A beyond the local potential approximation study for the dynamical chiral symmetry breaking in effective model of QCD

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4. Summary
Introduction
The lattice simulation has the sign problem.

Our aims:
• Approach to DχSB at finite temperature and density.
• Beyond the mean field approximation.
Analysis Method
Model

Quark-Meson model with O(4) symmetry

$$\Gamma_\Lambda = \int_x \left\{ Z_\psi (\Phi^2) \bar{\psi} (\gamma^\mu \partial_\mu + i \gamma_0 \mu) \psi + \frac{1}{2} Z_\phi (\Phi^2) (\partial_\mu \Phi)^2 + U(\Phi^2) + \frac{h(\Phi^2)}{\sqrt{2}} \bar{\psi} (\sigma + i \gamma_5 \tau^i \pi_i) \psi \right\}$$

- $$\Phi^2 = \sigma^2 + \vec{\pi}^2$$
- LPA: $$Z_\psi = 1, Z_\phi = 1$$ at all RG scale.
- The fermion field has degrees of freedom of color.
Background field method

- Split up the meson field into a background field $\phi$ and a fluctuation field $\varphi$, i.e. $\Phi = \phi + \varphi$.

- Expand couplings around the background field $\phi$:
  
  \[
  U(\Phi^2) = U(\phi^2) + U''(\phi^2)\varphi^2 + O(\varphi^4) \\
  \bar{h}(\Phi^2) = \bar{h}(\phi^2) + \bar{h}''(\phi^2)\varphi^2 + O(\varphi^4) \\
  Z_\psi(\Phi^2) = Z_\psi(\phi^2) + Z_\psi''(\phi^2)\varphi^2 + O(\varphi^4) \\
  Z_\phi(\Phi^2) = Z_\phi(\phi^2) + Z_\phi''(\phi^2)\varphi^2 + O(\varphi^4)
  \]

  ✚ Couplings depend on the background field $\phi$.
  ✚ Neglect the derivative terms of $\bar{h}$, $Z_\phi$, $Z_\psi$ for simplicity.
Meaning of $h(\phi^2)$

“Yukawa coupling $h(\phi^2)$” includes higher order operators:

$$\bar{h}(\phi^2) = \bar{h}^{(0)} + \bar{h}^{(2)} \phi^2 + \bar{h}^{(4)} \phi^4 + \cdots$$

Yukawa interactions

$$\frac{\bar{h}(\phi^2)}{\sqrt{2}} \bar{\psi}(\sigma + i\gamma_5 \sigma^i \pi_i) \psi = \frac{\bar{h}^{(0)}}{\sqrt{2}} \bar{\psi}(\sigma + i\gamma_5 \sigma^i \pi_i) \psi + \frac{\bar{h}^{(2)}}{\sqrt{2}} \phi^2 \bar{\psi}(\sigma + i\gamma_5 \sigma^i \pi_i) \psi + \cdots$$

$\bar{h}(\phi^2)$ $\bullet \cdots$ $= \bar{h}^{(0)} \cdot \cdots$ $+ \bar{h}^{(2)} \cdot \cdots$ $\cdots$
**RG equations**

- **Wetterich equation:**

\[
\partial_t \Gamma_\Lambda = \frac{1}{2} \text{tr} \left[ \left( \frac{\delta^2}{\delta \phi^2} \Gamma_\Lambda + R_\Lambda^\phi \right)^{-1} \cdot (\partial_t R_\Lambda^\phi) \right] - \text{tr} \left[ \left( \frac{\delta}{\delta \psi} \Gamma_\Lambda \frac{\delta}{\delta \psi} + R_\Lambda^\psi \right)^{-1} \cdot (\partial_t R_\Lambda^\psi) \right]
\]

- **Use 3d optimized cut-off function:**

\[
R_\Lambda^\phi = p^2 \left( \frac{\Lambda^2}{p^2} - 1 \right) \theta(1 - \frac{p^2}{\Lambda^2}) \quad R_\Lambda^\psi = p \left( \frac{\Lambda}{|p|} - 1 \right) \theta(1 - \frac{p^2}{\Lambda^2})
\]

- **Obtain four RG equations:**

\[
\partial_t A_\Lambda(\phi^2) = \beta_A(\phi^2, U, U', U'', Z_\phi, Z_\psi, h; \Lambda, T, \mu) \quad U' = \frac{\partial U}{\partial \phi^2}
\]

\[
A \in \{ U(\phi^2), h(\phi^2), Z_\phi(\phi^2), Z_\psi(\phi^2) \}
\]
Solving RG equations

How to solve the RG equations?

- We solve numerically RG equations as the coupled partial differential equations.

In the numerical calculations, we use the grid method:

- The derivatives $U'$ and $U''$: the 7-point formula
- The derivative by scale: fourth-order Runge-Kutta method

Setting initial values:

- The field renormalization factors $Z_\psi = Z_\phi = 1$ at $\Lambda = \Lambda_0$.
- The initial values of $\bar{h}$, $\Lambda_0$, $U(\phi^2)$ yield a vacuum pion decay constant of $f_\pi \sim 87$ MeV.
- Chiral limit
Numerical Results
RG evolution of potential in LPA without running of Yukawa coupling

\[ \Lambda/\Lambda_0 = 1 \]
\[ \Lambda/\Lambda_0 = 0 \]
\[ \Lambda/\Lambda_0 = 0.25 \]
\[ \Lambda/\Lambda_0 = 0.355 \]
\[ \Lambda/\Lambda_0 = 0.088 \]
\[ T = 140 \text{ MeV} \]
\[ \mu = 0 \]
RG evolution of potential in LPA without running of Yukawa coupling

\[ T = 45 \text{ MeV} \]
\[ \mu = 253.9 \text{ MeV} \]
Phase transition

2nd order

1st order

\( T = 143 \text{ MeV} \)
\( T = 142 \text{ MeV} \)
\( T = 140 \text{ MeV} \)

\(\langle \phi \rangle = 0\)
\(\langle \phi \rangle \neq 0\)

\( T = 45 \text{ MeV} \)

\(\mu = 254 \text{ MeV}\)
\(\mu = 253.9 \text{ MeV}\)
\(\mu = 253.5 \text{ MeV}\)

\(\langle \phi \rangle = 0\)
\(\langle \phi \rangle \neq 0\)
Chiral phase diagram in LPA case

\[ T_{cr} = 52 \text{ MeV} \]
\[ \mu_{cr} = 251 \text{ MeV} \]
Phase diagram in LPA vs. beyond

- The chiral restoration temperature and density become lower than LPA case.
- In our method, we could find the critical end point.
- However, we could not evaluate low temperature/high density region.
Anomalous dimension for boson

The beta function of $Z\phi$ doesn’t depend on $Z\phi$.

$\partial_t Z_\phi = \text{(fermion loop)}$

The beta function of $Z_\phi$ doesn’t depend on $Z_\phi$. 
We investigate Quark-Meson model with $O(4)$ at finite temperature and density.

- LPA
  - $T_{\text{cri}} = 52$ MeV $\mu_{\text{cri}} = 251$ MeV

- Beyond LPA
  - $T_{\text{cri}} = 61$ MeV $\mu_{\text{cri}} = 180$ MeV

We want to improve the analysis methods.

- How to investigate the low-temperature and high density region?

Extend Quark-Meson model to bosonized NJL model.