Black holes and Higgs stability

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Introduction

- The electroweak vacuum in the Standard Model is metastable.
- At very large field values

$$\mathcal{L}_{\rm eff} = \frac{1}{2}Z(h,\xi)(\partial_\mu h)^2 - \frac{1}{4}\lambda_{\rm eff}(h,\xi)h^4 = \frac{1}{2}(\partial_\mu h_{\rm can})^2 - \frac{1}{4}\lambda_{\rm can}(h_{\rm can},\xi)h_{\rm can}^4$$

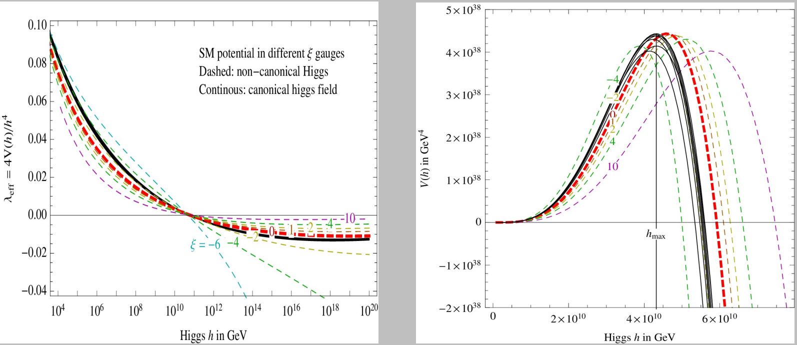

Figure: *The dashed curves show the effective quartic coupling (left) and effective SM potential (right) computed at next-to-leading order in a generic Fermi ξ-gauge. The thick red dashed curve corresponds to the Landau gauge, $\xi = 0$. The black continuous curves show the same potential expressed in terms of the canonical field $h_{\rm can}$.*

- The tunnelling rate to the new vacuum is exponentially suppressed by the action of the instanton, which is the configuration that minimizes the Euclidean action

$$S = \int d^4x \sqrt{g} \left[\frac{(\partial_\mu h)(\partial^\mu h)}{2} + V(h) - \frac{\mathcal{R}}{2\kappa} - \frac{\xi}{2}\mathcal{R}h^2 \right],$$

where $\mathcal{R}$ is the Ricci scalar, $\kappa = 1/\bar{M}_{\rm Pl}^2 = 8\pi G$ with $\bar{M}_{\rm Pl} = M_{\rm Pl}/\sqrt{8\pi}$, $M_{\rm Pl} \approx 1.22 \times 10^{19}\,{\rm GeV}$.

- The instanton is an $O(4)$-symmetric configuration on a space with metric

$$ds^2 = dr^2 + \rho(r)^2 d\Omega^2,$$

where $d\Omega$ is the volume element of the unit 3-sphere.

- A. Salvio, A. Strumia, N. Tetradis and A. Urbano, arXiv:1608.02555 [hep-ph], JHEP 1609 (2016) 054

SM with $M_h = 114$ GeV, $M_t = 173.34$ GeV, $\alpha_3(M_Z) = 0.1184$

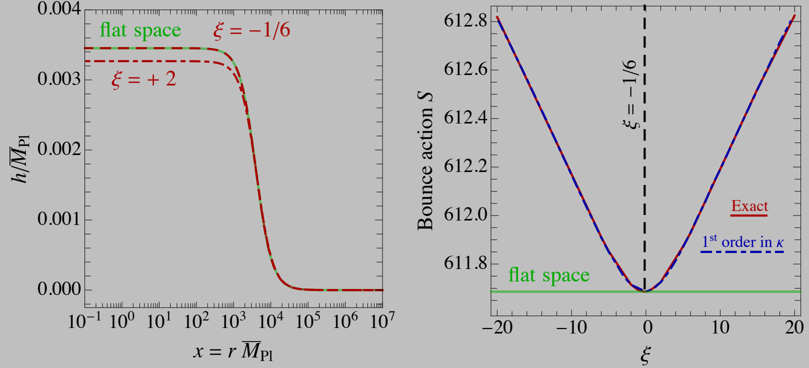

Figure: *SM bounce solutions for different values of ξ (left panel), and their action (right panel). We consider $M_h = 114$ GeV, which is the value that saturates the metastability bound for the central value of the top mass.*

SM with $M_h = 125.09$ GeV, $M_t = 173.34$ GeV, $\alpha_3(M_Z) = 0.1184$

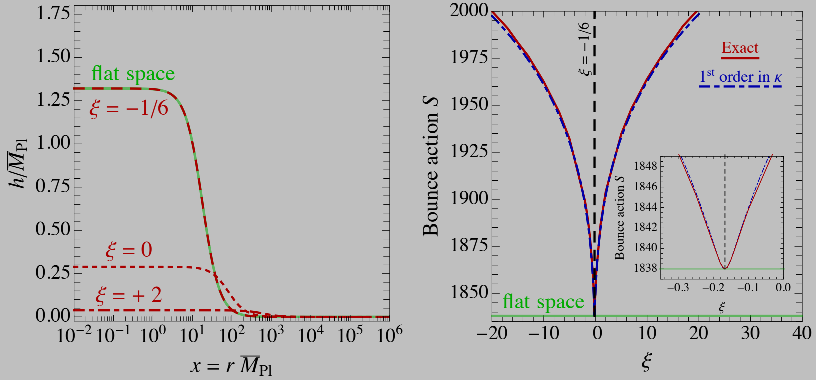

Figure: *SM bounce solutions for different values of ξ (left panel), and their action (right panel). We consider here the best fit Higgs mass $M_h = 125.09$ GeV, for which the vacuum decay rate is negligibly small.*

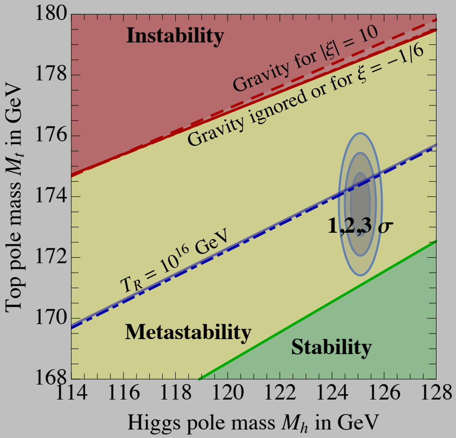

Figure: *SM phase diagram for $\alpha_3(M_Z) = 0.1184$. Continuous red line:no gravitational corrections or $\xi = -1/6$; almost coincident dot-dashed line: $\xi = 0$; dashed line: $|\xi| \sim 10$. Ellipses: Higgs and top mass at $1, 2, 3\sigma$. Blue lines: bound from thermal tunneling, for a reheating temperature of 10^{16} GeV.*

- There are also constraints on the scale of inflation.
- In the absence of a large Higgs mass term, the evolution of the long wavelength modes of the Higgs field h is controlled by the Langevin equation

$$\frac{dh}{dt} + \frac{1}{3H}\frac{dV(h)}{dh} = \eta(t),$$

where η is a Gaussian random noise with

$$\langle \eta(t)\eta(t') \rangle = \frac{H^3}{4\pi^2}\delta(t-t').$$

- J.R. Espinosa *et al.*, arXiv:1505.04825 [hep-ph], JHEP 1509 (2015) 174

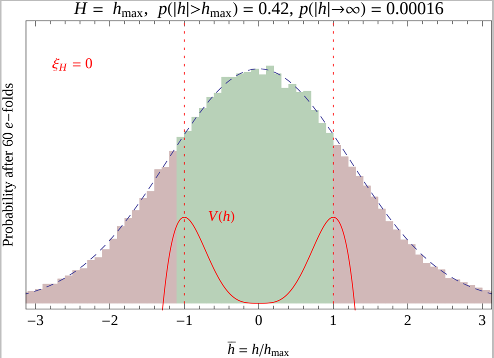

Figure: *Random distribution of the Higgs field $\bar{h} = h/h_{\max}$ after $N = 60$ e-folds of inflation with Hubble constant equal to the Higgs instability scale, $H = h_{\max}$. The red curve is the SM Higgs potential $\bar{V}(\bar{h})$, in arbitrary units.*

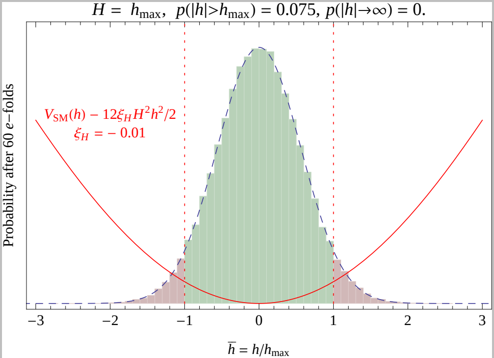

Figure: *Random distribution of the Higgs field $\bar{h} = h/h_{\rm max}$ after $N = 60$ e-folds of inflation with Hubble constant equal to the Higgs instability scale, $H = h_{\rm max}$, and for a non-minimal Higgs coupling $\xi_H = -0.01$. The red curve is the Higgs potential bar$V_{\rm SM}(\bar{h}) - 12\xi_H H^2 \bar{h}^2/2$, in arbitrary units.*

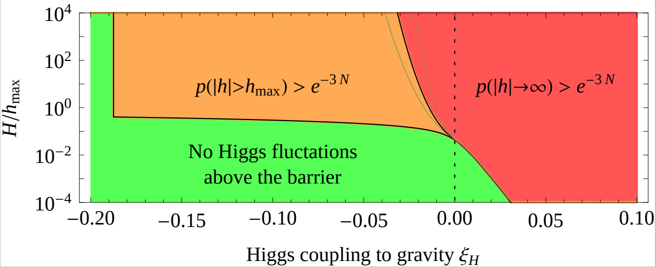

Figure: *As a function of ξ_H and the Hubble constant in units of the instability scale h_{max} (and for $N = 60$ e-folds of inflation), we show the three regions where: the probability for the Higgs field to end up in the negative-energy true minimum is larger than e^{-3N} (red); the probability for the Higgs field to fluctuate beyond the potential barrier is larger than e^{-3N} (orange); the latter probability is smaller than e^{-3N} (green). Higgs fluctuations are damped for $\xi_H < -3/16$.*

- Burda, Gregory and Moss (2015,2016) argued that the quantum nucleation of bubbles of true vacuum can be enhanced around black holes that act as impurities in the false vacuum.
- In the context of the Standard Model, the presence of black holes may destabilize the standard electroweak vacuum by reducing drastically the barrier for quantum fluctuations.
- A natural question is whether the creation of black holes during inflation or later periods is accompagnied by the appearance of AdS bubbles around them that arise as classical fluctuations in the high-temperature or density environment.
- N. Tetradis, arXiv:1606.04018 [hep-ph], to appear in JCAP

Matching the geometries

- Spherical AdS-Schwarzschild bubble within asymptotically flat or dS space, separated by a thin wall with surface tension σ.
- The space inside the bubble has a metric

$$ds^2 = -f_{\rm in}(r)\, d\eta^2 + \frac{dr^2}{f_{\rm in}(r)} + r^2 d\Omega_2^2, \qquad r < R,$$

  with $f_{\rm in}(r) = 1 + r^2/\ell^2 - 2Gm/r$, $1/\ell^2 = 8\pi G|V|/3$.
- The space outside the bubble is described by the metric

$$ds^2 = -f_{\rm out}(r)\, dt^2 + \frac{dr^2}{f_{\rm out}(r)} + r^2 d\Omega_2^2, \qquad r > R,$$

  with $f_{\rm out}(r) = 1 - r^2/\ell'^2 - 2GM/r$, $1/\ell'^2 = 8\pi GV'/3$.
  M is the ADM mass of the bubble.
- The two regions are separated by a domain wall with metric

$$ds^2 = -d\tau^2 + R^2(\tau) d\Omega_2^2.$$

- The Israel junction conditions give

$$\epsilon_2(f_{\rm out}+\dot{R}^2)^{1/2}-\epsilon_1(f_{\rm in}+\dot{R}^2)^{1/2}=-4\pi G\sigma R,$$

where $\epsilon_1=\pm 1$, $\epsilon_2=\pm 1$ are possible sign choices.

- The square of this equation can be put in the form

$$2GM=2Gm+\left(\kappa^2-\frac{1}{\ell^2}-\frac{1}{\ell'^2}\right)R^3+2\epsilon_2\kappa R^2\left(1-\frac{2GM}{R}-\frac{R^2}{\ell'^2}+\dot{R}^2\right)^{1/2}$$

with $\kappa=4\pi G\sigma$. The time-derivative is with respect to τ.

- Alternatively

$$2GM=2Gm-\left(\frac{1}{\ell^2}+\frac{1}{\ell'^2}+\kappa^2\right)R^3+2\epsilon_1\kappa R^2\left(1-\frac{2Gm}{R}+\frac{R^2}{\ell^2}+\dot{R}^2\right)^{1/2}$$

- For $\epsilon_1=1$, $\dot{R}\ll 1$ and $G\to 0$, the above expression has a Newtonian interpretation.

- By squaring the junction equation twice, we can express the equation of motion for the bubble wall as the equation for the one-dimensional motion of a particle of constant 'energy' in an effective 'potential'.
- For fixed values of ℓ, ℓ', κ, m, the 'energy' depends on the total mass M of the configuration.
- A multitude of wall trajectories are possible for various values of M, describing shrinking or expanding bubbles.
- For a given set of parameters there is a critical configuration that separates small bubbles that tend to collapse from large bubbles that tend to grow and engulf the external space.
- The mass M of these bubbles characterizes the energy barrier for transitions towards the deeper AdS vacuum.

Asymptotically flat space, positive mass

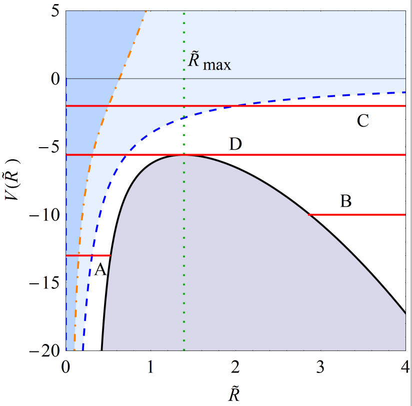

Figure: *The 'potential' for $1/\ell'^2 \to 0$, $M > 0$, and $m/M = 0.5$.*

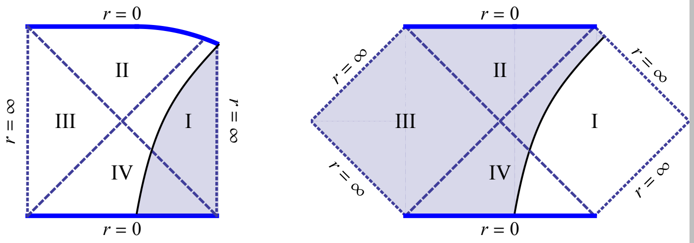

Figure: *The Penrose diagram for the wall trajectory C of the previous figure. The total spacetime is constructed by joining the two diagrams, after the elimination of the shaded areas.*

Asymptotically flat space, negative mass

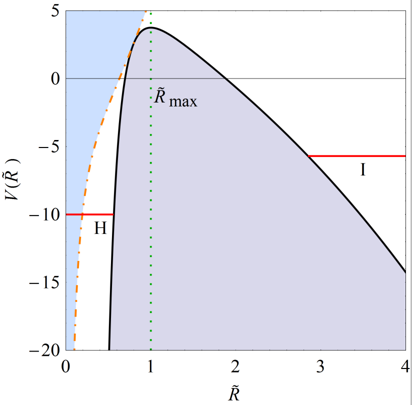

Figure: *The 'potential' for $1/\ell'^2 \to 0$, $M < 0$, and $m/|M| = 0.5$.*

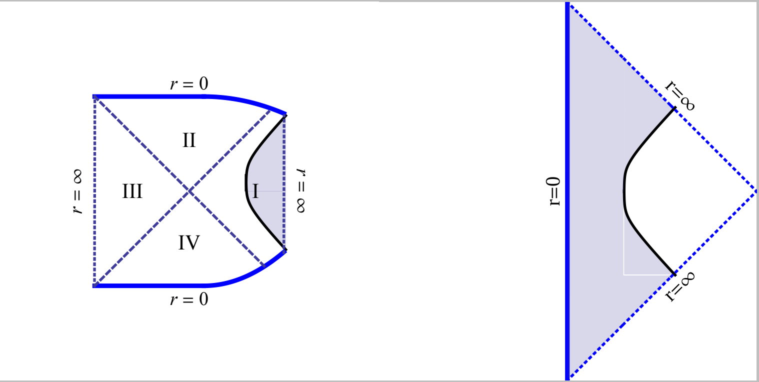

Figure: *The Penrose diagram for the wall trajectory I of the previous figure. The total spacetime is constructed by joining the two diagrams, after the elimination of the shaded areas.*

Asympotically dS space

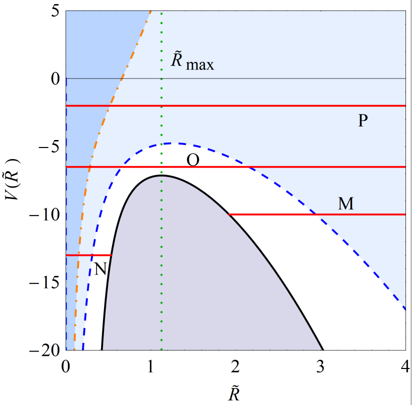

Figure: *The 'potential' for $1/\ell' > 0$, $M > 0$, and $m/M = 0.5$.*

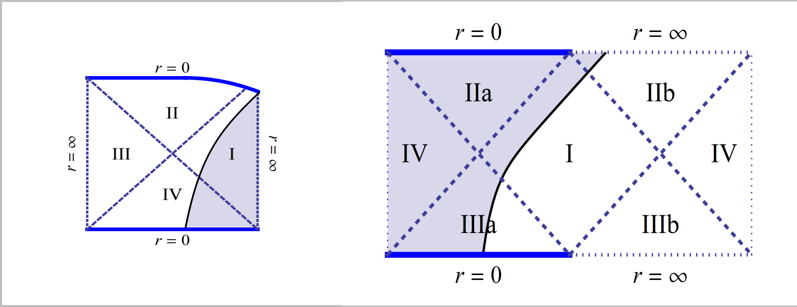

Figure: *The Penrose diagram for the wall trajectory O of the previous figure. The total spacetime is constructed by joining the two diagrams, after the elimination of the shaded areas.*

The AdS crunch

- The evolution after the wall reaches the timelike boundary of AdS, cannot be determined without additional boundary conditions. There is a Cauchy horizon.
- A spacelike singularity (the AdS 'crunch' of Coleman and De Luccia) must develop in the bubble interior.
- The coordinate change

$$r = \ell_{\rm in} \sin\frac{\hat{t}}{\ell_{\rm in}} \sinh\psi \qquad \cos\frac{\hat{t}}{\ell_{\rm in}} = \left(1+\frac{r^2}{\ell_{\rm in}^2}\right)^{1/2} \cos\frac{\eta}{\ell_{\rm in}}$$

  puts the AdS metric in the form

$$ds^2 = -d\hat{t}^2 + \ell_{\rm in}^2 \sin^2\frac{\hat{t}}{\ell_{\rm in}} \left(d\psi^2 + \sinh^2\psi\, d\Omega_2^2\right).$$

  This metric describes an homogeneous FRW universe that is born with a big 'bang' and collapses in a big 'crunch'.
- The coordinate singularity becomes a true physical singularity in the presence of a fluctuating Higgs field.

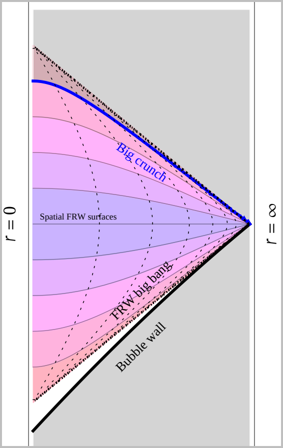

Figure: *The AdS interior of the bubble in conformal coordinates, showing the crunch and (in color) a patch in FRW coordinates.*

The critical bubbles

- The mass of the bubble configuration is

$$2GM = 2Gm - \left(\frac{1}{\ell^2} + \frac{1}{\ell'^2} + \kappa^2\right) R^3 + 2\epsilon_1 \kappa R^2 \left(1 - \frac{2Gm}{R} + \frac{R^2}{\ell^2} + \dot{R}^2\right)^{1/2}$$

- We concentrate on the bubble evolution for $\kappa^2 = (G\sigma)^2 \ll 1/\ell^2$.
- It is reasonable to assume that at the time of production of an AdS bubble the wall has small velocity ($\dot{R} \simeq 0$).
- We set $\epsilon_1 = 1$. Configurations with $\epsilon_1 = -1$ correspond to shrinking bubbles.
- For a central black hole with mass parameter m, the presence of an initially static AdS bubble of radius R results in the modification of the asymptotic ADM mass by an amount equal to $\delta M(R, m) = M(R, m) - m$.

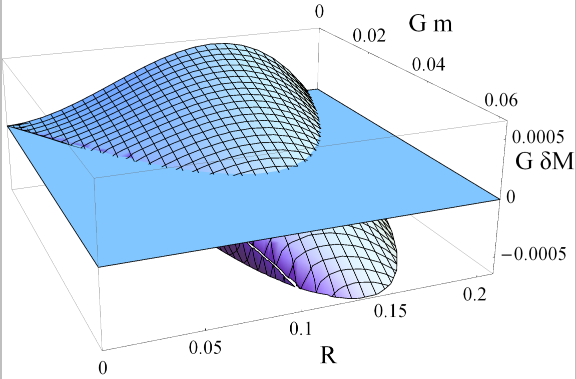

Figure: The mass difference $\delta M = M - m$ for a bubble with $\dot{R} = 0$ and $1/\ell' \to 0$, as function of its radius R and the mass m of the central black hole. All quantities are given in units of ℓ.

- There is a critical value $m_{\rm cr}$, above which $\delta M(R,m)$ is negative for all R. This value and the corresponding bubble radius $R_{\rm cr}$ are

$$Gm_{\rm cr} = \frac{1}{3}R_{\rm cr} = \frac{2}{3\sqrt{3}}\frac{\kappa}{\left[\frac{1}{\ell'^2}+(\frac{1}{\ell}+\kappa)^2\right]^{\frac{1}{2}}\left[\frac{1}{\ell'^2}+(\frac{1}{\ell}-\kappa)^2\right]^{\frac{1}{2}}} \simeq \frac{2}{3\sqrt{3}}\frac{\kappa}{\frac{1}{\ell^2}+\frac{1}{\ell'^2}}.$$

- The bubble radius $R_{\rm cr}$ is always larger than the horizon radius of the black hole.
- There are no bubbles with radii below a certain value. For $\dot{R}=0$, the minimal radius $R_{\rm h}$ satisfies

$$1-\frac{2Gm}{R_{\rm h}}+\frac{R_{\rm h}^2}{\ell^2}=0. \qquad (1)$$

If the bubble is located within the horizon, with vanishing wall velocity, it cannot grow.

- The mass parameter m is not a properly defined physical quantity.
- A geometrical quantity that can be used to characterize the energy content of the central region is the horizon radius $R_{\rm h}$.
- The difference $\delta M'(R,m) = M(R,m) - R_{\rm h}(m)/(2G)$ provides an alternative estimate for the energy barrier associated with the bubble.
- $R_{\rm h}(m)/(2G)$ coincides with m only for $2Gm/\ell \ll 1$, while it is much smaller than m for $2Gm/\ell \gg 1$. As a result, $\delta M'$ may provide an overestimate of the energy barrier for large m.

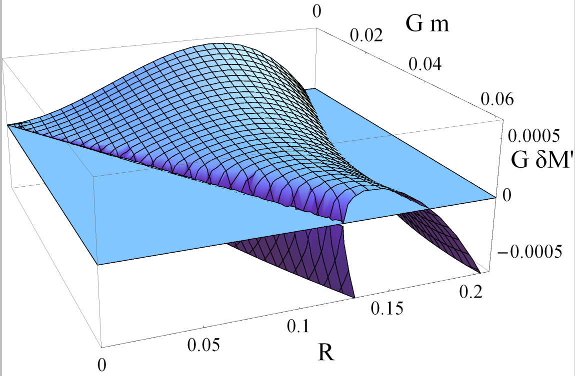

Figure: The mass difference $\delta M' = M - R_{\rm h}/(2G)$ for a bubble with $\dot{R} = 0$ and $1/\ell' \to 0$, as function of its radius R and the mass m of the central black hole. All quantities are given in units of ℓ.

- For small $\kappa\ell$ and $1/\ell' \to 0$, the minimal critical value of $\delta M'$ is obtained for

$$Gm'_{\rm cr} = \frac{1+\sqrt{13}}{12} R'_{\rm cr} \simeq \frac{\sqrt{16-\sqrt{13}}\left(1+\sqrt{13}\right)}{36}\ell^2\kappa.$$

- The points $(R_{\rm cr}, m_{\rm cr})$, $(R'_{\rm cr}, m'_{\rm cr})$ are close:
$Gm_{\rm cr} = R_{\rm cr}/3 \simeq 0.385\,\ell^2\kappa$ and $Gm'_{\rm cr} = 1.151\; R_{\rm cr}/3 \simeq 0.450\,\ell^2\kappa$.
- The quantity $\delta M'(R'_{\rm cr}, m'_{\rm cr})$ can be compared with the barrier in the absence of the black hole, estimated by $M(R_0, 0)$, with $\partial M(R_0, 0)/\partial R = 0$. For small $\kappa\ell$ and $1/\ell' \to 0$

$$\frac{\delta M'(R'_{\rm cr}, m'_{\rm cr})}{M(R_0, 0)} = \frac{1}{32}\left(\sqrt{220+47\sqrt{13}} - \sqrt{1492-397\sqrt{13}}\right) \simeq 0.373.$$

- For $1/\ell' \gg 1/\ell$, the quantity $\delta M'(R, m)$ turns negative for

$$Gm_{\rm cr} = \frac{1}{3}R_{\rm cr} \simeq \frac{2}{3\sqrt{3}}\,\frac{\kappa}{\frac{1}{\ell^2}+\frac{1}{\ell'^2}}.$$

Complete instability is expected for a strong dS background.

- The critical bubble mass can be estimated as

$$Gm_{\rm cr} \sim \frac{\kappa}{\frac{1}{\ell^2}+\frac{1}{\ell'^2}}$$

for all values of $1/\ell$, $1/\ell'$.

Standard Model Higgs

- The Higgs potential has the approximate form $V \sim \lambda(h)\, h^4/4$ for values of the Higgs field h above 10^6 GeV.
- The quartic coupling λ varies from 0.02 to -0.02 for Higgs values between 10^6 GeV and 10^{20} GeV, respectively.
- The maximum of the potential is located at a value $h_{\rm max} \sim 5 \times 10^{10}$ GeV.
- Near the maximum the potential can be approximated as

$$V(h) \simeq -b \ln\left(\frac{h^2}{h^2_{\rm max}\sqrt{e}}\right) \frac{h^4}{4},$$

with $b \simeq 0.16/(4\pi)^2$.
- For Higgs values within the range of interest around $h_{\rm max}$, we have $|\lambda| = \mathcal{O}(10^{-3})$.
- In order to avoid the destabilization of the standard electroweak vacuum because of Higgs fluctuations during inflation, one requires that the scale $H_{\rm inf}$ of inflation satisfies $H_{\rm inf} \lesssim 0.04\, h_{\rm max}$ (for a minimally coupled Higgs field).

- In the presence of a black hole, the energy barrier to be overcome in order to produce an AdS bubble is reduced significantly.
- A primordial black hole can form when the density fluctuations are sufficiently large for an overdense region of horizon size to collapse.
- Its maximal mass is of order the total mass within the particle horizon $m_{\rm bh} \sim M_{\rm Pl}^2/H$, while its maximal radius is $R_{\rm bh} \sim 1/H$.
- These estimates are also valid for black holes that are pair-produced during inflation.
- We can estimate

$$\frac{m_{\rm cr}}{m_{\rm bh}} \sim \frac{|V|}{V' + |V|} \frac{H}{\sqrt{\hat{\lambda}} h},$$

  with $\hat{\lambda} = \mathcal{O}(10^{-3})$, $V \sim -\hat{\lambda} h^4$, and V' equal to the inflaton vacuum energy $V_{\rm inf}$ during inflation, or to zero after its end.
- This ratio is always smaller than 1.

Beyond the thin-wall approximation

- For the metric

$$ds^2 = -N(r)\, e^{2\delta(r)} dt^2 + N^{-1}(r)\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

with $N(r) = 1 - 2GM(r)/r$, the equations of motion become

$$\begin{aligned} M' &= 4\pi r^2 \left(\frac{1}{2} N h'^2 + V(h)\right) \\ \delta' &= 4\pi G r h'^2 \\ h'' + \left(\frac{2}{r} - 8\pi G \frac{r}{N} V(h) + 2G\frac{M}{Nr^2}\right) h' &= \frac{1}{N}\frac{dV(h)}{dh}. \end{aligned}$$

- On the horizon: $2GM(R_\mathrm{h}) = R_\mathrm{h}$. For a finite ADM mass:

$$h'(R_\mathrm{h}) = \frac{R_\mathrm{h}}{1 - 8\pi G R_\mathrm{h}^2\, V(h(R_\mathrm{h}))} \frac{dV(h(R_\mathrm{h}))}{dh}.$$

$$h(r) \to 0 \quad \text{for } r \to \infty, \qquad \delta(r) \to 1 \quad \text{for } r \to \infty.$$

- $\delta M' = M(\infty) - R_\mathrm{h}/(2G)$ is the energy barrier.

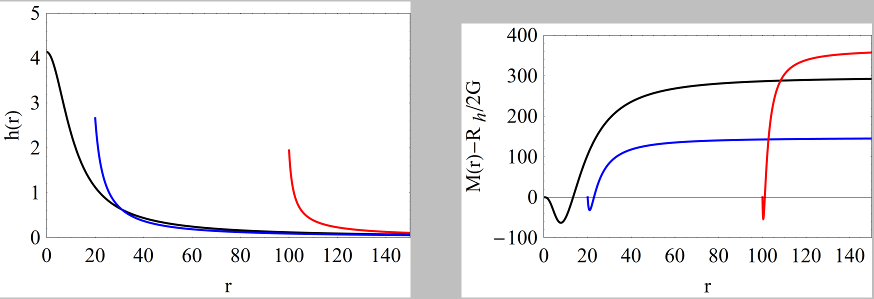

Figure: *The Higgs field* $h(r)$ *(left plot) and the mass function* $M(r)$ *(right plot) outside a black hole with horizon radius* $R_h = 0.1$*, 20 and 100 (lines from from left to right). All quantities are given in units of* h_{max}.

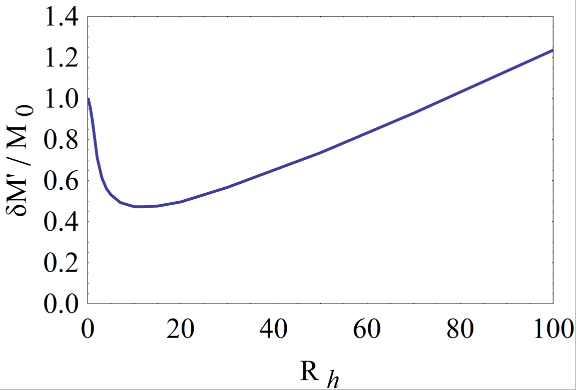

Figure: The ratio of the energy barrier in the background of a black hole relative to the barrier in the absence of a black hole.

- The ratio $\delta M'/M_0$ has a minimum $\delta M'_{\rm max} \simeq 0.473$ at $R_{\rm h} \simeq 11$.
- There is a reduction of the energy barrier by approximately a factor of 2, instead of the factor of 3 estimated through the thin-wall approximation.
- This reduction still has a profound effect on the nucleation rate. The energy barrier drops from approximately 300 to 150 in units of $h_{\rm max}$.
- The characteristic scale of the solutions is set by $h_{\rm max}$. This means that gravitational corrections are not relevant, as they are suppressed by powers of $h^2_{\rm max}/M^2_{\rm Pl}$.
- For our solutions $\kappa\ell \sim \sqrt{G}h_{\rm max} \sim h_{\rm max}/M_{\rm Pl} \ll 1$.
- The energy $\delta M'$ associated with the bubble is much smaller than the mass of the central black hole, as estimated by $R_{\rm h}/(2G)$. The gravitational background is induced mainly by the black hole, with the bubble being only a small perturbation.

Higgs potential and primordial black holes

- Is the presence of primordial black holes consistent with the Standard Model Higgs?
- The barrier for classical transitions to the AdS vacuum is reduced by roughly a factor of 2 in the presence of a black hole for an *asymptotically flat false vacuum*.
- For an *asympotically dS false vacuum*, the barrier is eliminated by a sufficiently big black hole, indicating complete instability.
- Consider a high-temperature environment with $T \sim h_{\rm max}$ in asymptotically flat space. The bubble nucleation probability per unit time is $dP/dt = T \exp(-\delta M'/T)$.
- The smallest time interval that can be associated with the scale T is the Hubble time $\sim M_{\rm Pl}/T^2$.
- The number of causally independent regions, which are currently within our horizon, is roughly $N \sim 10^{34}(T/{\rm GeV})^3$.
- Assume that a black hole can be produced within each of these regions with probability p.

- The total nucleation probability becomes $P \sim p\,\left(T/(10^{11}\mathrm{GeV})\right)^2\,\exp(173 - \delta M'/T)$.
- The reduction of the energy barrier $\delta M'$ from ~ 300 to ~ 150 in units of $h_{\max}$ means that the exponental suppression is eliminated.
- It must be emphasized, however, that the probability p for the creation of a primordial black hole may be very small, resulting in the suppression of the rate.

Comments

- One may ask if the typical black holes can be in equilibrium with the thermal background or affect it. The tunnelling rate becomes maximal for a black hole with Schwarzschild radius $R_{\rm h} \simeq 10/h_{\rm max}$ and Hawking temperature $T_{\rm H} \simeq h_{\rm max}/(40\pi)$. We expect that the bubble nucleation rate will be most efficient for $T \sim h_{\rm max}$. The black holes are not in equilibrium, while they have only minor influence on the background.
- The Higgs field couples through the Yukawa couplings to particles that contribute to density flucuations. The transition can take place in an environment that is out of thermal equilibrium if the density flucuations are sufficiently strong.
- Similar results are expected for a wide range of potentials, resulting from physics beyond the Standard Model at zero and nonzero temperature or density, as long as the standard electroweak vacuum is metastable.