Understanding hysteresis - same universality class of a system with disorder in and out of equilibrium

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Collaboration

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What is hysteresis?

Where does it appear?

- Ferromagnetic amorphous alloys  
  O'Brien, Weissman: PRE, 50, 3446

- Capillary condensation of helium in pores of an aerogel  
  Lilly, Finley, Hallock: PRL, 71, 4186

- Structural martensic transition (Cu-Zn-Al alloys)  
  Vives et al.: PRL, 72, 1694

- Charge density wave systems  
  Middleton: PRL, 68, 670

- “SOC” self organized criticality  
  Bak, Tang: PRL 59 381
Barkhausen noise

Heinrich Barkhausen  Z.Physik 20, 401 (1919)

- noninvasive testing and characterization of samples
  - residual stress  Curr. Appl. Phys. 4, 308
  - grain sizes  J. Appl. Phys. 61, 3199, Acta Mat. 49, 3019

Dobrinevski: arXiv:1312.7156
What is in common?
- interplay of two phases
- discontinuous phase transition in the background
- thermal activation is a secondary effect
- non-equilibrium
- disorder

Is there a theoretical model?
Can it be solved?

random field Ising model (RFIM):
- Hamiltonian $H = \sum \langle i, j \rangle - J \sigma_i \sigma_j - \sum_i h_i \sigma_i - h \sum_i \sigma_i$
- Gaussian disorder distribution: $P[h_i] = \frac{1}{\sqrt{2\pi\Delta^2}} e^{-\frac{h_i^2}{2\Delta}}$
Can RFIM explain all the concepts? - YES

Sethna & Dahmen (1990-2000): “plain old criticality...”
- power law behavior
- critical exponents
- ...

Phase transition by changing disorder strength $\Delta$:

$\Delta > \Delta_c$

$\Delta = \Delta_c$

$\Delta < \Delta_c$

GOAL: rigorously understand the phase transition in hysteresis
What next?

random field Ising model

⇓

Field theory

⇓

Non-Perturbative Renormalization Group

⇓

Result
A) formulate the field theory for RFIM (Hubbard-Stratonowich):

\[ H \rightarrow \int_x \left\{ \frac{1}{2} (\nabla_x \phi_x)^2 + \frac{r}{2} \phi_x^2 + \frac{u}{4!} \phi_x^4 - (h_x + J_B) \phi_x \right\} \]

B) sum over solutions of the stationary stochastic equation (+ source \( \hat{J} \)):

\[ Z = \int D\phi \delta[\frac{\delta S}{\delta \phi} - h_x - J_B] |\frac{\delta^2 S}{\delta \phi_x \delta \phi_y}| e^{\int_x \hat{J}_x \phi_x} \]

C) write \( Z \) in exponential form (superfield f. at \( T = 0 \) Parisi, Sourlas: PRL 43 744)

\[ Z = \int D(\phi, \hat{\phi}, \psi, \bar{\psi}) e^{\int_x \left\{ -\hat{\phi}_x(\frac{\delta S}{\delta \phi_x} - h_x - J_B) + \hat{J}_x \phi_x + \psi_x \bar{K}_x + \bar{\psi}_x K_x \right\} + \int_x \int_y \bar{\psi}_x \frac{\delta^2 S}{\delta \phi_x \delta \phi_y} \psi_y} \]

D) introduce a sum over infinite replicas Tarjus, Mouhanna: PRE 81, 051101 (2010)

\[ Z^n = \int \Pi_a D\Phi_a e^{\sum_a \left\{ \int_x \left\{ -\hat{\phi}_x^a(\frac{\delta S}{\delta \phi_x} - h_x - J_B) + \hat{J}_x^a \phi_x + \psi_x^a \bar{K}_x + \bar{\psi}_x^a K_x \right\} + \int_x \int_y \bar{\psi}_x^a \frac{\delta^2 S}{\delta \phi_x \delta \phi_y} \psi_y \right\}} \]

E) average over disorder:

\[ \overline{Z^n} = \int \Pi_a D\Phi_a e^{\sum_a \left\{ \int_x \left\{ -\hat{\phi}_x^a(\frac{\delta S}{\delta \phi_x} - J_B) + \hat{J}_x^a \phi_x + \psi_x^a \bar{K}_x + \bar{\psi}_x^a K_x \right\} + \int_x \int_y \cdots \right\}} + \frac{A}{4} \sum_{a,b} \int_x \hat{\phi}_a \hat{\phi}_b \]
Steps A)-E) define free energy

\[ \overline{Z^n} = e^{W[\{J_a, J_b, \cdots \}]} = \text{Exp}\left\{ \sum_a W_a[\{J_a\}] + \frac{1}{2} \sum_{a,b} W_{a,b}[\{J_a, J_b\}] + \cdots \right\} \]

F) Legendre transform \([W[\{J_a, J_b, \cdots \}]] \Rightarrow \Gamma[\{M_a, M_b, \cdots \}]\]

⇒ functional of the effective average action
Steps A)-E) define free energy

\[ \tilde{Z}^n = e^{W[\{ \mathcal{J}_a, \mathcal{J}_b, \cdots \}]} = \text{Exp}\left[ \sum_a W_a[\{ \mathcal{J}_a \}] + \frac{1}{2} \sum_{a,b} W_{a,b}[\{ \mathcal{J}_a, \mathcal{J}_b \}] + \cdots \right] \]

F) Legendre transform \[ W[\{ \mathcal{J}_a, \mathcal{J}_b, \cdots \}] \Rightarrow \Gamma[\{ \mathcal{M}_a, \mathcal{M}_b, \cdots \}] \]

⇒ functional of the effective average action

⇒ use NPRG

- introduce a proper infrared regulator
- set the Wetterich equation:

\[ \partial_t \Gamma = \frac{1}{2} \text{Tr} \int_q \]
Why is all this about hysteresis?
A) hysteresis

Auxiliary source $\hat{J}$ (introduced in steps A)-F):

- enters the partition function as $Z = \int D\phi e^{-\Gamma'} + \int_x \hat{J}_x \phi_x$

![Diagram showing hysteresis loop and expectation value $m = \langle \phi \rangle$]
A) hysteresis

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- $\hat{J} \to \infty$
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- $\hat{J} \to \infty$

\[ m = \langle \phi \rangle \]
a problem of choosing the ground state!
introduce auxiliary temperature $T_a$: $Z = \int D\phi e^{\int x \frac{1}{T_a} S[\phi]} + \ldots$

lifts the ground state degeneracy!
B) equilibrium RFIM Tarjus & Tissier 2004-2012

- Introduce auxiliary temperature $T_a$: $Z = \int D\phi e^{\int_x \frac{1}{T_a} S[\phi]} + \cdots$
- Equations do not depend on $T_a$ ($T_a \to 0$)
B) equilibrium RFIM Tarjus & Tissier 2004-2012

- introduce auxiliary temperature $T_a$: $Z = \int D\phi e^{\int_x \frac{1}{T_a} S[\phi]} + \cdots$
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A) hysteresis ($\hat{J} \to \pm \infty$)

B) equilibrium RFIM ($\hat{J} \to 0$ + auxiliary temperature $T_a$)
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B) equilibrium RFIM ($\hat{J} \to 0 +$ auxiliary temperature $T_a$)

\[ \partial_t \Gamma = \beta \Gamma \]

formally identical flow equation

Are we done???
A) hysteresis (\( \hat{J} \to \pm \infty \))

B) equilibrium RFIM (\( \hat{J} \to 0 + \) auxiliary temperature \( T_a \))

\[ \partial_t \Gamma = \beta \Gamma \]

formally identical flow equation

Are we done???

NO: what are the initial conditions?
What is the meaning of the eff. average action? \((\mathcal{M} = (M, \hat{M}))\):

\[
\Gamma[\{M_a, M_b, \cdots\}] = \sum_a \Gamma_1[M_a] - \frac{1}{2} \sum_{a,b} \Gamma_2[M_a, M_b] + \cdots
\]

Derivatives by \(\hat{M}\) have a physical meaning:

- \[
\frac{\delta \Gamma_1[M_a]}{\delta \hat{M}_\alpha} = \frac{\delta}{\delta M_\alpha} \left( \frac{1}{2} (\nabla M_\alpha)^2 Z_k[M_\alpha] + U_k[M_\alpha] \right)
\]
- \[
\frac{\delta^2 \Gamma_2[M_a, M_b]}{\delta \hat{M}_\beta \delta \hat{M}_\alpha} = \Delta_k[M_\alpha, M_\beta]
\]
- higher terms in the sum \(= 0\)
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higher terms in the sum = 0

- \( Z_k \) - field renormalization (critical exponent \( \eta \))
- \( \Delta_k \) - renormalized disorder strength (critical exponent \( \tilde{\eta} \))
- \( U_k \) - effective potential
$U''_{k_1}$
move $m_{k_1-\delta k} = m_{k_1} + \delta m_{k_1,0}$ so that $U_{k_1}'''[0] = 0 \forall k$!
move $m_{k_1-\delta k} = m_{k_1} + \delta m_{k_1,0}$ so that $U'''[0] = 0 \; \forall k$!

Solve in dimensionless quantities (gives me critical exponents):

$$\Rightarrow M = k^{d-4+\bar{\eta}} m; \; Z[M] = k^{-\eta} z[m]; \; U''[M] = k^{2-\eta} u''[m]; \; \Delta[M_1, M_2] = k^{-2\eta + \bar{\eta}} \delta[m_1, m_2]$$
a) $d = 5.5$, b) $d = 4$
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Conclusions

♦ Z2 symmetry is asymptotically restored in $k \to 0$

♦ equations + renormalization functions at fixed point + critical exponents equal as in equilibrium RFIM = SAME UNIVERSALITY CLASS

♦ “COROLLARY”: fluid gas transition in the pores of aerogel is in the same universality class (binary system with an asymmetric initial condition!)
♦ study relaxation in the vicinity of the critical point

♦ implications on the physics of structural and spin glasses

♦ RFI model under external driving field
Thanks for your attention!