The inhomogeneity of the Universe and the cosmological model

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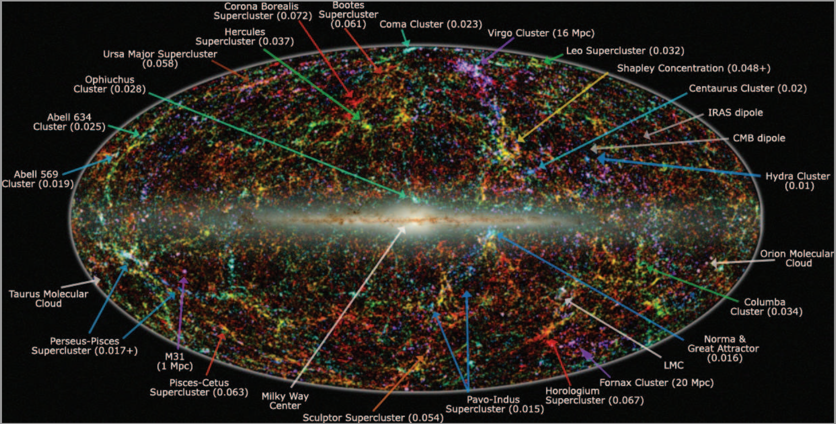


Figure: *2MASS Galaxy Catalog (more than 1.5 million galaxies).*

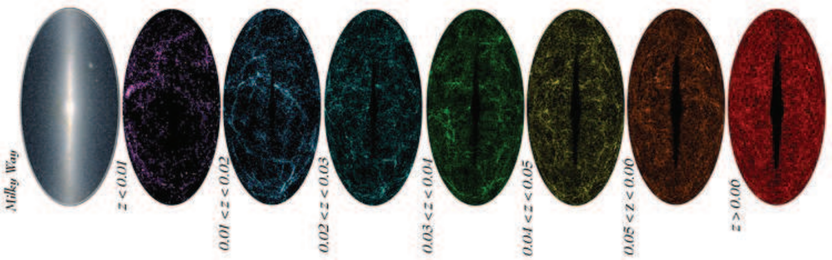


Figure: *2MASS Galaxy Catalog: Various redshifts.*

Outline

- Effect of inhomogeneities on the average expansion
- A similar problem in brane cosmology
- Large-scale structures in quintessence cosmology
- Non-linear spectrum of matter perturbations
    1. Coupled quintessence
    2. Variable equation of state
    3. Growing neutrino quintessence
- Conclusions

Standard framework

- Basic assumptions: Homogeneity and isotropy

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- Inhomogeneities can be treated as small perturbations of this background
- Indications for the acceleration of the cosmological expansion
  1. Distant supernovae
  2. Power spectrum of the galaxy distribution
  3. Cosmic microwave background
- For acceleration: $p < -\rho/3$

Question

- Assumed, but not observed directly, dominant contributions to the energy content of the Universe:
  1. Dark matter: $p = 0$ ($\sim 25\%$)
  2. Dark energy: $p < 0$ ($\sim 70\%$)
- Could the acceleration of the cosmological expansion be related to the appearance of inhomogeneities in a pressureless cosmological fluid (dark matter)?

Our approach

- All the information about the expansion of the Universe is obtained through light signals.
- Study light propagation in an exact background that mimics a Universe with structure.
- Calculate observables: Luminosity distance of a light source a function of its redshift.
- P. Apostolopoulos, N. Brouzakis, N. T., E. Tzavara
  astro-ph/0603234, JCAP 0606: 009 (2006)
  N. Brouzakis, N. T., E. Tzavara
  astro-ph/0612179, JCAP 0702: 013 (2007)
  astro-ph/0703586, JCAP 0804: 008 (2008)
  N. Brouzakis, N. T.
  arXiv:0802.0859 [astro-ph], Phys. Lett. B 665: 344-348 (2008)

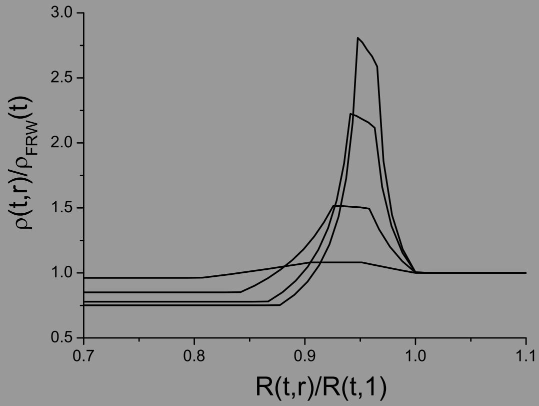


Figure: *The evolution of the density profile for a central underdensity surrounded by an overdensity.*

Luminosity distance and redshift

- Consider photons emitted within a solid angle Ω_s by an isotropic source with luminosity L. These photons are detected by an observer for whom the light beam has a cross-section A_o.
- The redshift factor is

$$1+z=\frac{\omega_s}{\omega_o}=\frac{k_s^0}{k_o^0},$$

- The energy flux f_o measured by the observer is

$$f_o=\frac{L}{4\pi D_L^2}=\frac{L}{4\pi}\frac{\Omega_s}{(1+z)^2 A_o}.$$

- Integrating the Sachs optical equations allows the determination of the luminosity distance D_L as a function of the redshift z.

Observer at the center of a large hole

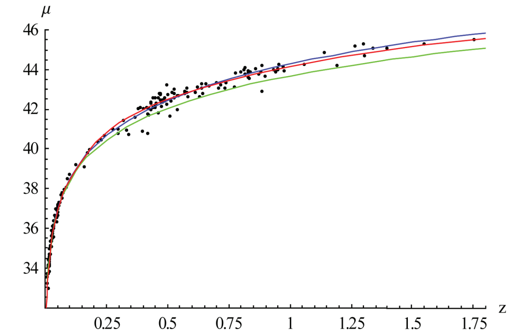


Figure: *The distance modulus* $\mu = m - M = 5\log(D_L/\mathrm{Mpc}) + 25$ *as a function of redshift z.*
a) Green line: FRW cosmology with $\Omega_m = 1$, $\Omega_\Lambda = 0$.
b) Blue line: FRW cosmology with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$.
c) Red line: LTB cosmology with the observer at the center of an underdense region of present size $\sim$ *800 Mpc.*

Observer and source at random positions

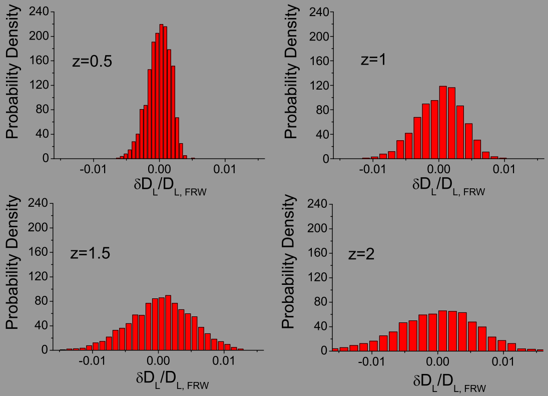


Figure: *The distribution of luminosity distances for various redshifts in the LTB Swiss-cheese model for inhomogeneities with length scale 40 h^{-1} Mpc.*

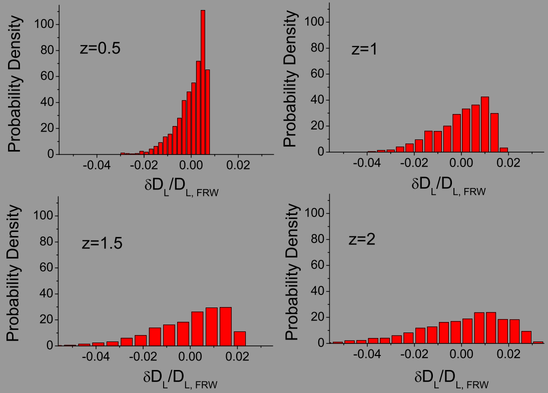


Figure: *Same as before for a characteristic scale of* $400\,h^{-1}$ *Mpc.*

A similar problem in brane cosmology

- Identify the Universe with a hypersurface (brane) in five-dimensional space-time. Low-energy gravity is localized near the brane (Randall, Sundrum (1999)).
- Assume an inhomogeneous energy distribution along the fourth spatial dimension. Is accelerated expansion possible along the brane?
- For an arbitrary energy distribution, accelerated expansion requires negative pressure either on the brane or in the bulk.
- This holds even when corrections, such as an induced gravity term on the brane, or a Gauss-Bonnet term in the bulk, are taken into account.
- P. Apostolopoulos, N. T.
  astro-ph/0604014, Phys. Rev. D 74: 064021 (2006)
  P. Apostolopoulos, N. Brouzakis, N. T., E. Tzavara
  arXiv:0708.0469, Phys. Rev. D 76: 084029 (2007)

Quintessence cosmology

- It seem unlikely that the acceleration of the cosmological expansion can be attributed to the growth of inhomogeneities.
- Negative pressure is needed.
- The simplest scenario assumes the presence of a cosmological constant.
- The quintessence scenario attempts to provide a dynamical explanation for the smallness of the present value of the vacuum energy.
- We shall discuss coupled quintessence: a quintessence field coupled with dark matter (or neutrinos).
- What kind of new structures can appear in such cosmologies?
- Are they observable?

Basic relations

- Action

$$\mathcal{S} = \int d^4x \sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{2}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - U(\phi)\right) - \sum_i \int m(\phi(x_i))d\tau_i,$$

with $d\tau_i = \sqrt{-g_{\mu\nu}(x_i)dx_i^\mu dx_i^\nu}$ and the second integral taken over particle trajectories.

- Equation of motion

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, g^{\mu\nu}\frac{\partial\phi}{\partial x^\nu}\right) = \frac{dU}{d\phi} - \frac{d\ln m(\phi(x))}{d\phi}\,(T_M)^\mu_{\ \mu}\,.$$

Cosmological evolution

- Homogeneous background

$$\ddot{\phi} + 3H\dot{\phi} = \frac{dU}{d\phi} + \frac{d\ln m(\phi)}{d\phi}(\rho - 3p)$$

$$\dot{\rho} + 3H\rho = -\frac{d\ln m(\phi)}{d\phi}(\rho - 3p)\dot{\phi}$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + U(\phi) + \rho\right)$$

Static spherically symmetric configurations

- Metric:

$$ds^2 = -B(r)dt^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + A(r)dr^2.$$

- Fermi-Dirac distribution at every point in space:

$$f(p) = \left[\exp\left(\frac{\sqrt{p^2 + m^2(\phi(r))} - \mu(r)}{T(r)}\right) + 1\right]^{-1}.$$

- The Einstein equations give:

$$T(r) = T_0/\sqrt{B(r)}, \qquad \mu(r) = \mu_0/\sqrt{B(r)}.$$

- N. T.
  hep-ph/0507288, Phys. Lett. B 632: 463-466 (2006)
  N. Brouzakis, N. T.
  astro-ph/0509755, JCAP 0601: 004 (2006)
  N. T., J.D. Vergados, A. Faessler
  hep-ph/0609078, Phys. Rev. D 75: 023504 (2007)

Dark matter in galaxy haloes

- The coupling between DM and the quintessence field generates an attractive force between DM particles.
- The typical DM velocity is larger than in the decoupled case.
- Implications for DM detection.

Compact astrophysical objects made of dark matter

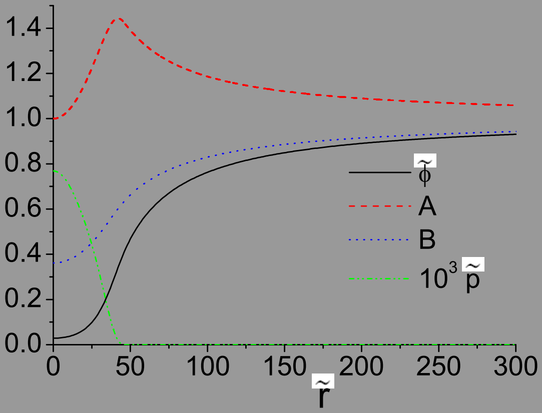


Figure: *A typical configuration*

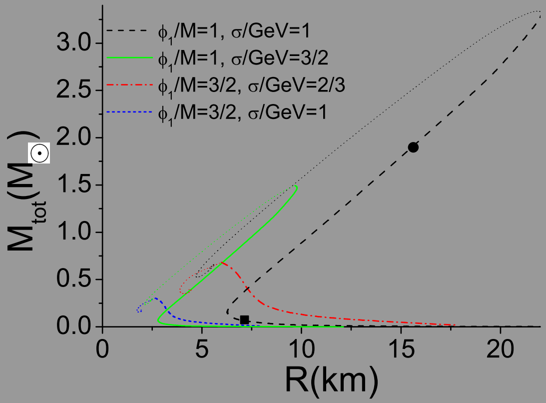
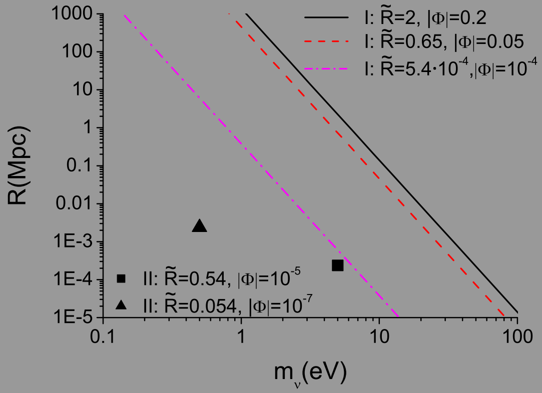


Figure: *The mass to radius relation.*

Astrophysical objects made of neutrinos



N. Brouzakis, N. T., C. Wetterich
e-Print: arXiv:0711.2226 [astro-ph] , Phys. Lett. B 665: 131 (2008)

Link with observations

- Study the formation of structure in the distribution of dark (and baryonic) matter.
- Dark energy does not cluster.
- The evolution of inhomogeneities depends on the cosmological background.
- The matter spectrum at various redshifts reflects the detailed structure of the cosmological model.
- Comparison with observations of the galaxy distribution can differentiate between models.
- Baryon acoustic oscillations: 100 Mpc range.
- Analytical calculation of the matter spectrum beyond the linear level. Crocce, Scoccimarro (2005)

Sloan Digital Sky Survey (2005)

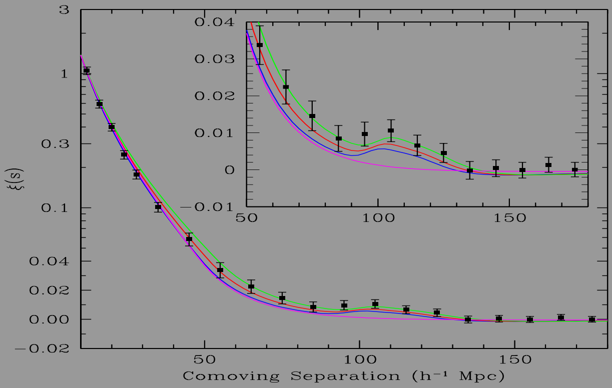


Figure: Galaxy correlation function.

Formalism: Time renormalization group (Pietroni 2008)

- Action for quintessence and non-relativistic fluid

$$\mathcal{S} = \int d^4x \sqrt{-g}\left(\frac{1}{2M^2}R - \frac{1}{2}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - U(\phi)\right) - \sum_i \int m(\phi(x_i))d\tau_i,$$

with $d\tau_i = \sqrt{-g_{\mu\nu}(x_i)dx_i^\mu dx_i^\nu}$ and the second integral taken over particle trajectories.

- Equation of motion

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, g^{\mu\nu}\frac{\partial\phi}{\partial x^\nu}\right) = \frac{dU}{d\phi} - \frac{d\ln m(\phi(x))}{d\phi}\,(T_{CDM})^\mu_{\ \mu}.$$

- $M = 1$
- $\beta(\phi) = -d\ln m(\phi)/d\phi$.

- **Ansatz for the metric**

$$ds^2 = a^2(\tau)\left[(1+2\Phi(\tau,\vec{x}))\,d\tau^2 - (1-2\Phi(\tau,\vec{x}))\,d\vec{x}\,d\vec{x}\right],$$

  with $\Phi \ll 1$.

- **Scalar field**

$$\phi(\tau,\vec{x}) = \bar{\phi}(\tau) + \delta\phi(\tau,\vec{x}),$$

  with $\delta\phi/\bar{\phi} \ll 1$. In general, $\bar{\phi} = \mathcal{O}(1)$ in units of M.

- **Density field**

$$\rho(\tau,\vec{x}) = \bar{\rho}(\tau) + \delta\rho(\tau,\vec{x}),$$

  with $\delta\rho/\bar{\rho} \lesssim 1$.

- **Velocity field**

$$|\delta\vec{v}| \ll 1.$$

Hierarchy of scales

- For subhorizon perturbations with momenta $k \gg \mathcal{H} = \dot{a}/a$, the linear analysis predicts $|\delta\vec{v}| \sim (k/\mathcal{H})\Phi \sim (\mathcal{H}/k)(\delta\rho/\bar{\rho})$.
- We assume the hierarchy of scales: $\Phi, \delta\phi/\bar{\phi} \ll |\delta\vec{v}| \ll \delta\rho/\bar{\rho} \lesssim 1$.
- As we are dealing with subhorizon perturbations, we assume that the spatial derivatives of $\Phi, \delta\phi, \delta\vec{v}$ dominate over their time derivatives. We also assume that a spatial derivative acting on Φ, $\delta\phi$ or $\delta\vec{v}$ increases the position of that quantity in the hierarchy by one level: $\vec{\nabla}\Phi$ is comparable to $\mathcal{H}\delta\vec{v}$, while $\nabla^2\Phi$ is comparable to $\mathcal{H}^2\delta\rho/\bar{\rho}$.

F. Saracco, M. Pietroni, N. T., V. Pettorino, G. Robbers
arXiv:0911.5396[astro-ph], Phys. Rev. D 82: 023528 (2010)

Equations of motion for several non-relativistic species

- The evolution of the homogeneous background is described by

$$\mathcal{H}^2 = \frac{1}{3}\left[a^2 \sum_{i=1,2} \bar{\rho}_i + \frac{1}{2}\dot{\bar{\phi}}^2 + a^2 U(\bar{\phi})\right] \equiv \frac{1}{3} a^2 \rho_{tot}$$

$$\dot{\bar{\rho}}_i + 3\mathcal{H}\bar{\rho}_i = -\,\beta_i\,\dot{\bar{\phi}}\bar{\rho}_i$$

$$\ddot{\bar{\phi}} + 2\mathcal{H}\dot{\bar{\phi}} = -\,a^2\left(\frac{dU}{d\phi}(\bar{\phi}) - \sum_{i=1,2}\beta_i\,\bar{\rho}_i\right),$$

  with $\rho_{tot} \equiv \sum_i \bar{\rho}_i + \dot{\bar{\phi}}^2/(2a^2) + U(\bar{\phi})$.
- For the CDM we set $\beta_1 = \beta$, while for BM, because of strong observational constraints , we set $\beta_2 = 0$.

Equations for the perturbations

- Poisson equations

$$\nabla^2\delta\phi = -\,a^2\sum_i \beta_i\delta\rho_i \equiv -3\sum_i \beta_i\mathcal{H}^2\Omega_i\delta_i$$

$$\nabla^2\Phi = \frac{1}{2}a^2\sum_i \delta\rho_i \equiv \frac{3}{2}\mathcal{H}^2\sum_i \Omega_i\delta_i,$$

with $\Omega_i(\tau) \equiv \bar{\rho}_i\, a^2/(3\mathcal{H}^2)$.

- Continuity and Euler equations

$$\delta\dot{\rho}_i + 3\mathcal{H}\delta\rho_i + \vec{\nabla}[(\bar{\rho}_i + \delta\rho_i)\delta\vec{v}_i] = -\,\beta_i\dot{\bar{\phi}}\delta\rho_i$$

$$\delta\dot{\vec{v}}_i + (\mathcal{H} - \beta_i\dot{\bar{\phi}})\delta\vec{v}_i + (\delta\vec{v}_i\cdot\vec{\nabla})\delta\vec{v}_i = -\,\vec{\nabla}\Phi + \beta_i\vec{\nabla}\delta\phi.$$

Time renormalization group (Pietroni (2008))

- Fourier-transformed density contrast and velocity field: $\delta_i \equiv \delta\rho_i(\mathbf{k},\tau)/\bar{\rho}_i$ and $\theta_i(\mathbf{k},\tau) \equiv \vec{\nabla}\cdot\delta\vec{v}_i(\mathbf{k},\tau)$.
- They obey

$$\dot{\delta}_i(\mathbf{k},\tau)+\theta_i(\mathbf{k},\tau) = -\int d^3\mathbf{k}_1\, d^3\mathbf{k}_2\, \delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\, \tilde{\alpha}(\mathbf{k}_1,\mathbf{k}_2)\, \theta_i(\mathbf{k}_1,\tau)\, \delta_i(\mathbf{k}_2,\tau$$

where $\tilde{\alpha}(\mathbf{k}_1,\mathbf{k}_2) = \mathbf{k}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)/k_1^2$, and

$$\dot{\theta}_i(\mathbf{k},\tau)+(\mathcal{H}-\beta_i\dot{\bar{\phi}})\theta_i(\mathbf{k},\tau) + \frac{3\mathcal{H}^2\sum_j \Omega_j(2\beta_i\beta_j+1)\delta_j(\mathbf{k},\tau)}{2}$$
$$= -\int d^3\mathbf{k}_1\, d^3\mathbf{k}_2\, \delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\, \tilde{\beta}(\mathbf{k}_1,\mathbf{k}_2)\, \theta_i(\mathbf{k}_1,\tau)\, \theta_i(\mathbf{k}_2,\tau),$$

where $\tilde{\beta}(\mathbf{k}_1,\mathbf{k}_2) = (\mathbf{k}_1+\mathbf{k}_2)^2\mathbf{k}_1\cdot\mathbf{k}_2/(2k_1^2k_2^2)$.

- Define the quadruplet

$$\begin{pmatrix} \varphi_1(\mathbf{k},\eta) \\ \varphi_2(\mathbf{k},\eta) \\ \varphi_3(\mathbf{k},\eta) \\ \varphi_4(\mathbf{k},\eta) \end{pmatrix} = e^{-\eta} \begin{pmatrix} \delta_{CDM}(\mathbf{k},\eta) \\ -\frac{\theta_{CDM}(\mathbf{k},\eta)}{\mathcal{H}} \\ \delta_{BM}(\mathbf{k},\eta) \\ -\frac{\theta_{BM}(\mathbf{k},\eta)}{\mathcal{H}} \end{pmatrix},$$

where $\eta = \ln a(\tau)$.

- The equations of motion become

$$\partial_\eta \varphi_a(\mathbf{k},\eta) + \Omega_{ab}\varphi_b(\mathbf{k},\eta) = e^{\eta}\gamma_{abc}(\mathbf{k},-\mathbf{k}_1,-\mathbf{k}_2)\varphi_b(\mathbf{k}_1,\eta)\varphi_c(\mathbf{k}_2,\eta).$$

The indices a, b, c take values $1, \ldots, 4$. Repeated momenta are integrated over, while repeated indices are summed over.

The non-zero components of the effective vertices γ are

$$\gamma_{121}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \frac{\tilde{\alpha}(\mathbf{k}_1,\mathbf{k}_2)}{2}\delta_D(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2) = \gamma_{112}(\mathbf{k},\mathbf{k}_2,\mathbf{k}_1)$$
$$\gamma_{222}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \tilde{\beta}(\mathbf{k}_1,\mathbf{k}_2)\,\delta_D(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2)$$
$$\gamma_{343}(\mathbf{k},\mathbf{k}_3,\mathbf{k}_4) = \frac{\tilde{\alpha}(\mathbf{k}_3,\mathbf{k}_4)}{2}\delta_D(\mathbf{k}+\mathbf{k}_3+\mathbf{k}_4) = \gamma_{334}(\mathbf{k},\mathbf{k}_4,\mathbf{k}_3)$$
$$\gamma_{444}(\mathbf{k},\mathbf{k}_3,\mathbf{k}_4) = \tilde{\beta}(\mathbf{k}_3,\mathbf{k}_4)\,\delta_D(\mathbf{k}+\mathbf{k}_3+\mathbf{k}_4).$$

The Ω-matrix, that defines the linear evolution, is

$$\Omega(\eta) = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -\frac{3}{2}\Omega_{CDM}(2\beta^2+1) & 2-\beta\bar{\phi}'+\frac{\mathcal{H}'}{\mathcal{H}} & -\frac{3}{2}\Omega_{BM} & 0 \\ 0 & 0 & 1 & -1 \\ -\frac{3}{2}\Omega_{CDM} & 0 & -\frac{3}{2}\Omega_{BM} & 2+\frac{\mathcal{H}'}{\mathcal{H}} \end{pmatrix}.$$

- Define the spectra, bispectra and trispectra as

$$
\begin{aligned}
\langle \varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{q},\eta)\rangle \equiv& \delta_D(\mathbf{k}+\mathbf{q})P_{ab}(\mathbf{k},\eta) \\
\langle \varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{q},\eta)\varphi_c(\mathbf{p},\eta)\rangle \equiv& \delta_D(\mathbf{k}+\mathbf{q}+\mathbf{p})B_{abc}(\mathbf{k},\mathbf{q},\mathbf{p},\eta) \\
\langle \varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{q},\eta)\varphi_c(\mathbf{p},\eta)\varphi_d(\mathbf{r},\eta)\rangle \equiv& \delta_D(\mathbf{k}+\mathbf{q})\delta_D(\mathbf{p}+\mathbf{r})P_{ab}(\mathbf{k},\eta)P_{cd}(\mathbf{p},\eta) \\
&+\delta_D(\mathbf{k}+\mathbf{p})\delta_D(\mathbf{q}+\mathbf{r})P_{ac}(\mathbf{k},\eta)P_{bd}(\mathbf{q},\eta) \\
&+\delta_D(\mathbf{k}+\mathbf{r})\delta_D(\mathbf{q}+\mathbf{p})P_{ad}(\mathbf{k},\eta)P_{bc}(\mathbf{q},\eta) \\
&+\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q}+\mathbf{r})Q_{abcd}(\mathbf{k},\mathbf{p},\mathbf{q},\mathbf{r},\eta).
\end{aligned}
$$

- Essential approximation: Neglect the effect of the trispectrum on the evolution of the bispectrum.

- In this way we obtain

$$
\begin{aligned}
\partial_\eta P_{ab}(\mathbf{k},\eta) = & -\Omega_{ac} P_{cb}(\mathbf{k},\eta) - \Omega_{bc} P_{ac}(\mathbf{k},\eta) \\
& + e^\eta \int d^3q \left[\gamma_{acd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) B_{bcd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) \right. \\
& \left. + \gamma_{bcd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) B_{acd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) \right], \\
\partial_\eta B_{abc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) = & -\Omega_{ad} B_{dbc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) - \Omega_{bd} B_{adc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) \\
& - \Omega_{cd} B_{abd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) \\
& + 2e^\eta \int d^3q \left[\gamma_{ade}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) P_{db}(\mathbf{q},\eta) P_{ec}(\mathbf{k}-\mathbf{q},\eta) \right. \\
& + \gamma_{bde}(-\mathbf{q},\mathbf{q}-\mathbf{k},\mathbf{k}) P_{dc}(\mathbf{k}-\mathbf{q},\eta) P_{ea}(\mathbf{k},\eta) \\
& \left. + \gamma_{cde}(\mathbf{q}-\mathbf{k},\mathbf{k},-\mathbf{q}) P_{da}(\mathbf{k},\eta) P_{eb}(\mathbf{q},\eta) \right].
\end{aligned}
$$

- Similarity with the Exact Renormalization Group (Wilson 1971).
- Path integral representation.
- Vertex expansion of the effective action.

  J. Berges, N. T., C. Wetterich, Phys. Rep. 363: 223 (2002)

Coupled quintessence

- The field has a potential $V(\phi) \sim \phi^{-\alpha}$, with $\alpha = 0.143$.
- The present-day energy content of the Universe has $\Omega_{DE} = 0.743$, $\Omega_{CDM} = 0.213$, $\Omega_{BM} = 0.044$.
- The Universe is assumed to have vanishing spatial curvature ($\Omega_k = 0$), current expansion rate $H_0 = 71.9$ km s^{-1} Mpc^{-1}.
- The mass variance is taken $\sigma_8 = 0.769$, as calculated from the linear spectrum.

F. Saracco, M. Pietroni, N. T., V. Pettorino, G. Robbers
arXiv:0911.5396[astro-ph], Phys. Rev. D 82: 023528 (2010)

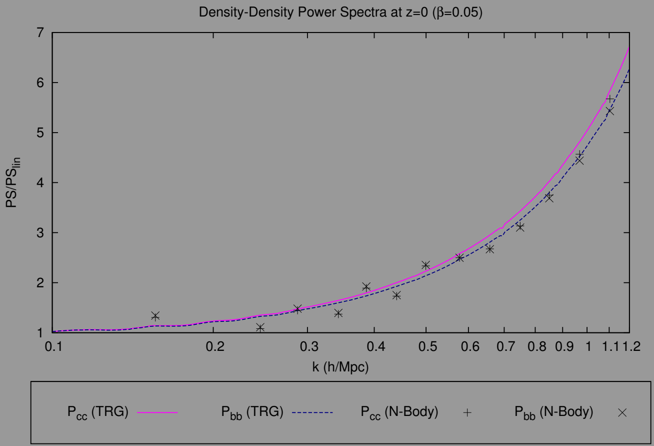


Figure: Comparison of results from N-body simulations and our calculation ($\beta = 0.05$). We display the ratio of the non-linear and linear spectra for $z = 0$.

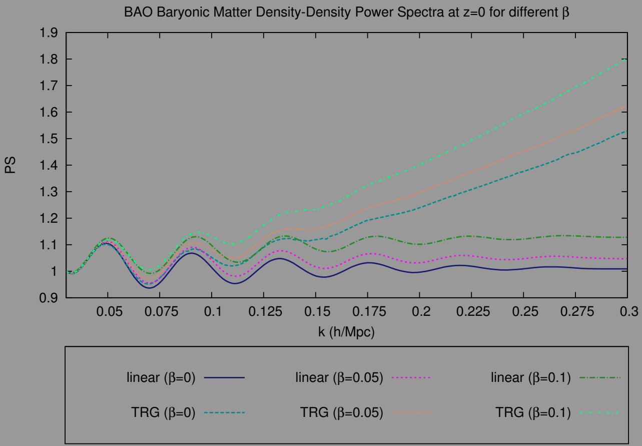


Figure: Baryonic matter density-density spectra for various β at $z = 0$, normalized with respect to the a smooth spectrum.

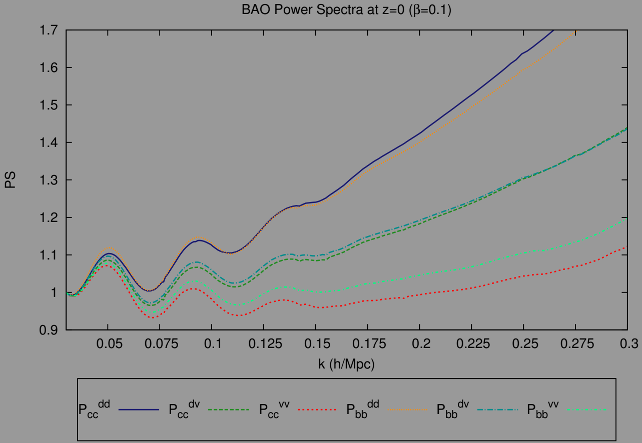


Figure: Spectra for $\beta = 0.1$ at $z = 0$, normalized with respect to a smooth spectrum.

Variable equation of state

- No coupling between dark matter and dark energy.
- One massive species: baryonic+dark matter
- Dark energy fluctuations are negligible.
- Equation of state $p/\rho = w(z)$
- $$w(a) = \frac{a\,\tilde{w}(a)}{a + a_{\rm trans}} - \frac{a_{\rm trans}}{a + a_{\rm trans}}$$
  $a = 1/(1+z)$ is the scale factor.
  $\tilde{w}(a) = \tilde{w}_0 + (1-a)\tilde{w}_a$
  $a_{\rm trans}$ corresponds to the "transition epoch"
- At small redshifts
  $$w(a) = w_0 + (1-a)w_a.$$
- We describe the various models through $w_0 \equiv w(z=0)$, $w' \equiv dw/dz|_{z=0}$, and $a_{\rm trans}$.
  N. Brouzakis, N. T.
  arXiv:1002.3277[astro-ph], JCAP 1101: 024 (2011)

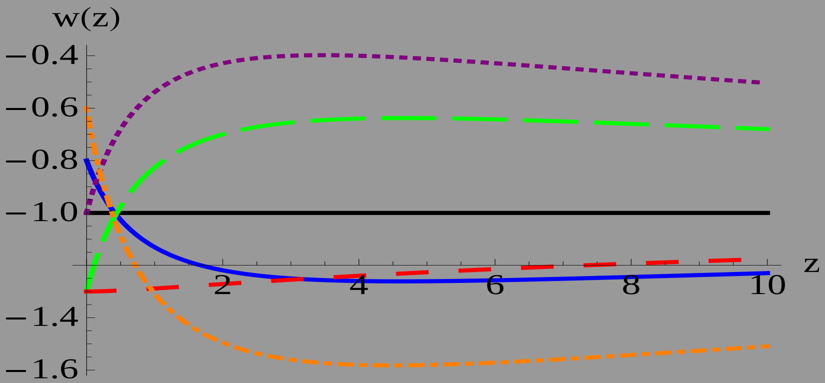


Figure: The form of the equation of state $w(z)$ for ΛCDM and a variety of models: $w_0 = -1$, $w' = 1$ (dotted), $w_0 = -1.3$, $w' = 1$ (long-dashed), $w_0 = -1.3$, $w' = 0$ (short-dashed), $w_0 = -0.8$, $w' = -0.7$ (continuous), $w_0 = -0.6$, $w' = -1.5$ (dash-dotted).

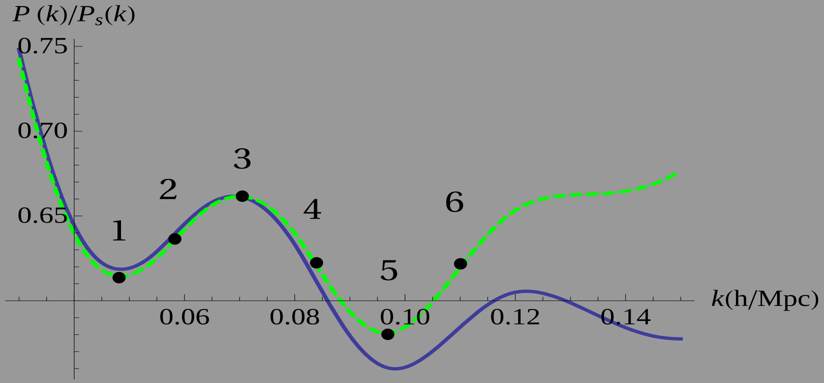


Figure: Linear and non-linear spectra at $z = 0$, for $w_0 = -0.8$, $w' = -0.7$, normalized with respect to a smooth spectrum.

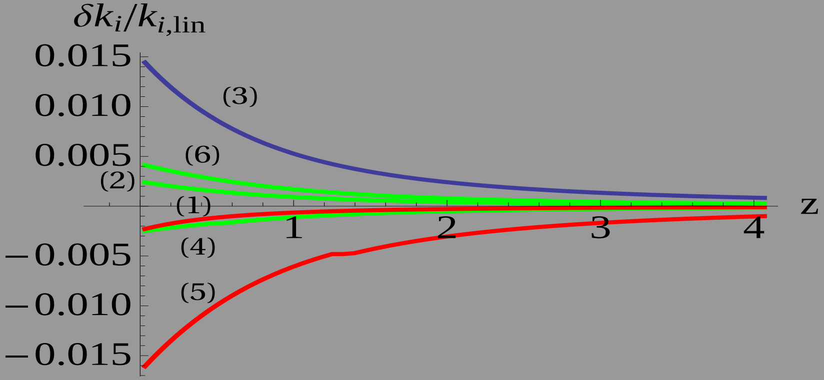


Figure: The fractional shift of the maximum, minima and nodes of the non-linear spectrum, as a function of redshift, for $w_0 = -0.8$, $w' = -0.7$.

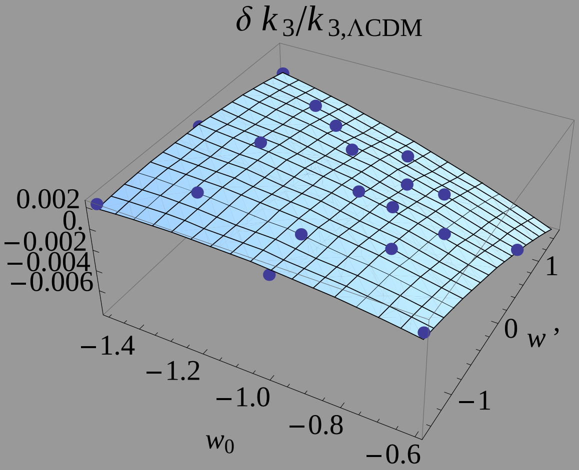


Figure: The fractional shift of the first maximum from its location for ΛCDM, as a function of w_0 and w', at a redshift $z = 0.366$.

Growing neutrino quintessence

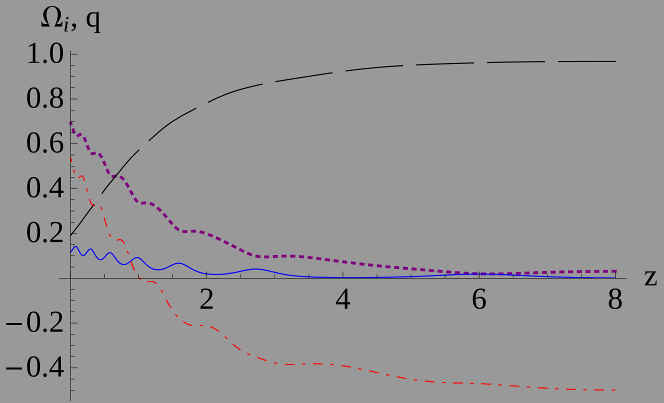


Figure: The fractional energy density in neutrinos (solid), CDM+BM (dashed) and dark energy (dotted). The acceleration parameter $q = a\ddot{a}/\dot{a}^2$ (dot-dashed) is also depicted.

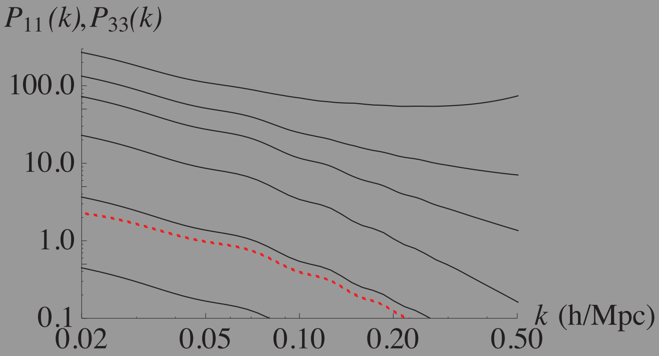


Figure: The neutrino density power spectrum $P_{11}(k,\eta)$ (solid lines) and the CDM+BM density spectrum $P_{33}(k,\eta)$ (dotted lines) at redshifts $z = 4.70$, 4.08, 3.04, 2.77, 2.69, 2.60 (starting from below).

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Conclusions

- The inhomogeneities in the matter distribution have a very small effect on the average acceleration.
- Novel large-scale structures can appear in quintessence cosmology.
- The spectrum of matter perturbations is a very useful tool in order to differentiate between cosmological models. The non-linear corrections to the spectrum must be evaluated carefully in order to compare with astrophysical data.
- The resummation of cosmological perturbations permits the comparison with observations in the BAO range. At smaller length scales, the spectrum can be deduced through numerical simulations.
- Required accuracy: 1%. A real challenge.