Composite chiral fermions from the renormalization group

Stefan Flörchinger (CERN)

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Remaining problems of the standard model

- Standard model of elementary particle physics works surprisingly well.
- Seems to describe all measurements at the LHC so far.
- Contains 18 free parameters (without neutrino masses)
  - 3 gauge couplings for U(1), SU(2) and SU(3)
  - 1 Higgs field vacuum expectation value
  - 1 Higgs field self coupling
  - 3 lepton masses
  - 6 quark masses
  - 3 CKM mixing angles + 1 phase

13 out of 18 parameters are determined by the Yukawa couplings.

- Open questions are:
  - Why are there three generations?
  - What explains the Yukawa-coupling hierarchy between generations?
  - What gives mass to neutrinos?
  - What determines the Higgs VEV? (Hierarchy problem)
Are leptons and quarks composite?

- It seems plausible that there is some structure underlying the standard model that explains the Yukawa couplings.
- Quarks and leptons before electroweak symmetry breaking are chiral: left-handed and right-handed fields in different gauge representations.
- Chiral symmetry forbids a mass term.
- Can chiral fermions be composite?
- In principle yes, there is at least no good argument against it.
- Some constrains come from anomaly matching ['t Hooft (1979)].
- However, a formalism to describe this and to determine whether chiral bound states form in a given theory, is lacking.
- For example it is clear that Schrödinger equation cannot be used.
Constituents have not been found so far...

- If leptons and quark consist of more elementary constituents the question arises why these have never been found.
- In principle a confining theory with strong interactions at a very high energy scale could do the job.
- Can only work if this theory has unbroken chiral symmetry in contrast to QCD.

There is no obvious candidate for a theory underlying the standard model so let us sharpen knifes by asking some questions on the standard model itself.
Right-handed fermions and scalar bosons

Start from

- right-handed lepton $\psi_R$: SU(2) singlet, U(1)$_Y$ charge $g'$
- mass-less scalar boson $\phi$: SU(2) doublet, U(1)$_Y$ charge $-\frac{1}{2}g'$
- gauge fields $B_\mu$ for U(1)$_Y$ and $A^a_\mu$ for SU(2)

Quantum fluctuations induce fermion-boson vertex $\lambda_{\phi R}$

- all particles in the loop are mass-less
- perturbative one-loop contributions linearly infrared divergent
What can be composite particles of $\psi_R$ and $\phi$?

Or: What substructures can fermion-boson vertex $\lambda \phi_R$ have?

- left handed lepton $\psi_L$: SU(2) doublet, U(1)$_Y$ charge $\frac{1}{2}g'$
- left-handed fermion $f_L$: SU(2) doublet, U(1)$_Y$ charge $\frac{3}{2}g'$
- vector boson of $B_\mu$ type

$\psi_R$ and $\phi$ have opposite U(1)$_Y$ charge or attractive interaction, in favor of bound state $\psi_L$
Fermionic Hubbard-Stratonovich transformation

- perform Hubbard-Stratonovich transformation with respect to the attractive channel
- field for $\psi_L$ is introduced as auxiliary field with quadratic “Lagrangian”

$$\mathcal{L}_{HS} = i(\bar{\psi}_L - \bar{\xi}_L) \bar{\sigma}^\mu D_\mu q_L (-D_\nu D^\nu) (\psi_L - \xi_L)$$

- $D_\mu$ is covariant derivative appropriate for $\psi_L$
- $\xi_L$ is quadratic in right-handed fermion and scalar fields, $\xi_L \sim \phi \psi_R$
- the function

$$q_L(p^2) = 1 + \nu_L^2/p^2$$

contains a non-local mass $\nu_L$
- for large $\nu_L$ the fermion $\psi_L$ is heavy and plays no role
Effective theory after HS transformation

- Right-handed fermions as before, standard kinetic term.
- Left-handed fermions with kinetic term and non-local mass term $\nu_L$

\[
\mathcal{L}_{\psi_L} = i (\bar{\psi}_L)\dot{a} \left( \bar{\sigma}^\mu \right)^{\dot{a}b} \left( \partial_\mu - i A_\mu^a t_L^a - i B_\mu y_L \right) (\psi_L)_b \\
+ i \nu_L^2 (\bar{\psi}_L)\dot{a} \left( [\sigma^\mu D_\mu]^{-1} \right)^{\dot{a}b} (\psi_L)_b
\]

- Yukawa interactions

\[
\mathcal{L}_{\text{Yukawa}} = -h \left[ (\bar{\psi}_L)\dot{a} \phi (\psi_R)^{\dot{a}} + (\bar{\psi}_R)^a \phi^{\dagger} (\psi_L)_a \right].
\]

- Boson-Fermion interaction vertex

\[
\mathcal{L}_{\phi_R} = i (\bar{\psi}_R)^a \phi^{\dagger} \lambda_{\phi_R} (-D_\nu D^\nu) (\sigma^\mu)^{ab} D_\mu \phi (\psi_R)^b
\]

- Kinetic terms for scalars and gauge fields as before.
Adapting parameters

- Boson-fermion vertex has two contributions

\[ \lambda_{\phi R} = (\lambda_{\phi R})_{\text{loops}} - \frac{h^2}{p^2 + \nu_L^2} \]

- first term generated by radiative corrections / loops
- second term from HS transformation
- Idea is now to adapt \( h \) and \( \nu_L \) such that \( \lambda_{\phi R} = 0 \).
- One-loop calculation with IR cutoff \( \Lambda \) gives

\[ (\lambda_{\phi R})_{\text{loops}} = \frac{g'^4}{16\pi^2} \left[ \frac{1}{4\Lambda^2} - p^2 \frac{7}{12\Lambda^4} + \mathcal{O}(p^4) \right]. \]

which cancels to the given order in \( p^2 \) for

\[ h_\Lambda^2 = \frac{3g'^4}{448\pi^2}, \quad \nu_{L,\Lambda}^2 = \frac{3}{7}\Lambda^2. \]

- for \( g'^2 = \alpha \frac{4\pi}{\cos^2 \theta_W} \) with the fine structure constant \( \alpha(M_Z) = 1/128 \)
  and \( \sin^2 \theta_W(M_Z) = 0.23126 \) one finds \( h_\Lambda = 0.0033 \)
- surprisingly close to Yukawa coupling of \( \tau \)-lepton \( h_\tau = 0.0072 \)
- non-local mass \( \nu_L \) vanishes for \( \Lambda \to 0 \)
Exact flow equation with HS transformation

For functional RG study one needs flow equation that implements $k$-dependent HS transformation [Floerchinger & Wetterich, PLB 680, 371 (2009), see also Gies & Wetterich (2002), Pawlowski (2007)]

$$\partial_k \Gamma_k = \frac{1}{2} \text{STr} \left\{ \left( \Gamma_k^{(2)} + R_k \right)^{-1} \left( \partial_k R_k - R_k (\partial_k Q^{-1}) R_k \right) \right\}$$

- exact flow equation that generalizes Wetterich equation
- $\Gamma_k^{(1)}$ is functional derivative with respect to the composite field
- $\partial_k Q^{-1}$ can be chosen arbitrary
- works also for fermionic composite fields
Regulator functions

- all relevant diagrams are UV finite
- simple IR regulators are sufficient

\[ \Delta \mathcal{L}_k = -i k^2 (\bar{\psi}_L) \dot{a} \left( [\sigma^\mu \partial_\mu]^{-1} \right)^{\dot{a}b} (\psi_L)_b \\
- i k^2 (\bar{\psi}_R)^a \left( [\bar{\sigma}^\mu \partial_\mu]^{-1} \right)_{\dot{a}b} (\psi_R)^b \\
+ k^2 \phi^\dagger \phi \\
- k^2 \frac{1}{2} (A^{a\mu} A^{\mu}_a + B^{\mu} B_{\mu}) + k^2 \bar{c}^a c^a \]

- regulator functions break gauge invariance
- results presented in the following are for fixed gauge: Feynman gauge
Flow equations for anomalous dimensions

- anomalous dimension right-handed fermions

\[ (\eta_R)_{\text{loops}} = \frac{1}{16\pi^2} \left[ 4g'^2 + 2h^2 \frac{k^2}{\nu^2_L} \ln \left( \frac{k^2 + \nu^2_L}{k^2} \right) \right] \]

- anomalous dimension left-handed fermions

\[ (\eta_L)_{\text{loops}} = \frac{1}{16\pi^2} \left[ (3g^2 + g'^2) \frac{k^2}{\nu^2_L} \ln \left( \frac{k^2 + \nu^2_L}{k^2} \right) + 2h^2 \right] \]
Flow equations Yukawa coupling

Yukawa coupling at vanishing momentum

\[
(\partial_t h)_{\text{loops}} = \frac{1}{16\pi^2} \left[ -h (3g^2 - g'^2) \frac{k^2}{\nu_L^2} \ln \left( \frac{k^2 + \nu_L^2}{k^2} \right) \\
- 2h g'^2 - 8h g'^2 \frac{k^2}{\nu_L^2} \ln \left( \frac{k^2 + \nu_L^2}{k^2} \right) \right]
\]

First derivative with respect to fermion momentum \( p^2 \)

\[
(\partial_t h')_{\text{loops}} = \frac{1}{16\pi^2} \left[ h \left( \frac{3}{4} g^2 - \frac{1}{4} g'^2 \right) \left[ -12 \frac{k^2}{\nu_L^4} + 6 \frac{2k^4 + k^2 \nu_L^2}{\nu_L^6} \right. \right. \\
\times \ln \left( \frac{k^2 + \nu_L^2}{k^2} \right) + \frac{1}{2} h g'^2 \frac{1}{k^2} \left. \right]
\]
Flow equation boson-fermion vertex

at vanishing momentum

\[
(\partial_t \lambda_{\phi R})_{\text{loops}} = \frac{1}{16\pi^2} \left[ -\frac{1}{2} g'^4 \frac{1}{k^4} + 8h^2 g'^2 \frac{1}{k^2 + \nu_L^2} - 3h^4 \frac{k^2}{\nu_L^4} \ln \left( \frac{(2\nu_L^2 + k^2)k^2}{(\nu_L^2 + k^2)} \right) 
- h^2 \left( \frac{3}{2} g^2 + \frac{1}{2} g'^2 \right) \left[ \frac{3k^2 + 2\nu_L^2}{\nu_L^2(\nu_L^2 + k^2)} - \frac{3k^2}{\nu_L^4} \ln \left( \frac{k^2 + \nu_L^2}{k^2} \right) \right] \right]
\]

first derivative with respect to fermion momentum \( p^2 \)

\[
(\partial_t \lambda'_{\phi R})_{\text{loops}} = \frac{1}{16\pi^2} \left[ \frac{7}{3} g'^4 \frac{1}{k^4} + 2h^2 g'^2 \frac{k^2 + 2\nu_L^2}{(k^2 + \nu_L^2)^2 k^2} - h^2 \left( \frac{3}{2} g^2 + \frac{1}{2} g'^2 \right) 
\times \left[ - \frac{24k^2}{\nu_L^6} - \frac{2}{k^2 \nu_L^2} + \frac{2}{k^2(k^2 + \nu_L^2)} + \frac{12k^2(2k^2 + \nu_L^2)}{\nu_L^8} \ln \left( \frac{k^2 + \nu_L^2}{k^2} \right) \right] \right]
\]
Scale-dependent HS transformation

- choose parameters of $k$-dependent HS transformation such that
  \[
  \partial_k \lambda_{\phi_R}(p^2)\big|_{p^2=0} = 0, \quad \partial_k \lambda'_{\phi_R}(p^2)\big|_{p^2=0} = 0.
  \]

- choose also $p$-dependent wave-function renormalization for composite field $\psi_L(p)$ such that
  \[
  \partial_k h(p^2)\big|_{p^2=0} = 0.
  \]

- that gives final flow equations for non-local mass
  \[
  \partial_t \nu_L^2 = (\eta_L)_{\text{loops}} \nu_L^2 + \frac{\nu_L^4}{h^2} (\partial_t \lambda_{\phi_R})_{\text{loops}} + \frac{\nu_L^6}{h^2} (\partial_t \lambda'_{\phi_R})_{\text{loops}}
  \]
  \[
  + \frac{2\nu_L^4}{h} (\partial_t h')_{\text{loops}}
  \]

  and the Yukawa coupling
  \[
  \partial_t h^2 = 2h (\partial_t h)_{\text{loops}} + h^2 [(\eta_R)_{\text{loops}} + (\eta_L)_{\text{loops}}]
  \]
  \[
  + 2\nu_L^2 (\partial_t \lambda_{\phi_R})_{\text{loops}} + \nu_L^4 (\partial_t \lambda'_{\phi_R})_{\text{loops}} + \nu_L^2 2h (\partial_t h')_{\text{loops}}
  \]
for fixed gauge couplings $g(M_Z) = 0.651$ and $g'(M_Z) = 0.807$

fixed point approximately at

$$h^* = \frac{3g'^4}{448\pi^2} \approx 0.000011, \quad \tilde{\nu}_L^* = \frac{\nu_L^2}{k^2} = \frac{3}{7} \approx 0.43$$

non-local mass parameter $\nu_L$ vanishes with $k$

Yukawa coupling related to $U(1)_Y$ gauge coupling

numerical value $h^* = 0.0033$ close to $h_{\tau\text{-lepton}} = 0.0072$
Flow of gauge couplings

- One loop perturbative flow equations

\[ \partial_t g = - \frac{\frac{22}{3} - \frac{1}{3}(n_{l_L} + 3n_{q_L}) - \frac{1}{6} g^3}{16\pi^2}, \]

\[ \partial_t g' = \frac{\frac{2}{3} \left( \frac{1}{2} n_{l_L} + n_{l_R}^e + \frac{1}{6} n_{q_L} + \frac{4}{3} n_{q_R}^u + \frac{1}{3} n_{q_R}^d \right) + \frac{1}{6} g'^3}{16\pi^2}, \]

where the fermion content is
- \( n_{l_L} \) left-handed leptons,
- \( n_{l_R}^e \) right-handed leptons of electron type,
- \( n_{q_L} \) left-handed quarks,
- \( n_{q_R}^u \) right-handed quarks of up-type,
- \( n_{q_R}^d \) right-handed quarks of down-type

- For the standard model with complete fermion content

\[ g^2(k) = \frac{1}{g^2(k_0)} + \frac{19}{96\pi^2} \ln(k/k_0), \]

\[ g'^2(k) = \frac{1}{g'^2(k_0)} - \frac{41}{96\pi^2} \ln(k/k_0). \]
Flow with flowing gauge couplings

\[ 1 \over g^2, \ 1 \over g_s^2, \ 3 \over 5 g^2, \ k @ \text{GeV} \]

\[ h^2, \ k @ \text{GeV} \]

\[ \bar{v}_L^2, \ k @ \text{GeV} \]

\[ h^2 - \frac{3 g_s^4}{448 \pi^2}, \ k @ \text{GeV} \]
Remarks on anomalies

- it is known that theories with only right-handed fermions (or only left-handed fermions) lead to gauge anomalies
- on first sight this seems to make an initial theory with only right-handed fermions inconsistent
- on the other side, the auxiliary fields that are added by the Hubbard-Stratonovich transformation can also contribute to the anomaly and might even cancel it
- quite generally, theories with composite chiral fermions must fulfill anomaly matching conditions ['t Hooft (1979)]
- these issues need more study
Composite right-handed fermions

- also right-handed fermions might be composite
  \[ \psi_L \phi \in \psi_R \nu_R \psi'_R \nu'_R \]

- combinations of left-handed fermions $\psi_L$ and scalars $\phi$
  - right-handed fermion $\psi_R$: SU(2) singlet, U(1)$_Y$ charge $g'$
  - right-handed fermion $\nu_L$: SU(2) singlet, U(1)$_Y$ charge 0
  - right-handed fermion $\psi'_R$: SU(2) triplet, U(1)$_Y$ charge $g'$
  - right-handed fermion $\nu'_L$: SU(2) triplet, U(1)$_Y$ charge 0
  - vector boson of $B_\mu$ type
  - vector boson of $A^a_\mu$ type

- $\psi_L$ and $\phi$ can be bound by U(1)$_Y$ or SU(2) interactions
- attractive U(1)$_Y$ interaction favors right-handed neutrino type $\nu_R$
Conclusions

- Left-handed $\tau$-lepton could be composite of scalar doublet and right-handed $\tau$-lepton!
- Yukawa coupling can be predicted and agrees up to factor $\sim 2$ with experimental value but good agreement could be partly accidental.
- Theoretical uncertainties still high:
  - Fierz ambiguities in Hubbard-Stratonovich transformation
  - Effect of scalar field self interaction and vacuum expectation value
- Flow equation with scale-dependent Hubbard-Stratonovich transformation can be used to investigate this interesting physics.
- More detailed analysis needed to investigate possibilities for other bound states (right-handed neutrinos ?).
- Question of anomalies needs further studies.