Suppression of Quantum Fluctuations by Classical Backgrounds

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The cubic Galileon theory describes the dynamics of the scalar mode that survives in the decoupling limit of the DGP model (Dvali, Gabadadze, Porrati).

The action contains a higher-derivative term, cubic in the field \( \pi(x) \), with a dimensionful coupling that sets the scale \( \Lambda \) at which the theory becomes strongly coupled.

\[
\nu = \frac{1}{\Lambda^3}
\]

\[
\Lambda \sim (m^2M_{Pl})^{1/3} \text{ with } m \sim H \sim M_5^3/M_{Pl}^2
\]
The action is invariant under the **Galilean transformation**
\[ \pi(x) \rightarrow \pi(x) + b_\mu x^\mu + c, \] up to surface terms.

In the **Galileon theory** additional terms can also be present, but the theory is **ghost-free**: EOM is second order (Nicolis, Rattazzi, Trincherini).

Nonlinearities become important below the **Vainshtein radius**
\[ r_V \sim (M/\Lambda^3 M_{Pl})^{1/3}. \]

**Does this construction survive quantum corrections?**

The DBI action
\[ S = \int d^4 x \mu \sqrt{1 + \partial_\mu \pi \partial^\mu \pi} \] corresponds to the simplest term of a theory of embedded surfaces.

The effective theory of **embedded surfaces** can be used in order to reproduce the **Galileon theory** at low energies \( (\partial \pi)^2 << 1 \) (de Rham, Tolley).
Outline

- Classical solutions and Vainshtein mechanism.
- Renormalization of the cubic Galileon theory, perturbative background.
- Heat-kernel method for nontrivial backgrounds.
- Suppression of quantum corrections by the Vainshtein mechanism.
- Classicalon.
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Classical solution, Vainshtein mechanism

- The classical EOM for cubic Galileon is
  \[
  \Box \pi - \frac{1}{\Lambda^3} (\Box \pi) + \frac{1}{\Lambda^3} \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi = T \delta^3(\vec{x})
  \]

- Spherically symmetric solution \((w = r^2)\)
  \[
  \pi'_{cl}(w) = \frac{1}{8\nu} \left( 1 - \sqrt{1 + \frac{16\nu c}{w^{3/2}}} \right).
  \]

- \(r_v \sim (c\nu)^{1/3}\)
- For \(r \ll r_v\) we have \(\pi \sim \sqrt{c/\nu} \sqrt{r}\).
- For \(r \gg r_v\) we have \(\pi \sim c/r\).
Renormalization of the Galileon theory

- Perturbative background.

\[ S = \int d^4 x \left\{ \frac{1}{2} (\partial \pi)^2 - \frac{\nu}{2} (\partial \pi)^2 \Box \pi + \frac{\kappa}{4} (\partial \pi)^2 \left( (\Box \pi)^2 - (\partial_\mu \partial_\nu \pi)^2 \right) + \ldots \right\}. \]

- If a momentum cutoff is used, of the order of the fundamental scale \( \Lambda \) of the theory, and the couplings are taken of order \( \Lambda \), the one-loop effective action of the Galileon theory is, schematically, (Luty, Porrati, Nicolis, Rattazzi)

\[ \Gamma_1 \sim \int d^4 x \sum_m \left[ \Lambda^4 + \Lambda^2 \partial^2 + \partial^4 \log \left( \frac{\partial^2}{\Lambda^2} \right) \right] \left( \frac{\partial^2 \pi}{\Lambda^3} \right)^m. \]

- Non-renormalization of the Galileon couplings (de Rham, Gabadadze, Heisenberg, Pirtskhalava, Hinterbichler, Trodden, Wesley).

- Explicit one-loop calculation using dimensional regularization (Paula Netto, Shapiro).
One-loop corrections to the cubic Galileon

- Tree-level action in Euclidean $d$-dimensional space

\[ S = \int d^d x \left\{ \frac{1}{2} (\partial \pi)^2 - \frac{\nu}{2} (\partial \pi)^2 \Box \pi \right\}. \]

- Field fluctuation $\delta \pi$ around the background $\pi$. The quadratic part is

\[ S^{(2)} = \int d^d x \left\{ -\frac{1}{2} \delta \pi \Box \delta \pi + \frac{\nu}{2} \delta \pi \left[ 2 (\Box \pi) \Box \delta \pi - 2 (\partial \mu \partial ^\nu \pi) \partial \mu \partial ^\nu \delta \pi \right] \right\}. \]

- Define

\[ K = -\Box \quad \Sigma_1 = 2\nu (\Box \pi) \Box \quad \Sigma_2 = -2\nu (\partial_\mu \partial ^\nu \pi) \partial ^\mu \partial ^\nu \]

- One-loop contribution to the effective action

\[ \Gamma_1 = \frac{1}{2} \text{tr} \log \left( K + \Sigma_1 + \Sigma_2 \right) = \frac{1}{2} \text{tr} \log \left( 1 + \Sigma_1 K^{-1} + \Sigma_2 K^{-1} \right) + \mathcal{N}. \]
Expanding the logarithm up to $O(\nu^2)$ we obtain

\[
\text{tr} \left( \Sigma_1 K^{-1} \Sigma_1 K^{-1} \right) = 4\nu^2 (2\pi)^d \int d^d k \ k^4 \tilde{\pi}(k) \tilde{\pi}(-k) \int \frac{d^d p}{(2\pi)^d}
\]

\[
\text{tr} \left( \Sigma_1 K^{-1} \Sigma_2 K^{-1} \right) = -4\nu^2 (2\pi)^d \int d^d k \ k^4 \tilde{\pi}(k) \tilde{\pi}(-k) \frac{1}{d} \int \frac{d^d p}{(2\pi)^d}
\]

\[
\text{tr} \left( \Sigma_2 K^{-1} \Sigma_2 K^{-1} \right) = 4\nu^2 (2\pi)^d \int d^d k \ \tilde{\pi}(k) \tilde{\pi}(-k) \left\{ \frac{3}{d(d+2)} k^4 \int \frac{d^d p}{(2\pi)^d} + \frac{(d-8)(d-1)}{d(d+2)(d+4)} k^6 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} - \frac{(d-24)(d-2)(d-1)}{d(d+2)(d+4)(d+6)} k^8 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^4} \right\}.
\]
Putting everything together, we obtain in position space, the one-loop correction to the effective action

\[ \Gamma^{(2)}_1 = \nu^2 \int d^d x \pi(x) \left\{ - \frac{d^2 - 1}{d(d + 2)} \left( \int \frac{d^d p}{(2\pi)^d} \right) \Box^2 + \frac{(d - 8)(d - 1)}{d(d + 2)(d + 4)} \left( \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} \right) \Box^3 + \frac{(d - 24)(d - 2)(d - 1)}{d(d + 2)(d + 4)(d + 6)} \left( \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^4} \right) \Box^4 \right\} \pi(x). \]

- The momentum integrals are defined with UV and IR cutoffs.
- If dimensional regularization near \( d = 4 \) is used, the first two terms are absent. The third one corresponds to a counterterm \( \sim 1/\epsilon \) (Paula Netto, Shapiro).
- No corrections to the Galileon couplings.
- Terms outside the Galileon theory are generated.
Perturbation theory:

\[ \Gamma_1 \sim \int d^4 x \sum_m \left[ \Lambda^4 + \Lambda^2 \partial^2 + \partial^4 \log \left( \frac{\partial^2}{\Lambda^2} \right) \right] (\nu \partial^2 \pi)^m. \]

- Split the field as \( \pi = \pi_{cl} + \delta\pi. \)
- The action includes terms \( \sim \nu^2 \Lambda^4 (\nu \square \pi_{cl})^n (\square \delta\pi)^2 \)
- But \( \nu \square \pi_{cl} \sim (r_V/r)^{3/2} \gg 1 \) below the Vainshtein radius.
Heat-kernel approach around a nontrivial background

- Our task is to evaluate the one-loop effective action

\[ \Gamma_1 = \frac{1}{2} \text{tr} \log \Delta \]

with

\[ \Delta = -\Box + 2\nu (\Box \pi) \Box - 2\nu (\partial_\mu \partial_\nu \pi) \partial^\mu \partial^\nu \]

around the background \((w = r^2)\)

\[ \pi'_\text{cl}(w) = \frac{1}{8\nu} \left( 1 - \sqrt{1 + \frac{16\nu c}{w^{3/2}}} \right). \]

- The propagation of classical fluctuations in suppressed below the Vainshtein radius \(r_V \sim (\nu c)^{1/3}\), where \(\nu \Box \pi_{cl} \sim (r_V / r)^{3/2} \gg 1\).

- What about the quantum fluctuations?
Calculate the heat kernel

\[ h(x, x', \epsilon) = \int \frac{d^4 k}{(2\pi)^4} e^{-ikx'} e^{-\epsilon \Delta} e^{ikx} \]

The one-loop effective action can be obtained as

\[ \Gamma_1 = -\frac{1}{2} \int_{1/\Lambda^2}^{\infty} \frac{d\epsilon}{\epsilon} \int d^4 x \ h(x, x, \epsilon). \]

\[ h(x, x, \epsilon) = \int \frac{d^4 k}{(2\pi)^4} e^{-k^2} e^{\sqrt{\epsilon}X(k, \partial) + \epsilon Y(k, \partial)} \]  \hspace{1cm} (1)

Expand in powers of \( \sqrt{\epsilon} \). The result is the derivative expansion of the effective action.
The diagonal part of the heat kernel becomes

$$h(x, x, \epsilon) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{\epsilon^2} \exp \left\{ -k^2 + 2i\sqrt{\epsilon} k^\mu \partial_\mu + \epsilon \Box 
+ 2\nu \Box \pi \left(k^2 - 2i\sqrt{\epsilon} k^\mu \partial_\mu - \epsilon \Box \right)
- 2\nu \partial_\mu \partial_\nu \pi \left(k^\mu k^\nu - 2i\sqrt{\epsilon} k^\mu \partial^\nu - \epsilon \partial_\mu \partial^\nu \right) \right\}$$

- Expand in $\epsilon$ and $\nu$.
- The leading perturbative result is reproduced:

$$h(x, x, \epsilon) = \frac{15}{32 \pi^2 \epsilon^2 \nu^2 (\Box \pi)^2}$$

$$\Gamma_1^{(2)} = -\frac{1}{2} \int_{1/\Lambda^2}^{\infty} \frac{d\epsilon}{\epsilon} \int d^4 x \, h(x, x, \epsilon) = -\frac{15}{128 \pi^2 \nu^2 \Lambda^4} \int d^4 x \, (\Box \pi)^2.$$
Heat kernel

- The exponent of the heat-kernel is \( \pi = \pi_{cl} + \delta \pi \)

\[
F = -G_{\mu \nu} k^\mu k^\nu - (1 - 2\nu \Box \pi_{cl}) D_\epsilon(k) + 2\nu \partial_\mu \partial_\nu \pi_{cl} L_\epsilon^{\mu \nu}(k) \\
+ 2\nu \Box \delta \pi (k^2 + D_\epsilon(k)) + 2\nu \partial_\mu \partial_\nu \delta \pi (-k^\mu k^\nu + L_\epsilon^{\mu \nu}(k))
\]

with the "metric" \( G_{\mu \nu} = g_{\mu \nu} - 2\nu \Box \pi_{cl} g_{\mu \nu} + 2\nu \partial_\mu \partial_\nu \pi_{cl} \) and

\[
D_\epsilon(k) = -2i \sqrt{\epsilon} k^\mu \partial_\mu - \epsilon \Box \\
L_\epsilon^{\mu \nu}(k) = 2i \sqrt{\epsilon} k^\mu \partial^\nu + \epsilon \partial^\mu \partial^\nu.
\]

- Make the "metric" \( G_{\mu \nu} \) trivial by rescaling \( k^\mu = S^\mu_\nu k'^\nu \), with

\[
S^\mu_\rho G_{\mu \nu} S^\nu_\sigma = g_{\rho \sigma}.
\]

- The most divergent term quadratic in \( \delta \pi \) in the heat kernel is

\[
h(x, x, \epsilon) = \int \frac{d^4 k}{(2\pi)^4} (\det S) \frac{1}{2\epsilon^2} e^{-k^2} \left(2\nu \Box \delta \pi (Sk)^2 \\
+ 2\nu \partial_\mu \partial_\nu \delta \pi (-Sk^\mu Sk^\nu) \right)^2.
\]
On the background that realizes the Vainshtein mechanism

\[
\Gamma_1^{(2)} = -\frac{1}{128\pi^2} \nu^2 \Lambda^4 \int d^4x \left( (\Box \delta \pi)^2 P(r^2) - 2(\Box \delta \pi)(\partial_\mu \partial_\nu \delta \pi) V^{\mu\nu}(r^2) 
+ (\partial_\mu \partial_\nu \delta \pi)(\partial_\rho \partial_\sigma \delta \pi) W^{\mu\nu\rho\sigma}(r^2) \right).
\]

with \( P(r^2), V^{\mu\nu}(r^2), W^{\mu\nu\rho\sigma}(r^2) \sim (r/r_V)^6 \) and \( r_V \sim (\nu c)^{1/3} \).
Figure: \((\det S)\left(S^i_i\right)^4\) as a function of \(r\) with \(\nu = 1, c = 10^6\). The solid, blue line corresponds to \(i = 0\), the dotted, red line to \(i = 1\) and the dashed, green line to \(i = 2\) or \(3\).
Higher order in $\epsilon$

- The heat-kernel for the cubic Galileon takes the form

$$h(x, x, \epsilon) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{\epsilon^2} \exp \left\{ -G_{\mu\nu} k^\mu k^\nu + 2i\sqrt{\epsilon} G_{\mu\nu} k^\mu \partial^\nu + \epsilon G_{\mu\nu} \partial^\mu \partial^\nu \right\},$$

- $X = -G_{\mu\nu} k^\mu k^\nu$, \hspace{0.5cm} $Y = 2i\sqrt{\epsilon} G_{\mu\nu} k^\mu \partial^\nu + \epsilon G_{\mu\nu} \partial^\mu \partial^\nu$.
- $e^{X+Y} = e^X \left(1 - \frac{1}{2} Y[X, Y] - \frac{1}{2} [X, Y] + \ldots\right)$. 
The general structure of the effective action is

\[
\Gamma^{(2)}_1 = \nu^2 \int d^4x \left[ \Lambda^4 \left( c_0 \frac{r^6}{R_V^6} (\delta \pi \partial^4 \delta \pi) \right) + \Lambda^2 \left( c_{1a} \frac{r^{5/2}}{R_V^{9/2}} (\delta \pi \partial^4 \delta \pi) + c_{1b} \frac{r^{7/2}}{R_V^{9/2}} (\delta \pi \partial^5 \delta \pi) + c_{1c} \frac{r^{9/2}}{R_V^{9/2}} (\delta \pi \partial^6 \delta \pi) \right) 
+ \log(\Lambda/\mu) \left( c_{2a} \frac{1}{r R_V^3} (\delta \pi \partial^4 \delta \pi) + c_{2b} \frac{1}{R_V^3} (\delta \pi \partial^5 \delta \pi) 
+ c_{2c} \frac{r}{R_V^3} (\delta \pi \partial^6 \delta \pi) + c_{2d} \frac{r^2}{R_V^3} (\delta \pi \partial^7 \delta \pi) + c_{2e} \frac{r^3}{R_V^3} (\delta \pi \partial^8 \delta \pi) \right) \right].
\]
Classicalon

We repeat the same procedure for the Classicalon field.

\[ S = \int d^4 x \left( \frac{1}{2} \partial_\mu \pi \partial^\mu \pi + \frac{1}{\Lambda^4} (\partial_\mu \pi \partial^\mu \pi)^2 \right). \]

\[ G_{\mu\nu} = g_{\mu\nu} \left( 1 + \frac{\nu}{2} \partial_\rho \pi \partial^\rho \pi \right) + \nu \partial_\mu \pi \partial_\nu \pi. \]

\[ r_c = \frac{1}{\Lambda} \left( \frac{M}{\Lambda} \right)^{\frac{1}{2}} \]

\[ h(x, x, \epsilon) = \frac{1}{16\pi^2 \epsilon^2} \det S \]
Figure: \((\det S) (S_i^i)^4\) as a function of \(r\) with \(\Lambda = 1\), \(r_c = 30\).
Conclusions

- The couplings of the Galileon theory do not get renormalized. However, the Galileon theory is not stable under quantum corrections. Additional terms are generated.
- Quantum corrections are suppressed below the Vainshtein radius.
- The Classicalon model possibly shares the same properties.