

Byzantine Fault Tolerant Symmetric-Persistent Circle Evacuation

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Abstract. We consider (n, f) -evacuation on a circle, an evacuation problem of a hidden exit on the perimeter of a unit radius circle for $n > 1$ robots, f of which are faulty. All the robots start at the center of the circle and move with maximum speed 1. Robots must first find the exit and then move there to evacuate in minimum time. The problem is considered complete when all the honest robots know the correct position of the exit and the last honest robot has evacuated through the exit. During the search, robots can communicate wirelessly.

We focus on symmetric-persistent algorithms, that is, algorithms in which all robots move directly to the circumference, start searching the circle moving in the same direction (cw or ccw), and do not stop moving around the circle before receiving information about the exit. We study the case of $(n, 1)$ and $(n, 2)$ evacuation. We first prove a lower bound of $1 + \frac{4\pi}{n} + 2 \sin(\frac{\pi}{2} - \frac{\pi}{n})$ for one faulty robot, even a crash-faulty one. We also observe an almost matching upper bound obtained by means of an earlier search algorithm. We finally study the case with two Byzantine robots and we provide an algorithm that achieves evacuation in time at most $3 + \frac{6\pi}{n}$, for $n \geq 9$, or at most $3 + \frac{6\pi}{n} + \delta(n)$, for $n < 9$, where $\delta(n) \leq 2 \sin(\frac{3\pi}{2n}) + \sqrt{2 - 4 \sin(\frac{3\pi}{2n}) + 4 \sin^2(\frac{3\pi}{2n})} - 2$.

1 Introduction

An important, extensively studied family of problems in mobile agent computing concerns situations where a group of robots need to find one or more targets that are located in unknown points of a territory. In a particular case of interest, the target(s) are *exit(s)* and the goal of the robots is either to locate the exit(s) (*Search* problem) or to leave the territory (*Evacuation* problem), as fast as possible. In this paper we focus on the latter problem.

1.1 Model and Preliminaries

We detail below the particular setting that we are going to study in this work.

Location and Movement

The starting position of the robots is the center of a unit radius disk. The exit is located at the perimeter of the disk. All robots move with the same speed 1. During their movement, they can recognize and move along the perimeter of the disk, and they can find the exit if they are at its location. Robots are allowed to take shortcuts moving through chords in the interior of the disk.

Communication

The robots can communicate wirelessly and with no delay at any time and distance. The messages may contain information about their location, whether or not they found the exit, how far they have moved from their starting position etc. Messages are tagged with the sender's unique identifier that cannot be modified in any way. All robots can deduce their relative position from each other messages. They are also equipped with a pedometer in order to measure distances.

Fault Types

Some robots may display faulty behaviour. A crash-faulty robot may stop functioning at any time, meaning that they fail to communicate any messages, and remains idle. A Byzantine robot behaves maliciously, it can alter its trajectory and provide or hide information in order to confuse the rest of the honest robots on the location of the exit. A Byzantine robot can also behave as a crash-faulty one.

Adversary

For the worst case analysis of our algorithms, we consider an adversary who controls the location of the exit and the behaviour of the malicious robots (its trajectories as well as the messages they will broadcast) so

as to maximize the resulting search and evacuation completion time. An evacuation is complete if the exit is found (has been visited by a non-faulty robot and the rest of them, if any, can be convinced (provably) of the (correct) location of the exit) and all the non-faulty robots reach the exit.

1.2 Related Work

Evacuation is closely related to Search, in fact the two problems coincide in the case of a single robot. A long line of research focused on the line search (aka Cow Path) problem (see [1–3] and references therein); more recent works include Kao et. al [4], in the randomized setting, and Demaine et. al. [5] who takes into account the turn cost.

The problem was studied under the fault-tolerance perspective by Czyzowicz et. al [6, 7] who considered the problem under the presence of either crash or Byzantine failures.

In circular topologies, evacuation was the first problem to be studied, in [8], who dealt with both the wireless and face-to-face communication models. Additional work on circle evacuation followed shortly afterwards, e.g. in [9] for the face-to-face model, and in [10] for equilateral triangles (for a comprehensive survey we refer the reader to [11]).

Regarding fault-tolerant evacuation in circles, a work closely related to this paper is [12], where they study the problem with 3 robots, one of which is faulty, and provide upper and lower bounds for both cases of crash-faulty and Byzantine-faulty robot. They also introduce the notion of symmetric-persistent algorithms that we make use of in the present paper.

For the search problem in circles, under crash or Byzantine failures, a recent study [13] has shown a tight bound of $1 + \frac{4\pi}{n}$ to find the exit where n is the number of robots, one of which is Byzantine.

1.3 Results of the paper

In Section 2 we consider the Evacuation problem for n robots one of which is Byzantine and we prove a lower bound for the symmetric persistent algorithms of $1 + \frac{4\pi}{n} + 2 \sin(\frac{\pi}{2} - \frac{\pi}{n})$. The lower bound is almost matching to our upper bound. In Section 3 we present our symmetric persistent algorithm for the case of Evacuation of n robots with 2 Byzantine faults

and we provide an upper bound of $3 + \frac{6\pi}{n}$, for $n \geq 9$ and $3 + \frac{6\pi}{n} + \delta(n)$ for $n < 9$, where $\delta(n) \leq 2 \sin(\frac{3\pi}{2n}) + \sqrt{2 - 4 \sin(\frac{3\pi}{2n}) + 4 \sin^2(\frac{3\pi}{2n})} - 2$.

2 Evacuation with One Byzantine Fault

We define (n, f) - Evacuation, to mean evacuation for $n > 1$ robots, of which f are faulty. In this work, we consider only Byzantine faults.

2.1 Lower bound for symmetric-persistent algorithms

As defined in [12], symmetric-persistent algorithms are family of natural algorithms that force all robots to immediately go to the disk perimeter and only allow a robot to stop its exploration of the assigned sector if it receives information about the exit. Symmetric-persistent algorithms force all the robots to move in the same direction (either clockwise or counter-clockwise).

Theorem 1. *Any symmetric-persistent algorithm requires time at least*

$$1 + \frac{4\pi}{n} + 2 \sin\left(\frac{\pi}{2} - \frac{\pi}{n}\right) \geq 3 + \frac{4\pi}{n} - \frac{\pi^2}{2n^2}$$

for evacuation of n robots, one of which is crash-faulty, from a circle of radius 1.

Proof. Note that if $n = 2$ the result is trivial, so we assume $n \geq 3$. Let us denote by $f(n)$ the quantity displayed in the statement.

Fix a 0 on the unit circle and denote by x_i the length of the arc between robot α_i and 0 in the ccw direction. Let ψ_i denote the length of the arc between robots α_i and α_{i+2} . Since $\sum \psi_i = 4\pi$, there exists i such that $\psi_i \geq 4\pi/n = 2\theta$. Without loss of generality, let $x_2 = \psi_0 \geq 2\theta$ and denote it by ψ . For any $\epsilon > 0$, if the adversary places the exit at distance $\psi - \epsilon$ from 0 and robot α_1 is faulty, then the exit will be discovered by robot α_0 in time $1 + \psi - \epsilon$.

We now consider two cases on ψ .

First, suppose $2\theta \leq \psi < \pi$. By the maximality of ψ , there is at least one robot at distance x from 0 such that $x \in [\pi - \psi/2, \pi + \psi/2]$. The total time this robot will require to reach the exit is at least

$$1 + \psi - \epsilon + 2 \sin\left(\frac{\pi - \psi/2}{2}\right) \geq 1 + 2\theta + 2 \sin\left(\frac{\pi - \theta}{2}\right) - \epsilon = f(n) - \epsilon,$$

where the inequality follows because $\psi \geq 2\theta$ and the left-hand side is increasing in ψ .

Next, we consider the case $\pi \leq \psi$. In this case we will bound the time robot α_2 will need, which is at least

$$1 + \psi - \epsilon + 2 \sin(\psi/2)$$

time units. Note that this is increasing in ψ . It follows that it is at least $1 + \pi - \epsilon + 2$, which for $n \geq 4$ is greater than $f(n)$. If $n = 3$, then this is at least $1 + 2\theta - \epsilon + 2 \sin \theta = f(n) - \epsilon$. The equality holds since $\sin \theta = \sin(2\pi/3) = \sin(\frac{\pi}{2} - \frac{\pi}{6}) \geq \sin(\frac{\pi}{2} - \frac{\pi}{3})$.

Since the above hold for any ϵ , the first bound in the statement follows. The second bound follows from the inequality $\cos(x) \geq 1 - x^2/2$.

Theorem 2. *There exists a symmetric-persistent algorithm that requires time at most*

$$3 + \frac{4\pi}{n}$$

for evacuation of n robots, one of which is Byzantine, from a circle of radius 1.

Proof. We employ the optimal Search algorithm proposed in [13], which has a time bound of $1 + \frac{4\pi}{n}$ to find the exit, and add the length of the diameter for the furthest robot to evacuate.

Remark 1. Note that above upper bound is within $O(1/n^2)$ from the lower bound of Theorem 1.

3 Evacuation with Two Byzantine Faults

3.1 Algorithm for $(n, 2)$ - Evacuation

We will now present an algorithm for Evacuation of n robots, 2 of which are faulty, both Byzantine, and then analyze its time requirements.

Consider n robots a_0, a_1, \dots, a_{n-1} and set $\theta := 2\pi/n$. Each robot a_k moves along a radius to the point $k\theta$ of the perimeter of the unit circle. We call the arc $[k\theta, (k+1)\theta)$ sector S_k ; that is, after 1 time unit, robot a_k will be located at the beginning of sector S_k . Robots make announcements if they find the exit and confirm/disprove the announcements of

other robots accordingly. Every robot searches one sector in each round, moving counter clockwise (ccw). At any moment, if an announcement is confirmed by two other robots, that announcement is correct. Also, an announcement disproved by three other robots is invalidated (announcing a different exit also counts as a disproof). When three robots make different announcements, we can deduce that two of them are Byzantine and as a result the silent ones are honest. All honest robots move through a chord to the exit to evacuate the circle and the algorithm terminates in time $E(n, 2)$. Details of the main algorithm are as follows.

Algorithm 1 $(n, 2)$ -Evacuation

- 1: Robot a_k moves along a radius of the circle to the point $k\theta$ of the unit circle.
 - 2: Robot a_k searches ccw and makes an announcement if it finds the exit. It also disproves faulty announcements concerning sectors it has visited.
 - 3: At time $1 + 3\theta$:
 - 4: **if** there is no consensus regarding the position of the exit **then**
the robot next to the contested announcements in clock wise order (called inspector robot and is uniquely determined as shown in the analysis), moves through a chord to the nearest announcement. If it is not the exit, it moves through a chord to the other announcement(s) to evacuate. A second inspector (the robot next to first inspector clockwise) may be utilized simultaneously, if it is known that the findings of the first inspector are still not enough for a consensus in the worst case.
 - 5: All other honest robots moves through a chord to the farthest announcement to evacuate, and may change trajectory according to inspector's findings.
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We define as t the time beyond the $1 + 3\theta$ needed to learn the position of the exit. If $t \leq 1$, evacuation time is unaffected and equals to $3 + 3\theta$. If $t > 1$, the evacuation time is increased by a function $\delta(n)$. For the geometric proof and the calculation of $\delta(n)$ please refer to the appendix.

Theorem 3 $((n, 2) - \text{Evacuation})$. *The worst-case time for $(n, 2) - \text{Evacuation}$ for $n \geq 9$ by Algorithm 1 satisfies*

$$E(n, 2) \leq 3 + \frac{6\pi}{n}$$

and $E(n, 2) \leq 1 + \frac{6\pi}{n} + 2 \sin(\frac{3\pi}{2n}) + \sqrt{2 - 4 \sin(\frac{3\pi}{2n}) + 4 \sin^2(\frac{3\pi}{2n})}$ for $n < 9$.

Proof. If after 3 rounds only one announcement is made, that announcement is correct because in these 3 rounds every point in the circle has

been searched by at least one honest robot. All robots will move through a chord towards the exit, and evacuation will be completed in time $3 + 3\theta$. For any other outcome, we consider the following cases depending on the number of announcements made during the execution of the algorithm.

Case 1: No announcement at the end of round 1.

Subcase 1-a: No announcement at the end of round 2.

- *1-a-i:* Two announcements at the end of round 3.
Assume that they are made by robots a_0 and a_{n-1} . Robots a_1 and a_2 searched the sector with the announcement of a_0 in the previous rounds and made no announcements. Also a_0 disagrees with a_{n-1} as he made an announcement elsewhere. Three robots disagree with a_0 so that is the Byzantine one. As a result the correct exit is the one a_{n-1} announced. The exit will be known in time $1 + 3\theta$. Similar argument can be used for announcements not in consecutive sectors.
- *1-a-ii:* Three announcements at the end of round 3.
Announcements must be in consecutive sectors (in any other case, we would have earlier announcements). Lets say announcements made by robots a_0 , a_{n-1} and a_{n-2} . Then we know that the rest of the robots are honest. Robot a_1 visited the sectors with the announcements of a_0 and a_{n-1} in the previous rounds. As a result a_{n-2} is honest and the exit is on his announcement. The exit will be known in time $1 + 3\theta$.

Subcase 1-b: One announcement at the end of round 2.

Then at round 3 there will be one or two new announcements.

- *1-b-i.* One new announcement at the end of round 3.
If the new announcement is in a different sector than the previous one, refer to case (1-a-i). Else (two announcements in the same sector), suppose that the first announcement was made by a_0 and the second by a_{n-1} . Then we can deduce that a_1 is the one Byzantine robot (as he disagrees with both announcements, and at the end of round 3, no new announcement is made) and as a result a_{n-2} is an honest one. a_{n-2} as an inspector robot will travel through a chord to the nearest announcement. We will know the exit in time $1 + 3\theta + 2 \sin(\pi/n)$. See Figures 1-4.
- *1-b-ii:* Two new announcements at the end of round 3.
If the three announcements made until the end of round 3 are in different sector, refer to case (1-a-ii). If one of the new announcements

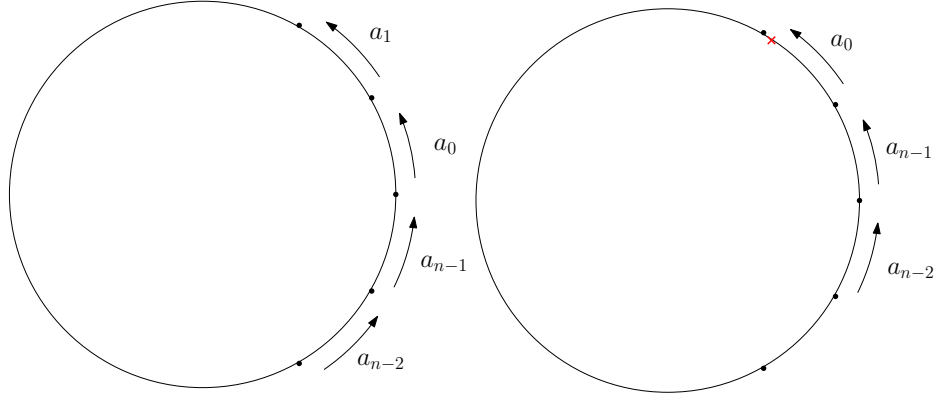


Fig. 1. time $1 + \theta$

Fig. 2. time $1 + 2\theta$

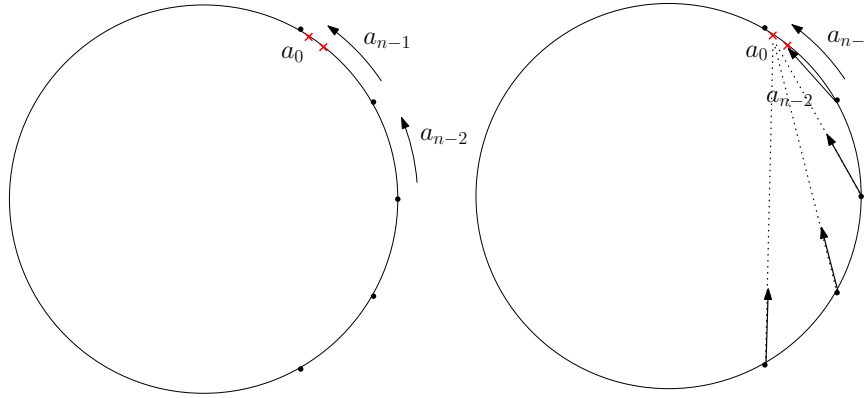


Fig. 3. time $1 + 3\theta$

Fig. 4. Inspection and Evacuation

is in the same sector with the previous one, consider the following cases:

- Assume that a_0 made the announcement in the second round and a_{n-1} , a_1 made an announcement in the third round. We can deduce that a_1 is the Byzantine robot as he disagrees with the other two announcements and also with robots a_2 and a_3 that searched that sector in previous rounds. As a result a_{n-2} is honest. Inspector robot a_{n-2} will travel through a chord to the nearest announcement. We will know the exit in time $1 + 3\theta + 2 \sin(\pi/n)$.
- Assume that a_0 made the announcement in the second round and a_{n-1} , a_{n-2} made an announcement in the third round. In that case we can deduce that all other robots are honest. Robot a_1 visited

the sector with the announcements of a_{n-1} and a_{n-2} . The exit is the announcement of a_{n-2} and will be known in time $1 + 3\theta$.

Subcase 1-c: Two announcements at the end of round 2.

If there is no new announcement at round 3, refer to case (1-a-i). If there is one new announcement at the end of round 3 in a different sector, refer to case (1-a-ii). Else, if there there is one new announcement at the end of round 3 in the same sector with a previous announcement refer to case (1-b-ii).

Subcase 1-d: Three announcements at the end of round 2.

Refer to case (1-a-ii).

Case 2: One announcement at the end of round 1.

Subcase 2-a: No new announcement at the end of round 2.

– *2-a-i:* One new announcement at the end of round 3.

If announcements in different sector, refer to case (1-a-i). If two announcements are in the same sector, say by a_0 , a_{n-2} , then after 3 rounds we will have no consensus about the exit. In the worst case, a_{n-1} will agree with a_0 and the inspector robot a_{n-3} will agree with a_{n-2} . In such a case, a second inspector robot a_{n-4} is needed to move to the nearest announcement. The exit will be known in time $1 + 3\theta + 2 \sin(3\pi/2n)$. *This is the worst search time.* See Figures 5-8.

– *2-a-ii:* Two new announcements at the end of round 3.

If all announcements in different sectors, refer to case (1-a-ii). Else consider the following cases:

- Assume the announcements were made by a_0 in the first round and by a_{n-2} and a_{n-4} in the third round. We know that all the other robots are honest. Robot a_{n-1} searched the sector with the announcements of a_0 and a_{n-2} in the second round. If one of them is correct, that must be the one a_0 announced (if a_{n-2} would be correct, a_{n-1} would announced it in the second round). If a_{n-1} disagrees with both of them, the exit is announced correctly by a_{n-4} and will be known in time $1 + 3\theta$.
- Assume the announcements were made by a_0 in the first round and by a_{n-1} and a_{n-2} in the third round. We know that all the other robots are honest. Inspector robot a_{n-3} will visit the nearest announcement and the exit will be known in time $1 + 3\theta + 2 \sin(\pi/n)$.

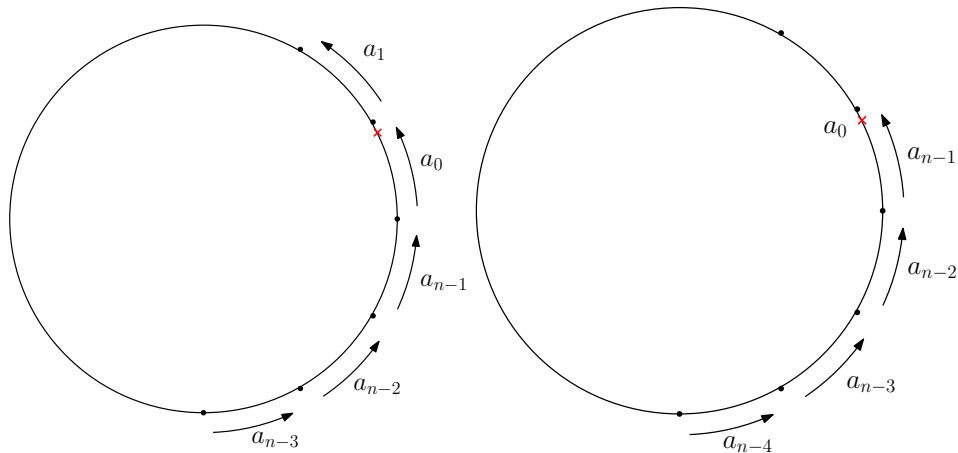


Fig. 5. time $1 + \theta$

Fig. 6. time $1 + 2\theta$

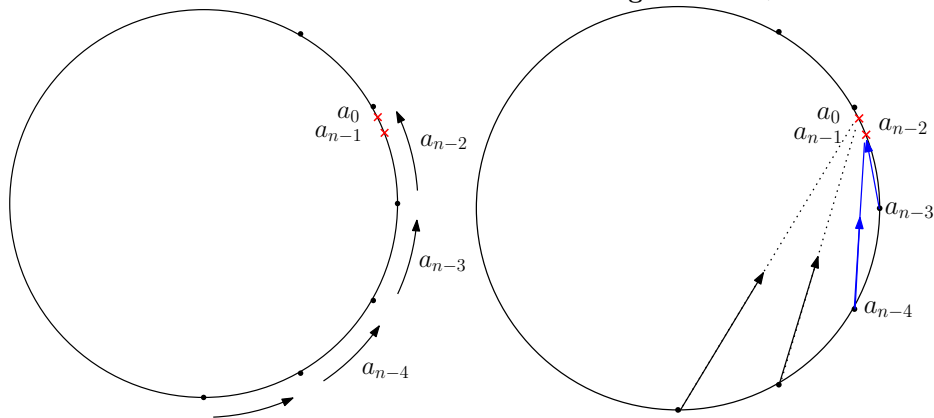


Fig. 7. time $1 + 3\theta$

Fig. 8. Inspection and Evacuation

Subcase 2-b: One new announcement at the end of round 2.

- *2-b-i:* No new announcement at the end of round 3.

If we can't already differentiate the announcements, inspector robot will find us the exit at time $1 + 3\theta + 2 \sin(\pi/n)$. Refer to case (2-a-i).

- *2-b-ii:* One new announcement at the end of round 3.

If all three announcements are in different sectors, refer to case (1-a-ii). If two of the three announcements are in the same sector refer to case (2-a-ii). If all three announcements are in the same sector made by, say, a_0 , a_{n-1} and a_{n-2} then after the third round, honest inspector robot a_{n-3} must visit 2 of these announcements, The exit will be known in time $1 + 3\theta + 4 \sin(\pi/3n)$. See Figures 9-12.

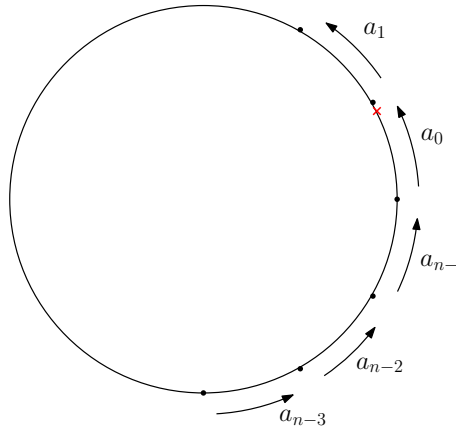


Fig. 9. time $1 + \theta$

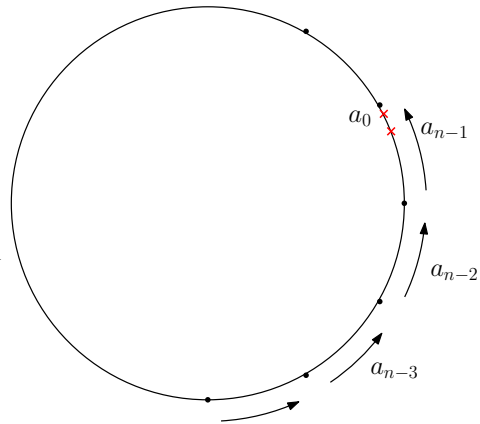


Fig. 10. time $1 + 2\theta$

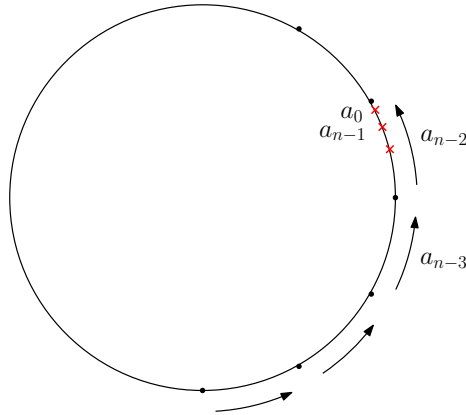


Fig. 11. time $1 + 3\theta$

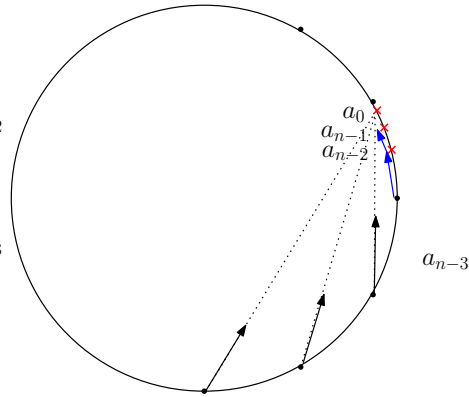


Fig. 12. Inspection and Evacuation

Subcase 2-c: Two new announcements at the end of round 2.

Refer to case (2-a-ii).

Case 3: Two announcements at the end of round 1.

Subcase 3-a: No new announcement at the end of round 2.

– *3-a-i.* No new announcement at the end of round 3.

Refer to case (1-a-i).

– *3-a-ii.* One announcement at the end of round 3.

If all three announcements are in different sectors, refer to case (1-a-ii).

If two announcements are in the same sector, consider the following cases:

- Assume that a_0 and a_{n-4} made the announcements in the first round and a_{n-2} in the third round. All other robots are honest. Robot a_{n-1} for example have visited all the locations of the announcements and we will know the correct exit in time $1 + 3\theta$.
- Assume that a_0 and a_{n-1} made the announcements in the first round and a_{n-2} in the third round. That means that a_3 is honest, and if the announcement of a_{n-1} is not correct, becoming an inspector robot, will move to the nearest announcement through a chord and the exit will be known in time $1 + 3\theta + 2 \sin(\pi/n)$.

Subcase 3-b: One new announcement at the end of round 2.

Refer to case (3-a-ii).

Case 4: Three announcements at the end of round 1.

Refer to case (1-a-ii).

This completes the proof of the claimed time bound. \square

Some calculations follow that give (for $4 \leq n \leq 9$) the bounds obtained by the above algorithm for the $(n, 2)$ case in comparison with the lower bound obtained for the $(n, 1)$ case (which holds of course for 2 Byzantine robots as well):

n	$(n, 1)$: LB	$(n, 2)$: $3 + 3\theta$	$(n, 2)$: $\delta(n)$	$(n, 2)$: UB
4	5.5558	7.7124	0.5687	8.2811
5	5.1313	6.7699	0.2361	7.0060
6	4.8264	6.1415	0.0881	6.2297
7	4.5971	5.6927	0.0318	5.7246
8	4.4186	5.3561	0.0095	5.3657
9	4.2756	5.0944	0	5.0944

4 Conclusion

We studied the evacuation problem of n robots with one or two Byzantine faults in the wireless model and provided a lower bound for the $(n, 1)$ -evacuation case and an upper bound for the $(n, 2)$ -evacuation case. An interesting possible direction after that would be to tighten our bounds, consider other communication models (like the face-to-face model) or generalize for f Byzantine robots. In particular, we conjecture that $3 + 3\theta$ is a lower bound for the $(n, 2)$ evacuation problem for infinitely many n .

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A Appendix - $\delta(n)$ calculation

As shown in Figure 13, suppose at the end of round 3 the exit is not yet known, and possible exits are in D and E. Robot a_k placed at A, moves to evacuate to its farthest announcement D, at distance 2 (diameter, $r=1$). After $t \leq r$ the inspector robot moving from F will determine the correct exit and robot a_k may need to change direction to E, but the new path that will travel is not larger than the diameter of the circle.

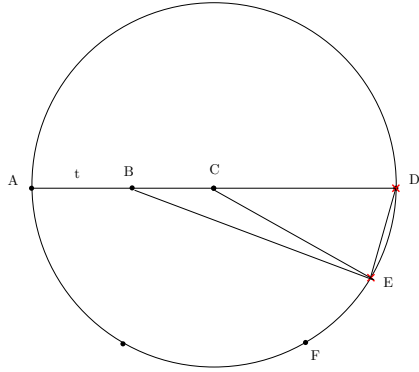


Fig. 13. $t \leq 1$

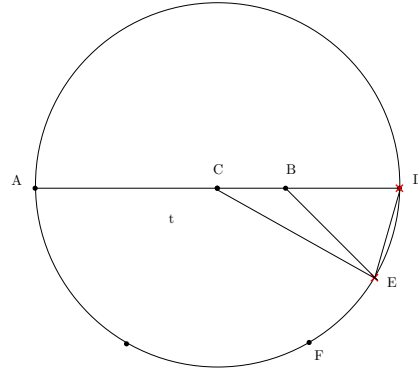


Fig. 14. $t > 1$

We must show that $BE \leq BD$. Triangle CED is equilateral ($CE = CD = r = 1$) and angle $\widehat{CED} = \widehat{CDE}$. As a result angle $\widehat{BED} \geq \widehat{CED}$. In triangle BED , $\widehat{BED} \geq \widehat{BDE}$ meaning that $BD \geq BE$.

If $t > 1$, evacuation time is increased by $\delta(n)$.

As we can see in Figure 14, we must calculate the distance of path ABE. We know that $AB = t$ so we continue to determine BE .

In triangle CBE , $CE = 1$, $CB = 1 - BD = 1 - (2 - t) = t - 1$. In the worst case regarding evacuation, angle $\widehat{ECD} = \theta/2$. Now we can calculate BE :

$$BE^2 = CB^2 + CE^2 - 2 \cdot CB \cdot CE \cdot \cos(\pi/n)$$

$$BE^2 = t^2 - 2(t - 1)(\cos(\pi/n) + 1).$$

The total distance the robot will travel to evacuate is $AB + BE = t + BE$ and that surpasses the diameter by $\delta(n)$ defined below:

$$\delta(n) = \begin{cases} 0 & \text{if } t \leq 1 \\ t + \sqrt{t^2 - 2(t - 1)(\cos(\pi/n) + 1)} - 2, & \text{if } t > 1 \end{cases}$$