Analysis of the Spontaneous Mass Generation by Using an Iterative Method

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Introduction

The standard method to discuss the spontaneous mass generation is to formulate a coupled system of self-consistent equations.

- The non-trivial solution is interpreted as a sum of an infinite number of bubble diagrams.
- Dose actually the sum of an INFINITE number of diagrams generate the FINITE (non-vanishing) mass?
- Those equations are no more than the *necessary* condition and it is needed to examine solutions to select correct one by using another mean. (e.g. by referring to the free energy of each solution.)

We adopt the Nambu–Jona-Lasinio(NJL) model and give a new iteration method that directly sums up an infinite number of diagrams in the ladder approximation.

Iteration Method

Introduce a bare mass

$$\mathscr{L}_{\text{NJL}} = \bar{\psi}\partial\!\!\!/\psi + \frac{2\pi^2 g}{N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right] - m_0\bar{\psi}\psi$$

- NJL model has four-fermion interactions among the massless fermions with the chiral invariance.
- Adding the bare mass m_0 to the Lagrangian to make the standard perturbation theory work well.
- N is the number of fermion flavors. We consider $1/N$ leading contribution to the mass.

Iteration Method

Introduce "node length"

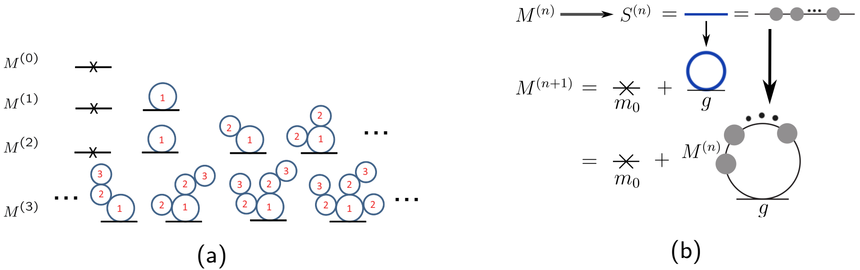


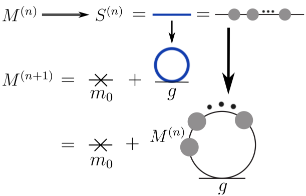
Fig. 1. Node length. (a) Definition. (b) Node length iteration.

- We classify diagrams using the node length of each diagram.
- Node length of a diagram is defined by the maximum number of loops in a continuous route towards the edge loop.
- We define $M^{(n)}$ is a sum of diagrams whose node length is no greater than n.

Transformation function

$$M^{(n+1)} = F\left(M^{(n)}\right),$$
$$\textit{where } F(M) = m_0 + gM\left(1 - M^2 \log\left(1 + M^{-2}\right)\right).$$

- The transformation function is a one loop integral and we denote it by F.
- The total sum of the tree diagrams is obtained by $M^{(\infty)}$, infinitely many times of transformation of the same F.



Iterative steps

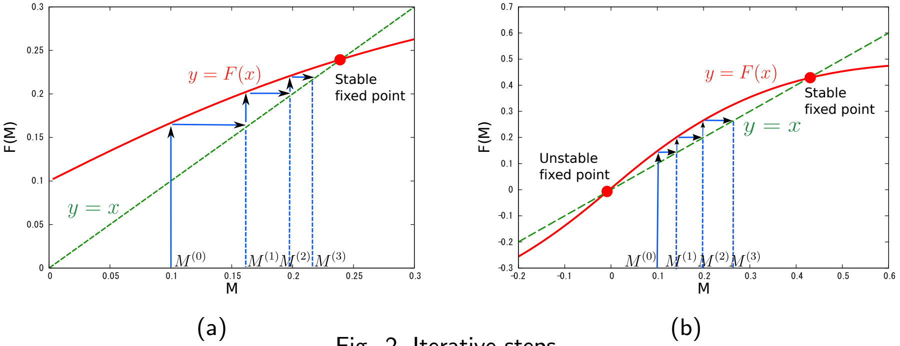


Fig. 2. Iterative steps.

(a) Weak coupling : $g = 0.7$. (b) Strong coupling : $g = 1.5$.

- In any case the iterative transformation finally reaches a stable fixed point.
- In the weak coupling region, there is only one fixed point.
- In the strong coupling region, there appear pair creation of fixed points.

Fixed Point Structure

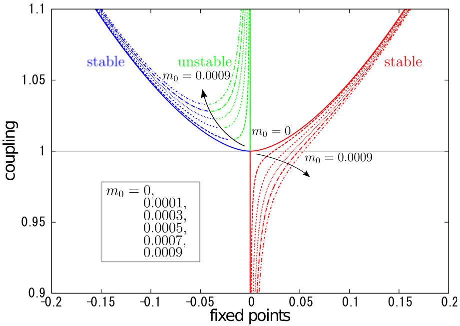
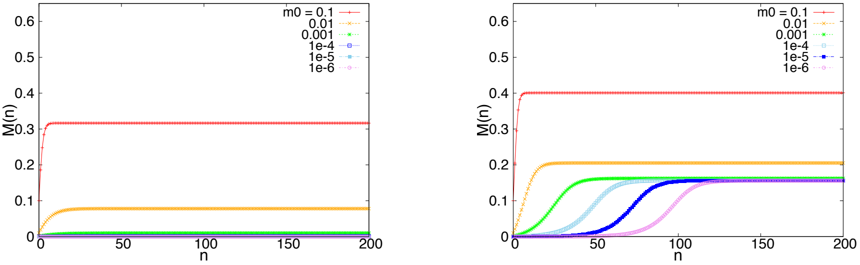


Fig. 3. Fixed point structure with the coupling constant and the bare mass.

- We set a positive value for the bare mass, then the initial point is in the territory of the right-hand side stable fixed point.
- Therefore for all region of the coupling constant, the physical result is controlled by the right most stable fixed point.
- The critical coupling constant for the change of fixed point structure depends on the bare mass and it unifies in the vanishing bare mass limit.

Mass Generation



(a) (b)

Fig. 4. Mass generation by the node length iteration.
(a) Weak coupling : $g = 0.9$. (b) Strong coupling : $g = 1.1$.

- In the weak coupling, the dynamical mass is generated rather quickly at low n and becomes constant.
- In the strong coupling, the generation of the dynamical mass depends strongly on the bare mass, and it is mainly generated at some narrow range of node length.

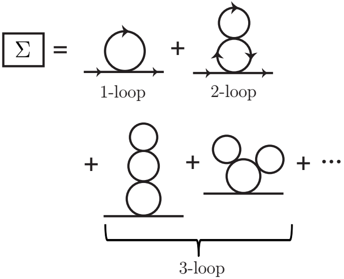
Summary

- Adding a bare mass m_0 to NJL model, we define the node length iteration.
- Infinite diagrams are summed up explicitly by using iterative method according to the node length.
- The results correctly give the physical mass.

References

- Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); **124**, 246 (1961).
- K-I. Aoki, M. Bando, T. Kugo, M.G. Mitchard and H. Nakatani, *Prog. Theor. Phys.* **84**, 683 (1990).
- K-I. Aoki, S. Onai and D. Sato in preparation.

We calculate the self-energy Σ in perturbation theory.



$$I = \mathrm{Tr}\int d^4 dp \frac{1}{\not p}$$

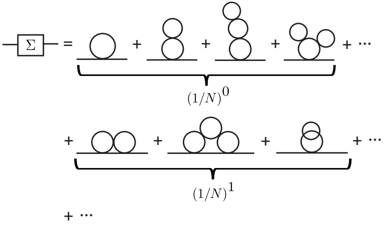
The loop part is not affected by the momentum from external line.

$$I = \mathrm{Tr}\Omega_4 \int d^3 dp \frac{\not p}{p^2}, \ \mathrm{Tr}\gamma^\mu = 0$$

For the reason of containing at least one I at the edge of every diagrams, each part becomes zero in the end. It means that all part of diagrams do not receive perturbative contributions.

⇒ Mass cannot be generated in the perturbation theory.

Perturbation Theory



- A $1/N$ factor comes out through the interaction at the vertex as the coupling constant is proportional to $1/N$.
- On account of the flavor number, a factor N appears for each loop.