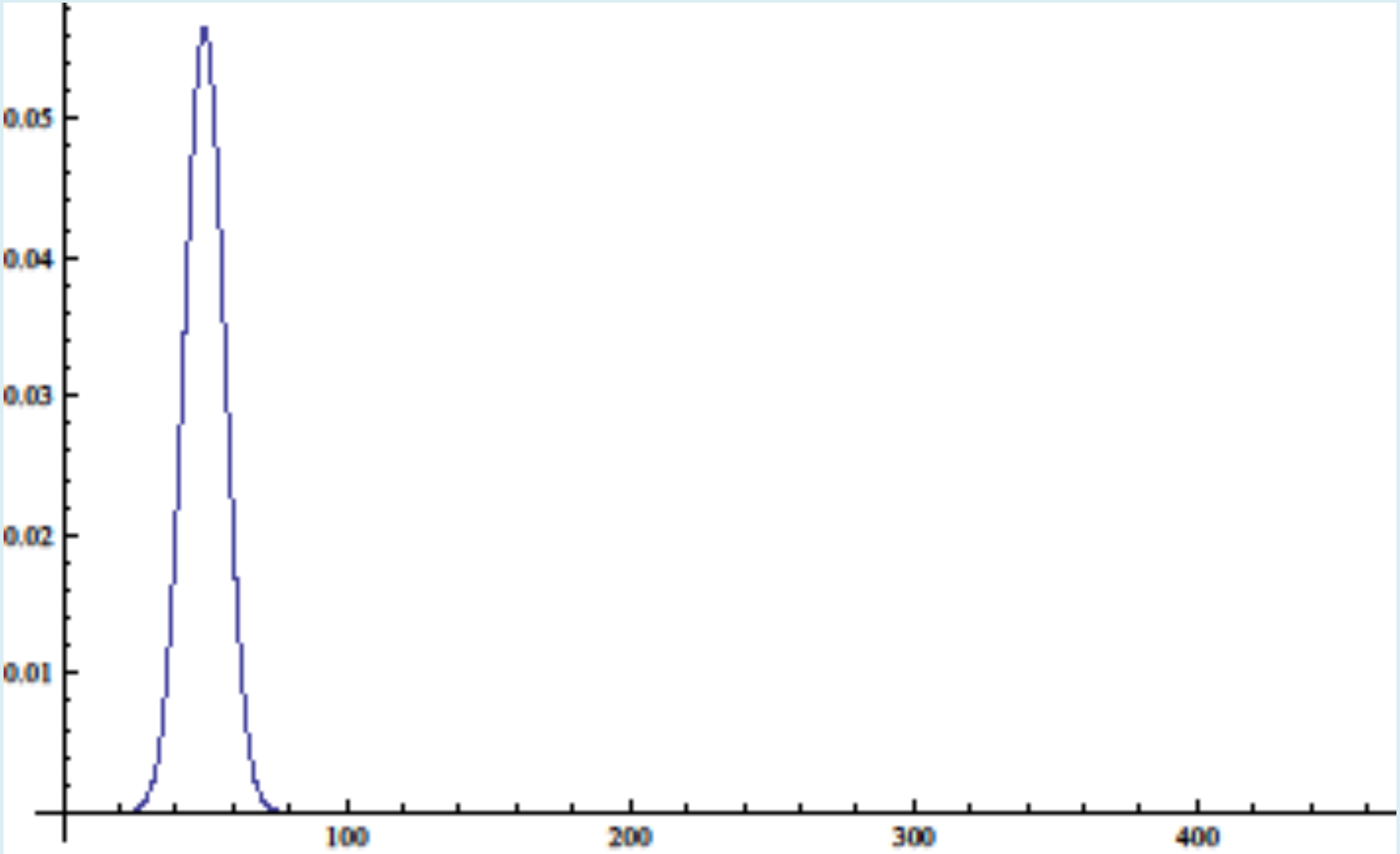
- GROUND STATE AMPLSQUARE DF



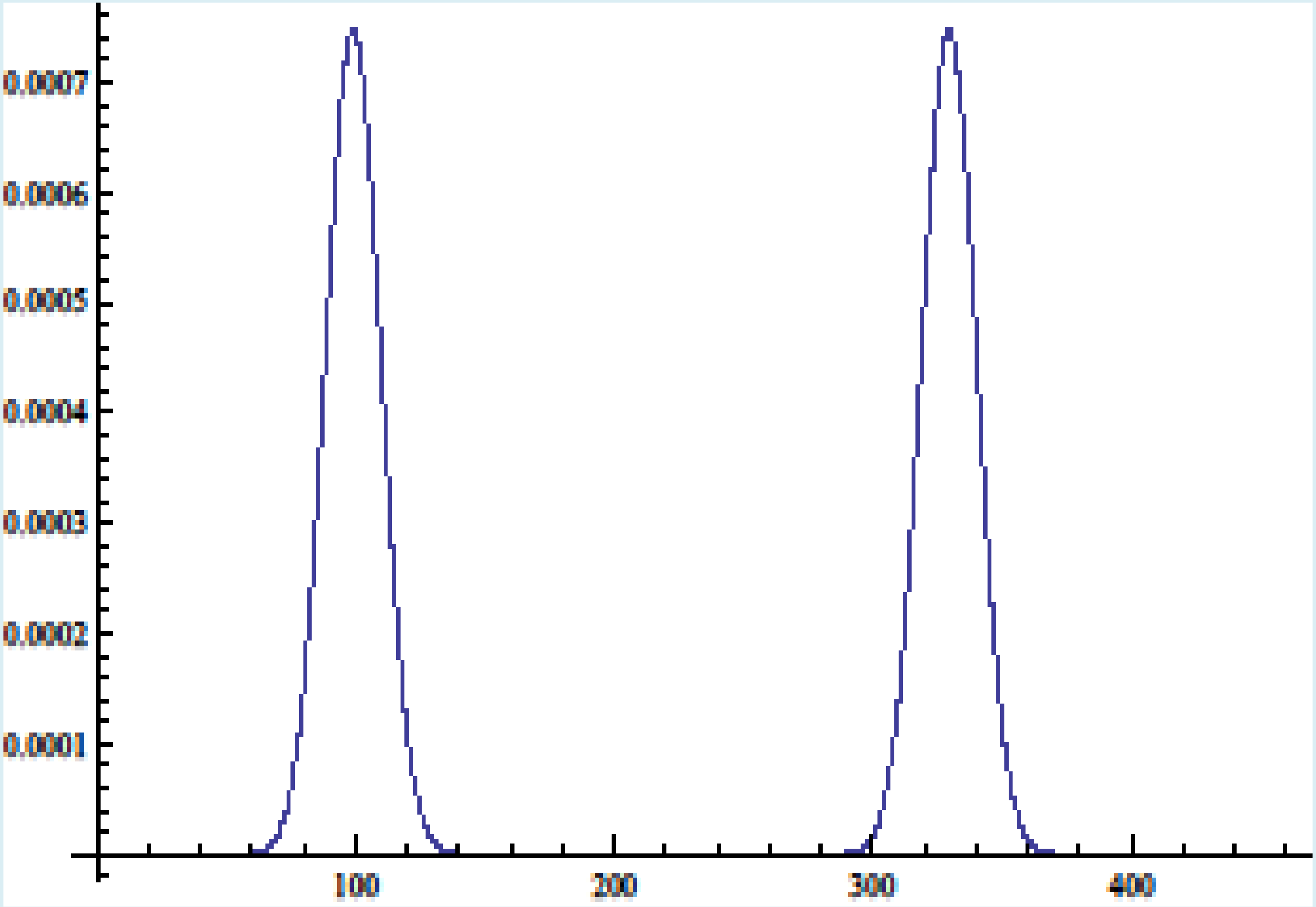
- GROUND STATE PHASE DF



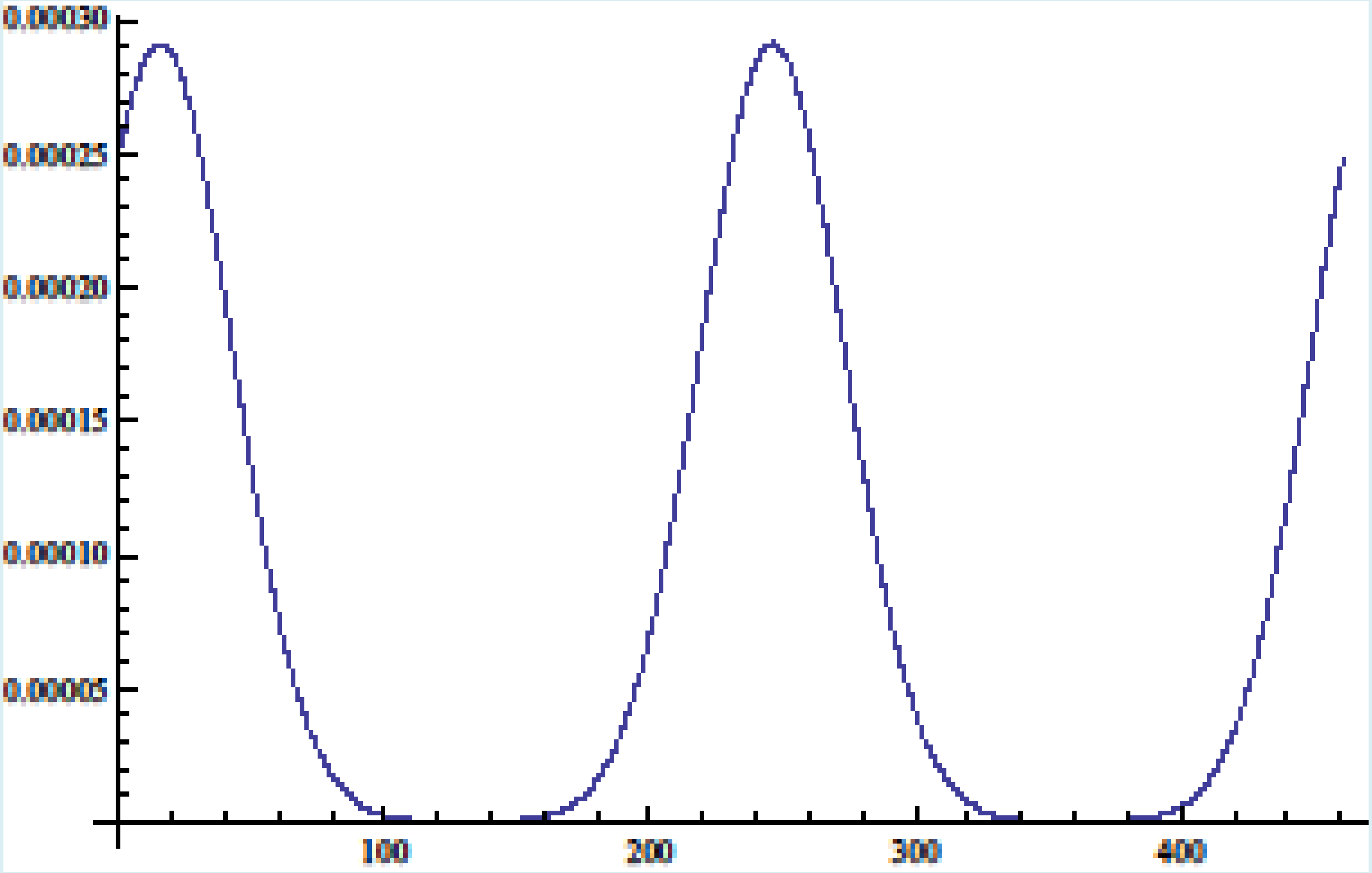
- SCATTERING EXPERIMENT, N=p=461
- GAUSSIAN WF AT T=0



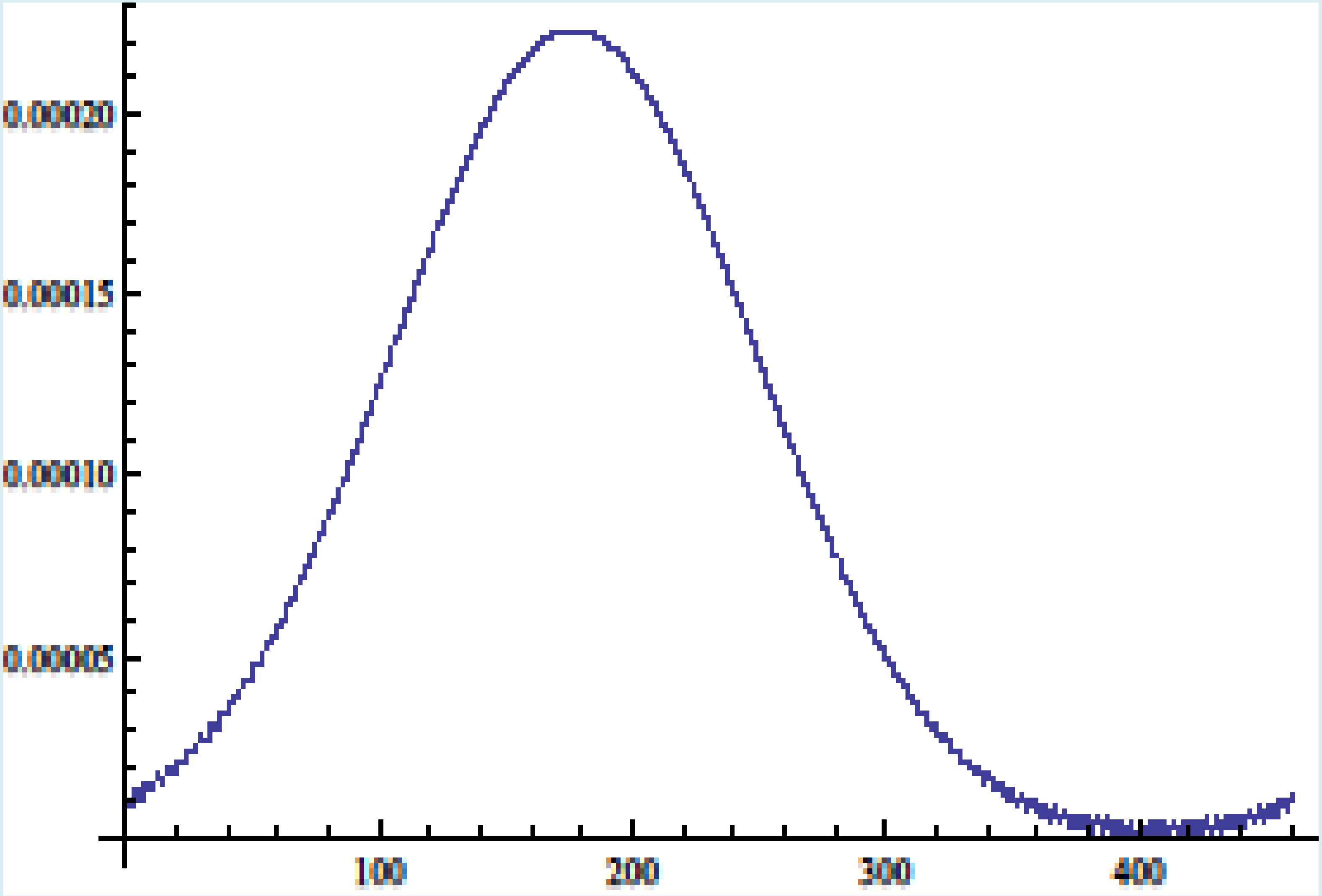
- TIME=1



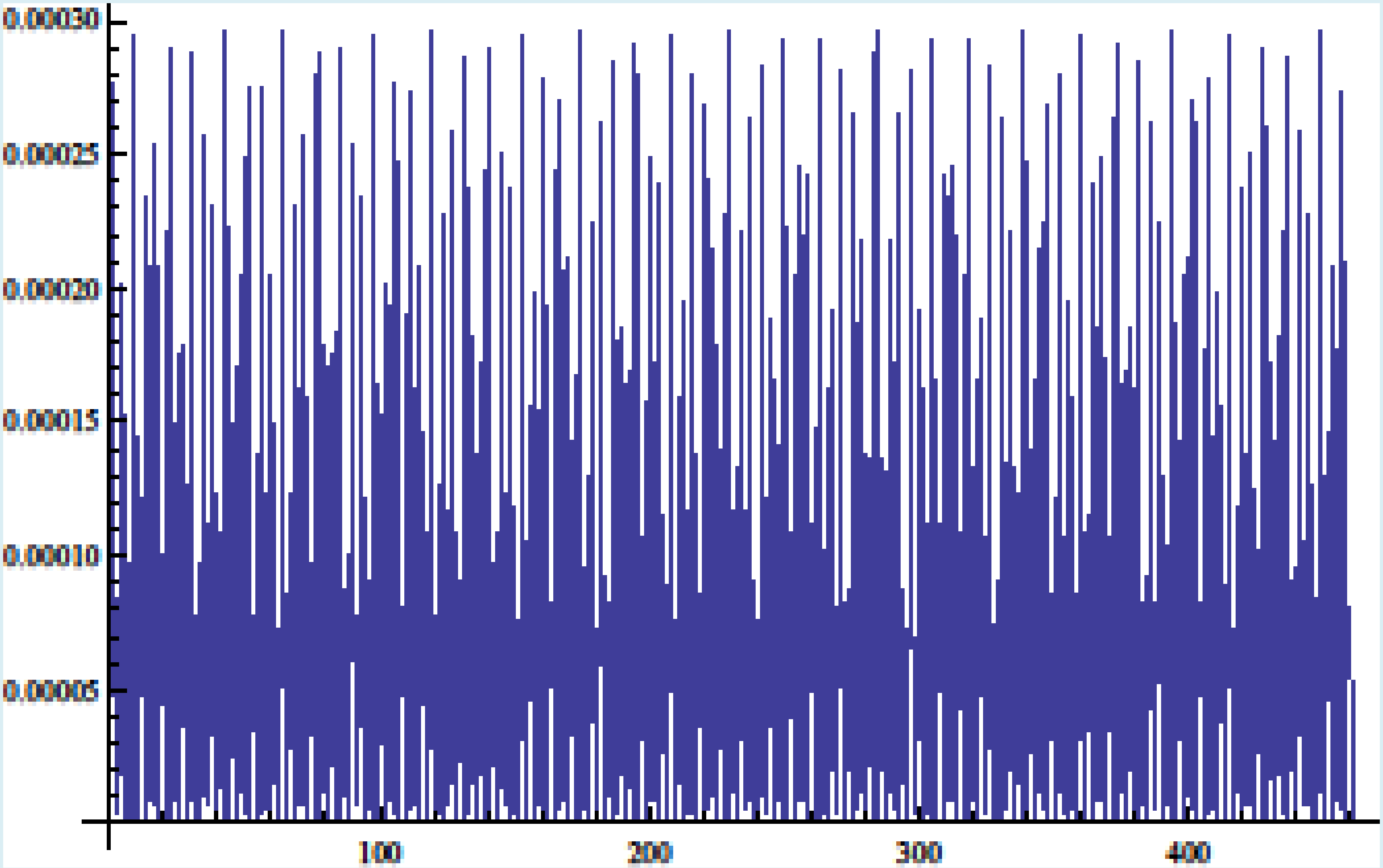
- T=3



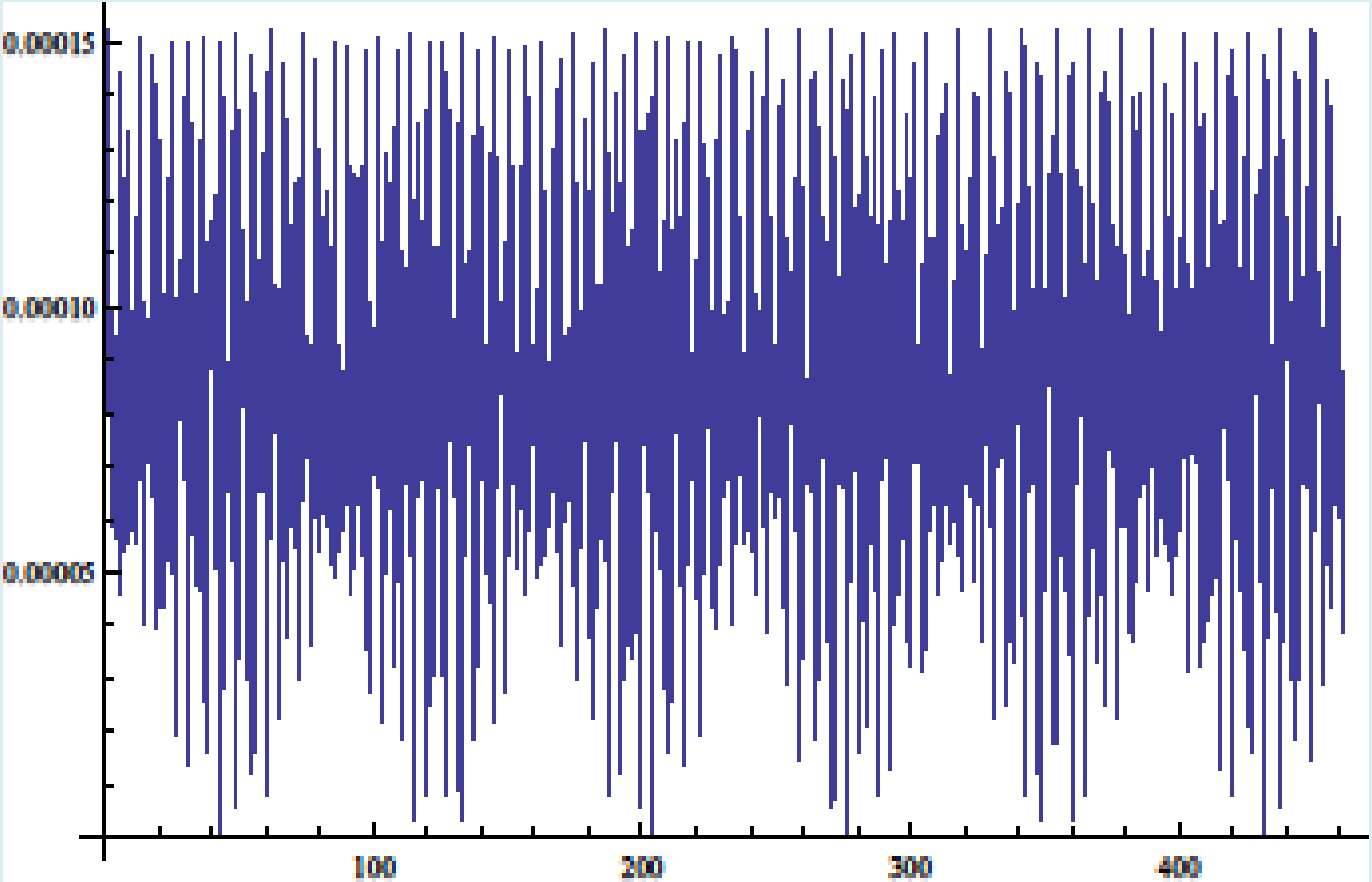
- T=3



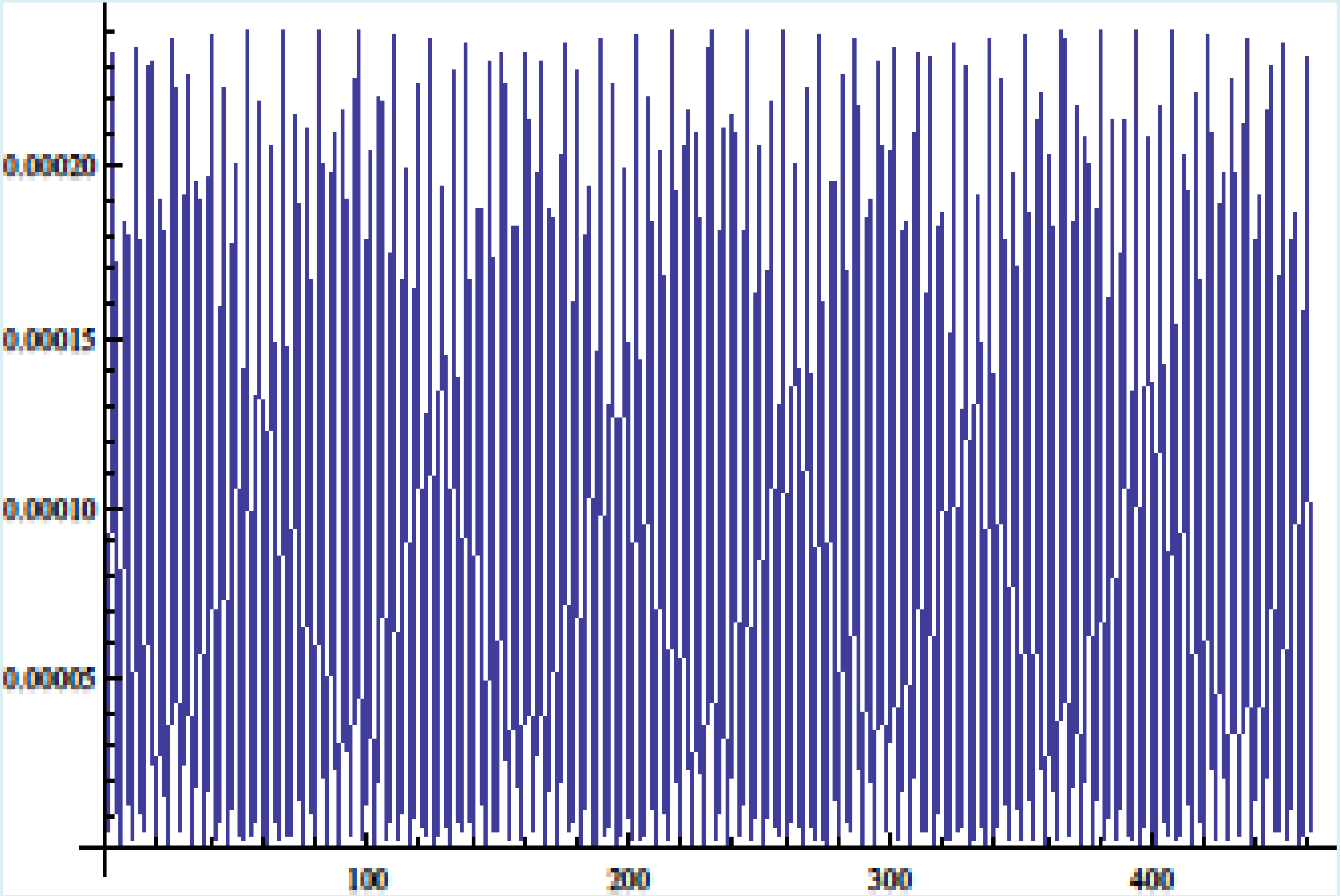
- T=4



- T=6



- T=7



CONCLUSIONS

DISCRETIZING AdS2 RADIAL AND TIME NEAR HORIZON

GEOMETRY OF EXTREMAL BLACKHOLES

AND AT THE SAME TIME PRESERVING THE ALGEBRAIC STRUCTURE OF THE ISOMETRIES

NECESSARILY LEADS TO THE MOD[N] ARITHMETIC DISCRETIZATION WITH HOLOGRAPHY AdS2[N]/CFT1[N],

THIS CLASSICAL STRUCTURE IS LIFTED TO THE QUANTUM LEVEL

THROUGH FINITE QUANTUM MECHANICS

THE QUANTUM CAT MAP (QACM) CHAOTIC DYNAMICS ON THE DISCRETIZED HORIZON AdS2[N] ,

DUE TO ITS RANDOM EIGENSTATES,(ETH),

THERMALIZES IN LOGARITHMIC TIME

SINGLE PARTICLE WAVE PACKETS,

THUS IT SATURATES

THE SCRAMBLING TIME BOUND OF HAYDEN-PRESKILL

SEKINO-SUSSKIND

THE COMPLEXITY OF THE QACM GROWS AS

$n^2=\text{Log}[S]^2$