Phase structure, Thermodynamics and Fluctuations in QCD

Bernd-Jochen Schaefer

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Agenda

• Phase transitions and QCD
• QCD-like model studies
  ➔ chiral and deconfinement aspects
• Significance of Fluctuations
Experiments: Heavy-Ion Collision

aim: create hot and dense QCD matter ➔ understanding strongly correlated systems

QCD under extreme conditions: very active field ➔ see e.g. FAIR construction (2014)

Goals of HIC Experiments: learn QCD matter Equation of State

Understanding fundamental phenomena:

• color confinement
• nature of chiral & deconfinement transition
• early Universe history
• nuclear matter
• properties of stars
• ...
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FAIR construction start 2012

Aug.2014
Quantum Chromodynamics

Strongly-interacting matter: non-Abelian SU(3)_c gauge theory

Lagrangian (without gauge fixing):

\[
\mathcal{L}_{\text{QCD}} = \bar{\psi} \left( i \slashed{D}_a T_a - m - \mu f \gamma_0 \right) \psi - \frac{1}{4} F_{\mu \nu}^a F_{\mu \nu, a}
\]

quark masses (input Electroweak)\hspace{1cm} chemical potentials

\[
D^\mu_a = \partial^\mu + ig A^\mu_a
\]

quark fields\hspace{1cm} gauge fields

covariant derivative: \hspace{1cm} gauge field tensor:

\[
F_{\mu \nu}^a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - g f^{abc} A^\mu_b A^\nu_c
\]

Partition function:

\[
Z(T, \mu_f) = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi e^{-\int_0^\beta d\tau d^3 x \mathcal{L}_{\text{QCD}}(\bar{\psi}, \psi, \phi)}
\]

[S. Bethke, 2012]
Quantum Chromodynamics

QCD at finite temperatures and densities

→ “transitions” partial deconfinement & partial chiral symmetry restoration

For physical quark masses: smooth phase transitions → deconfinement: analytic change of d.o.f.

→ associated global QCD symmetries only exact in two mass limits:

1.) infinite quark masses → center symmetry:

Order parameter: VEV of traced Polyakov loop

(alternatives: dual observables, e.g. dressed Polyakov loop)

\[ \Phi = \langle l(x) \rangle = \exp(-\beta F_q) \quad ; \quad \bar{\Phi} = \langle l^\dagger(x) \rangle = \exp(-\beta F_{\bar{q}}) \]

Free energy \( F_q \) of a static quark (anti-quark) in hot gluonic medium

confined (disordered) phase
- free energy diverges
- Polyakov loop vanishes
- correlations vanishes
deconfined (ordered) phase
- free energy finite
- Polyakov loop non-vanishing
- correlations finite

[ Gattringer et al. 06/07]
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\[ SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A \]

\[ SU(N_f)_{L+R\equiv V} \times U(1)_B \]

→ \( N_f^2 - 1 \) massless Nambu-Goldstone bosons

\[ \langle \bar{q}q \rangle \neq 0 \]

broken (ordered) phase

2.) massless quarks → chiral symmetry:

Order parameter: chiral condensate

\[ \langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \]

\[ SU(N_f)_L + SU(N_f)_R \times U(1)_B \]

broken explicitly to \( Z_{2N_f} \) by quantum effects

\[ \bar{q}_L q_R \]

Right-handed:

\[ q_R \]

\[ p \]

\[ S \]

\[ \bar{q}_L \]

Left-handed:

\[ q_L \]

\[ p \]

\[ S \]

\[ \bar{q}q = 0 \]

symmetric (disordered) phase

\[ \langle \bar{q}q \rangle = 0 \]
Quantum Chromodynamics

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1.) infinite quark masses → center symmetry: Order parameter: VEV of traced Polyakov loop
2.) massless quarks → chiral symmetry: Order parameter: chiral condensate

for finite quark masses: both symmetries explicitly broken
Quantum Chromodynamics

QCD at finite temperatures and densities
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for finite quark masses: both symmetries explicitly broken

still conflicting lattice results!

open issue: \( N_f=2 \): O(4)?
\( U(2)_L \times U(2)_R / U(2)_V \)?
→ crit. exp. similar
or even 1\(^{\text{st}}\) order?
dep. on strength of axial anomaly!

\( m_\pi = 0 \)
\( m_\pi, c \)
\( m_\pi = \infty \)
\( 1^{\text{st}} \) order
\( Z(2) \) crossover

2\(^{\text{nd}}\) order crossover

\( m_\pi \infty \)
Conjectured QCD phase diagram

- early universe
- LHC
- RHIC
- SPS
- FAIR/JINR
- AGS
- SIS
- quark–gluon plasma
- hadronic fluid
- vacuum
- nuclear matter
- neutron star cores
- quark matter
- superfluid/superconducting phases
- CFL

Temperature

- Lattice simulations

→ can one improve the model calculations?
→ remove model ambiguities

QCD lattice simulations: no final answer

HISQ/tree: quadratic in $N_t^{-2}$
Asqtad: quadratic in $N_t^{-2}$

Physical $m/m_s$

Combined continuum extrapolation

courtesy of F. Karsch
Theoretical questions:

- **CEP**: existence/location/number
- **Quarkyonic phase**: coincidence of both transitions at $\mu = 0$ & $\mu > 0$?
- relation between chiral & deconfinement?
  - chiral CEP/deconfinement CEP?
- finite volume effects? ➔ lattice comparison
- inhomogeneous phases? ➔ more favored?
- role of fluctuations? so far mostly mean-field results ➔ effects of fluctuations are important
e.g. size of critical region around CEP
- axial anomaly restoration around chiral transition?
- good experimental signatures?
  ➔ higher moments more sensitive to criticality deviation from HRG model

⇒ can one improve the model calculations?
⇒ remove model ambiguities

[Braun, Janot, Herbst 12/14]
Conjectured QC$_3$D phase diagram

Theoretical questions: chiral & deconfinement transition

- CEP: existence/location/number
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  chiral CEP/deconfinement CEP? [Braun, Janot, Herbst 12/14]

- finite volume effects? $\rightarrow$ lattice comparison
- inhomogeneous phases? $\rightarrow$ more favored?
- role of fluctuations? so far mostly mean-field results
  $\rightarrow$ effects of fluctuations are important
e.g. size of critical region around CEP
- axial anomaly restoration around chiral transition?
- good experimental signatures?
  $\rightarrow$ higher moments more sensitive to criticality
deviation from HRG model

$\Rightarrow$ can one improve the model calculations?
$\Rightarrow$ remove model ambiguities
non-perturbative continuum functional methods (DSE, FRG, nPI)
$\Rightarrow$ complementary to lattice
$\Rightarrow$ no sign problem $\mu > 0$ $\Rightarrow$ chiral symmetry/fermions/small masses/chiral limit
Conjectured QC$_3$D phase diagram

Theoretical questions: chiral & deconfinement transition

- CEP: existence/location/number
- Quarkyonic phase: coincidence of both transitions at $\mu = 0$ & $\mu > 0$ ?
- relation between chiral & deconfinement? chiral CEP/deconfinement CEP? [Braun, Janot, Herbst 12/14]
- finite volume effects? $\Rightarrow$ lattice comparison
- inhomogeneous phases? $\Rightarrow$ more favored?
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- axial anomaly restoration around chiral transition?
- good experimental signatures?
  $\Rightarrow$ higher moments more sensitive to criticality deviation from HRG model

Method of choice: Functional Renormalization Group
e.g. (Polyakov)-quark-meson model truncation

- good description for chiral sector
- implementation of gauge dynamics (deconfinement sector)
Chiral transition

Fluctuations of order parameter $\to \infty$ at 2\textsuperscript{nd} order transition
critical fluctuations $\to$ phase boundary

How can we probe a transition?

- singular behaviour in $\frac{\partial^n p(X)}{\partial X^n}$ with $X = T, \mu, \ldots$
- higher order cumulants $c_n = \frac{\partial^n p(T, \mu)}{\partial(\mu/T)^n}$

... more sensitive to criticality

freeze-out close to chiral crossover line
Hadron Resonance Gas Model

HRG model: good lattice data description

HRG model: no critical fluctuations

HRG model versus experiment

[Andronic et al. 2011]

[Karsch, Redlich 2010]
Agenda

• Phase transitions and QCD

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  → chiral and deconfinement aspects

• Significance of Fluctuations
Vacuum Fluctuations

Partition function:

\[
Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi e^{-\int d^4x \mathcal{L}(\bar{\psi}, \psi, \phi)}
\]

Grand potential in Mean-field approximation

\[
\Omega(T, \mu; \sigma) = \Omega_{\text{vac}} + \Omega_T + V_{\text{MF}}(\sigma) + \mathcal{U}_{\text{Poly}}(\Phi)
\]

vacuum term: regularize e.g. with sharp three-momentum cutoff

\[
\Omega_{\text{vac}}(\Lambda) = -4 \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m^2_q}
\]

for each cutoff: adjust model parameters like \( f_\pi, m_\sigma, m_\pi \)

standard MFA: \( \Lambda = 0 \)
Role of fluctuations in (P)QM models

Fluctuations of higher moments exhibit strong variation from HRG model

- turn negative
- higher moments: $R_{n,m}^q = c_n/c_m$  \( c_n \): Taylor expansion coefficients of pressure
- regions where $R_{n,2} < 0$ along crossover in the phase diagram

unquenched PQM MFA

QM MFA w/o vacuum

role of vacuum term in (P)QM models see

[Karsch, Redlich, Friman, Koch et al. 2011]
Role of fluctuations in (P)QM models

Fluctuations of higher moments exhibit strong variation from HRG model

- turn negative
- higher moments: \( R_{n,m}^q = c_n / c_m \)  
  \( c_n \): Taylor expansion coefficients of pressure
- regions where \( R_{n,2} < 0 \) along crossover in the phase diagram

role of vacuum term in (P)QM models see: [BJS, Wagner 2011/12]

[Karsch, Redlich, Friman, Koch et al. 2011]
Mean-Field PQM \( N_f=2+1 \)

Improvement of pure (YM) Polyakov-loop potential: matter back-coupling on gluodynamics

An effective unquenching

\[ U_{\text{glue}}(t_{\text{glue}}) = U_{\text{YM}}(t_{\text{YM}}) \]

with \( t_{\text{YM}}(t_{\text{glue}}) = 0.57 t_{\text{glue}} \)

[Herbst, Mitter, Stiele, Pawlowski, BJS 2014]
Functional Renormalization Group

\( \Gamma_k[\phi] \) scale dependent effective action

\[ t = \ln(k/\Lambda) \quad R_k \text{ regulators} \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} \]

FRG (average effective action)

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right) \]

\[ k \partial_k \Gamma_k[\phi] \sim \frac{1}{2} \]

[Regulator]

Ansatz for \( \Gamma_k \): Leading order derivative expansion

\[ \Gamma_k = \int d^4x \bar{q} [i\gamma_\mu \partial^\mu - g(\sigma + i\tau_5 \gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 + V_k(\phi^2) \]

\[ V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \pi^2 - v^2)^2 - c\sigma \]

solutions with grid/polynomial techniques

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]
FRG and QCD

- full dynamical QCD FRG flow:

  fluctuations of \textit{gluon}, \textit{ghost}, \textit{quark} and (via hadronization) \textit{meson}

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} - \text{dashed} + \frac{1}{2} \]

in presence of \textit{dynamical quarks}:
\textit{gluon propagator} is modified

pure Yang Mills flow + matter back-coupling

[Braun, Haas, Pawlowski 2009/12]
First step: flow for \textit{quark-meson} model truncation: neglect \textit{YM contributions}

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \]

\[ \text{without bosonic fluctuations: MFA} \]
Phase diagram $N_f=2$ QM

$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!

$O(4)$ universality class

$\sigma \neq 0$ $\sigma = 0$

tricritical point

[BJS, J Wambach 2005]
Phase diagram $N_f=2$ QM

[BJS, J Wambach 2005]
pure Yang Mills flow:
fluctuations of **gluon, ghost**

\[
\partial_t \Gamma_k[\phi] = \frac{1}{2} - \quad \text{Diagram}
\]

**Glue potential**

\[
\beta^4 V_{\text{glue}}[A_0]
\]

(Haas, Stiele, Braun, Pawlowski, Schaffner-Bielich, (2013))
Polyakov-loop improved quark-meson flow:

fluctuations of Polyakov-loop, quark and meson

\[ \partial_t \Gamma_k[\phi] = \rightarrow U_{\text{Pol}}(\Phi) \]

Yang-Mills flow replaced by
effective Polyakov-loop potential

\[ \rightarrow U_{\text{Pol}}(\Phi) \]

fitted to lattice Yang-Mills thermodynamics
FRG: Quark-Meson with Polyakov

\[ T \text{ [MeV]} \]

\[ \mu \text{ [MeV]} \]

\[ m_\pi = 138 \text{ MeV} \]

- \( \chi \) crossover
- \( \Phi \) crossover
- \( \Phi \) crossover
- \( \chi \) 1st order
- \( \sigma(T=0)/2 \)

\[ \text{with } T_0(\mu) \]

\[ \text{w/o } T_0(\mu) \]

[Herbst, Pawlowski, BJS 2010,2013]
FRG: Quark-Meson with Polyakov

Pressure and interaction measure in comparison with lattice data (polynomial Polyakov-loop potential)

\[ N_f = 2 \]

[Herbst, Mitter, Stiele, Pawlowski, BJS 2014]
FRG: Quark-Meson with Polyakov

$N_f = 2+1$

[Herbst, Mitter, Stiele, Pawlowski, BJS 2014]
Influence axial anomaly

![Graphs showing masses vs. T and µ]
Critical Endpoint

Location of CEP not accessible with lattice, FRG & DSE

so far:

we can exclude CEP for small densities

but no baryons!

[C. Fischer, J. Lücker, C. Welzbacher 2014]
$N_c=2$ : diquark condensation

- no low-$T 1^{st}$ order transition, no CEP at $\mu \sim 2.5 \, m_\pi$ !
**Outlook: Inhomogeneities**

**inhomogeneous chiral symmetry breaking:**
phases characterized by spatially varying chiral condensate $\sigma(x)$ which breaks translational variance

allowing for inhomogeneous phases $\Rightarrow$ cooper pairs with non-vanishing total momentum near Fermi surface

only one- and two-dimensional condensates (here, in this context, first work beyond mean-field approximation)

quark-meson model (renormalizable):
include vacuum term
in grand potential
( Dirac-sea contribution)

example:
Gross-Neveu 1+1
$\Rightarrow$ chiral spirals
favored solution
for $\mu>0$
Outlook: Inhomogeneities

QM model: Phase diagram (two flavor, extended MFA)

Influence Dirac sea (left: $\Lambda=0$ middle: $\Lambda=600$ MeV right: $\Lambda=5$ GeV)

LP: Lifshitz point (two homogeneous phases meet one inhomogeneous phase)

CP: Critical point (endpoint of 1$^{st}$ order transition)

For $m_\sigma = 2M_q$  LP=CP

outlook: full FRG treatment....
Summary & Conclusions

- QCD-like model studies for two and three flavors
- Effects of quantum and thermal fluctuations on QCD phase structure
- Existence of critical points in phase diagram

Functional approaches (e.g. FRG) are suitable and controllable tools to investigate the QCD phase diagram and its boundaries.