On IR Fixed Points of Quantum Gravity.

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Figure 1: Wilsonian flows for scale-dependent effective actions $\Gamma_k$ in the space of all action functionals (schematically); arrows point towards smaller momentum scales and lower energies $k \to 0$.

a) Flow connecting a fundamental classical action $S$ at high energies in the ultraviolet with the full quantum effective action $\Gamma$ at low energies in the infrared ("top-down").

b) Flow connecting the Einstein-Hilbert action at low energies with a fundamental fixed point action $\Gamma^*$ at high energies ("bottom-up").

- Integral representation.
  The physical theory described by $\Gamma$ can be defined without explicit reference to an underlying path integral representation, using only the (finite) initial condition $\Gamma_\Lambda$, and the (finite) flow equation (3.2)
  \[ \Gamma = \Gamma_\Lambda + \int \frac{\Lambda}{k^2} \text{Tr} \left( \Gamma^{(2)} + R_k \right)^{-1} k \partial_k R_k. \]
Example: Einstein-Hilbert Action.

\[ \Gamma_k = \int d^4x \sqrt{g} \left[ \frac{1}{16\pi G_k} (-R + 2\Lambda_k) \right] \]

\[ g = G(k) k^2 \quad \lambda = \Lambda(k) k^{-2} \quad \eta = k \partial_k \log G_k \]

Anomalous Dimension.
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Figure 4: The Type IIIa trajectory realized in Nature and the separatrix. The dashed line is a trajectory of the canonical RG flow.

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Classical Gravity.

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IR Fixed Point Hypothesis.

\[ G_k = \frac{g_*}{k^2} \]

Strong Gravity in the IR.

\[ \Lambda_k = \lambda_* k^2 \]

High Redshift Type Ia Supernovae.

- \( m_B \): Apparent Magnitude.
- \( z \): Red Shift.

FIG. 1: The measured apparent magnitudes of the supernovae as a function of their redshift. The continuous line represents the prediction of the IRFP cosmology, the dashed one is the best-fit FRW model, and the dot-dashed line is a flat FRW model with zero cosmological constant.

E. Bentivegna, A. Bonanno, M. Reuter, JCAP 0401 (2004) 001
Approximations.
Approximation (Leading order in g).

$$\eta = -g\alpha_1$$

Close-up on C.
Approximation - Nullclines.

**Definition:**

NULLCLINES are integral curves where the beta functions vanish.

\[
\beta_g(\lambda, g_g(\lambda)) = 0 \quad \beta_\lambda(\lambda, g_\lambda(\lambda)) = 0
\]

- The intersection of two nullclines is a Fixed Point.
Nullclines - Approximation.

A (UVFP)  

B (GFP)  

C  

$g_b$  

$g_g(\lambda)$  

$g_\lambda(\lambda)$
Nullclines - Approximation.

\[
\eta = -2 \quad \text{A (UVFP)}
\]

\[
1/\eta = 0 \quad \text{B (GFP)}
\]

\[
\eta = 0 \quad \text{C}
\]
Nullclines – Approximation.

A (UVFP)

B (GFP)

DEGENERACY

\[ \eta = -2 \]

\[ 1/\eta = 0 \]

\[ \eta = 0 \]

\[ g / g^* \]

\[ g_b \]

\[ g_g(\lambda) \]

\[ g_\lambda(\lambda) \]
Introduce a small parameter $\delta$ to lift the degeneracy.
Lifting the degeneracy.
Lifting the Degeneracy-Approximation.
Lifting the Degeneracy-Approximation.
Lifting the Degeneracy-Approximation.

\[ \eta = 2 \]

\[ \frac{1}{\eta} = 0 \]

\[ \eta = 0 \]

New Fixed Point \( C' \).

\( C' : g_\ast \neq 0, \quad \lambda_\ast \neq 0, \quad \eta = -2 \rightarrow \) Candidate for IRFP.
Lifting the Degeneracy-Approximation.

\[ \eta = -2 \]

\[ \frac{1}{\eta} = 0 \]

\[ \eta = 0 \]

New Fixed Point C.

\[ C : \ g_* = 0, \ \lambda_* \neq 0, \ \eta = 0 \rightarrow \text{Classical Gravity.} \]
Approximation 2: Hartree-Fock Resummation.
Stability Analysis of C and C.

• **Critical Exponents:** - Eigenvalues of the Stability Matrix.

1. **IRFP C':**

\[
\theta^{1}_{C'} = 4 - \frac{16\sqrt{2}}{3} \delta^{1/2} + \frac{80}{3} \delta - \frac{160\sqrt{2}}{3} \delta^{3/2}
\]

\[
\theta^{2}_{C'} = -2\sqrt{2} \delta^{-1/2} - \frac{8}{3} + \frac{40\sqrt{2}}{9} \delta^{1/2} - \frac{256}{9} \delta + \frac{6056\sqrt{2}}{81} \delta^{3/2}
\]

2. **IRFP C:**

\[
\theta_{C}^{1} = -\frac{4}{\sqrt{3}} \delta^{-1/2} - \frac{8}{3} - \frac{14}{3\sqrt{3}} \delta^{1/2} + 8\delta
\]

\[
\theta_{C}^{2} = -2
\]
• One special trajectory (separatrix (red)) will hit the FP C’ without feeling the effects of C. And another similar but connecting C.
• Trajectories between the separatrices (blue) will be dragged abruptly towards C’, spending some time in its vicinity (strong gravity). After that, they will be pushed smoothly to C where it will finish.
Conclusions.

* Deep Infrared regime of the flow contain a degenerated FP.

* We have lifted the degeneracy and found new FP.

* $C'$: $g_* \neq 0 \quad G_k = \frac{g_*}{k^2}$

* $C$: $g_* = 0 \quad G_k \rightarrow constant$

1. Use the result in Cosmology (Transition to FP epoch, Accelerated Expansion without Dark Matter?).

2. Find a dynamical way to lift the degeneracy.
Thank you!

Plaudite, cives.
Poles in the flow.

★ The graviton propagator displays a pole around:

\[ \sim \frac{1}{1 - 2\lambda} \quad \text{or} \quad \sim \frac{1}{1 - 2\alpha \lambda} \]

★ Then, fixed point solutions must obey

\[ \lambda_* \leq \lambda_{\text{bound}} = \min \left\{ \frac{1}{2}, \frac{1}{2\alpha} \right\} \]
For $0 \leq \alpha \leq 1$, we computed the mean value and the standard deviation for FP and Critical Exponents:

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_A$</th>
<th>$g_A$</th>
<th>$\theta_{A}^R$</th>
<th>$\theta_{A}^I$</th>
<th>$\lambda_C$</th>
<th>$g_C$</th>
<th>$\theta_{C}^1$</th>
<th>$\theta_{C}^2$</th>
<th>$\lambda_C'$</th>
<th>$g_C'$</th>
<th>$\theta_{C'}^1$</th>
<th>$\theta_{C'}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle X_{LO} \rangle$</td>
<td>0.1972</td>
<td>0.9124</td>
<td>1.3943</td>
<td>2.5287</td>
<td>0.4316</td>
<td>0</td>
<td>-27.0797</td>
<td>-2</td>
<td>0.4513</td>
<td>0.0424</td>
<td>3.5088</td>
<td>-38.1576</td>
</tr>
<tr>
<td>$\langle \Delta X_{LO} \rangle$</td>
<td>0.0040</td>
<td>0.0053</td>
<td>0.1264</td>
<td>0.0355</td>
<td>0.0010</td>
<td>0</td>
<td>0.3064</td>
<td>0</td>
<td>0.001</td>
<td>0.0003</td>
<td>0.0338</td>
<td>0.2687</td>
</tr>
<tr>
<td>$\langle X_{HF} \rangle$</td>
<td>0.1651</td>
<td>0.8362</td>
<td>1.909</td>
<td>2.5061</td>
<td>0.4635</td>
<td>0</td>
<td>-31.7670</td>
<td>-2</td>
<td>0.4692</td>
<td>0.0126</td>
<td>3.5554</td>
<td>-38.1802</td>
</tr>
<tr>
<td>$\langle \Delta X_{HF} \rangle$</td>
<td>0.0018</td>
<td>0.0547</td>
<td>0.0926</td>
<td>0.0752</td>
<td>0.0003</td>
<td>0</td>
<td>0.2688</td>
<td>0</td>
<td>0.0003</td>
<td>0.0001</td>
<td>0.0319</td>
<td>0.3591</td>
</tr>
</tbody>
</table>

The relative standard deviation ranges:

- **LO (d = 1/50):** 0.22 % for $\lambda_C$ to 9.06% for $\text{Re}(\theta_A)$
- **HF (d = 1/300):** 0.06% for $\lambda_C'$ to 6.54% for $g_A$