DIMENSIONAL REDUCTION IN ASYMPTOTICALLY SAFE GRAVITY

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Motivation

Renormalisation Group

Gravity
- Einstein-Hilbert Theory
- Asymptotic Safety Scenario
- Flow Equation

Dimensional Reduction
- Kaluza-Klein modes
- Effective Coupling
- Results

Conclusions & Outlook
Motivation I

• Asymptotically Safe Quantum Gravity might be tested at the LHC.

• Functional Renormalisation Group:
  Employed to investigate the Asymptotic Safety Scenario.

• “Large” extra dimensions (ADD model):
  Introduced to solve hierarchy problem of Particle Physics.
Motivation II

• "Large" extra dimensions (ADD model):
  - At short distances: Gravity not tested below $10^{-4}$ m, ample space for 'new' physics, e.g. extra dimensions as in the ADD model [1] to unify 'true' Planck with e.w. scale.
  - Low-scale Quantum Gravity:
    No hierarchy problem as $m_{EW} \approx$ gravity scale in $D=4+n$.
  - "Large" extra dimensions: $L^n \approx M_{Pl}^2/m_{EW}^{2+n}$
  - Appearance of Kaluza-Klein modes: Experimentally testable at LHC [2].
  - Is the ADD model UV complete?

**Renormalisation Group**

- Different resolution scales: \( k \) (momentum) or \( \ell \) (distance).

Start from \( M_{Pl} = (\hbar c/G_N)^{1/2} = 1.22 \times 10^{19} \text{ GeV} \), resp.,

\[ \ell_{Pl} = 1.62 \times 10^{-35} \text{ m} \]
Gravity I

Einstein-Hilbert Theory

- Einstein Gravity: metric field $g_{\mu\nu}(x)$
  - curvature (Riemann) tensor $R_{\mu\nu\rho\sigma}$
  - Ricci tensor $R_{\mu\nu} = R^\rho_{\mu\rho\nu}$
  - curvature scalar $R = R_{\mu\mu}$

- Einstein-Hilbert action (Euclidean signature):

  $$S_{EH} = \int d^Dx \sqrt{\det g_{\mu\nu}} \left( \frac{-R + 2\Lambda}{16\pi G_N} \right) + S_{\text{matter}}$$

Asymptotically safe!


- **Non-trivial UV fixed point** (A).
- **Gaussian IR fixed point** (B).
**Gravity III**

**Exact Functional Identity**

- **Wetterich equation** [3] for QEG [4]:

\[
\frac{d}{dk} \Gamma_k[g_{\mu\nu}; \bar{g}_{\mu\nu}] = \frac{1}{2} \text{Tr} \left[ \left( \Gamma_k^{(2)}[g_{\mu\nu}; \bar{g}_{\mu\nu}] + R_k \right)^{-1} k \frac{d}{dk} R_k \right]
\]

- **Effective action:**

\[
\Gamma_k = \int d^D x \, \sqrt{\text{det} g_{\mu\nu}} \left( \frac{-R + 2\Lambda_k}{16\pi G_k} + \ldots \right) + S_{\text{matter},k} + S_{gf,k} + S_{\text{ghosts},k}
\]

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\[ \partial_t g_k = \beta_g (g_k, \lambda_k) = (D - 2 + \eta_{Nk}) g_k \]

\[ \partial_t \lambda_k = \beta_\lambda (g_k, \lambda_k) = \eta_{Nk} \lambda_k - 2 \lambda_k + g_k \left( A_0 (\lambda_k) - \eta_{Nk} A_1 (\lambda_k) \right) \]

\[ \eta_{Nk} = \frac{g_k B_0 (\lambda_k)}{1 + g_k B_1 (\lambda_k)} \]

\[ \partial_t = k \frac{\partial}{\partial k} \]

\[ g_k \equiv k^{D-2} G_k \]

\[ \lambda_k \equiv k^{-2} \Lambda_k \]
**Dimensional Reduction I**

- **4+1 ADD model**: Choose one extra dimension to be compact (periodic boundary conditions), sum over Kaluza-Klein modes.

\[
\int \frac{d^D q}{(2\pi)^D} \to \frac{1}{L} \sum_n \int \frac{d^{D-1} q}{(2\pi)^{D-1}}, \quad q_D \to \frac{2\pi n}{L}
\]

- After many and 20: Expressions for the $\beta$-functions, e.g., in an approximate Background Field Flow:

\[
A_0 (\lambda_k ; kL) = \frac{1}{8 \pi} \left[ \frac{15}{\sqrt{1 - 2 \lambda_k}} \coth \left( \frac{kL\sqrt{1 - 2 \lambda_k}}{2} \right) - 10 \coth \left( \frac{kL}{2} \right) \right]
\]
**Effective Coupling**

Consistency of limits requires: \( g_k^{4D} = \frac{g_k^{5D}}{kL} \) for \( kL \ll 1 \).

Follows also from the identification \( G_{4N}^{4D} = G_{5N}^{5D} / L \)

Define **effective coupling** such that
- it is well-defined and finite in both limits \( L \to \infty \) and \( L \to 0 \).
- it connects smoothly both limits
- it behaves like \( k^2 \) for \( k \ll 1/L \) and \( k^3 \) for \( k \gg 1/L \), semi-class. regime.
- it displays the 4D to 5D crossover at \( k \approx 1/L \).

\[
g_{k,\text{eff}} = g_k B_0(\lambda_k; kL) / B_0^\infty \]

with \( B_0^\infty = \lim_{L \to \infty} B_0(\lambda_k; kL) \)
Conclusions & Outlook

- Asymptotic Safety Scenario to Einstein-Hilbert quantum gravity in four extended + one compact dimensions.
- Explicit example for an UV completion of the ADD model!!!
- 4D-5D crossover identified.

☐ Include several compact dimensions.
☐ Improve on truncation, e.g., f(R) gravity.
☐ Include matter.
☐ ...

Thank you!
Backup Slide

Result: Exact Functional Identity