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Organization and Sensitivity of the Antarctic Circumpolar Current

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Abstract

It has been recently argued that the governor of the Earth's climate may lie in the southern seas, where the strongest ocean current on Earth, the Antarctic Circumpolar Current (ACC).

This study utilized novel theoretical results and constructed a simplified, yet realistic, model of the ACC dynamics that enables the comprehensive investigation of the ACC climatology and its dependence on external parameters.

The study concluded that the ACC could be in two climate states. In the first the structure of the ACC is principally determined by the mean atmospheric forcing, while the principal determinant of the ACC structure in the second state are the small scale eddies supported by the current. For given external parameters the climate states were shown to be unique. If there were multiple climate states the ACC could, under the same external conditions, transition from one state to the other and induce itself abrupt climate change. The uniqueness of the states excludes this possibility. The main parameter that determines the climate state was found to be the variability in the atmospheric wind stress, which is not included in current climate models. A main conclusion of the study is that wind gustiness should be included in future models in order to obtain realistic assessment of the climatic impact of the ACC.

The project is also available at http://web.cc.uoa.gr/~pji/acc/

Περίληπη

Πρόσφατα υποστηρίχθηκε ότι ο ρυθμιστής του κλίματος της Γης μπορεί να βρίσκεται στις νότιες θάλασσες, όπου ανάμεσα στις προερχόμενες του Νότου και της Ανταρκτικής, βρίσκεται το πιο ισχυρό οικείο ρεύμα στη Γη, το Ανταρκτικό Κύκλικο Μερικό Ποταμό (CC). Ωστόσο, το ACC έχει κατανοηθεί ελάχιστα, καθώς στροβίλοι μικρής κλίμακας που δεν προσομοιώνονται από κλιματικά μοντέλα καθορίζουν τις ιδιότητές του. Επομένως, υπάρχει ανάγκη για μία θεωρητική μελέτη της κλιματολογίας του ρεύματος ώστε να κατανοηθεί η δυναμική του και να επιτευχθεί η ακριβής μοντελοποίησή του.

Στη μελέτη χρησιμοποιήσαμε νέα θεωρητικά αποτελέσματα και κατασκευάσαμε ένα απλοποιημένο αλλά πραγματικό μοντέλο της δυναμικής του ACC ώστε να δείξουμε τη δυνατότητα μελέτης της εξάρτησης της κλιματολογίας του ACC από εξωτερικούς παράγοντες.

Η μελέτη κατέληξε ότι υπάρχουν δύο κύριες κλιματικές καταστάσεις του ACC. Στην πρώτη κατάσταση η δομή του ACC καθορίζεται από τη μέση ατμοσφαιρική κυκλοφορία, ενώ η δεύτερη προσδιορίζεται από τη δυναμική των στροβίλων που αναπτύσσονται στο ρεύμα. Δεδομένων εξωτερικών παραμέτρων οι κλιματικές καταστάσεις αυτές είναι μοναδικές. Αν οι καταστάσεις δεν ήταν μοναδικές τότε το ACC θα μπορούσε, κάτω από ιδίες εξωτερικές συνθήκες, να μεταβαίνει από τη μία κατάσταση στην άλλη και να δημιουργήσει αφειτωμένο σε απότομες κλιματικές αλλαγές. Η μοναδικότητα των καταστάσεων δείχνει ότι αυτή η συμπεριφορά αποκλείεται.

Η βασική παράμετρος που καθορίζει την κλιματική κατάσταση του ρεύματος βρέθηκε ότι είναι η μεταβλητικότητα του ανέμου, η οποία δεν περιορίζεται στα σημερινά κλιματικά μοντέλα. Συμπέσαμε της μελέτης μας είναι ότι ο παράγοντας αυτός είναι καθοριστικός και πρέπει να συμπεριληφθεί για να επιτευχθεί ακριβής περίγραφη της επίπτωσής του ACC στο κλίμα.

Η μελέτη είναι επίσης διαθέσιμη στο http://web.cc.uoa.gr/~pji/acc/
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Chapter 1

Introduction

The Antarctic Circumpolar Current (ACC) is the world’s strongest ocean current. It flows between the Southern continents and Antarctica and transports a mean volume of 133 Sv with a standard deviation of 11 Sv (1 Sv = $10^6$ m$^3$s$^{-1}$) (Whitworth et al. 1982, Whitworth and Peterson 1985). The flow of the ACC is concentrated in several jets associated with regions of strong horizontal gradients in water mass properties (Deacon 1937, Orsi et al. 1995). There are three main fronts extending through the depth of the ocean: the subantarctic front (SAF), the polar front (PF), and the southern ACC front (sACCf). Their gross position and meridional structure are shown in Fig. 1.1-1.2 taken from the observational studies of Orsi et al. (1995) and Firing et al. (2011). The structure of the fronts is complex and each front consists of multiple branches, which merge and diverge along the circumpolar path, as illustrated in Fig. 1.3.

ACC plays a crucial role in the Earth’s climate system for three main reasons. The first is that it links the Atlantic, Pacific and Indian Oceans. It therefore serves as a conduit of passive oceanic tracers, salt and heat, permitting a global overturning circulation that dominates the ocean heat transport. The second is that it insulates Antarctica from the heat of the subtropics and it regulates the
Figure 1.1: Circumpolar distributions of the SAF, PF and southern ACC front. Taken from Orsi et al. (1995).
temperature of the ocean in the vicinity of Antarctica, which is responsible for maintaining the massive ice sheets of the continent. The third is that the ACC’s meridional density structure influences the carbon dioxide uptake of the ocean and consequently the atmospheric CO\textsubscript{2} concentration that is a major driver of the Earth’s climate system (Thompson 2008).

Despite its essential role in the climate, the ACC remains poorly understood. The reason is that the ACC is dynamically distinct from ocean basin currents because it is lacking any meridional boundaries. In the ocean basins that are zonally blocked by continents, momentum that is imparted to the ocean surface by the atmospheric zonal winds, causes a meridional Ekman transport near the surface that is balanced by zonal pressure gradients. These pressure gradients are related in this case to a slope in sea surface height across the basin caused by the boundaries. In the ACC however, there are no such boundaries above the highest sill depth, which is approximately at 2500 m. The Ekman transport of light Antarctic surface water has to be balanced through other mechanisms, if a constant density structure, and by thermal wind balance a constant current structure, is to be maintained in steady state.

There are three mechanisms proposed in the literature. The first mechanism is dyapicnal mixing that converts dense water into light water, replenishing the amount of light water lost in the Ekman transport. In this case, the zonal transport of the circumpolar current is determined by the relationship between the density
Figure 1.3: (a) Schematic path of the ACC. Regions shaded blue have depths shallower than 3500 m. (b) Snapshot of the surface current speed from the OCCAM 1/128 ocean model, where the filamentary structure of the ACCs jets is clear (courtesy of Andrew Coward).

structure and the buoyancy forcing (Gnanadesikan 1999, Gent et al. 2001). The second mechanism is to balance the momentum imparted by the winds and the related Ekman transport, through a momentum and heat transport by stationary eddies excited by major topographic features such as the Scotia Arc, Kerguelen Ridge, and the Campbell Plateau (Gille 1997, McWilliams et al. 1978). In this case, the stationary eddies reflect the topography vertically and deform the internal interface. Pressure forces acting on the deformed interface result in a form stress that transfers momentum to the lower layer, where it is dissipated. The third mechanism is a similar transport by transient eddies that produce the interfacial form stress, fluxing momentum in the vertical (Straub 1993, Marshall et al. 1993). These eddies are typically generated by baroclinic instability developing in the ACC. In the real world, all three processes probably play a role, but their contribution towards the observed structure of the ACC, as well as their interconnections and interactions are yet unknown. In addition, the eddy length scales are too small to be resolved in climate models. Instead, their effects are incorporated
in the models through parameterizations. To achieve accurate parameterizations, a theory that explains and predicts the systematic organization of the eddy fluxes maintaining both the current and the meridional heat circulation is needed.

What is also needed and has become a priority in climate research, is a comprehensive understanding of how the complicated balance between baroclinic instability, eddy transport and topographic steering determines the sensitivity of different aspects of the ACC to changes in the wind forcing. The goal is to provide more realistic model parametrizations and to improve predictions of the ACC’s behaviour in a changing climate. This is important in a climate change context since the strengthening and poleward shifting of the atmospheric winds is a robust feature of climate projections obtained from climate models under increasing atmospheric CO2 concentrations (Fyfe et al. 2007). High resolution numerical simulations have shown that such changes in the wind stress can cause a poleward shift of the ACC (Gille 2008), or a change in meridional heat transport (Hog et al. 2008), both of which cause a warming of the Southern Ocean with possible dire consequences for the Antarctic ice sheets. Changes in the wind stress can also weaken the CO2 uptake leading to a steep increase of atmospheric CO2 and to abrupt climate changes, as well as transitions into and out of glacial climates (Toggweiler et al. 2006). However, a definitive answer remains elusive (Thompson 2008). The goal of this project is to provide the theory for systematic organization of the eddy fluxes in the ACC and to test the sensitivity of the eddy-mean current equilibrium to changes in the wind stress.

To achieve this goal, we build on results from linear stochastic turbulence modeling that has led to a novel framework for investigating the organization of eddy fluxes in turbulent zonal flows and is called Stochastic Structural Stability Theory (SSST; Farrell and Ioannou (2003)). SSST has three building blocks. The first is to neglect the eddy-eddy interactions and retain only the interaction between the
eddies with the instantaneous mean flow. This results in a quasi-linear approximation of the dynamics producing accurate quadratic eddy statistics and mean flows (Schoeberl and Lindzen 1984, DelSole and Farrell 1996, O’Gorman and Schneider 2007, Marston et al. 2008). The second building block is to form, based on the quasi-linear approximation, the dynamical equations for the joint evolution of the eddy statistics and the mean flow. Since the eddy-eddy non-linearity is not retained explicitly, this constitutes a second order closure for the eddy statistics (Farrell and Ioannou 2003, Marston et al. 2008). The third building block is to parameterize the eddy-eddy nonlinearity or the external excitation of the eddies occurring at small temporal and spatial scales (as is for example the case in convective forcing), as stochastic forcing and enhanced dissipation. This has been shown to be accurate explicitly in baroclinic turbulence through analysis of non-linear interactions with linear inverse modeling (DelSole and Farrell 1996, DelSole 1996; 2004) and indirectly by showing that the eddy statistics at midlatitudes are accurately obtained as the steady state response of the stochastically forced stable climatological mean flow (Farrell and Ioannou 1993a;b;c, Newman et al. 1997, Whitaker and Sardeshmukh 1998, Zhang and Held 1999).

Under these three assumptions, a nonlinear system for the joint evolution of the mean flow and the second order eddy statistics is obtained. These statistical equations have fixed points that correspond to equilibria with stationary eddy statistics that maintain the steady mean flow against dissipation. The fixed points can accurately approximate in this way the structure of stationary climate states (DelSole and Farrell 1996, Farrell and Ioannou 2008). The SSST equations can also have limit cycles, that describe periodic phenomena like the QBO (Farrell and Ioannou 2003), and time dependent chaotic solutions. In barotropic, baroclinic or drift-wave turbulence, these chaotic solutions are unlikely (Farrell and Ioannou 2003; 2007; 2009a;b), but in three dimensional boundary layer flows these time-
dependent solutions characterize transition to the state of fully developed three dimensional shear turbulence (Farrell and Ioannou 2011). Furthermore, stability of the non-trivial SSST equilibria can give rise to a new type of a turbulence-mean flow cooperative instability that has been called structural, because when the SSST equilibrium becomes unstable, bifurcation to a new type of equilibria results.

Since SSST is a promising tool to investigate the eddy-mean flow equilibria, the goal of the project is to apply SSST methods to study the bifurcation properties of the eddy-ACC equilibrium and to assess the sensitivity of the ACC properties (i.e. strength and meridional heat transport) to changes in wind forcing. We consider a quasi-geostrophic, two layer baroclinic model with simplified topography and surface wind stress forcing. Such a model of the Southern Ocean is the simplest possible that retains the dynamics that are relevant to the ACC and has been successfully used in previous ACC modeling studies. Under these assumptions, we formulate the SSST system for the evolution of the mean current under the influence of its consistent field of turbulent eddies, realizations of which are seen in numerical simulations. We calculate the stationary solutions that are equilibria corresponding to the ACC and to the average eddy statistics and we compare their characteristics to existing results from numerical simulations and observations. Subsequently, we obtain the bifurcation properties of the equilibria with respect to the wind forcing and finally we assess the sensitivity of the results to physically important parameters in the model.
Chapter 2

Description of the dynamics of the ACC

2.1 Formulation

The ACC is located in the Southern Ocean in the region between the Drake passage, the Cape of Good Hope, Australia to the North and Antarctica to the South. We will consider that the geometry of the ocean basin is a re-entrant channel of meridional size (in the $y$ direction) $L = 2000$ km bounded to the North and the South by solid walls. The ACC flows on average in the zonal ($x$) direction and we define as the ACC this zonal mean flow. Departures of the flow from the zonal mean are perturbations to the mean and will be called eddies. The effects of the sphericity of the planet are represented in this planar representation of the geometry, by including in the dynamics the meridional variation $\beta = \frac{df}{dy}$ of the Coriolis parameter $f = 2\Omega \sin \theta$. Here, $\Omega$ is the rate of rotation of the planet and $\theta$ is the latitude, so changes in the meridional direction $\delta y$ are equal to $R_e \delta \theta$ where $R_e$ is the Earth’s radius. The average value of $\beta$ in the region of the ACC is $\beta = 1.4 \times 10^{-11}$ m$^{-1}$s$^{-1}$. In the vertical ($z$ direction) we will make, for the most
2.1. Formulation

part of this study, the traditional two-layer approximation, in which the ocean is
considered to be well represented by two stably stratified layers of constant density.
A well mixed warmer and lighter upper layer (layer 1) above the thermocline of
depth $H_1$, which will be typically taken to be $H_1 = 500$ m, and below the ther-
mocline a colder and denser water mass of depth $H_2$, which will be typically taken
to be $H_2 = 3500$ m. Bottom topography can also be included. This two-layer fluid
represents both the barotropic and the baroclinic structure of the flow and is the
minimal configuration in which turbulence can self-sustain. We will also consider
a single layer ocean in which turbulence will be maintained by exogenous forcing.
This single layer captures the barotropic dynamics of the ACC and will be con-
sidered in order to obtain an understanding of the validity and the implications of
the theory that we advance for the ACC.

The dynamics of the ACC will be taken, as in previous studies (Treguier and
McWilliams 1990, Treguier and Panetta 1994, Thompson 2008; 2010), to be quasi-
geostrophic. Under these assumptions the field equations for the evolution of the
potential vorticity in each layer, $q_i$, is given by:

$$
\partial_t q_i + J(\psi_i, q_i) = -\delta_{i1} \frac{T_0}{\rho H_1} \partial_y f - \delta_{i2} E \Delta \psi_2 - r \Delta \psi_i,
$$

(2.1)

where $i = 1, 2$ for the two layer representation and $i = 1$ for the single layer. In
the above, $J(A, B) = (\partial_x A)(\partial_y B) - (\partial_y A)(\partial_x B)$ is the Jacobian operator, and
$\Delta \equiv \partial_{xx}^2 + \partial_{yy}^2$ is the horizontal Laplacian. We denote with $\psi_i$ the streamfunction
at the $i - th$ layer, and the zonal and meridional velocities in the corresponding
layer are respectively given by:

$$
    u_i = -\partial_y \psi_i \quad , \quad v_i = \partial_x \psi_i .
$$

The potential vorticity of the single layer ocean is simply:

$$
    q = \Delta \psi + \beta y ,
$$
Figure 2.1: Global Surface Velocity Analysis of the surface currents in the South Seas. This plot is based on ship-drift estimates of sea surface velocities that are mostly available along major shipping routes. The ACC is poorly represented because of the lack of data. The typical mean velocities are of the order of 20 cm s$^{-1}$ (Data averaged over 2001-2008. Source: CIMAS).
while for the two layer fluid a baroclinic contribution is included and the potential vorticity in the two layers is:

\[ q_1 = \Delta \psi_1 + \beta y - \frac{1}{2\lambda^2} (\psi_1 - \psi_2) , \quad q_2 = \Delta \psi_2 + \beta y + \frac{f_0}{H_2} h + \frac{r}{2\lambda^2} (\psi_1 - \psi_2) , \]

where \( r = H_1/H_2 \) and \( h \) is the height of the bottom topography. The deformation radius in the upper layer is \( 2\lambda^2 = g' H_1/f_0^2 \), where \( f_0 \) is the Coriolis parameter at the center of the channel, and \( g' \) is the reduced gravity. Further in Eq. (2.1), \( r_e \), is the coefficient of linear dissipation that acts equally in all layers, and \( E \) is the coefficient of Ekman drag. We will take the value \( E = 1.15 \times 10^{-7} \) s\(^{-1}\) which produces decay with an e-folding of 100 days.

The energy source for the flow is the wind stress at the surface, that appears in the above equations as an exogenous excitation of the vorticity of the top layer given by

\[ -\frac{T_0}{\rho H_1} \partial_y f , \]

where \( \tau_0 \) is the surface stress, \( \rho \) the density of the water, and \( \partial_y f \) the meridional derivative of the structure of the forcing. The structure of the wind stress \( f(y, x, t) = \mathcal{F} + f'(x, y, t) \) is separated in its zonal and time mean component, and a variable component. The mean component represents the climatological value of the wind stress. The variable component is traced to the variations in the wind that occur with the passage of cyclones and frontal systems. We will assume that the winds associated with the time varying component of the stress are also varying in the zonal direction. Consequently, the time mean equations will be assumed forced by a time invariant stress. The value of the mean stress will be taken in the bulk of the study to be \( T_0/\rho = 10^{-4} \) m\(^2\)s\(^{-2}\).

We nondimensionalize lengths with the deformation radius \( \lambda \), velocities with a velocity scale \( U \), and time with the advective time scale \( \lambda/U \). We will take as velocity scale \( U = 1 \) m s\(^{-1}\), and \( \lambda = 20 \) km. This is a typical value for the
deformation radius in the Southern ocean, although values as low as 13 km have also been observed in the most southern part of the current (Inoue 1984). With these non dimensional values, the unit of time is 5.5 hours. Using tildes to denote the nondimensional variables, we have \( t = (\lambda/U)\tilde{t}, \psi = U\lambda\tilde{\psi}, q = (U/\lambda)\tilde{q}, \beta = (U/\lambda^2)\tilde{\beta}, \text{ and } h = \lambda\tilde{h}. \) The nondimensional Ekman parameter becomes

\[ \epsilon = \frac{E\lambda}{U}, \]

and the nondimensional stress parameter is

\[ \tau_0 = \frac{T_0\lambda}{\rho H_1 U^2}. \]

With these parameters the non dimensional equations of motion (2.1), having dropped all the tildes, are:

\[ \partial_t q_i + J(\psi, q_i) = -\delta_1\tau_0 f - \delta_2\epsilon\Delta \psi_2 - r\epsilon\Delta \psi_i, \quad (2.2) \]

with the non-dimensional potential vorticities given by:

\[ q_1 = \Delta \psi_1 + \beta y - \frac{1}{2}(\psi_1 - \psi_2), \quad q_2 = \Delta \psi_2 + \beta y + r\eta + \frac{r}{2}(\psi_1 - \psi_2), \]

where

\[ \eta = \frac{f_0\lambda^2}{H_1 U} h \]

and \( h \) is the nondimensional height of the bottom topography.

We decompose the fields into their zonal mean component, denoted with capital letters, and perturbations from this mean, denoted with primes. For example \( U_i(y, t) \) is the zonal mean velocity (because of the boundary conditions of no flow past the channel walls \( V_i = 0 \)) and \( Q_i(y, t) \) the zonal mean potential vorticity. The operation of zonal averaging will be denoted with an overline, i.e. \( U_i \equiv \overline{U}_i \).

The perturbation vorticities are

\[ q'_1 = \Delta \psi'_1 - \frac{1}{2}(\psi'_1 - \psi'_2), \]
2.1. Formulation

\[ q'_2 = \Delta \psi'_2 + \frac{r}{2} (\psi'_1 - \psi'_2), \]

and the linear operator that transforms \( \psi' = [\psi'_1, \psi'_2]^T \) to \( q' = [q'_1, q'_2]^T \) is given by

\[
\Lambda = \begin{pmatrix}
\Delta - 1/2 & 1/2 \\
r/2 & \Delta - r/2
\end{pmatrix}.
\]

The inverse operator \( \Lambda^{-1} \) will be rendered unique by imposing the boundary conditions that there is no flow across the channel walls. The meridional gradients of the zonally averaged potential vorticities are:

\[
Q_{1y} = \beta - \partial^2_{yy} U_1 + \frac{1}{2} (U_1 - U_2),
\]

\[
Q_{2y} = \beta + r \partial_y \eta - \partial^2_{yy} U_2 - \frac{r}{2} (U_1 - U_2).
\]

The evolution of the perturbations \( q'_i \), which are obtained by taking the zonal average of Eq. (2.2), are:

\[
(\partial_t + U_1 \partial_x) q'_1 + Q_{1y} \partial_x \psi'_1 = -r \Delta \psi'_1 + f_e + f_{1nl}, \tag{2.3}
\]

\[
(\partial_t + U_2 \partial_x) q'_2 + Q_{2y} \partial_x \psi'_2 = -r \Delta \psi'_2 - \epsilon \Delta \psi'_2 + f_{2nl}, \tag{2.4}
\]

where \( f_e = -\tau_0 \partial_y f'(y, t) \) is the time dependent component of the stress and

\[
f_{int} = \overline{J(\psi'_i, q'_i)} - J(\psi'_i, q'_i), \tag{2.5}
\]

is the forcing term from the nonlinear interactions among perturbations that redistribute energy among scales and produce the turbulent cascading process. The corresponding equations for the evolution of the mean currents are

\[
\partial_t U_1 = \overline{v'_1 q'_1} - r_u U_1 + \tau_0 \overline{f}, \tag{2.6}
\]

\[
\partial_t U_2 = \overline{v'_2 q'_2} - r_u U_2 - \epsilon U_2. \tag{2.7}
\]

Equations (2.3), (2.4), (2.6), (2.7) form the nonlinear system of equations which can be solved if the wind stress on the surface \( f \) is fully prescribed. The ACC
emerges as the mean flow $U_i$. Without any further approximations, direct numerical simulation (DNS) of these equations has been undertaken in the past (Treguier and McWilliams 1990, Treguier and Panetta 1994, Thompson 2008; 2010) and it has been shown that the framework described can produce the main features of the ACC. Whenever we integrate this set of nonlinear equations we will refer to the integration with the acronym NL.

2.2 The quasi-linear dynamics of the ACC

We have proposed to go further than the DNS simulations and develop a theory of the ACC that will determine both the important mechanisms that govern its dynamics and make quick and reliable predictions for its structure under diverse conditions. This is important because it can guide us to reach understanding of climatic regimes that are substantially removed from our present climate. Development of such a theory will also guide us to determine the sensitivity of the ACC to external changes that our climate system may undergo under anthropogenic or other influences.

The first approximation is to parameterize the eddy-eddy interaction terms $f_{int}$ as a stochastic vorticity forcing. This the basis for constructing the stochastic turbulence models (STM) (Farrell and Ioannou 1993d;a). We also consider that the time dependent component of the wind stress is also a stochastic variable, being a principally unknowable field, with only known components the statistical moments of the wind stress. As a result we will consider in the sequel $f_1 = f_e + f_{int}$ as a single stochastic field, without distinguishing the random wind stress component.
This STM approximation leads to the approximate nonlinear system:

\[
\begin{aligned}
(\partial_t + U_1 \partial_x) q_1' + Q_{1y} \partial_x \psi_1' &= -r \epsilon \Delta \psi_1' + \xi_1, \\
(\partial_t + U_2 \partial_x) q_2' + Q_{2y} \partial_x \psi_2' &= -r \epsilon \Delta \psi_2' - \epsilon \Delta \psi_2' + \xi_2,
\end{aligned}
\]

\[
\begin{aligned}
\partial_t U_1 &= \frac{v_1 q_1'}{\nu} - r U_1 + \tau_0 \overline{f}, \\
\partial_t U_2 &= \frac{v_2 q_2'}{\nu} - r U_2 - \epsilon U_2.
\end{aligned}
\]

The excitation $\xi_i$ is a zonally uniform random field that will be taken white in time, with two point correlation between two points with coordinates $(x_\alpha, y_\alpha)$ and $(x_\beta, y_\beta)$ given by

\[
\langle \xi_i(x_\alpha, y_\alpha, t_1) \xi_j(x_\beta, y_\beta, t_2) \rangle = \delta_{ij} \delta(t_2 - t_1) Q(x_\alpha, x_\beta, y_\alpha, y_\beta),
\]

where the brackets denote an ensemble average and $Q$ is a function of the difference $\tilde{x} = x_\alpha - x_\beta$. We have also assumed that there is no correlation between the forcing in the top and bottom layers. These equations comprise a significant simplification of the dynamics of the ACC. In this nonlinear system, the ACC interacts with the eddy field and the eddy field interacts with the ACC, but there is no explicit calculation of the eddy-eddy interactions which have been parameterized as a stochastic excitation. This system, in the sense that the eddy-eddy interactions are not explicitly calculated, is a quasi-linear approximation of the turbulence. If the quasi-linear STM model is accurate then it is implied that

- the dynamics of the turbulent field at the energetic scales is essentially the dynamics of the two way interaction between the turbulent field and the mean flow. This is in contrast with the dynamics of classical isotropic and homogeneous turbulence.

- the emergence of the ACC results from the non-local in wavenumber space direct interaction between the ACC and the eddies. This is also in contrast to classical upscale cascade theories for the formation of jets in rotating planets.
It will be of particular interest to investigate the accuracy of the quasi-linear set of equations, because the implications are far reaching for understanding the climate in planetary environments. Whenever integrations are performed with this quasi-linear set of equations the results will be reported with the acronym QL.

### 2.3 The SSST formulation of the ACC

While considerable simplification has been achieved with the QL equations, this set of equations is a non-autonomous set of equations with stochastic inputs. We wish to develop deterministic equations governing the evolution of the statistics of the eddy field in the presence of the ACC as well as evolution equations governing the structure of the ACC. To achieve this goal, we first derive an equation for the evolution of two-point eddy correlation functions, and then relate the eddy vorticity fluxes $\overline{v'_i q'_i}$ that drive the mean flow to these functions.

We start by assuming, as we have done previously, that the stochastic forcing has a two-point, two-time correlation function of the form

$$
\langle \xi_i(x_\alpha, y_\alpha, t_1) \xi_j(x_\beta, y_\beta, t_2) \rangle = \delta_{ij} \delta(t_2 - t_1) Q(x_\alpha, x_\beta, y_\alpha, y_\beta),
$$

where the brackets denote an ensemble average and $Q$ is a function of the difference $\tilde{x} = x_\alpha - x_\beta$. We now use the shorthand $A_\alpha = A_\alpha(x_\alpha, t)$, to refer to the value of the variable or the operator $A$ at the point $x_\alpha = (x_\alpha, y_\alpha)$ and $Q_{\alpha\beta} = Q(x_\alpha, x_\beta, y_\alpha, y_\beta)$.

In order to calculate the equation for the evolution of the potential vorticity correlation function

$$
C_{\alpha\beta}(t) \equiv C(x_\alpha, x_\beta, t) = \left( \begin{array}{c} \langle q'_1 q'_1 \rangle \\ \langle q'_1 q'_2 \rangle \\ \langle q'_2 q'_1 \rangle \\ \langle q'_2 q'_2 \rangle \end{array} \right),
$$

we write Eq. (2.8) and Eq. (2.9) in the compact form:

$$
\partial_t q'_\alpha = A_\alpha q'_\alpha + \xi_\alpha,
$$

(2.12)
where \( q'_\alpha = [q'_{1\alpha}, q'_{2\alpha}]^T \) is the total state vector consisting of the potential vorticities in both layers at the point \( x_\alpha = (x_\alpha, y_\alpha) \) and similarly \( \xi \) is the total state vector of the forcing at the same point. The cumulative operator acting on the total vorticity is:

\[
A_\alpha = - \begin{pmatrix}
U_{1\alpha} \partial_{x_\alpha} & 0 \\
0 & U_{2\alpha} \partial_{x_\alpha}
\end{pmatrix}
- \begin{pmatrix}
Q_{1\alpha} \partial_{x_\alpha} & 0 \\
0 & Q_{2\alpha} \partial_{x_\alpha}
\end{pmatrix} \Lambda_{\alpha}^{-1}
- \begin{pmatrix}
\epsilon \Delta_{\alpha} & 0 \\
0 & (r_e + \epsilon) \Delta_{\alpha}
\end{pmatrix} \Lambda_{\alpha}^{-1}
\]

Multiplying (2.12) for \( \partial_t q'_\alpha \) by \( q'_\beta \) and (2.12) for \( \partial_t q'_\beta \) by \( q'_\alpha \), adding the two equations and taking the ensemble average, we obtain:

\[
\partial_t C_{\alpha\beta} = (A_\alpha + A_\beta) C_{\alpha\beta} + \langle \xi_\alpha q'_\beta + \xi_\beta q'_\alpha \rangle.
\]  

(2.13)

It is worth noting that the operators \( A_\alpha \) and \( A_\beta \) commute, since the derivatives are taken at different points. Note also that because the forcing has been assumed \( \delta \) correlated in time, the average injection of perturbation potential vorticity is independent of the state of the system and is equal to

\[
\langle \xi_\alpha q'_\beta + \xi_\beta q'_\alpha \rangle = Q_{\alpha\beta} = Q_{\beta\alpha}.
\]

Therefore, (2.13) becomes the deterministic evolution equation:

\[
\partial_t C_{\alpha\beta} = (A_\alpha + A_\beta) C_{\alpha\beta} + Q_{\alpha\beta}.
\]  

(2.14)

Note also, that \( C_{\alpha\beta} \), apart from being a symmetric function of \( \alpha \) and \( \beta \), is also a function of \( x_\alpha - x_\beta \), since \( A_\alpha, A_\beta, Q_{\alpha\beta} \) are all invariant under translations in the zonal, \( x \), direction. The ensemble average potential vorticity fluxes \( \langle v'_i q'_i \rangle \) can be obtained from calculation of

\[
\langle v'_i q'_i \rangle = \begin{pmatrix}
\langle v'_{1\alpha} q'_{1\beta} \rangle & \langle v'_{1\alpha} q'_{2\beta} \rangle \\
\langle v'_{2\alpha} q'_{1\beta} \rangle & \langle v'_{2\alpha} q'_{2\beta} \rangle
\end{pmatrix},
\]
at the same point $x_\alpha = x_\beta$. This is related to $C_{\alpha\beta}$ through

$$
\langle v'q' \rangle = \langle v'_\alpha q'_\beta \rangle_{x_\alpha = x_\beta} = \left( \partial_{x_\alpha} \Lambda^{-1}_{\alpha} C_{\alpha\beta} \right)_{x_\alpha = x_\beta}.
$$

The zonal average of the zonal momentum equations (2.6), (2.7), and equation (2.14) that advances the ensemble average second order statistics of the perturbation field, can form a closed deterministic system by making the ergodic assumption that the ensemble average of the eddy momentum fluxes is equal to the zonal average, that is $\overline{v'q'} = \langle v'q' \rangle$. Discussion of the validity of this assumption can be found in Farrell and Ioannou (2003). Under this assumption, the mean flow $U = [U_1, U_2]^T$ evolves according to:

$$
\frac{dU}{dt} = \text{diag} \left( \left( \partial_{x_\alpha} \Lambda^{-1}_{\alpha} C_{\alpha\beta} \right)_{x_\alpha = x_\beta} \right) + \begin{pmatrix} r_c U_1 + \tau_0 f \\ -(r_c + \epsilon) U_2 \end{pmatrix},
$$

(2.15)

where diag obtains the diagonal of a matrix. Equations (2.14) and (2.15) form a closed deterministic system describing the evolution of the eddy statistics and the mean flow associated with the ACC and constitutes a second order closure for the statistics of the nonlinear equations. This coupled system is the basis of the Stochastic Structural Stability Theory (SSST) of Farrell and Ioannou (2003) (see also the recent reformulation of the SSST equations by Srinivasan and Young (2011)). The deterministic SSST system can be solved numerically by discretizing the above equations on a grid. Details of the methods involved can be found in Farrell and Ioannou (2003) and Bakas and Ioannou (2011).

The importance of the SSST equations is that they are deterministic and they define a new type of equilibria, which are mean flows in statistical equilibrium with the turbulence. The fixed points of the SSST define stationary climate states of the system and investigation of the attractor of these states determines the sensitivity of the equilibria. For the case of the ACC the SSST system will determine the equilibrium climates that characterize it and the attractor basin of these equilibria.
Also, and most importantly, we can study with the SSST equations the stability of the equilibria. When these SSST equilibria become unstable the flow bifurcates to a different climate. Consequently, investigation of the SSST equations of the ACC will determine the crisis points of the ACC.
Chapter 3

Comparison of the SSST system with realistic nonlinear integrations

As discussed in the previous chapter, SSST is based on two assumptions: the neglect of the eddy-eddy interactions in the eddy dynamics that leads to a second order closure for the eddy statistics and the parametrization of the eddy-eddy non-linearity or the external excitation of the eddies as stochastic forcing and enhanced dissipation. In the case of the ACC, there is both non-linear scattering and a very robust external excitation. Above the surface of the ocean there are very strong wind gusts that have short time scales compared to the time scales in the ACC and are incoherent in time. The gusts have a length scale of the order of 80-100 km due to their frontal origin and the surface wind velocity during these events can reach as much as 8 times the mean velocity of the wind (Sura and Gille 2003, Yuan 2004). As a result, the stress that is proportional to the square of the velocity can reach as much as 70 times the mean value of the stress. We will therefore include a stochastic stress in the model due to these gusts.
3.1 Comparison with nonlinear simulations for a single layer ocean

The effective damping is due to small scale mixing by the eddy-eddy interactions that diffuses momentum, but can also result from mixing induced by topography in the ACC. Apart from form drag that provides a large sink of momentum in the ocean, breaking of small scale internal gravity waves focusing due to a sloped bottom and tides breaking over topography can induce a large amount of dyapircnal mixing that diffuses momentum and constituents (Wunsch and Ferrari 2004, Kuhlbrodt et al. 2007). This effective damping will be included in both layers as a Rayleigh friction, with coefficient $r_e$.

Although these assumptions have been tested in a series of studies addressing quasi-geostrophic turbulence in the atmosphere (DelSole and Farrell 1996, DelSole 1996; 2004, Farrell and Ioannou 1993a;b;c, Newman et al. 1997, Whitaker and Sardeshmukh 1998, Zhang and Held 1999), the validity of these approximations in the regime of parameters pertinent to the ACC remains. As a result, the first step is to test the accuracy of the SSST system against fully non-linear simulations of turbulence in the parameter regime of the ACC. We undertake this task for the case of a single layer ocean that represents the barotropic dynamics of the current and then show that the SSST model reproduces the ACC structure and the associated eddy field in fully nonlinear two-layer simulations.

3.1 Comparison with nonlinear simulations for a single layer ocean

We will first consider the case of a single layer ocean obeying vorticity dynamics on a $\beta$ plane. For simplicity, we will consider a channel that is doubly periodic. In this case the pertinent nonlinear equations are Eq. (2.6) and Eq. (2.3) with

$$Q_y = \beta - \partial^2_{yy} U,$$
where we have dropped the subscript indicating the layer index.

The corresponding quasi-linear equations of the single layer ocean are then:

\[
\partial_t U = \overline{v'} q' - r_e U + \tau_0 \overline{f},
\]

\[
(\partial_t + U \partial_x) q' + Q_y \partial_x \psi' = -r_e q' + \xi,
\]

where for the single layer ocean the vorticity and stream function fields are related simply by \( q' = \Delta \psi' \) and the stochastic term \( \xi \) is taken to be delta correlated in time, as was discussed in the previous chapter.

The SSST system will involve the single correlation function \( C_{\alpha\beta} = \langle q'_\alpha q'_\beta \rangle \) which obeys

\[
\partial_t C_{\alpha\beta} = (A_\alpha + A_\beta) C_{\alpha\beta} + Q_{\alpha\beta},
\]

with

\[
A_\alpha = -U_\alpha \partial_{x_\alpha} - (\beta - U_{y_\alpha y_\alpha}) \Delta^{-1}_\alpha \partial_{x_\alpha} - r_e,
\]

and the mean flow dynamics

\[
\partial_t U = \partial_{x_\alpha} \Delta^{-1}_\alpha C_{\alpha\beta} - r_e U + \tau_0 \overline{f}.
\]

We will test the SSST equations under probably the most demanding conditions. We will consider the case of \( \overline{f} = 0 \), a case in which there is no mean stress, and we will impose simply a random and a spatially homogeneous stress field with zero mean. The test of the SSST theory is demanding because in such a case there is no imposed inhomogeneity and it is to be expected in that case that eddy-eddy interactions, which are absent in SSST, dominate the dynamics. If SSST works under these conditions, it is incumbent that an infinitesimal mean flow inhomogeneity can organize the eddy field so as to grow through eddy-mean flow interaction into a finite mean flow.

The question we ask is: if we increase the amplitude of the fluctuations of the wind stress will there be a critical value of stress fluctuations for which a mean
3.1. Comparison with nonlinear simulations for a single layer ocean

Figure 3.1: The ratio $\bar{E}_m/(\bar{E}_m + \bar{E}_p)$ of the mean flow kinetic energy $E_m$ over the total kinetic energy of the flow as a function of the forcing amplitude $f/f_c$, where $f_c$ is the critical forcing amplitude for which according to SSST theory jets emerge. Solid line asterisks: nonlinear integration, Dashed line circles: quasilinear integration, Solid line dots: SSST integration. The forcing is temporally white and spatially homogeneous. It imparts equal power at each zonal wavenumber in the range $k_x = 1 - 14$ and has a decorelation scale in the meridional direction of $\delta = 0.2$ in the $y$ direction. The dissipation coefficient is $r_e = 0.1$. For these integrations we have imposed periodic boundary conditions at the channel walls. A pseudospectral code has been used with $256 \times 256$ harmonics and hyperdiffusion for numerical stability. (Unpublished)
ACC current will emerge? If this occurs, we have a symmetry breaking of the homogeneous turbulent state. This question can be rather easily addressed using the SSST equations and as we will see the answer is affirmative. SSST will predict a critical stress fluctuation level which, when surpassed, leads to the formation of mean ACC current. We want to compare the statistical predictions of the SSST theory with both non-linear calculations and quasi-linear calculations.

It is clear that if a single nonlinear simulation produces results that are consistent with the SSST calculations, then by necessity the quasi-linear simulation, which comprises of a single realization of the ensemble SSST system, will in all probability produce agreement. Running the quasi-linear simulations next to the SSST, addresses the realizability of the ergodic assumption that has been employed in order to make the transition from the quasi-linear system to the ideal SSST system. The SSST produces ideal trajectories of the climate of the system, which may differ from individual realizations of the motions of the ocean. Therefore agreement of the SSST, the quasi-linear and the non-linear is necessary for assessing the validity and the realizability of the proposed theory. The SSST theory is negated when there will be substantial disagreement between quasi-linear and non-linear simulations. If the quasi-linear and non-linear simulations agree, but do not reveal the properties of the SSST trajectory, then the SSST still underlies the now elusive climate dynamics and it can be a very useful vehicle for obtaining an understanding of its stability.

We consider a doubly periodic square channel of size $2\pi \times 2\pi$ nondimensionized here to correspond to the 2000 km channel of the ACC. We will show results for nondimensional $\beta = 10$. We have considered other parameter values and similar results have been obtained. The variable stress excites with equal variance the first 14 gravest zonal harmonics in the channel. The stress is considered to be correlated in the meridional direction in the shape of a Gaussian with half width
3.1. Comparison with nonlinear simulations for a single layer ocean

Figure 3.2: Comparison of nonlinear and quasi-linear simulation of the emergent ACC flow in a single layer ocean. Left: Nonlinear instantaneous flow; shown are the vorticity field and the associated zonal wind (center, solid blue). Right: Quasi-linear instantaneous flow; shown are the vorticity field and the associated zonal wind (center, dashed red). The emergent ACC is almost identical. The flow is forced at \( f/f_c = 20 \) times the critical amplitude needed for the emergence of an ACC. Other parameter as in Fig. 3.1. (Unpublished)

The size of the width of the forcing correlation is important in this barotropic problem with infinite radius of deformation. We have investigated both numerically and analytically the dependence on the shape of the forcing. The results we obtain are typical of the regime in which the region of decorrelation is not infinitesimal. Because, the wind stress on the ocean surface decorrelates over large distances compared to the ocean’s deformation radius, the results we show are in the physically relevant regime. We will show the case with \( r_e = 0.1 \). The numerical calculations were obtained using a discretization of \( 256 \times 256 \) grids in the channel.

First, under periodic boundary conditions and homogeneous forcing the SSST equations Eq. (3.3) and Eq. (3.4) have as an equilibrium the state:

\[
U^E = 0 , \quad C^E = \frac{Q}{2r_e} .
\]
Figure 3.3: Comparison of the development of mean zonal velocity of the ACC, $U(y,t)$, in a single layer ocean from a state of rest using nonlinear (top panel), quasilinear (middle panel) and SSST integrations (bottom panel). The ACC is averaged over 1.5 time units. The structure of the ACC that emerges has the structure of the most unstable eigenfunction that is calculated from stability analysis of the SSST operator about a state with no mean flow and homogeneous turbulence. The parameters are as in Fig. 3.2. (Unpublished).
In checking that the above is an equilibrium, we make use of the translation invariance in the zonal direction of $Q$ which implies that $\partial_{x_\alpha} C_{\alpha\beta} + \partial_{x_\beta} C_{\alpha\beta} = 0$.

We can determine by using the perturbation form of Eq. (3.3) and Eq. (3.4) that there is a critical wind stress amplitude $f_c$ at which the motionless equilibrium becomes unstable and a mean flow emerges. The stability analysis predicts the structure and the meridional wavenumber of the most unstable mean flow. For wind stress forcing $f > f_c$, the emergent mean flows eventually equilibrate to a steady mean zonal flow at a statistical equilibrium with a statistically steady eddy field. We use as a bifurcating parameter the ratio of $E_m/(E_m + E_p)$, where $E_m$ is the kinetic energy of the mean zonal flow and $E_p$ the kinetic energy of the eddy field. We have performed SSST calculations, quasi-linear calculations and non-linear calculations and obtained the value of $E_m/(E_m + E_p)$ in each case when conditions became steady. The resulting bifurcation diagram is shown in Fig. 3.1. The agreement is remarkable. In Fig. 3.2 we show the eddy field associated with the nonlinear and the quasi-linear simulation for supercritical forcing $f/f_c = 20$. In Fig. 3.3 we show the time history of the mean flow in the three integrations and finally in Fig. 3.4 we show that while the bifurcation properties of the climate states have been captured accurately, the resulting equilibrated climatological mean ACC is also obtained accurately by the SSST.

These results demonstrate the validity of the SSST approximation. It is the first time that such agreement has been demonstrated.

3.2 Comparison with nonlinear simulations for a two layer ocean

Consider a two layer ocean, whose dynamics are described by (2.1). Treguier and Panetta (1994), hereafter TP94, have performed a fully non-linear integration of
Figure 3.4: The corresponding to Fig. 3.3 time average mean flow $U(y)$ that obtains at supercriticality $f/f_c = 20$ in the nonlinear (NL), quasilinear (QL) and SSST integration. Agreement between the NL, the QL and the SSST integrations demonstrates that the dynamics of the ACC are accurately captured by the SSST dynamics. (Unpublished)

(2.3)-(2.7). The model consists of a flat bottom, zonally reentrant channel that is 3000 km long and 2000 km wide in the meridional direction, which corresponds roughly to the region between Antarctica and the surrounding continents. The Rossby radius of deformation is taken to be $\lambda = 20$ km, the depth of the two layers is $H_1 = 500$ m and $H_2 = 3500$ m and the model is forced by a wind stress $\tau = 10^{-4}$ m$^2$s$^{-2}$ that is constant over a central region of width 1600 km. The stress is relaxed to zero close to the boundaries of the channel, on which zero normal velocity boundary conditions are imposed. The Ekman drag in the lower layer yields an e-folding time of the order of 100 days. After a short spin up, the baroclinic shear that develops due to the stress becomes unstable, the eddies start to grow and sustain a fully developed baroclinic turbulence. After a long integration, TP94 obtain a statistical steady equilibrium. The time and zonal mean velocities for the two layers are shown in Fig. 3.5, while the time mean
3.2. Comparison with nonlinear simulations for a two layer ocean

momentum balance for the two layers is shown in Fig. 3.6. The flow is equivalent barotropic, with two maxima in velocity occurring at 600 and 1400 km.

![Figure 3.5: Meridional profile of time and zonally averaged zonal flow, \(u\,\text{ms}^{-1}\) in each layer. Taken from Treguier and Panetta (1994).](image)

It is worth noting that the meridional scale of the jet does not seem to agree with the Rhines scale, as is typically found in jets in baroclinic turbulence. However, a reason for this discrepancy is not discussed in their study. In the upper layer, the momentum imparted by the stress is mainly balanced by the thermal fluxes, while the Reynolds stress has a small contribution in the momentum balance. In the lower layer, the Reynolds stress is insignificant and the heat fluxes transport the momentum in the lower layer that is then dissipated by Ekman drag. Compared to the observations, the current transport is overestimated, due to the lack of topography that provides through form drag a significant sink of momentum. On the other hand, the vertical structure of the velocities, as well as the
Figure 3.6: Momentum balance as a function of latitude in the upper (a) and lower (b) layers. Shown are the wind stress $W$, the Reynolds stress $R$, the interfacial form stress $D$ and bottom friction $B$, all in $\text{m}^2\text{s}^{-2}$. Taken from Treguier and Panetta (1994).
3.2. Comparison with nonlinear simulations for a two layer ocean momentum balance are in reasonable agreement with the observations.

![Figure 3.7: Meridional profile of the zonally averaged zonal flow, $U$ (ms$^{-1}$) in the upper (solid line) and lower (dash-dotted line) layer, as obtained by the SSST system. The depths of the two layers are $H_1 = 0.5$ km and $H_2 = 3.5$ km and $\lambda = 20$ km. The model is forced by a mean wind stress $\tau/\rho = 10^{-4}$ m$^2$s$^{-2}$ and a stochastic stress with amplitude $e_f = 30\tau/\rho$. The variable stress excites with equal variance the first 12 gravest zonal harmonics in the channel. The stress is considered to be correlated in the meridional direction in the shape of a Gaussian with a correlation scale of 80 km. The stochastic forcing is confined in the same central region of 1600 km that the mean stress in the TP94 model is. The Ekman drag is $E = 1.15 \times 10^{-7}$ s$^{-1}$ and the effective damping is $r_e = 8E$. (Unpublished)

We consider the SSST model with the same geometry and external parameters of TP94. We take the stochastic forcing to correspond to surface stress excitation from frontal gusts. The variable stress excites with equal variance the first 12 gravest zonal harmonics in the channel. The stress is considered to be correlated in the meridional direction in the shape of a Gaussian with a correlation scale of 80 km. The stochastic forcing is confined in the same central region of 1600 km that the mean stress in the TP94 model is. We choose a forcing amplitude that
corresponds to gusty winds that are 5 times larger than the value of the mean velocity that forces the current and an effective damping in both layers that is 8 times larger than the Ekman drag. The model is run from rest. After a few years of integration, the system reaches a fixed point that corresponds to a statistical equilibrium with steady current velocity and stationary eddy statistics. The zonal mean velocities for the two layers and the momentum balance for the two layers are shown in Fig. 3.7-3.8. Comparison of this equilibrium with TP94 (c.f Fig. 3.5-3.6), shows a very good agreement of the SSST model with the TP94 fully non-linear model. The velocity current structure as well as the momentum balance in both layers is essentially the same with small differences in the maximum velocities and in the maximum values of the momentum and heat flux divergencies. This agreement offers evidence that the SSST model provides a very good approximation of the fully non-linear dynamics, regarding both the mean current equilibria of the ACC and the corresponding stationary eddy statistics.
3.2. Comparison with nonlinear simulations for a two layer ocean

Figure 3.8: Upper (panel a) and lower layer (panel b) momentum balance as a function of latitude. Shown are the wind stress $W$, the Reynolds stress $R$, the interfacial form stress $D$ and bottom friction $B$, all in $m^2s^{-2}$. The parameters are as in Fig. 3.7. (Unpublished)
Chapter 4

Characteristics of the ACC equilibria

Having established that the SSST system can yield realistic mean current equilibria and eddy statistics compared to nonlinear simulations, we perform a large number of integrations of the SSST model and explore the sensitivity of the ACC characteristics to the physically relevant parameters in the model. These include the magnitude, $\tau_0$, and the meridional profile, $f(y)$, of the wind stress, the magnitude, $e_f$, and scale of the stochastic forcing (as defined by the retained zonal harmonics and the meridional correlation of the forcing), the first baroclinic radius of deformation, $\lambda$, the effective damping, $r_e$, the depth of the two layers and finally the form and the height of topography, $h$. Focus will be on the strength and the meridional and vertical structure of the jet, on the momentum balance in both layers and on the eddy characteristics, such as the RMS eddy velocity, $u_{\text{rms}}$, in both layers.

All integrations are performed on a square channel that is 100 baroclinic radii of deformation wide and long, with zero normal velocity boundary conditions at the lateral boundaries. The meridional grid on which we discretize the equations
has \( N = 64 \) points and numerical convergence of the results was checked by doubling the number of grid points. The Ekman drag and the meridional gradient of planetary vorticity are equal to \( E = 100 \text{ days}^{-1} \) and \( \beta = 1.4 \times 10^{-11} \text{ m}^{-1} \text{s}^{-1} \) respectively. For most experiments the layers have depths of \( H_1 = 500 \text{ m} \) and \( H_2 = 3500 \text{ m} \) and the baroclinic radius of deformation is \( \lambda = 20 \text{ km} \). The wind stress is constant across the central region of the channel and smoothly decays to zero close to the boundaries and has an amplitude \( \tau/\rho = 10^{-4} \text{ m}^2\text{s}^{-2} \), unless stated otherwise. For most experiments, the stochastic forcing is correlated over a distance of 80 km and has a magnitude \( e_f = 30\tau/\rho \) and the effective damping is \( r_e = 8E \). For the experiments involving topography, a sinusoidal profile were considered with variable height and scale. Relative details will be given in the corresponding section. The SSST model, as defined by Eqs. (2.14) and (2.15), is integrated from rest until it reaches a fixed point, in the cases in which such a fixed point exists. The fixed points were found to be unique for each set of parameter values, although a formal proof for their uniqueness has not been found at this point in time. However, by performing a large number of integrations starting with different initial conditions, we were unable to find multiple equilibria for the same parameter values. This is important because it indicates that the ACC is not expected to make a rapid transition to another state under the same external parameters. The equilibrated states can be qualitatively classified in three regimes.

### 4.1 Time dependent regime

This regime, that will be called regime I, corresponds to a stochastic forcing amplitude \( e_f < 14\tau/\rho \). For these low values of stochastic forcing, the SSST system reached a time dependent state rather than a fixed point. Figure 4.1 shows the current velocities in both layers as a function of latitude and time. We observe
Figure 4.1: Contours of upper (panel a) and lower layer (panel b) current zonal mean velocities as a function of latitude and time for the time dependent regime with $e_f = \tau/\rho$. Velocity units are ms$^{-1}$. The rest of the parameters are as in Fig. 3.7. (Unpublished)
that velocity maxima are reenforced or weakened in time, with the variations in the velocity of the lower layer being larger. The vertical shear varies with time as well, but remains at low values, so that the flow can be characterized as equivalent barotropic. The amplitude of the forcing, below which we could not find any fixed points, was found to be insensitive to changes in the parameters of the model. As a result, sufficient stochastic forcing is necessary for obtaining time independent mean flows. This stochastic excitation may result from endogenous sources such as the eddy-eddy interactions that are inherent in non-linear models of the ACC, but can also result in this case due to the random wind stresses imposed by wind gusts at the surface of the ocean, a process that is currently not included in the ACC models. The robust result found in this project that a random wind stress component can have significant impact in the ACC dynamics, indicates that this process should be incorporated in realistic models of the ACC.

## 4.2 Stress dominated regime

This regime, that will be called regime II, corresponds to a stochastic forcing amplitude $14\tau/\rho < \varepsilon_f < 45\tau/\rho$. For these intermediate values of stochastic forcing, the SSST system equilibrates to a steady current in statistical equilibrium with the eddy fluxes. Examples of these equilibria are shown in Fig. 3.7-3.8. The flow is equivalent barotropic with a vertical shear averaged over the channel of $0.1 \text{ m s}^{-1}$, in agreement with the observed structure of the ACC (Orsi et al. 1995, Firing et al. 2011). However, the current transport is overestimated compared to the observed, due to the simplified setting of the model.

The scale of the jet is mainly sensitive to the meridional profile of the wind stress. This was tested by taking a curved stress profile $f(y) = \sin(n\pi y/L_y)$ and calculating the corresponding equilibrated currents, that are illustrated in Fig. 4.2
Figure 4.2: Upper layer zonal mean velocity (solid line) as a function of latitude for a sinusoidal wind stress profile $f(y) = \sin(n\pi y/L_y)$ with $n = 1$ (panel a) and $n = 2$ (panel b). The corresponding wind stress profile $f(y)$ with its amplitude normalized to have the same maximum value as the zonal mean velocity is also shown for reference (dashed lines). The rest of the parameters are as in Fig. 3.7. (Unpublished)
4.3. Eddy dominated regime

for \( n = 1 \) and \( n = 2 \). While both the flat stress profile and the curved stress profile with \( n = 1 \) produce two maxima (although at different latitudes), the curved profile with \( n = 2 \), produces a jet with 2 maxima. The same holds for profiles with higher values of \( n \). Consequently, the jet scale does not bear any relation to the Rhines scale typically found in studies of baroclinic turbulence. The maxima in current speed for a flat stress profile can be pronounced with a difference between maximum to minimum zonal mean velocities of the order of 35 cm s\(^{-1}\), or they can be smooth undulations over the profile of the wind stress with a difference between maximum to minimum zonal mean velocities of the order of 15 cm s\(^{-1}\), depending on the effective damping and the strength of the forcing. An increase in the effective damping and in the strength of the forcing, yield more smoothed undulations around the profile imposed by the wind stress. This is illustrated in Fig. 4.3 showing the zonal mean velocity in the upper layer for the same parameters as in Fig. 3.7, but with a double effective damping.

In both layers the Reynolds stresses do not have a significant contribution in the momentum balance, in agreement with the observations. Instead, the thermal fluxes transport the momentum imparted by the surface stress to the bottom of the ocean, where it is dissipated by the Ekman drag. The rms velocity of the eddies is around 5 % of the current velocity in the upper layer and 1 % in the lower layer. Such low velocities are a result of the small deformation radius, as will be argued in the next section, and are another indication that momentum transport by the eddies plays a secondary role in the eddy-mean flow dynamics at equilibrium.

4.3 Eddy dominated regime

This regime, that will be called regime III, corresponds to a stochastic forcing amplitude \( 45\tau/\rho < e_f \). For these large values of stochastic forcing, the equilibria
Figure 4.3: Upper layer zonal mean velocity as a function of latitude for effective damping $r_e = 8 E$ (solid line) and $r_e = 16 E$ (dashed line) and for a flat bottom ocean. The corresponding velocity for effective damping $r_e = 8 E$ and a sinusoidal zonal ridge $h = 10 \beta \sin(2y/Ly)$ is also shown (dash-dotted line). The rest of the parameters are the same as in Fig. 3.7. (Unpublished)
are shown in Fig. 4.4-4.5. The flow is almost barotropic with a vertical shear averaged over the channel of just 0.06 m s$^{-1}$. There is a single strong eastward jet, followed by two weaker westward flanks. The scale of the jet is now insensitive to the profile of the wind stress and is also much lower than the Rhines scale.

The momentum that is imparted by the stress is now balanced in the upper layer by both heat and momentum fluxes that are comparable in magnitude. In the lower layer, the heat and momentum fluxes are of the same order, while momentum is dissipated by bottom friction. The rms velocity of the eddies is now increased to around 25 % of the current velocity in the upper layer and 8 % in the lower layer.

The sensitivity of the equilibria to changes in the wind stress were then tested, as we expect a transition from one regime to the other. The amplitude of the wind stress was changed and keeping all other parameters the same, including the

**Figure 4.4:** Meridional profile of the zonally averaged zonal flow, $U$ (ms$^{-1}$) in the upper (solid line) and lower (dash-dotted line) layer, as obtained by the SSST system for a forcing strength $e_f = 60	au/\rho$. The rest of the parameters are as in Fig. 3.7. (Unpublished)
Figure 4.5: Upper (panel a) and lower layer (panel b) momentum balance as a function of latitude for the parameters in Fig. 4.4. Shown are the wind stress $W$, the Reynolds stress $R$, the interfacial form stress $D$ and bottom friction $B$, all in $\text{m}^2\text{s}^{-2}$. (Unpublished)
amplitude of the stochastic forcing, the corresponding equilibria were obtained. Figure 4.6 shows the maximum velocity in the upper layer and the vertical shear averaged over the channel as a function of the mean wind stress amplitude. We observe that both the current transport and the vertical shear increase linearly with the wind stress in regime II, while they are rather insensitive to changes in the wind stress in regime III. Figure 4.7 shows the rms velocity in the upper layer as a function of the mean wind stress amplitude. We observe that the rms velocity stays almost constant in regime III, while it increases rapidly in regime II. We therefore conclude that regime II is controlled by the wind stress, while regime III has the characteristics of an eddy saturated regime.

In addition, we performed a second set of integrations, by keeping the mean stress constant and changing the amplitude of the stochastic forcing. A similar transition between the two regimes occurs. The only difference is that the eddy
fluxes change in amplitude. Figure 4.8 shows the rms velocity as a function of the forcing amplitude. We can see that the rms velocity increases as $u_{\text{rms}} \sim \epsilon_f^4$.

### 4.4 Sensitivity to other parameters

In this section we explore the sensitivity of the obtained results to changes in the rest of the parameters in the model. Meridional and zonal coherence of the forcing does not influence the obtained results, apart from small quantitative differences. On the other hand, the amplitude of the effective damping, $r_e$, influences both the structure of the ACC and the eddy statistics. In regime II, it controls the maximum to minimum velocity ratio of the jet in the case of a constant stress profile, as seen in the previous section. It also controls the vertical shear, with small $r_e$ yielding almost barotropic flows. The reason is that for this low value of deformation radius, a baroclinic flow becomes unstable for very low values of shear.
4.4. Sensitivity to other parameters

Figure 4.8: Rms velocity at the upper layer as a function of the stochastic forcing amplitude (circles). A line of slope 4 is also shown for reference. The rest of the parameters are as in Fig. 3.7. (Unpublished)

This can be seen by a linear stability analysis of the baroclinic mean flow. We consider the simple case of a baroclinic flow with constant velocities in both layers $U_1$ and $U_2$. Small perturbations evolve according to (2.3)-(2.4), with the forcing terms on the right hand side taken to be zero. By considering modal solutions of the form $q'_i = Ae^{ikx+ily-ikct}$, we obtain the non-dimensional dispersion relation (Pedlosky 1990):

$$c = U_2 + U_s \frac{K^2(K^2 + 1) - \beta(2K^2 + 1)}{2K^2(K^2 + 1)} \pm \frac{\sqrt{\beta^2 - K^4U_s^2(1 - K^4)}}{2K^2(K^2 + 1)}, \quad (4.1)$$

where $U_s = U_1 - U_2$ is the vertical shear and $K = \sqrt{k^2 + l^2}$ is the total wavenumber. The flow is unstable if small perturbations grow exponentially, that is if $c$ has a positive imaginary part. It can readily shown that the maximum growth rate is obtained for $K = 2^{-1/4}$. For this total wavenumber, the flow is unstable for $U_s \geq 2\beta$ that corresponds to a shear of 0.01 ms$^{-1}$. In the absence of effective
damping and a meridional structure in the flow, the vertical shear at equilibrium cannot exceed this value, as the statistical equilibrium state is perturbation stable. A non-zero value of effective damping allows for a larger vertical shear, as the stability boundary is pushed towards larger values of $U_s$. However, for low values of $r_e$, the critical shear does not change much, and as a result the flow at equilibrium is almost barotropic. This result, also implies that the baroclinic shear observed in the non-linear integrations of TP94, requires an effective damping with a dissipation time scale of the order of 12 days (c.f. Fig. 3.7). Finally, it is worth noting that the value of the effective damping also influences the rms velocity of the eddies, as larger values of effective damping dissipate the eddies and lower their rms velocity, as well as the Reynolds stress contribution in the momentum balance.

The first baroclinic radius of deformation changes the static stability of the ocean. As a result, a larger $\lambda$ yields a more stably stratified ocean that is also more stable to baroclinic instability. As a result, the vertical shear at equilibrium increases, along with the maximum velocity, as shown in Fig. 4.9 that illustrates the maximum velocity and equilibrium and the vertical shear as a function of the radius of deformation. The eddies at equilibrium are also more energetic. As a result both the $u_{\text{rms}}$ and the contribution of the Reynolds stresses in the momentum balance increase. Figure 4.10 shows the rms velocity of the eddies as a function of $\lambda$ that follows the power law $u_{\text{rms}}/\max(U) \sim \lambda$. It is worth noting that both this power law and the additional power law $u_{\text{rms}} \sim e^{4f}$ can be traced to the non-normality of the flow (Farrell and Ioannou 2008) and are generic to strongly turbulent equilibria (Held and Larichev 1996, Barry et al. 2002, Zurita-Gotor 2007).

Finally, we performed a large set of experiments with simplified zonally invariant topography $h = a \sin(ny/L_y)$, where $a$ is the amplitude of these zonal
4.4. Sensitivity to other parameters

Figure 4.9: Zonal mean velocity at the upper layer as a function of the baroclinic radius of deformation. The rest of the parameters are as in Fig. 3.7. (Unpublished)

Figure 4.10: Rms velocity at the upper layer as a function of the baroclinic radius of deformation. The rest of the parameters are as in Fig. 3.7. (Unpublished)
ridges and \( n \) defines the variation with latitude. Thompson (2010) found that in non-linear simulations of baroclinic turbulence in a doubly-periodic channel that is driven by a given shear flow, topography of a scale larger than the baroclinic radius of deformation had significant effects. For a certain set of parameters and a flat bottom ocean, Thompson (2010) found statistically steady jets. However, when a zonally invariant topography was included, the system experiences instead continuous jet formation and merger events contributing to a braided jet structure. This phenomenon was attributed to the local change of the planetary vorticity gradient that is caused by topography. We varied the amplitude \( a \) in the range \([0.01\beta, 10\beta]\) and \( n \) in the range \([1, 6]\). However, we were unable to find time dependent states, except for parameters that are close to the boundary between regimes I and II. Instead, we obtain an asymmetric equilibrium, whose zonal mean velocity is enhanced in the region of the ridge. This is illustrated in Fig. 4.3, showing the upper layer velocity. We therefore conclude that we must be in a different range of parameters compared to Thompson (2010).
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Chapter 5

Conclusions

The Antarctic Circumpolar Current (ACC) is the world’s strongest ocean current. Despite its essential role in the climate system, the ACC remains poorly understood. The reason is that a comprehensive understanding of the dynamics governing the intricate balance between the small scale eddies and the mean current, on which the structure of the current sensitively depends, is currently lacking. In this project, a theory of the ACC that determines both the important mechanisms that govern its dynamics and makes computationally inexpensive but nevertheless reliable predictions for its structure under diverse conditions was developed. This is important because this theory can guide us to determine the sensitivity of the ACC to external changes that our climate system may undergo under anthropogenic or other influences.

This theory was built on results from stochastic turbulence modeling that is called Stochastic Structural Stability Theory (SSST). In the context of SSST, interaction between the eddies and the mean flow can be well approximated using a Stochastic Turbulence Model (STM) in which the eddies draw most of their energy from the mean flow while their sources are represented as stochastic forcing and effective eddy dissipation. The STM provides an analytic method to obtain the
quadratic statistics of the eddy field for a given mean flow structure. The average large scale flow is then forced by the momentum flux divergence, obtained from the STM, producing a closed set of eddy-mean flow equations. The equilibria of the SSST system are the steady large scale flow and the stationary eddy statistics.

A quasi-geostrophic, two layer baroclinic model with simplified topography and surface wind stress forcing, that is the simplest possible while retaining the dynamics that are relevant to the ACC, was considered. The SSST system for this model was formulated and a code for its numerical integration was developed. The accuracy of the approximations inherent in the SSST model was then extensively verified for an ocean with one and two layers. In addition, the SSST model was found to produce realistic statistical equilibria of the ACC that correspond to its structure and its stationary eddy statistics, when compared to fully non-linear simulations. As a result, we have obtained a model with the following advantages: the eddy-mean flow dynamics in the model can be comprehensively understood as the eddy-eddy nonlinearity that might render these dynamics opaque are conveniently parameterized; the model is computationally inexpensive compared to a fully non-linear simulation and allows for a full exploration of its behavior under changes in its parameters yet producing realistic equilibria, comparable to the corresponding non-linear simulations.

We then obtained the equilibria of the SSST model for a large number of simulations, covering a very wide range of the physically relevant parameters in the model, thus capturing the behavior of the ACC under very diverse conditions. This led to a better understanding of the eddy-mean flow dynamics underlying the structure of the ACC and to a full exploration of the response of the ACC under different surface stress conditions. The main result is that there are three regimes that depend on the ratio of the amplitude of the time variation over the amplitude of time mean stress. A statistical equilibrium is obtained only when this ratio has
moderate or large values. We have found that for the same parameter values there are no multiple equilibria, which rules out abrupt changes of the climate of the ACC ceteris paribus.

For moderate values of this ratio, we obtain an ACC structure that is sensitive to the wind stress. The meridional structure follows or is close to the wind stress profile and the transport, as well as the vertical shear increase linearly with the magnitude of the stress for a given stochastic forcing amplitude. The flow is equivalent barotropic or even barotropic, depending on the amplitude of the effective damping and mostly the eddy heat fluxes balance the imposed stress, while the eddies have low amplitude and their momentum fluxes do not contribute significantly to the momentum transport. For large values of this ratio, the ACC structure is weakly sensitive to the wind stress. Both the magnitude of the transport and the scale of the jet do not depend significantly on the magnitude or the meridional profile of the jet. Furthermore, the eddies have large kinetic energy and their momentum fluxes is equal important to their heat fluxes in balancing the imposed stress.

While the details of the jet and the eddy statistics are determined by the values of the rest of the parameters, like the effective damping, the baroclinic radius of deformation and the inclusion of topography, the qualitative characteristics of these two regimes remain unchanged.


