Massive renormalization scheme and perturbation theory at finite temperature

ERG 2014
Lefkada, Greece
September 26, 2014

Based on work done with N. Wschebor, arXiv 1409.4795

Jean-Paul Blaizot, IPhT- Saclay
QCD at finite temperature

Perturbation theory is ill behaved

Perturbation theory:

$g^2$: Shuryak; Chin (1978)
$g^3$: Kapusta (1979)
$g^4$: In $g$: Toimela (1983)
$g^4$: Arnold, Zhai (1994)
$g^5$: Zhai, Kastening (1995),
      Braaten, Nieto (1996)
$g^6$: In $g$: Kajantie, Laine,
      Rummukainen, Schröder (2002)
$g^6$ (partly): Di Renzo, Laine,
      Miccio, Schröder, Torrero (2006)

Lattice data: G. Boyd et al. (1996); M. Okamoto et al. (1999).
Scalar field theory with quartic coupling
Scalar field theory with quartic coupling
How can this be improved?
How can this be improved?

- **Calculate higher orders**....

  Pressure in scalar theory is known up to order $O(g^8 \ln g)$

How can this be improved?

- Calculate higher orders....
  Pressure in scalar theory is known up to order $O(g^8 \ln g)$

- reorganize perturbation theory, resum, 2PI, NPRG, etc)
\[ p = T^4 \left[ c_0 + c_2 g^2 + c_3 g^3 + (c'_4 \ln g + c_4) g^4 + c_5 g^5 + c_6 g^6 \right] \]

\[ c_6 = N_c^3 \frac{\frac{N_c^2}{(4\pi^4)} - 1}{(\frac{215}{12} - \frac{805}{768} \pi^2) \ln \frac{1}{g} + 8 \delta} \]

Figure 3: (Color online) Left: Pressure as a function of the mass. The same curves are shown as in 3, but for the scale-independent function of pressure as a function of the thermal mass. The perturbative results are shown through order $g_6 \log g$. Right: Pressure as a function of the coupling including results of screened perturbation theory [19]. The curves labelled SPT2, SPT3, and SPT4 correspond to the two-loop, three-loop, and four-loop pressure, respectively.

It turns out to be available only for not too large couplings $g(2\pi T) \ll 0.6$, so that a scheme-independent comparison like in Fig. 3 (left panel) is not available for SPT3 and SPT4 at larger couplings. For SPT2, the results for screening mass and pressure almost coincide with the results of 2PI. Nevertheless, the plot of pressure versus coupling shows that SPT leads to a stable calculation scheme, even at large couplings, and does not deviate too much from the 2PI and BMW results. However, the systematics of the results as one moves from SPT2 to SPT3 to SPT4 remains unclear to us.

We turn now to more technical aspects of the calculations, namely the dependence of the results displayed above on the choice of the temperature, or on the choice of the regulator. Consider first the dependence on the temperature, which is measured by its ratio $T/\xi$ to the microscopic scale $\xi$. As $T/\xi$ increases and becomes close to 1, our numerical calculations loose accuracy for a variety of reasons, the main one being the following: If $T$ is too close to $\xi$, $2\pi T$ may become bigger than $\xi$ and the whole procedure eventually collapses. One indeed assumes that the beginning of the flow is not affected by the temperature (so as to use the 4-dimensional integration procedures), and this assumes $\xi > 2\pi T$ so that there is...
......perseverare diabolicum!
......perseverare diabolicum!

Why is pertubation theory so bad?
Expansion parameter and thermal fluctuations

\[ \langle \varphi^2 \rangle_{\kappa} \approx \int_{\kappa}^{\infty} \frac{d^3 p}{(2\pi)^3} \frac{n_p}{E_p} \approx T \kappa \]

\[ n_p = \frac{1}{e^{E_p/T} - 1} \]

\[ \gamma_\kappa \sim \frac{g^2 \langle \varphi^2 \rangle_\kappa}{\kappa^2} \sim \frac{g^2 T}{\kappa} \]

Suggests a breakdown of perturbation theory when \( \kappa \lesssim g^2 T \)
But!

- Dimensional reduction at high temperature

\[ \kappa \frac{d\gamma_k}{d\kappa} = -\gamma_k + \frac{3}{16} \gamma_k^2 \]

- Dynamical generation of a thermal mass

\[ m \sim gT \]
Massive, decoupling, scheme

\[ S[\varphi] = \int_0^\beta d\tau \int d^dx \left\{ \frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi + \frac{m_B^2}{2} \varphi^2 + \frac{g_B^2}{4!} \varphi^4 \right\} \]

Renormalization conditions

\[ m^2 = \Gamma^{(2)}(p = 0, \omega = 0, T) \]
\[ 1 = \frac{d\Gamma^{(2)}}{dp^2}(p^2 = \mu^2, \omega = 0, T) \]
\[ g^2 = \Gamma^{(4)}(p_{sym}^2 = \mu^2, \omega_i = 0, T) \]
One-loop running in massive scheme

\[ g^2 = 1 \]

\[ t = \ln(\mu/\Lambda) \]

\[ g^2 = 12 \]
Leading order calculation

\[ \Gamma^{(2)}(p, \omega, T) = m^2 + \delta m^2 + p^2 + \frac{g^2 T}{2} \sum_n \int \frac{d^d q}{(2\pi)^d} \frac{1}{\omega_n^2 + q^2 + m^2} \]

\[ I(m) \equiv T \sum_n \int q \frac{1}{\omega_n^2 + q^2 + m^2} = \int_q \frac{1 + 2n_q}{2E_q} \equiv I_0(m) + I_T(m) \]

\[ \Gamma^{(2)}(p = 0, \omega = 0, T) = m^2 + \delta m^2 + \frac{g^2}{2} I(m) \]

The renormalization condition implies

\[ \delta m^2 = -\frac{g^2}{2} I(m) \]
Relate thermal mass to zero temperature mass

\[ \Gamma^{(2)}(p = 0, \omega = 0, T = 0) = m^2 + \delta m^2 + \frac{g^2}{2} I_0(m) = m^2 - \frac{g^2}{2} I_T(m) \]

Note: unusual calculation!

Self-consistent equation for the thermal mass

\[ m_0^2 = m^2 - \frac{g^2}{2} I_T(m) \]
The diagram shows a plot with the $m^2/T^2$ on the y-axis and $g^2$ on the x-axis. The plot includes different schemes:

- **Massless scheme order $g^2$**
- **Massive thermal scheme**
- **2PI**
- **Massless scheme order $g^3$**

Each scheme is represented by a different symbol and color, allowing for a comparative analysis of their behaviors as $g^2$ increases.
The graph shows the ratio $P/P_0$ as a function of $g^2$. The legend indicates four different schemes:

- Massive thermal scheme
- 2PI
- Massless scheme order $g^2$
- Massless scheme order $g^3$
Summary

• An appropriate choice of renormalization scheme can greatly improve perturbation theory at finite temperature
• The proposed massive scheme leads to a well behaved perturbative expansion
• The idea of expanding around a massive theory is not new (screened perturbation theory, optimized perturbation theory, etc), but the present implementation is conceptually and technically simpler.