Quantum Gravity within Quantum Field Theory

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   ○ ensures the absence of UV-divergences
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d) question of unitarity
   - information loss in black holes?
The phase diagram of Asymptotic Safety

The phase diagram of Causal Dynamical Triangulations

J. Ambjørn, J. Jurkiewicz, R. Loll; D. Benedetti, J. Cooperman, . . .
Once upon a time there was a . . . puzzle

FRGE and Dynamical Triangulations investigate the same path integral
continuum functional renormalization group (FRGE):

- covariant computation, Euclidean signature
  - non-Gaussian fixed point (NGFP)
  - classical general relativity recovered at $\ell \approx 10\ell_{Pl}$
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Monte Carlo Simulation of gravitational partition sum

- Causal Dynamical Triangulations (CDT)
  - second order phase transition line
  - “classical universes” at $\ell \approx 10\ell_P$

- Euclidean Dynamical Triangulations (EDT)
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How does a causal structure influence Asymptotic Safety?
Functional Renormalization Group Equation

for foliated spacetimes
Foliation structure via ADM-decomposition

Preferred “time”-direction via foliation of space-time

- foliation structure $\mathcal{M}^{d+1} = S^1 \times \mathcal{M}^d$ with $y^\mu \mapsto (\tau, x^a)$:

\[ ds^2 = N^2 dt^2 + \sigma_{ij} (dx^i + N^i dt) (dx^j + N^j dt) \]

- fundamental fields: $g_{\mu\nu} \mapsto (N, N_i, \sigma_{ij})$

\[ g_{\mu\nu} = \begin{pmatrix} N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix} \]
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- foliation structure $\mathcal{M}^{d+1} = S^1 \times \mathcal{M}^d$ with $y^\mu \mapsto (\tau, x^a)$:

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$$g_{\mu\nu} = \begin{pmatrix}
\epsilon N^2 + N_i N^i & N_j \\
N_i & \sigma_{ij}
\end{pmatrix}$$

Allows to include signature parameter $\epsilon = \pm 1$
Foliated functional renormalization group equation

Flow equation: formally the same as in covariant construction

\[ k \partial_k \Gamma_k [h, h_i, h_{ij}; \bar{\sigma}_{ij}] = \frac{1}{2} \mathrm{STr} \left[ \left( \Gamma_k^{(2)} + R_k \right)^{-1} k \partial_k R_k \right] \]

- covariant: \( \mathcal{M}^4 \)

\[ \mathrm{STr} \approx \sum_{\text{fields}} \int d^4 y \sqrt{g} \]

- foliated: \( S^1 \times \mathcal{M}^3 \)

\[ \mathrm{STr} \approx \sqrt{\epsilon} \sum_{\text{component fields}} \sum_{\text{KK–modes}} \int d^3 x \sqrt{\bar{\sigma}} \]

structure resembles: quantum field theory at finite temperature!
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- structure resembles: quantum field theory at finite temperature!

Advantages of the foliated flow equation:

- \( \epsilon \)-dependence: keep track of signature effects
- structure: same as in Causal Dynamical Triangulations
Comparison: phase diagrams for ADM-variables

\[ \Gamma_k^{\text{ADM}} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int d\tau d^3x \sqrt{\sigma} \left[ \epsilon^{-1} \left( K_{ij} K^{ij} - K^2 \right) - R(3) + 2\Lambda_k \right] + S_{gf} + S_{gh} \]
It’s all about choosing a gauge:

**covariant formulation:**

\[ g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu} \]

perform covariant gauge-fixing (e.g., harmonic gauge)

\[ F_\mu = \tilde{D}^\nu h_{\mu\nu} - \frac{1}{2} \tilde{D}_\mu h_{\nu\nu} = 0. \]

**foliated formulation with ADM-fields** \( g_{\mu\nu} \mapsto \{ N, N_i, \sigma_{ij} \} \)

\[ N = \tilde{N} + h, \quad N_i = \tilde{N}_i + h_i, \quad \sigma_{ij} = \tilde{\sigma}_{ij} + h_{ij} \]

perform temporal gauge-fixing (non-covariant):

\[ h = 0, \quad h_i = 0 \]

- fluctuations in the metric on the spatial slice only
It’s all about choosing a gauge:

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- fluctuations in the metric on the spatial slice only

ADM fields in temporal gauge

No fluctuations in stacking spatial slices!
Symmetries conserved by the foliated FRGE

fundamental fields: \( \{ \tilde{N}(\tau, x), \tilde{N}_i(\tau, x), \tilde{\sigma}_{ij}(\tau, x) \} \)

symmetry: general coordinate invariance inherited from \( \gamma_{\mu\nu} : \)

\[ \delta \gamma_{\mu\nu} = \mathcal{L}_v(\gamma_{\mu\nu}), \quad v^\alpha = (f(\tau, x), \zeta^a(\tau, x)) \]

induces

\[ \delta \tilde{N} = f \partial_\tau \tilde{N} + \zeta^k \partial_k \tilde{N} + \tilde{N} \partial_\tau f - \tilde{N} \tilde{N}^i \partial_i f, \]

\[ \delta \tilde{N}_i = \tilde{N}_i \partial_\tau f + \tilde{N}_k \tilde{N}^k \partial_i f + \tilde{\sigma}_{ki} \partial_\tau \zeta^k + \tilde{N}_k \partial_i \zeta^k + f \partial_\tau \tilde{N}_i + \zeta^k \partial_k \tilde{N}_i + \epsilon \tilde{N}^2 \partial_i f \]

\[ \delta \tilde{\sigma}_{ij} = f \partial_\tau \tilde{\sigma}_{ij} + \zeta^k \partial_k \tilde{\sigma}_{ij} + \tilde{N}_j \partial_i f + \tilde{N}_i \partial_j f + \tilde{\sigma}_{jk} \partial_i \zeta^k + \tilde{\sigma}_{ik} \partial_j \zeta^k \]

- Non-linearity of ADM-decomposition: symmetry realized **non-linearly**
Symmetries conserved by the foliated FRGE

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- Non-linearity of ADM-decomposition: symmetry realized non-linearly

- in ADM it is impossible to combine:
  - linear background field method
  - regulator $\Delta_k S$ quadratic in fluctuation fields
  - background Diff($\mathcal{M}$)-symmetry
Symmetries conserved by the foliated FRGE

background symmetry respected by FRGE:

• subgroup of linear transformations

\[
\delta \tilde{N} = f \partial_\tau \tilde{N} + \zeta^k \partial_k \tilde{N} + \tilde{N} \partial_\tau f - \tilde{N} \tilde{N}^i \partial_i f, \\
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\]

• foliation-preserving diffeomorphisms: \( \text{Diff}(\mathcal{M}, \Sigma) \subset \text{Diff}(\mathcal{M}) \)

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\delta \gamma_{\mu\nu} = \mathcal{L}_v(\gamma_{\mu\nu}), \quad v^\alpha = (f(\tau), \zeta^\alpha(\tau, x))
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Symmetries conserved by the foliated FRGE

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symmetry group of Hořava-Lifshitz gravity
Wetterich Equation

for projectable Hořava-Lifshitz gravity

[M. Baggio, J. de Boer and K. Holsheimer, arXiv:1112.6416]
central idea: find a perturbatively renormalizable quantum theory of gravity

fundamental fields: \( \{ N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x) \} \)

symmetry: \( \text{Diff}(\mathcal{M}, \Sigma) \subset \text{Diff}(\mathcal{M}) \)

- spatial higher-derivative terms make theory power-counting renormalizable
- anisotropic dispersion relation breaks Lorentz-invariance
projective Hořava-Lifshitz gravity in a nutshell


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Can construct the effective average action for projectable HL-gravity

- scale-dependence governed by functional renormalization group equation

\[
k \partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2} \text{Str} \left[ \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k \partial_k \mathcal{R}_k \right]
\]

- Complication: anisotropic models have two correlation lengths
Relation between Asymptotic Safety and Hořava-Lifshitz gravity

Theory space: Hořava-Lifshitz
Symmetry: foliation preserving

Subspace: Quantum Einstein Gravity
Symmetry: diffeomorphisms

also see: talk by G. D’Odorico tomorrow
Proposals for UV fixed points (incomplete...)

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in $G_N$
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RG-flows of Hořava-Lifshitz gravity in the IR

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

RG-flow of anisotropic $\lambda$-$R$ truncation

$$\Gamma^{\text{grav}}_k[N, N_i, \sigma_{ij}] = \frac{1}{16\pi G_k} \int d\tau d^3x N \sqrt{g} \left[ K_{ij} K^{ij} - \lambda_k K^2 - (^{(3)} R + 2\Lambda_k) \right]$$
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Fixed points of the beta functions:

- Wheeler-de Witt metric $\Rightarrow$ line of GFPs
  $$\tilde{G}_* = 0, \quad \tilde{\Lambda}_* = 0, \quad \lambda = \lambda_*$$
  - one IR attractive, one IR repulsive, one marginal direction

- NGFP:
  $$\tilde{G}_* = 0.49, \quad \tilde{\Lambda}_* = 0.17, \quad \lambda_* = 0.44$$
  - three UV-attractive eigen-directions
  - imprint of Asymptotic Safety
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anisotropic GFP providing UV-limit of HL-gravity not in truncation
Hořava-Lifshitz gravity: recovering general relativity in the IR
Scale-dependence of dimensionful couplings

![Graphs showing scale-dependence of dimensionful couplings](image)
Scale-dependence of dimensionful couplings

GFP governs IR-behavior of HL-gravity
small value of cosmological constant makes $\lambda$ compatible with experiments
Summay

Asymptotic Safety Program

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  - predictive: finite number of relevant parameters
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Connecting the FRG to CDT

- Constructed FRG probing CDT theory space
- prospects of comparing RG flows
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Connection to Hořava-Lifshitz gravity

- use different RG fixed points for continuum limit
- FRGE: key tool for establishing renormalizability
Outlook

many proposals for quantum gravity within QFT:

- Asymptotic Safety
- (Causal) Dynamical Triangulations
- Hořava-Lifshitz gravity
- first order formalism
- shape dynamics

differences:

- field content (metric, vielbein, ADM-variables, . . .)
- symmetry group (diffeomorphisms, foliation preserving diff.)

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RG techniques crucial in all models!