Vertex Functions in Quantum Gravity

N.C., Litim, Pawlowski, Rodigast (PLB29813)
N.C., Knorr, Pawlowski, Rodigast (1403.1232)
N.C., Knorr, Pawlowski, Rodigast (on arxiv next week...or so)

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Outline

- Introduction

- Vertex expansions for flows in quantum gravity: General setup

- Global phase structure: ultraviolet and infrared fixed points

- The three point function

- Outlook
Quantum gravity and asymptotic safety in a nutshell

- Classical gravity is described by **general relativity** on all scales observed so far

$$10^{-4}\text{m} \rightarrow 10^{26}\text{m}$$

- In $d = 4$: $[G_N] = \text{energy}^{-2}$

  - **Newton's Constant**

  - Perturbative quantization fails

- Non-perturbative quantization: asymptotic safety: **UV fixed point**

  $$\lim_{k \rightarrow \infty} g_i(k) = g_{i,*}$$

  - Dimensionless coupling constant

  - Predictive theory, finite on all scales

  - Finite observables!
Einstein-Hilbert Truncation

- basic approximation scheme:

$$\Gamma_k = \frac{Z_k}{16\pi G_N} \int \omega_d(g)(-R(g) + 2\Lambda_k)$$

scale dependent, running couplings:

$$G_N \rightarrow G_k = \frac{G_N}{Z_k}$$

$$\Lambda \rightarrow \Lambda_k$$

- parametrization of the effective action
- calculate $\partial_t \Gamma$
- non-perturbative beta-functions for the dimensionless couplings

$$g_k = G_k k^2 \quad \text{and} \quad \lambda_k = \frac{\Lambda_k}{k^2}$$
Asymptotic Safety in basic Einstein-Hilbert

- Einstein Hilbert approximation leads to UV fixed point
- singularity in the IR

confirm this!

change this!
The background field and all that

- usual background field approach:

\[
k \frac{d}{dk} \Gamma_k[\bar{g}, h] = \frac{1}{2} \text{Tr} \left( \frac{\delta^2}{\delta h^2} \Gamma_k + \mathcal{R} \right)^{-1} \frac{d}{dk} \mathcal{R}
\]

evaluated at \( h = 0 \) with \( g = \bar{g} + h \)

- flow is not closed since

\[
\frac{\delta^2}{\delta h^2} \Gamma_k \neq \frac{\delta^2}{\delta \bar{g}^2} \Gamma_k
\]

- background approximation: using equality

- this can change the sign of the 1-loop YM-beta function!

- couplings of fluctuation fields \( \neq \) couplings of the background field

related to fluctuation correlators related to background correlators

note: linear split is not the most general form!!!
Hierarchy of flow equations

- calculate the flow of the propagator itself!

\[
\partial_t \frac{\delta^2 \Gamma}{\delta h^2} \bigg|_{h=0} = \text{Flow}^{(2)}[\Gamma^{(2)}, \Gamma^{(3)}, \Gamma^{(4)}]
\]

\[\Gamma^h = \frac{\delta^n \Gamma}{\delta h^n}\]

- no reason to stop at the two point function!

- flow of higher order vertex functions:
  - \(\partial_t \Gamma^{(3)}\)
  - \(\partial_t \Gamma^{(4)}\)
  - .......

- generates the full hierarchy of flow equations for the vertex functions \(\Gamma^{(n)}\)
Structure of the vertex functions and approximations

- Structure of the vertices

\[ \Gamma^{(n)} = \prod_{i=1}^{n} Z_{k}^{1/2}(p_i) G_{k}^{n/2-1} \mathcal{T}^{(n)}_{k} \left( p_i; \Lambda^{(n)}_{k} \right) \]

- two point function:

\[ \Gamma^{(2)} = Z_{k}(p^2) \left( p^2 + M_{k}^{2} \right) \Pi_{TT} \]

- three point function:

\[ \Gamma^{(3)} = \sqrt{G_{k}} \sqrt{Z_{k}(p_1)} \sqrt{Z_{k}(p_2)} \sqrt{Z_{k}(p_3)} \left( \mathcal{T}_{1}(p) + \mathcal{T}_{2} \Lambda^{(3)}_{k} \right) \]

Fully momentum dependent!

Momentum independent parts

\[ M_{k}^{2} = -2 \Lambda^{(2)}_{k} \]
Flow of the propagator

- flow equation for the inverse propagator

\[
\frac{\partial_t}{\partial h^2} \delta^2 \Gamma_k[\bar{g}; h] \bigg|_{h=0} = -\frac{1}{2} \left( \begin{array}{c}
\text{LHS contains: } (\partial_t Z, \partial_t M^2) \\
\text{RHS contains: } (Z, M^2, G, \Lambda^{(3)}, \Lambda^{(4)}) \\
\partial_t G \text{ from geometrical flow equations} \\
\Lambda^{(3)}, \Lambda^{(4)} \text{ constrained via scaling analysis}
\end{array} \right)
\]

\equiv \text{Flow}^{(2)}
The Phase Diagram

- The phase diagram in the \((g, \mu)\) plane:
  \[ g = G k^2 \quad \mu = M^2 k^{-2} \]

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Running of couplings

\[ \lambda \sim k^{-2} \]

\[ g \sim k^2 \]

\[ G = \text{const.} \quad \Lambda = \text{const.} \]

classical gravity in the IR!

\[ G = \frac{g}{k^2} \]

\[ \Lambda = k^2 \lambda \]
The three point function

- coupling constants of the three point function: \((G, \Lambda^{(3)})\)

\[
\frac{\partial_t \frac{\delta^3 \Gamma[\bar{g}, \phi]}{\delta h^3}}{\delta h^3} \bigg|_{\phi=0} (p_1^2, p_2^2, p_3^2) = -\frac{1}{2}
\]

\[
\begin{align*}
\text{+ 3} & & \text{+ 6} \\
\text{+ 3} & & \text{+ 6}
\end{align*}
\]
The phase diagram II

- phase diagram with couplings \((g, \lambda^{(3)}, \mu)\)

UV FP
Summary

- dynamical vs background couplings
- full momentum dependence of the propagator
- UV - IR stability with classical IR fixed point
- flow of the three point function: UV fixed point

Outlook:

- higher derivative tensor structures
- fermions
- gluons
- curved backgrounds
- four point function

see poster: J.Meibohm, M.Reichert
Thank you!