Age-based restrictions: An alternative to horizontal lockdowns?

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Abstract

During an epidemic, such as the COVID-19 pandemic, policy-makers are dealing with the decision of effective, but socioeconomically costly interference, such as horizontal lockdowns, including school and workplace closure, physical distancing e.t.c. Investigating the role of the age of the individuals in the evolution of epidemiological phenomena, we propose a scheme for the comparison of certain epidemiological strategies. Then, we put the proposed scheme to the test by employing an age-based epidemiological compartment model introduced in [4], coupled with data from the literature, in order to compare the effectiveness of age-based and horizontal interventions. In general, our results suggest that these two are comparable mainly at low or medium level of austerity.

Keywords: Epidemiological strategy, Comparison scheme, Basic reproductive number, Horizontal restrictions, Aged-based interventions, Asymptomatic infectious, Numerical simulations

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1 Introduction

The recent COVID-19 pandemic brought to the fore the disastrous for the economy consequences of horizontal lockdowns. Economically costly horizontal measures during the COVID-19 pandemic have been the closure of workplaces and schools, the cancellation of public events and general stay-at-home restrictions (see [5, 10, 11, 20] and many references therein).

This fact highlights the need for a more sophisticated managing of epidemiological crises. In this context, many countries, especially after the spasmodic first response, have looked for more flexible intervention policies. Multiple combinations of interventions were deployed by policy-makers in order to combat the spread of SARS-CoV-2 and minimize their financial ramifications [2, 19, 21].

The aim of the present paper is to investigate the role of the age of the individuals in the evolution of epidemiological phenomena, in order to verify age-based restrictions as a potential alternative to horizontal lockdowns. Using as a case study the COVID-19 outbreak and data from the literature, in order to address the following questions:

- How can we systematically model and compare the effect of lockdowns and testing?
- How does the age of individuals affect the outcome of the epidemic?
- Can age-based interventions offer a replacement to horizontal lockdowns?

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We answer the above questions by firstly proposing a scheme for the comparison of certain epidemiological strategies, and secondly, by utilizing the aforementioned scheme and the age-based epidemiological compartment model proposed and studied in [4] for the comparison of a horizontal lockdown policy with various age-based interventions.

This study is organized as follows. In §2, we introduce the notion of (epidemiological) strategy with their potential gradations, and we present a framework for comparing the effectiveness of certain strategies. In §3, we present the age-based SVEAIR model derived in [4], that incorporates, among others, the ambiguous (see §4.1) variable of asymptomaticity of infectious individuals for the spread of COVID-19 disease, and we estimate the model parameters. In §4, we compare the effect of a horizontal lockdown with varying level of austerity with certain age-based counter measures of similar epidemiological, but less social hence financial, impact. We conclude in §5 with a summary and discussion of the results.

2 A framework for comparing the effectiveness of different strategies

Let us divide a population into two classes, the infectious, \mathcal{I} , and the non-infectious, \mathcal{I}^c . Each of these classes can be divided to further sub-compartments, e.g. $A \in \mathcal{I}$ and $B \in \mathcal{I}^c$.

The transmission rate from compartment B to compartment A is defined as

$$\beta_{B \to A} \coloneqq \frac{c_B \cdot \varpi_{B \to A}}{N} , \qquad (1)$$

where c_B is the average number of close contacts of an individual belonging in B with other individuals, $\varpi_{B\to A}$ is the probability of a contact to be effective in turning an individual of compartment B to an individual of compartment A, and

$$N \coloneqq \mathcal{I} + \mathcal{I}^c$$

is the total number of the population. The *removal rate* from compartment A to compartment B is defined as

$$\gamma_{A \to B} \coloneqq \frac{1}{P_{A \to B}} , \qquad (2)$$

where $P_{A \to B}$ is the average period an individual spends on compartment A before moving into compartment B. A diagram for the above definitions is shown in Figure 1.



Figure 1: Flows between the classes of infectious, \mathcal{I} , and non-infectious, \mathcal{I}^{c} , individuals of a population.

These parameters, probably among some others, are involved into the formulation of an epidemiological model that describes an epidemiological problem under study. However, these parameters are special because the interventions of exterior factors acting for the control of the studied epidemiological phenomenon (e.g. policy makers) can be described as certain changes of their values.

Now, enumerating all the different transmission and removal rates of a particular model, i.e. $\beta_1, \ldots, \beta_{n_1}$ and $\gamma_1, \ldots, \gamma_{n_2}$, respectively, we can write

$$\boldsymbol{\beta} = (\beta_i)_{i=1}^{n_1} \text{ and } \boldsymbol{\gamma} = (\gamma_i)_{i=1}^{n_2}.$$

Throughout the present section we assume a well-posed global (with respect to time) epidemiological compartmental problem,

$$\mathscr{P} = \mathscr{P}(\mathscr{M}),$$

which is described by a (differential/difference equations, agent-based, etc.) model,

$$\mathcal{M} = \mathcal{M}(\boldsymbol{x}, (\boldsymbol{\beta}(\boldsymbol{x}), \boldsymbol{\gamma}(\boldsymbol{x})), \boldsymbol{\delta}(\boldsymbol{x})),$$

where

$$\boldsymbol{x} = (x_i)_{i=1}^m \in \mathcal{X} \subseteq \mathbb{R}^m$$

is the vector of the independent variables,

$$(\boldsymbol{\beta}, \boldsymbol{\gamma}) \in F(\mathcal{X}; \mathcal{P}_{\mathrm{tr},\mathrm{r}} \subseteq \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}) = \{ \boldsymbol{f}: \mathcal{X} \to \mathcal{P}_{\mathrm{tr},\mathrm{r}} \}$$

is the vector-valued function of the parameters of interest of $\mathcal M$ and

$$\boldsymbol{\delta} \in F(\mathcal{X}; \mathcal{P}_{other} \subseteq \mathbb{R}^{n_3})$$

is the vector-valued function of the rest of parameters of $\mathcal{M}.$

2.1 Strategies and substrategies

First, we introduce the notion of a *strategy* of \mathscr{P} , which is of pivotal importance for the following analysis. In the epidemiology framework, a strategy can be considered as the mathematical description of a set of epidemiological interventions made by potential external factors, such as policy makers, experts e.t.c., in order to restrict the epidemiological phenomenon. The structure of these interventions consists first on fixing a value of the parameters, $(\beta_0, \gamma_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$, chosen as a reference value, and second, on scaling each element of the set in terms of the fixed value.

Definition 1 (strategy & strategic scale of an element). Let $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$.

1. A set $S = S(\beta_0, \gamma_0) \subseteq F(\mathcal{X}; \mathcal{P}_{tr,r})$ is called strategy (of \mathscr{P}) with respect to (β_0, γ_0) iff

$$\forall \boldsymbol{y} \in S \exists \boldsymbol{r} = \boldsymbol{r}(\cdot; (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0), \boldsymbol{y}) \in F(\mathcal{X}; \mathbb{R}^{n_1 + n_2}) \text{ s.t. } \boldsymbol{y} = \boldsymbol{r} \odot (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0),$$

where \odot stands for the Hadamard product.

2. Let

i. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective strategy and *ii.* $\boldsymbol{y} \in S$.

A function $\mathbf{r} \in F(\mathcal{X}; \mathbb{R}^{n_1+n_2})$ as in 1. is called strategic scale of \mathbf{y} .

We observe that every subset of a strategy is a strategy itself, as it is referred in the following result, the elementary proof of which is omitted.

Proposition 1. Let

- 1. $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r}),$
- 2. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective strategy and
- 3. $S_0 \subseteq S$.

Then S_0 is a strategy with respect to $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$.

In view of Proposition 1, we give the definition of a substrategy of a given strategy. In the epidemiology framework, a *substrategy* can be considered as the mathematical description of a subset of a given set of epidemiological interventions.

Definiton 2 (substrategy). Let

- *i.* $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r}),$
- ii. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective strategy and
- *iii.* $S_0 \subseteq S$.

We call S_0 a substrategy of S.

In fact, we can define a substrategy by setting limitations to the choice of a strategic scale of each of its elements. Below we name certain such examples.

Definition 3 (horizontal and x_i -based strategy). Let

- *i.* $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r}),$
- ii. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective strategy and
- *iii.* $S_0 \subseteq S$.

We name the following substrategies.

1. Let $i \in \{1, \ldots, m\}$. S₀ is called horizontal with respect to x_i iff

 $\boldsymbol{r}(\boldsymbol{x}; (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0), \boldsymbol{y}) = \boldsymbol{r}(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_m; (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0), \boldsymbol{y}), \quad \forall \boldsymbol{x} \in \mathcal{X}, \quad \forall \boldsymbol{y} \in S_0,$

- *i.e.* $\forall y \in S_0$ a respective strategic scale is independent of x_i , otherwise we call it x_i -based.
- 2. S_0 is called horizontal iff it is horizontal with respect to $x_i, \forall i \in \{1, \ldots, m\}$.

In the epidemiology framework, a x_i -based substrategy can be considered as the mathematical description of a subset of epidemiological interventions, which targets a certain group of a population partitioned with respect to x_i variable.

We also observe that every union of strategies is a strategy itself, as it is referred in the following elementary result.

Proposition 2. Let

- 1. $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ and
- 2. $\{S_j = S_j(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)\}_{j \in \mathcal{J}}$ be a family of respective strategies.

Then $\bigcup_{j \in \mathcal{J}} S_j$ is a strategy with respect to $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$.

In view of Proposition 2, we give the following definition.

Definiton 4 (the largest strategy). Let

- 1. $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ and
- 2. S be the family of all the respective strategies.

 $We \ call$

$$\hat{S} = \hat{S}(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) = \bigcup_{S \in \mathcal{S}} S$$

the largest strategy with respect to $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$.

Of course, whatever result holds for the largest strategy also holds for an abstract strategy, like the following direct one.

Proposition 3. Let $(\beta_0, \gamma_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$. If

$$\left(\underbrace{0,\ldots,0}_{\#\ n_1+n_2}\right) \notin \left(\boldsymbol{\beta}_0(\boldsymbol{\mathcal{X}}), \boldsymbol{\gamma}_0(\boldsymbol{\mathcal{X}})\right),$$
(3)

then

 $\forall \boldsymbol{y} \in \hat{S} \exists ! strategic scale of \boldsymbol{y}.$

Under the light of Proposition 3, the next notion is well-defined.

Definition 5 (strategic scale of a strategy). Let

- 1. $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ satisfy (3) and
- 2. $S = S(\beta_0, \gamma_0)$ be a respective strategy.

We call the function

$$S \ni \boldsymbol{y} \mapsto \boldsymbol{r}(\cdot; (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0), \boldsymbol{y}) \in F(\mathcal{X}; \mathbb{R}^{n_1 + n_2})$$

the strategic scale of S.

We can then easily deduce the following result.

Proposition 4. Let

- *i.* $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ satisfy (3),
- ii. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective strategy,
- iii. $\mathfrak{P}_j, \forall j \in \{1,2\}$ be mathematical statements with respect to the strategic scale of S such that $\mathfrak{P}_1 \Rightarrow \mathfrak{P}_2$ and
- iv. $S_j \subseteq S, \forall j \in \{1, 2, 3\}$, be substrategies of S such that

$$S_{j} = \left\{ \boldsymbol{y} \in S \left| \mathfrak{P}_{i}(\boldsymbol{r}(\cdot; (\boldsymbol{\beta}_{0}, \boldsymbol{\gamma}_{0}), \boldsymbol{y})) \right\}, \quad \forall j \in \{1, 2\} \quad \& S_{3} = \left\{ \boldsymbol{y} \in S \left| \neg \mathfrak{P}_{1}(\boldsymbol{r}(\cdot; (\boldsymbol{\beta}_{0}, \boldsymbol{\gamma}_{0}), \boldsymbol{y})) \right\} \right\}$$

Then

- 1. $S_1 \subseteq S_2$ and
- 2. $S_3 = S \smallsetminus S_1$.

For example, for a given $(\beta_0, \gamma_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ that satisfies (3) and a given $i \in \{1, \ldots, m\}$, the horizontal with respect to x_i substrategy of $\hat{S} = \hat{S}(\beta_0, \gamma_0)$ is the set-theoretic complement with respect to \hat{S} of the x_i -based substrategy of \hat{S} . The scope of the present paper can be now stated as the comparison of the above substrategies for x_i being the age of an individual of a population.

2.2 Comparison of strategies

Here we introduce a scheme for the comparison of strategies, for which we need some preliminary notions, such as the basic reproductive number and the graded strategies.

2.2.1 \mathcal{R}_0 : the measure of comparison

An important epidemiological notion studied and used extensively in the epidemiological literature is the basic reproductive number, \mathcal{R}_0 , which is defined as the average number of infectious cases directly generated by one such case in a population where all individuals are susceptible to an infection. For every mathematical model, that describes a problem under study, corresponds the respective \mathcal{R}_0 , which can be calculated with several ways, such as with the next-generation method or the existence of the endemic equilibrium [13].

In general, \mathcal{R}_0 depends on both the independent variables and the parameters of a model, therefore it is considered as a function defined as

$$\mathcal{R}_{0}: \mathcal{X} \times F(\mathcal{X}; \mathcal{P}_{\mathrm{tr}}, \mathrm{r}) \times F(\mathcal{X}; \mathcal{P}_{\mathrm{other}}) \to (0, \infty)$$
$$(\boldsymbol{x}, (\boldsymbol{\beta}, \boldsymbol{\gamma}), \boldsymbol{\delta}) \mapsto \mathcal{R}_{0}(\boldsymbol{x}, (\boldsymbol{\beta}, \boldsymbol{\gamma}), \boldsymbol{\delta}).$$

Only for the sake of brevity and compactness of the exposition, in the present paper we assume that it is independent of \boldsymbol{x} , that is

$$\mathcal{R}_{0}: F(\mathcal{X}; \mathcal{P}_{tr}, r) \times F(\mathcal{X}; \mathcal{P}_{other}) \to (0, \infty)$$
$$((\boldsymbol{\beta}, \boldsymbol{\gamma}), \boldsymbol{\delta}) \mapsto \mathcal{R}_{0}((\boldsymbol{\beta}, \boldsymbol{\gamma}), \boldsymbol{\delta}).$$

In the proposed scheme, we check how one strategy measures against another of a special kind, via the calculation of the respective values of \mathcal{R}_0 . That special kind of strategies is described below.

2.2.2 Gradable and graded strategies

The notion of the graded strategies is the crux of the proposed scheme. But before its introduction, we first need the following one.

Definition 6 (gradable strategy). Let

- 1. $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ satisfy (3) and
- 2. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective strategy.

We call S gradable iff $\forall \boldsymbol{\delta} \in F(\mathcal{X}; \mathcal{P}_{other})$ the function $\mathcal{R}_0|_S(\cdot, \boldsymbol{\delta})$ is injective.

Since $\mathcal{R}_0(S, \delta) \subseteq (0, \infty)$, $\forall \delta \in F(\mathcal{X}; \mathcal{P}_{other})$, we can arrange any family of pairwise distinct elements of such a set in a strictly ascending order when S is gradable, hence the following notion is well defined.

Definition 7 (graded strategy). Let

- 1. $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{tr,r})$ satisfy (3),
- 2. $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ be a respective gradable strategy,
- 3. $\boldsymbol{\delta} \in F(\mathcal{X}; \mathcal{P}_{other})$ and
- 4. $\{\boldsymbol{y}_i\}_{i=1}^k \subseteq S$ be a family of pairwise distinct elements of S, such that

$$\underbrace{\mathcal{R}_0(\boldsymbol{y}_1,\boldsymbol{\delta})}_{=:G_1} < \cdots < \underbrace{\mathcal{R}_0(\boldsymbol{y}_k,\boldsymbol{\delta})}_{=:G_k}.$$

We call the pair $(S, \mathbf{G} = (G_i)_{i=1}^k)$ a graded strategy, while \mathbf{G} is called a gradation of S and each of the G_1, \ldots, G_k is called a grade of \mathbf{G} .

We note that the gradation of a graded strategy is a matter of choice. In what follows, for the sake of simplicity, we write S instead of (S, \mathbf{G}) for a graded strategy.

2.2.3 Comparison matrix and coverage

With the above toolbox at hand, we propose a scheme for the comparison of two strategies, only when one of them is graded. In addition, the scheme allows us to include many substrategies of the other strategy. Below follow the steps of the proposed scheme, which is governed by the construction of the respective *comparison matrix* and evaluated in terms of *epidemiological* and *social coverage*.

Construction of the comparison matrix.

1. Placing of the grades G_1, \ldots, G_k of the gradation $\boldsymbol{G} = (G_i)_{i=1}^k$ of a given graded strategy S_1 , in increasing order, to the top row:



2. Placing the under study substrategies $\{S_{2_i}\}_{i=1}^{\ell}$ of a second strategy S_2 to the left of the table, with the intend of comparing them against the first graded strategy.



3. Populating the comparison matrix with $\star,$ where

 $\star_{ij} = \begin{cases} \checkmark & \text{if the } \mathcal{R}_0 \text{ of } S_{2i} \text{ is greater than or equal to } G_j \\ \bigstar & \text{otherwise }, \end{cases} \quad \forall (i,j) \in \{1,\ldots,\ell\} \times \{1,\ldots,k\} \ .$

S_2 S_1	G_1	G_2		G_k
S_{2_1}	* 11	\star_{12}		\star_{1k}
$S_{2_{2}}$	\star_{21}	$*_{22}$		\star_{2k}
÷	÷	÷	۰.	÷
$S_{2_{\ell}}$	$\star_{\ell 1}$	$\star_{\ell 2}$		$\star_{\ell k}$

Next we present two ways to read the comparison matrix for extracting useful information.

Social overview of the comparison matrix: epidemiological coverage. Here we compare S_2 to S_1 . In particular, for every fixed substrategy of S_2 (social overview), we check how good of an alternative it is, compared to S_1 (epidemiological coverage).

4→. Calculating the epidemiological coverage of the gradation G of S_1 by each substrategy of S_2 , by calculating the average number of \checkmark in each row.

S_2 S_1	G_1	G_2		G_k	Epidemiological coverage (·100%)
S_{2_1}	*11	* 12		\star_{1k}	$\frac{\#\{\star_{1j}=\checkmark\}_{j=1}^k}{k}$
S_{2_2}	$\dot{*}_{21}$	\star_{22}		\star_{2k}	$\frac{\#\{\star_{2j}=\checkmark\}_{j=1}^n}{k}$
S_2	:	: */2	·. 	: * 01.	$: \\ \#\{\star_{\ell j} = \checkmark\}_{j=1}^k$
	~ ℓ1	~ ℓ2		~ <i>ℓĸ</i>	

 5_{\rightarrow} . Calculating the total coverage of the gradation G of S_1 by the whole S_2 , through the average value of all epidemiological coverages.

S_2 S_1	G_1	G_2		G_k	Epidemiological coverage (·100%)
S_{2_1}	*11	* ₁₂		* 1k	$\frac{\#\{\star_{1j}=\checkmark\}_{j=1}^k}{k}$
$S_{2_{2}}$	$*_{21}$	$*_{22}$		\star_{2k}	$\frac{\#\{\star_{2j}=\checkmark\}_{j=1}^n}{k}$
÷	÷	÷	۰.	÷	:
$S_{2_{\ell}}$	$\star_{\ell 1}$	$\star_{\ell 2}$		$\star_{\ell k}$	$\frac{\#\{\star_{\ell j}=\checkmark\}_{j=1}^k}{k}$
					$\frac{\#\{\star_{ij}=\checkmark\}_{i,j=1}^{\ell,k}}{\underline{\ell}\cdot\underline{k}}$

The takeaway of the above analysis is that if the total (epidemiological) coverage of G by the (respective sub-)strategy S_2 (S_{2_i} , for $i \in \{1, \ldots, \ell\}$) is satisfying, then S_2 (S_{2_i}) could be considered as an alternative to S_1 . We note that the quantification of the term "satisfying" is subjective.

Epidemiological overview of the comparison matrix: social coverage. Here we compare S_1 to S_2 . In particular, for every fixed grade of \boldsymbol{G} (epidemiological overview), we check how much of that grade S_2 is (social coverage).

S_2 S_1	G_1	G_2		G_k
	*11	$*_{12}$		\star_{1k}
S_{2_2}	\star_{21}	*22		\star_{2k}
÷	:	÷	۰.	÷
$S_{2_{\ell}}$	*ℓ1	$\star_{\ell 2}$		$\star_{\ell k}$
Social coverage $(\cdot 100\%)$	$\frac{\#\{\star_{i1}=\checkmark\}_{i=1}^\ell}{\ell}$	$\frac{\#\{\star_{i2}=\checkmark\}_{i=1}^\ell}{\ell}$		$\frac{\#\{\star_{ik}=\checkmark\}_{i=1}^\ell}{\ell}$

4. Calculating the social coverage of S_2 by each grade of \boldsymbol{G} of S_1 , by calculating the average number of \checkmark in each column.

 5_{\downarrow} . Calculating the total coverage of S_2 by the whole **G** of S_1 , through the average value of all social coverages.

S_2 S_1	G_1	G_2		G_k	
	*11	*12		\star_{1k}	
S_{2_2}	*21	\star_{22}		\star_{2k}	
	:	÷	۰.	:	
S_{2_ℓ}	$\star_{\ell 1}$	$\star_{\ell 2}$		$\star_{\ell k}$	
Social coverage $(\cdot 100\%)$	$\frac{\#\{\star_{i1}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\frac{\#\{\star_{i2}=\checkmark\}_{i=1}^\ell}{\ell}$		$\frac{\#\{\star_{ik}=\checkmark\}_{i=1}^\ell}{\ell}$	$\frac{\#\{\star_{ij}=\checkmark\}_{i,j=1}^{\ell,k}}{\ell\cdot k}$

Total overview of the comparison matrix. Here, we combine the social and the epidemiological overview of the camparison matrix.

6. Merging of the social and epidemiological overviews.

S_2 S_1	G_1	G_2		G_k	Epidemiological coverage $(.100\%)$
S_{2_1}	*11	*12		\star_{1k}	$\frac{\#\{\star_{1j}=\checkmark\}_{j=1}^k}{k}$
S_{2_2}	*21	\star_{22}		\star_{2k}	$\frac{\#\{\star_{2j}=\checkmark\}_{j=1}}{k}$
÷	÷	÷	۰.	÷	•
S_{2_ℓ}	$\star_{\ell 1}$	$\star_{\ell 2}$		$\star_{\ell k}$	$\frac{\#\{\star_{\ell j}=\checkmark\}_{j=1}^k}{k}$
Social coverage ($\cdot 100\%$)	$\frac{\#\{\star_{i1}=\checkmark\}_{i=1}^\ell}{\ell}$	$\frac{\#\{\star_{i2}=\checkmark\}_{i=1}^\ell}{\ell}$		$\frac{\#\{\star_{ik}=\checkmark\}_{i=1}^\ell}{\ell}$	$\frac{\#\{\star_{ij}=\checkmark\}_{i,j=1}^{\ell,k}}{\ell\cdot k}$

3 The basic epidemiological model and estimation of parameters

Here we use the epidemiological model \mathscr{M} along with the respective problem \mathscr{P} , introduced and studied in [4], as a means of utilization of the proposed scheme in answering the main question of the present paper. We choose this model as it incorporates both symptomatic and asymptomatic

infectious individuals, with the later playing an important role in the spread of the COVID-19 (see [14] and many references therein), as well as the age of the infected/infectious individuals.

After *scaling* the independent age-variable, θ , and turning it to another time-variable measured in the same units as t (see [4]) and using the relation N = S + V + E + A + I + R, we obtain the following model

$$\begin{cases} \frac{\mathrm{d}S}{\mathrm{d}t} = \mu N_0 - \left(p + \int_0^\infty \beta_A(\theta) a(\cdot, \theta) + \beta_I(\theta) i(\cdot, \theta) \,\mathrm{d}\theta + \mu \right) S\\ S(0) = S_0. \end{cases}$$
(4a)

$$\begin{cases} \frac{\mathrm{d}V}{\mathrm{d}t} = pS - \left(\zeta\epsilon + \int_{0}^{\infty} \beta_{A}(\theta)a(\cdot,\theta) + \beta_{I}(\theta)i(\cdot,\theta)\,\mathrm{d}\theta\,(1-\epsilon) + \mu\right)V \\ V(0) = V_{0}, \end{cases}$$
(4b)

$$\begin{cases} \frac{\partial e}{\partial t} + \frac{\partial e}{\partial \theta} = -(k+\mu)e\\ e(\cdot,0) = \int_{0}^{\infty} \beta_{A}(\theta)a(\cdot,\theta) + \beta_{I}(\theta)i(\cdot,\theta)\,\mathrm{d}\theta\,(S+(1-\epsilon)V) \end{cases}$$
(4c)

$$\begin{cases}
\frac{\partial a}{\partial t} + \frac{\partial a}{\partial \theta} = -(\gamma_A \xi + \chi (1 - \xi) + \mu) a \\
a(\cdot, 0) = \int_0^\infty k(\theta)q(\theta)e(\cdot, \theta) d\theta \\
a(0, \cdot) = a_0,
\end{cases}$$

$$\begin{cases}
\frac{\partial i}{\partial t} + \frac{\partial i}{\partial \theta} = -(\gamma_I + \mu) i \\
i(\cdot, 0) = \int_0^\infty k(\theta) (1 - q(\theta)) e(\cdot, \theta) + \chi(\theta) (1 - \xi(\theta)) a(\cdot, \theta) d\theta
\end{cases}$$
(4e)
$$i(0, \cdot) = i_0.$$

The flow diagram of the differential equations in (4) is shown in Figure 2, and the dimensional units of all variables and parameters appeared in \mathscr{P} (4) are gathered in Table 1.



Figure 2: Flow diagram of $\mathscr{P}(4)$.

Independent	Description	Units
variables		
t	Time	Т
θ	Age, i.e. time elapsed since, e.g., birth or infection	Θ
Conversion	Description	Units
factor		
ω	Conversion factor from the units of θ to the units of t	$T \Theta^{-1}$
Dependent variables	Description	Units
$\frac{1}{N}$	Number of total population of individuals	#
S	Number of susceptible individuals	#
V	Number of vaccinated-with-a-prophylactic-vaccine individuals	#
e	Age density of latent/exposed individuals	$\# \Theta^{-1}$
E	Number of latent/exposed individuals	#
a	Age density of asymptomatic infectious individuals	$\# \Theta^{-1}$
A	Number of asymptomatic infectious individuals	#
i	Age density of symptomatic infectious	$\# \Theta^{-1}$
Ι	Number of symptomatic infectious individuals	#
R	Number of recovered/removed individuals	#
Parameters	Description	Units
	Description	CIII05
N_0	Population size	#
$rac{N_0}{\mu}$	Population size Birth/Death rate	# T ⁻¹
$egin{array}{c} N_0 \ \mu \ eta_A \end{array}$	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals	$ \begin{array}{c} \# \\ T^{-1} \\ \#^{-1} T^{-1} \end{array} $
$egin{array}{c} N_0 \ \mu \ eta_A \ eta_I \end{array}$	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals	$ \begin{array}{c} \# \\ T^{-1} \\ \#^{-1} T^{-1} \\ \#^{-1} T^{-1} \\ \end{array} $
N_0 μ β_A β_I p	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate	$ \begin{array}{c} \# \\ T^{-1} \\ \#^{-1} T^{-1} \\ \#^{-1} T^{-1} \\ T^{-1} \end{array} $
$ \begin{array}{c} N_0 \\ \mu \\ \beta_A \\ \beta_I \\ p \\ \epsilon \end{array} $	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness	$ \begin{array}{c} \# \\ T^{-1} \\ \#^{-1} T^{-1} \\ \#^{-1} T^{-1} \\ T^{-1} \\ \end{array} $
$ \begin{array}{c} N_0 \\ \mu \\ \beta_A \\ \beta_I \\ p \\ \epsilon \\ \zeta \end{array} $	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate	$ \begin{array}{c} \# \\ T^{-1} \\ \#^{-1} T^{-1} \\ \#^{-1} T^{-1} \\ T^{-1} \\ \hline T^{-1} \\ \hline \end{array} $
$egin{array}{c} N_0 \ \mu \ eta_A \ eta_I \ p \ \epsilon \ \zeta \ k \end{array}$	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate Latent rate (rate of susceptible individuals becoming infectious)	$ \begin{array}{c} \# \\ T^{-1} \\ \#^{-1} T^{-1} \\ \#^{-1} T^{-1} \\ T^{-1} \\ \hline T^{-1} \\ T^{-1} \\ T^{-1} \\ \end{array} $
$ \begin{array}{c} N_0 \\ \mu \\ \beta_A \\ \beta_I \\ p \\ \epsilon \\ \zeta \\ k \\ q \end{array} $	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate Latent rate (rate of susceptible individuals becoming infectious) Proportion of the latent/exposed individuals becoming asymptomatic infectious	$ \frac{\#}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{-}{T^{-1}} \\ \frac{-}{T^{-1}} \\ \frac{-}{-} \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ -$
$egin{aligned} &N_0 & \mu & \ η_A & \ η_I & p & \ &\epsilon & \ &\zeta & k & \ &q & \ &\xi & \end{aligned}$	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate Latent rate (rate of susceptible individuals becoming infectious) Proportion of the latent/exposed individuals becoming asymptomatic infectious Proportion of the asymptomatic infectious individuals becoming recov-	$ \frac{\#}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{-}{T^{-1}} \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ -$
$egin{aligned} &N_0\ &\mu\ η_A\ η_I\ &p\ &\epsilon\ &\zeta\ &k\ &q\ &\xi \end{aligned}$	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate Latent rate (rate of susceptible individuals becoming infectious) Proportion of the latent/exposed individuals becoming asymptomatic infectious Proportion of the asymptomatic infectious individuals becoming recov- ered/removed (without developing any symptoms)	$ \frac{\#}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{-}{T^{-1}} \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ -$
$ \begin{array}{c} N_0 \\ \mu \\ \beta_A \\ \beta_I \\ p \\ \epsilon \\ \zeta \\ k \\ q \\ \xi \\ \chi \end{array} $	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate Latent rate (rate of susceptible individuals becoming infectious) Proportion of the latent/exposed individuals becoming asymptomatic infectious Proportion of the asymptomatic infectious individuals becoming recov- ered/removed (without developing any symptoms) Incubation rate (rate of a part of asymptomatic infectious individuals developing symptoms)	$ \frac{\#}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{-}{T^{-1}} \\ - \\ - \\ T^{-1} \\ - \\ T^{-1} \\ - \\ T^{-1} $
$ \begin{array}{l} N_0 \\ \mu \\ \beta_A \\ \beta_I \\ p \\ \epsilon \\ \zeta \\ k \\ q \\ \xi \\ \chi \\ \chi \\ \gamma_A \end{array} $	Population size Birth/Death rate Transmission rate of asymptomatic infectious individuals Transmission rate of symptomatic infectious individuals Vaccination rate Vaccine effectiveness Vaccine-induced immunity rate Latent rate (rate of susceptible individuals becoming infectious) Proportion of the latent/exposed individuals becoming asymptomatic infectious Proportion of the asymptomatic infectious individuals becoming recov- ered/removed (without developing any symptoms) Incubation rate (rate of a part of asymptomatic infectious individuals developing symptoms) Recovery rate of asymptomatic infectious individuals	$ \frac{\#}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{\#^{-1} T^{-1}}{T^{-1}} \\ \frac{-}{T^{-1}} \\ - \\ T^{-1} \\ - \\ T^{-1} \\ - \\ T^{-1} \\ - \\ T^{-1} \\$

Table 1: Description of the independent and dependent variables and parameters of \mathcal{M} , along with their units.

From the analysis conducted in [4], the *basic reproductive number*, \mathcal{R}_0 , of the model is

$$\mathbb{R}_{0}^{+} \ni \mathcal{R}_{0} \coloneqq \frac{\mu N_{0}}{p+\mu} \left(1 + \frac{p\left(1-\epsilon\right)}{\zeta\epsilon + \mu} \right) \left(\mathcal{R}_{A} + \mathcal{R}_{I} \right),$$

$$(5)$$

where

$$\mathbb{R}_0^+ \ni \mathcal{R}_A \coloneqq \int_0^\infty k(s)q(s) \mathrm{e}^{-\int_0^s k(\tau) + \mu \,\mathrm{d}\tau} \,\mathrm{d}s \int_0^\infty \beta_A(s) \mathrm{e}^{-\int_0^s \gamma_A(\tau)\xi(\tau) + \chi(\tau)(1-\xi(\tau)) + \mu \,\mathrm{d}\tau} \,\mathrm{d}s$$

and

$$\begin{split} \mathbb{R}_{0}^{+} \ni \mathcal{R}_{I} \coloneqq \left(\int_{0}^{\infty} k(s) \left(1 - q(s)\right) e^{-\int_{0}^{s} k(\tau) + \mu \, \mathrm{d}\tau} \, \mathrm{d}s + \right. \\ \left. + \int_{0}^{\infty} k(s)q(s) e^{-\int_{0}^{s} k(\tau) + \mu \, \mathrm{d}\tau} \, \mathrm{d}s \int_{0}^{\infty} \chi(s) \left(1 - \xi(s)\right) e^{-\int_{0}^{s} \gamma_{A}(\tau)\xi(\tau) + \chi(\tau)(1 - \xi(\tau)) + \mu \, \mathrm{d}\tau} \, \mathrm{d}s \right) \times \\ \left. \times \int_{0}^{\infty} \beta_{I}(s) e^{-\int_{0}^{s} \gamma_{I}(\tau) + \mu \, \mathrm{d}\tau} \, \mathrm{d}s. \end{split}$$

We now present parameter values fitting for the case of SARS-CoV-2. The chosen values are taken from the biological and medical literature. Bellow, we give a detailed explanation about the value of each parameter, whereas a summary of the parameter values can be found in Table 2.

Table 2: A list of parameters of \mathcal{M} , along with their value, units and value source.

Parameters	Value	Units	Source
N_0	$80\cdot 10^6$	individuals	Estimated from [20]
μ	$4.38356 \cdot 10^{-5}$	day $^{-1}$	Estimated from $[20]$
β_A	Figure 4	individual ⁻¹ \cdot day ⁻¹	Estimated from $[12]$
β_I	Figure 4	$individual^{-1} \cdot day^{-1}$	Estimated from $[12]$
p	10^{-3}	day^{-1}	Estimated from $[20]$
ϵ	0.7	-	Estimated from $[15]$
ζ	$\frac{1}{14}$	day^{-1}	Estimated from [9]
k	Equation 6	day^{-1}	Estimated from $[18, 30]$
q	Figure 5	-	Estimated from [25]
ξ	0.5	-	Estimated from $[16, 6]$
χ	Equation 7	day^{-1}	Estimated from $[16, 6]$
γ_A	$\frac{1}{8}$	day^{-1}	Estimated from [7]
γ_I	$\frac{1}{14}$	day^{-1}	Estimated from [7]

The size of the population $N_0 = 80 \cdot 10^6$ individuals is assumed to be that of a relative large country, such as Germany, Turkey or Thailand [20]. The birth/death rate, $\mu = 4.38356 \cdot 10^{-5} \text{ day}^{-1}$, is taken from data from [20]. The average

The birth/death rate, $\mu = 4.38356 \cdot 10^{-5} \text{ day}^{-1}$, is taken from data from [20]. The average birth/death rate of the world for the year 2021 is about 16 per 1000 individuals per year. Hence, we convert the aforementioned quantity from "per 1000 individuals per year" to "per day" to get

$$16\frac{1}{1000 \text{ individuals} \cdot \text{year}} \mapsto 16 \cdot 10^{-3} \frac{1}{365 \text{ days}} = 4.38356 \cdot 10^{-5} \text{ day}^{-1} = \mu.$$

For the transmission rate of asymptomatic and symptomatic infectious individuals, we firstly assume the probability of an exposed individual passing to the compartments of asymptomatic and symptomatic individuals to be $\varpi_{E \to A} = \frac{1}{8}$ and $\varpi_{E \to I} = \frac{1}{3}$, respectively. From [12], we have that the average number of daily contacts of any person, regardless its epidemiological status, of age θ , $c(\theta)$, follows the graph as seen in Figure 3. To digitize the data of the contacts, we use WebPlotDigitizer 4.6 [23] to manually extract data points from Fig. 2 of [12] and then interpolated them using a third order polynomial interpolation scheme through Mathematica 13.1 [17] and the function Interpolation. Subsequently, from (1) we deduce that the transmission rate of asymptomatic and symptomatic infectious individuals are the functions presented in Figure 4.

The vaccination rate, $p = 10^{-3} \text{ day}^{-1}$, is taken from data from [20], during the summer of 2021 in the USA, when the Delta variant of SARS-CoV-2 was the dominant variant. During the end of summer, the percentage of fully vaccinated USA citizens was about 54% whereas in the beginning of summer it was around 45%. Hence, we estimate the vaccination from that three-month period to be $p = \frac{54\%-45\%}{90} \text{ day}^{-1} = 10^{-3} \text{ day}^{-1}$.

The vaccine effectiveness, $\epsilon = 0.7$, is estimated from data from [15]. In [15], the authors find that with the BNT162b2 vaccine, the effectiveness of two doses is 88.0% among those with the



Figure 3: Age density (in years) of the average number of daily contacts, c, taken from [12].

delta variant, whereas with the ChAdOx1 nCoV-19 vaccine, the respective effectiveness of two doses was 67.0%. Hence, we assume $\epsilon = 0.7$.

The vaccine-induced immunity rate, $\zeta = \frac{1}{14} \text{ day}^{-1}$, is estimated from data from [9]. The authors of [9] report that, after two weeks of the second dose of the ChAdOx1 nCoV-19 vaccine, the percentage of study participants with detectable neutralizing antibodies reached 98.1%.

The latent rate, k, is found by estimating that the latent and incubation period differ by 1 day. In [18], the authors examined data from 93 Delta transmission pairs and estimated the latent period by fitting the data to the Weibull distribution, which made the best fit. They found the mean latent period to be 3.9 days. In [30], the authors performed a systematic review and meta-analysis of 141 articles and found that the incubation periods of COVID-19 caused by the Alpha, Beta, Delta, and Omicron variants were 5.00, 4.50, 4.41, and 3.42 days, respectively. Hence, assuming that the latent and incubation period vary by 1 day, we have that $k = \frac{\chi}{1-\chi}$, and by substituting χ as found later in the present section, we have that

$$k(\theta) = \begin{cases} \frac{1}{4} \operatorname{day}^{-1}, & \theta < 30 \\ \frac{1}{4.8} \operatorname{day}^{-1}, & 30 \le \theta < 40 \\ \frac{1}{4.8} \operatorname{day}^{-1}, & 40 \le \theta < 50 \\ \frac{1}{5.5} \operatorname{day}^{-1}, & 50 \le \theta < 60 \\ \frac{1}{3.1} \operatorname{day}^{-1}, & 60 \le \theta < 70 \\ \frac{1}{6} \operatorname{day}^{-1}, & 70 \le \theta , \end{cases}$$
(6)

where θ is measured in years.

The proportion of the latent/exposed individuals becoming asymptomatic infectious, q, is taken from [25], where the authors estimated the asymptomatic proportion by age, by performing a systematic review and meta-analysis of 38 studies involving 14850 individuals. The curve they estimated can be seen in Figure 5. To digitize the data, we use the same procedure we used for the age density of daily contacts described earlier in the present section.

The proportion of the asymptomatic infectious individuals becoming recovered/removed without developing any symptoms, $\xi = 0.5$, is estimated from data from [16, 6]. In [16], the authors performed a systematic review and meta-analysis from 41 studies containing the rate of asymptomatic COVID-19 infection before May 20, 2020, aggregating 50155 patients, and found that nearly half of the patients with no symptoms at the time of their detection, would develop symptoms later. In [6], the authors performed a systematic review and meta-analysis from 130 studies and reported the percentage of persistently asymptomatic individuals being between 14 to 50%.

The incubation rate, χ , is estimated from data from [28]. The authors of [28] found that the incubation period varies with age, based on data from Singaporean hospitals between January 23,

 $-\beta_A - \beta_I$



Figure 4: Estimation of the age density (in years) of the asymptomatic and symptomatic transmission rate, assuming $\beta_A = \frac{c_A \cdot \varpi_E \to A}{N_0}$ and $\beta_I = \frac{c_I \cdot \varpi_E \to I}{N_0}$ according to (1), where $c_A = c = c_I$.

2020 and April 2, 2020. The authors divided the participants based on their age to six groups (<30, 30–39, 40–49, 50–59, 60–69 and 70<) and presented their results through a box plot. Hence, we assume that χ is a piecewise function with its domain intervals being the six aforementioned age groups, and with the function being constant on each interval and equal to one over the median of the respective age group. Therefore, we have that

$$\chi(\theta) = \begin{cases} \frac{1}{5} \operatorname{day}^{-1}, & \theta < 30\\ \frac{1}{5.8} \operatorname{day}^{-1}, & 30 \le \theta < 40\\ \frac{1}{5.8} \operatorname{day}^{-1}, & 40 \le \theta < 50\\ \frac{1}{6.5} \operatorname{day}^{-1}, & 50 \le \theta < 60\\ \frac{1}{4.1} \operatorname{day}^{-1}, & 60 \le \theta < 70\\ \frac{1}{7} \operatorname{day}^{-1}, & 70 \le \theta \end{cases}$$
(7)

where θ is measured in years.

The recovery rate of asymptomatic infectious individuals, $\gamma_A = \frac{1}{8} \text{ day}^{-1}$, and recovery rate of the symptomatic infectious individuals, $\gamma_I = \frac{1}{14} \text{ day}^{-1}$, is estimated from [7]. In [7], the authors performed a rapid scoping review up to April 1, 2020 and found that the median infectious period for asymptomatic cases was 6.5–9.5 days, whereas time from symptom onset to two negative RT-PCR tests ranged from 10.9 to 15.8 days. Hence, we assume that the recovery period of asymptomatic and symptomatic infectious individuals to be 8 and 14 days respectively.

4 Horizontal lockdowns vs age-based interventions

In this section, we investigate whether age-based interventions can offer a replacement to horizontal lockdowns for the case of SARS-CoV-2, following the framework presented in §2. In §4.1, we categorize the parameters into δ, β and γ , and pick our choice of strategic scales, \mathbf{r} ; both (β, γ) and \mathbf{r} serve for the definition of the strategies under investigation. Additionally, we distribute the total population of \mathscr{P} (4) into five cohorts, based on age, θ , of each individual. In §4.3, we define the graded strategy of horizontal lockdowns and the strategy of age-based restrictions. Finally, in §4.4, we compare the aforementioned strategies.



Figure 5: Percentage of asymptomatic COVID-19 infection, by age (in years), taken from [25].

4.1 Choice of general strategy

The independent variables that appear in $\mathscr{P}(4)$ are t and θ , thus

$$\boldsymbol{x} = (t, \theta)$$
.

In order to define the strategies under investigation, we need to categorize the parameters appeared in $\mathscr{P}(4)$ into (β_0, γ_0) and δ , and consequently choose an appropriate strategic scale as discussed in §2.1.

The parameters which affect the strategies under investigation are β_A, β_I and γ_I . Therefore, we have that

$$(\boldsymbol{\beta}_0, \gamma_0) = (\beta_A, \beta_I, \gamma_I)$$
,

whereas

$$\boldsymbol{\delta} = (\mu, p, \epsilon, \zeta, k, q, \xi, \chi, \gamma_A) ,$$

with the parameter values being as in Table 2.

We note that regardless of the aforementioned (see §3), seemingly important role of asymptomaticity for the spread of the disease, the performance of the means of detection, such as the rapid antigen tests (Ag-RDTs), for the case of asymptomatic infectious individuals still remains ambiguous [8, 22, 24, 27]. In the light of the above we prefer not to incorporate such means to our general strategy, hence we exclude γ_A from (β_0, γ_0). Moreover we note that (3) holds.

We are now ready to construct our choice of general strategy along with its strategic scales, following the next steps.

- 1. We consider an interval $\Lambda \subseteq \mathbb{R}_0^+$ such that $0 \in \Lambda$, to be the average lifespan of an individual of the population under study, hence $\theta \in \Lambda$. Of course, $\sup \Lambda < \infty$.
- 2. We discretise Λ by considering a respective partition $\Delta_{\Lambda} \coloneqq \{\delta_j\}_{j=0}^n$, for a fixed $n \in \mathbb{N}$, i.e.

$$0 = \delta_0 < \delta_1 < \dots < \delta_n = \sup \Lambda$$

and we define the subintervals

$$\Lambda_j \coloneqq [\delta_{j-1}, \delta_j), \ \forall j \in \{1, \dots, n\}.$$

3. We set

$$\Lambda_W \coloneqq \bigcup_{j \in W} \Lambda_j, \ \forall W \in \mathcal{P}(\{1, \dots, n\}),$$

where \mathcal{P} stands for the powerset, as well as we define

$$\rho_W(\cdot; a) \colon \Lambda \to [0, 1]$$

$$\theta \mapsto \rho_W(\theta; a) \coloneqq \begin{cases} 1 & \theta \notin \Lambda_W & \forall (W, a) \in \mathcal{P}(\{1, \dots, n\}) \times [0, 1) \\ a & \theta \in \Lambda_W \end{cases},$$

and

$$g_W(\cdot; b) \colon \Lambda \to [1, \infty]$$

$$\theta \mapsto g_W(\theta; b) \coloneqq \begin{cases} 1 & \theta \notin \Lambda_W & \forall (W, b) \in \mathcal{P}(\{1, \dots, n\}) \times [0, 1), \\ \frac{1}{b} & \theta \in \Lambda_W, \end{cases}$$

where the choice of $(a, b) \in [0, 1)^2$ is left to be explained.

We note that in the extreme cases of $W \in \{\emptyset, \{1, ..., n\}\}\$ we have $\Lambda_{\emptyset} = \emptyset$ and $\Lambda_{\{1,...,n\}} = \Lambda$, as well as

$$\left(\rho_{\varnothing}(\,\cdot\,;a),g_{\varnothing}(\,\cdot\,;b)\right)=\left(1,1\right)\;,\;\forall\left(a,b\right)\in\left[0,1\right)^{2}$$

and

$$(\rho_{\{1,\ldots,n\}}(\cdot;a),g_{\{1,\ldots,n\}}(\cdot;b)) = (a,\frac{1}{b}), \ \forall (a,b) \in [0,1)^2.$$

Hence, the above functions are independent of θ iff $W \in \{\emptyset, \{1, \dots, n\}\}$, as well as they are equal to 1 iff $W = \emptyset$.

4. We define the strategic scales to be

$$\boldsymbol{r}_{W_{\boldsymbol{\beta}},W_{\gamma}}(\cdot;a,b) \coloneqq \left(\rho_{W_{\boldsymbol{\beta}}}(\cdot;a),\rho_{W_{\boldsymbol{\beta}}}(\cdot;a),g_{W_{\gamma}}(\cdot;b)\right),$$

$$\forall \left(W_{\boldsymbol{\beta}},W_{\gamma},a,b\right) \in \left(\mathcal{P}(\{1,\ldots,n\})\right)^{2} \times [0,1)^{2}.$$

5. The general strategy of interest S has the form

$$S \coloneqq \left\{ (\boldsymbol{\beta}, \gamma) = \boldsymbol{r}_{W_{\boldsymbol{\beta}}, W_{\gamma}}(\cdot; a, b) \odot (\boldsymbol{\beta}_{0}, \gamma_{0}) \middle| (W_{\boldsymbol{\beta}}, W_{\gamma}, a, b) \in (\mathcal{P}(\{1, \dots, n\}))^{2} \times [0, 1)^{2} \right\}$$

Regarding $a \in [0,1]$, in the light of (1), the effect of every $\rho_{W_{\beta}}(\cdot, a)$ to β can be interpreted as having the average number of close contacts of an (asymptomatic or symptomatic) infectious individual belonging in $\Lambda_{W_{\beta}}$ reduced by 1 - a, i.e.

$$(c_A, c_I)|_{\Lambda_{W_{\boldsymbol{\theta}}}} \mapsto a \cdot (c_A, c_I)|_{\Lambda_{W_{\boldsymbol{\theta}}}}.$$

Regarding $b \in (0, 1]$, in the light of (2), the effect of every $g_{W_{\gamma}}(\cdot, b)$ to γ can be interpreted as having the average period an individual belonging in $\Lambda_{W_{\beta}}$ spends on compartment I before moving into compartment R reduced by 1 - b, i.e.

$$P_{I \to R}|_{\Lambda_{W_{\mathcal{B}}}} \mapsto b \cdot P_{I \to R}|_{\Lambda_{W_{\mathcal{B}}}}$$

4.2 Choice of distribution of the population into age cohorts

We now specify the distribution of the whole population into cohorts with respect to the age of each individual, hence with respect to its occupational and social activity.

We divide the population into five (5) cohorts, thus n = 5, as seen in Figure 6, where $\Lambda = [0,90)$ years and $\Delta_{\Lambda} = \{0,6,18,24,65,90\}$ years (both non-scaled). The 1st cohort is made of toddlers and preschoolers, the 2nd is made of school students, the 3rd is primarily made of university students, the 4th is primarily made of the working class, and the 5th is primarily made of pensioners.

As an example of the strategic scale within the context of the cohorts presented in Figure 6, the reduction of the number of contacts of the 1st and 3rd cohort by 80% and not performing tests on any cohort is modeled by the strategic scale

$$\boldsymbol{r}_{\{1,3\},\varnothing}\left(\cdot;\frac{1}{5},\cdot\right) = \left(\rho_{\{1,3\}}\left(\cdot;\frac{1}{5}\right),g_{\varnothing}(\cdot;\cdot)\right),$$

Distribution of popullation into cohorts



Lifespan partition

Figure 6: The partition of the non-scaled lifespan and the respective distribution of the whole population into cohorts. The partition was made by taking into account the social profile connecting individuals in each cohort, such as going to school, working, or being pensioners.

where

$$\rho_{\{1,3\}}\left(\theta;\frac{1}{5}\right) = \begin{cases} \frac{1}{5}, & \text{if } \theta \in \Lambda_1\\ 1, & \text{if } \theta \in \Lambda_2\\ \frac{1}{5}, & \text{if } \theta \in \Lambda_3\\ 1, & \text{if } \theta \in \Lambda_4\\ 1, & \text{if } \theta \in \Lambda_5 \end{cases},$$

and

4.3 Choice of substrategies of general strategy

In this section, we define the two strategies under investigation. Furthermore, we utilize the strategic scale introduced in $\S4.1$ to model each strategy.

 $g_{\emptyset}(\theta; \cdot) = 1, \forall \theta \in \Lambda$.

4.3.1 Horizontal lockdowns substrategy

It is straightforward to check that the largest horizontal with respect to age substrategy of S is

$$\left\{ \left(\boldsymbol{\beta}, \boldsymbol{\gamma}\right) = \boldsymbol{r}_{W_{\boldsymbol{\beta}}, W_{\boldsymbol{\gamma}}}(\cdot; a, b) \odot \left(\boldsymbol{\beta}_{0}, \boldsymbol{\gamma}_{0}\right) \middle| \left(W_{\boldsymbol{\beta}}, W_{\boldsymbol{\gamma}}, a, b\right) \in \left(\mathcal{P}(\{1, \dots, 5\})\right)^{2} \times [0, 1)^{2} \text{ such that } \text{ such that } \left(W_{\boldsymbol{\beta}}, W_{\boldsymbol{\gamma}}\right) \in \left\{\varnothing, \{1, \dots, 5\}\right\}^{2} \right\},$$

which implies that every substrategy of the above strategy is horizontal with respect to age.

Thus, a choice of horizontal lockdowns substrategy can be made by considering $S_1 \subseteq S$ as

$$S_1 = \left\{ (\boldsymbol{\beta}, \gamma) = \boldsymbol{r}_{\{1, \dots, 5\}, \varnothing}(\,\cdot\,; a, \,\cdot\,) \odot (\boldsymbol{\beta}_0, \gamma_0) \,\middle|\, a \in [0, 1) \right\}$$

The austerity (that is the amount of contact reduction for every individual) can be varied but uniformly, i.e.

$$a \in [0,1)$$
 and $W_{\beta} = \{1,\ldots,n\}$,

respectively, in order to capture different scenarios. We also assume that no tests are performed in any of the five cohorts, i.e.

$$W_{\gamma} = \emptyset$$
.

Now, S_1 is gradable, since from (5) we get that

$$\mathcal{R}_0(\boldsymbol{\beta}, \gamma, \boldsymbol{\delta}) = a \cdot \mathcal{R}_0(\boldsymbol{\beta}_0, \gamma_0, \boldsymbol{\delta}), \forall (\boldsymbol{\beta}, \gamma) \in S_1$$

In particular, \mathcal{R}_0 is strictly increasing with respect to a, as it is depicted in Table 3.

a	Contact reduction	\mathcal{R}_0
0	100%	0
0.1	90%	0.285
0.2	80%	0.571
0.3	70%	0.856
0.4	60%	1.141
0.5	50%	1.427
0.6	40%	1.712
0.7	30%	1.998
0.8	20%	2.283
0.9	10%	2.569

Table 3: The value of \mathcal{R}_0 decreases linearly as the austerity of the stay-at-home restrictions increases, i.e. as *a* decreases.

To get a better understanding of how $a \in [0, 1)$ translates into the real life austerity of a stayat-home restriction policy, we firstly notice that when $W_{\beta} = \emptyset$ (or else $a \to 1^-$), we have that no stay-at-home restrictions are in effect. In that case, our model predicts an \mathcal{R}_0 of 2.854 (or else $\mathcal{R}_0 \to 2.854^-$), which is in line with various systematic reviews found in scientific literature, such as 2.87 (95% CI: 2.39–3.44) in [3] and 2.69 (95% CI: 2.40–2.98) in [1], which solidifies the validity of our model in predicting the \mathcal{R}_0 of SARS-CoV-2 pandemic. Furthermore, we see that the tight lockdown Italy enforced in early 2020 resulted in an 82% reduction in mobility [29], which would correspond to a being approximately equal to 0.2. During the same time frame in Germany, the authors of [26], report about a 50% drop in the average number of contacts, which corresponds to a = 0.5. Finally, in [31] the authors show that even a 20% reduction in mobility proved a good way of delaying the spread of the infection, which would correspond to a being approximately equal to 0.8.

Based on the aforementioned cases, we construct three different scenarios based on the austerity of the mobility restrictions:

- the low austerity scenario, \mathcal{L} , where the average number of contacts is reduced by 20% and $\mathcal{R}_{0\mathcal{L}} = 2.283$,
- the medium austerity scenario, \mathcal{M} , where the average number of contacts is reduced by 50% and $\mathcal{R}_{0\mathcal{M}} = 1.427$ and
- the high austerity scenario, \mathcal{H} , where the average number of contacts is reduced by 80% and $\mathcal{R}_{0\mathcal{H}} = 0.571$.

The above scenarios constitute a gradation G of S_1 with

$$G_1 = \mathcal{R}_{0\mathcal{H}}, \ G_2 = \mathcal{R}_{0\mathcal{M}} \text{ and } G_3 = \mathcal{R}_{0\mathcal{L}}.$$

Such a gradation is summarized in Table 4.

4.3.2 Aged-based substrategies

The largest aged-based substrategy of $S, S_2 \subseteq S$, is

Gradation	Austerity level	Contact reduction	\mathcal{R}_0
G_1	High (\mathcal{H})	80%	0.571
G_2	Medium (\mathcal{M})	50%	1.427
G_3	Low (\mathcal{L})	20%	2.283

Table 4: Summary of the three horizontal lockdowns austerity scenarios, Low (\mathcal{L}), Medium (\mathcal{M}) and High (\mathcal{H}), which constitute a gradation of S_1 .

$$S_{2} = \left\{ (\boldsymbol{\beta}, \boldsymbol{\gamma}) = \boldsymbol{r}_{W_{\boldsymbol{\beta}}, W_{\boldsymbol{\gamma}}}(\cdot; a, b) \odot (\boldsymbol{\beta}_{0}, \boldsymbol{\gamma}_{0}) \middle| (W_{\boldsymbol{\beta}}, W_{\boldsymbol{\gamma}}, a, b) \in (\mathcal{P}(\{1, \dots, 5\}))^{2} \times [0, 1)^{2} \text{ such that} \\ \text{ such that } (W_{\boldsymbol{\beta}}, W_{\boldsymbol{\gamma}}) \notin \{\emptyset, \{1, \dots, 5\}\}^{2} \right\},$$

which implies that every substrategy of S_2 is aged-based hence it can potentially be compared to the graded S_1 . For our simulations we choose the substrategies $\{S_{2_i}\}_{i=1}^{16}$ of S_2 summarized in Table 5 for the comparison to S_1 .

i	Age-based interventions	W_{β}	W_{γ}
1	Contact reduction: 1st, 2nd, 3rd cohorts Testing: 4th, 5th cohorts	$\{1, 2, 3\}$	$\{4, 5\}$
2	Contact reduction: 4th, 5th cohorts Testing: 1st, 2nd, 3rd cohorts	$\{4, 5\}$	$\{1, 2, 3\}$
3	Contact reduction: 1st cohort Testing: 4th, 5th cohorts	$\{1\}$	$\{4, 5\}$
4	Contact reduction: 4th, 5th cohorts Testing: 1st cohort	$\{4, 5\}$	{1}
5	Contact reduction: 2nd cohort Testing: 4th, 5th cohorts	$\{2\}$	$\{4, 5\}$
6	Contact reduction: 4th, 5th cohorts Testing: 2nd cohort	$\{4, 5\}$	$\{2\}$
7	Contact reduction: 3rd cohort Testing: 4th, 5th cohorts	$\{3\}$	$\{4, 5\}$
8	Contact reduction: 4th, 5th cohorts Testing: 3rd cohort	$\{4, 5\}$	$\{3\}$
9	Contact reduction: 1st cohort Testing: 2nd cohort	{1}	$\{2\}$
10	Contact reduction: 2nd cohort Testing: 1st cohort	$\{2\}$	{1}
11	Contact reduction: 4th cohort Testing: 5th cohort	$\{4\}$	$\{5\}$
12	Contact reduction: 5th cohort Testing: 4th cohort	$\{5\}$	<i>{</i> 4 <i>}</i>
13	Contact reduction: 2nd cohort Testing: 4th cohort	$\{2\}$	<i>{</i> 4 <i>}</i>
14	Contact reduction: 4th cohort Testing: 2nd cohort	$\{4\}$	$\{2\}$
15	Contact reduction: 2nd cohort Testing: 5th cohort	$\{2\}$	$\{5\}$
16	Contact reduction: 5th cohort Testing: 2nd cohort	$\{5\}$	$\{2\}$

Table 5: The age-based substrategies $\{S_{2_i}\}_{i=1}^{16}$ of S_2 which are chosen for the comparison to S_1 .

4.4 Simulations and results

Here we employ the scheme introduced in §2.2 for the comparison between S_1 of §4.3.1 and $\{S_{2_i}\}_{i=1}^{16}$ of §4.3.2. The simulations were performed using Mathematica 13.1 [17]. In the end of this section we summarize its results with the comparison matrix.



(a) \mathcal{R}_0 when social distancing is enforced on the 1st, 2nd and 3rd cohort and testing is enforced on the 5th and 6th cohort $(W_\beta = \{1, 2, 3\} \text{ and } W_\gamma = \{4, 5\}).$

(b) \mathcal{R}_0 when social distancing is enforced on the 5th and 6th cohort and testing is enforced on the 1st, 2nd and 3rd cohort $(W_\beta = \{5, 6\} \text{ and } W_\gamma = \{1, 2, 3\}).$

Figure 7: Two density plots of the grouping of the three younger cohorts and the two older cohorts together. In both cases all three of our horizontal lockdown scenarios \mathcal{L}, \mathcal{M} and \mathcal{H} , can be replaced by enforcing a wide range of austerity level restrictions to the different cohort groupings.

4.4.1 Social overview of the results, S_2 vs S_1

Throughout our simulations we let the (a, b) of each strategic scale to take values in the 2D interval $[0, 1)^2$ and illustrate the results in density plots, where in the *x*-axis and *y*-axis we have $a \cdot 100\%$ and $b \cdot P_{I \to R} = \frac{b}{\gamma_I} = b \cdot 14$ days, respectively.

 $S_{2_{1,2}}$ vs S_1 : We begin by examining whether restrictions on the younger or the older cohorts play a more important role in reducing \mathcal{R}_0 . In Figure 7a, we see that in order to achieve the same \mathcal{R}_0 as the scenario \mathcal{H} , the contact reduction of the first three cohorts needs to be at least 75% and the individuals of the last two cohorts need to be detected and removed at least before the twelfth day. Additionally, since the absolute value of the gradient of the contour lines is high, the younger cohorts influence the dynamics of \mathcal{R}_0 more when compared to the older cohorts. In Figure 7b, we see that the scenario \mathcal{H} , can be replaced by finding and removing from the community the people belonging in the first three cohorts at around the third day from symptom onset, whereas the contact reduction of the older age cohorts is almost irrelevant. Furthermore, since the gradient of the contour lines is almost zero, the younger cohorts play a far greater role in reducing \mathcal{R}_0 when compared to the older cohorts, especially the more austere the restrictions are. Overall, Figure 7 shows us that the younger cohorts are more influential in the dynamics of \mathcal{R}_0 , both when they are faced with social distancing restrictions, as well as mandatory testing.

It is now clear that the younger cohorts play a far more important role in the dynamics of \mathcal{R}_0 . We subsequently examine whether similar results as those presented in Figure 7 can be achieved, by restricting just one of the three younger cohorts instead of all three of them together.

 $S_{2_{3,4}}$ vs S_1 : Figure 8 illustrates restrictions on the 1st and the 4th - 5th cohorts. When social distancing on the 1st cohort and testing on the 4th and 5th cohorts are enforced, scenario \mathcal{M}



(a) \mathcal{R}_0 when social distancing is enforced on the 1st cohort and testing is enforced on the 5th and 6th cohort $(W_\beta = \{1\} \text{ and } W_\gamma = \{4, 5\}).$

(b) \mathcal{R}_0 when social distancing is enforced on the 5th and 6th cohort and testing is enforced on the 1st cohort $(W_{\beta} = \{5, 6\} \text{ and } W_{\gamma} = \{1\}).$

Figure 8: Two density plots of the 1st cohort and the grouping of the two older cohorts together. When social distancing is enforced on the 1st cohort scenario \mathcal{H} can only be achieved when the most austere restriction are enforced. On the other hand, when the symptomatic individuals of the 1st cohort are the ones getting tested all three of our horizontal lockdown scenarios \mathcal{L}, \mathcal{M} and \mathcal{H} , can be replaced by enforcing a wide range of austerity level restrictions to the 1st cohort and the grouping of the 4th and 5th cohort. The detection-and-removal day of asymptomatic individuals needs to be one day faster when compared to the simulation illustrated in Figure 8b, for the same results as scenario \mathcal{H} to apply.

can only be achieved with the strongest possible restrictions on the aforementioned cohorts, as we can see in Figure 8a. When the restrictions on the cohorts are reversed, Figure 8b shows that scenario \mathcal{H} can be achieved if the day that the symptomatic infectious individuals are detected and removed from the community is around the second day, with the contact reduction of the 4th and 5th cohort being almost irrelevant just like the case described by Figure 7. There is, however, a one day difference in the required detection day of asymptomatic individuals between the scenarios presented in Figure 7b and Figure 8b for them to have the same effect on R_0 , as scenario \mathcal{H} . In other words, the procedure of detection and removal of asymptomatic individuals from the community needs to be one day faster when only the 1st cohort is getting tested when compared to the grouping of the 1st, 2nd and 3rd cohorts, for them to have the same results on \mathcal{R}_0 as scenario \mathcal{H} .

 $S_{2_{5,6}}$ vs S_1 : Next, we examine the importance of the 2nd cohort to the dynamics of \mathcal{R}_0 , with the results being shown in Figure 9. Contrary to the simulation of Figure 8a, when social distance is enforced on the 2nd cohort, the results of scenario \mathcal{M} can be achieved with far less strict policies. In particular, as shown in Figure 9a, for scenario \mathcal{M} to be achieved, the contacts of the 2nd cohort need to be reduced by at least 80% and the symptomatic individuals of the 4th and 5th cohorts need to be detected and removed from the community at least before around the fifth day. When the 2nd cohort is the one being tested, Figure 9b shows that scenario \mathcal{M} can be achieved by removing symptomatic individuals from the community at around the fifth day, with the reduction in the average number of contacts of the 4th and 5th cohorts being almost irrelevant, much like the simulations illustrated in Figure 7b and Figure 8b. Additionally, none of the simulations of Figure 9 can act as a replacement measure to scenario \mathcal{H} .

 $S_{2_{7,8}}$ vs S_1 : Subsequently, we examine the contribution of the 3rd cohort to the dynamics of \mathcal{R} , with Figure 10 illustrating the results. As we can see from Figure 10, the 3rd cohort in combination with the grouping of the 4th and 5th cohort, seems to influence the reduction of \mathcal{R}_0 far less when



(a) \mathcal{R}_0 when social distancing is enforced on the 2nd cohort and testing is enforced on the 5th and 6th cohort $(W_\beta = \{2\} \text{ and } W_\gamma = \{4, 5\}).$

(b) \mathcal{R}_0 when social distancing is enforced on the 5th and 6th cohort and testing is enforced on the 2nd cohort $(W_{\beta} = \{5, 6\} \text{ and } W_{\gamma} = \{2\}).$

Figure 9: Two density plots of the 2nd cohort and the grouping of the two older cohorts together. When social distancing is enforced on the 2nd cohort scenario \mathcal{H} can only be achieved with laxer restriction compared to the respective restrictions on the 1st cohort. Neither of pictured simulations are able to offer a replacement to scenario \mathcal{H} . Much like the simulations of Figure 7b and Figure 8b, for the scenario \mathcal{M} to be achieved the testing of the younger cohorts dominates the dynamics of \mathcal{R}_0 , with the dynamics of the older cohorts being almost irrelevant.



(a) \mathcal{R}_0 when social distancing is enforced on the 3rd cohort and testing is enforced on the 5th and 6th cohort $(W_\beta = \{3\} \text{ and } W_\gamma = \{4, 5\}).$

(b) \mathcal{R}_0 when social distancing is enforced on the 5th and 6th cohort and testing is enforced on the 3rd cohort $(W_{\beta} = \{5, 6\} \text{ and } W_{\gamma} = \{3\}).$

Figure 10: Two density plots of the 3rd cohort and the grouping of the two older cohorts together. Neither of the simulations is able to offer a replacement to scenario \mathcal{M} and scenario \mathcal{H} . The influence of the 3rd cohort to the dynamics of \mathcal{R} is far weaker when compared to the influence of the 1st and 2nd cohort, as can be seen from Figure 8 and Figure 9.

compared to the younger cohorts. The only horizontal lockdown scenario that can be replaced with this combination of age-based interventions is scenario \mathcal{L} . Additionally, even though the 1st and 2nd cohort dominated the dynamics of \mathcal{R}_0 when the symptomatic individuals of those cohorts were getting tested, that is not the case with the 3rd cohort, as can be seen from Figure 10b. The same holds for the case when social distancing is enforced on the 3rd cohort, since the absolute value of the gradient of the contour lines of Figure 10a is about 2. Hence, out of the three younger cohorts, the 3rd one has the weakest influence on the dynamics of \mathcal{R}_0 .



(a) \mathcal{R}_0 when social distancing is enforced on the 1st cohort and testing is enforced on the 2nd cohort $(W_\beta = \{1\} \text{ and } W_\gamma = \{2\}).$

(b) \mathcal{R}_0 when social distancing is enforced on the 2nd cohort and testing is enforced on the 1st cohort ($W_{\beta} = \{2\}$ and $W_{\gamma} = \{1\}$).

Figure 11: Two density plots of the influence of the 1st cohort and 2nd cohort on the dynamics of \mathcal{R}_0 . In both cases all three of our horizontal lockdown scenarios \mathcal{L}, \mathcal{M} and \mathcal{H} , can be replaced by enforcing a wide range of austerity level restrictions to the 1st cohort and 2nd cohort. The 1st and 2nd cohorts are the most important cohorts at effecting the dynamics of \mathcal{R}_0 , since they influence the dynamics of \mathcal{R}_0 comparably to the influence of the combination of all of our cohorts as seen in Figure 8.

 $S_{29,10}$ vs S_1 : The 1st and 2nd cohort seem to be the two cohorts that influence the dynamics of \mathcal{R}_0 the most. Hence, we quantify the results of targeting only the aforementioned cohorts in Figure 11. As we can see from Figure 11, all three horizontal lockdown scenarios \mathcal{L}, \mathcal{M} and \mathcal{H} can be replaced with a combination of measures targeted at the 1st and 2nd cohort. This particular combination of age-based measures has similar dynamics as the scenarios presented in Figure 7 which combine all of our cohorts, and Figure 8b which includes measures regarding thee different cohorts. The vital role of the 1st and 2nd cohort is now undeniable. In Figure 11a we see that scenario \mathcal{H} can be replaced with the contacts of the 1st cohort being reduced by at least 50% and the infectious individuals of the 2nd cohort being found and removed from the community at least before the 4th day. When the restrictions are reversed, scenario \mathcal{H} can be replaced when the symptomatic individuals of the 1st cohort are detected and removed from the community at around the second day after symptom onset, as can be seen in Figure 11b, with minimal contribution from the 2nd cohort.

 $S_{2_{11,12}}$ vs S_1 : Up until now we examined the two older cohorts, namely the 4th and 5th cohort, grouping them together as a single cohort. In an attempt to study the result of the interactions of the aforementioned cohorts individually, we present Figure 12. As expected from the inability of the grouping of the 4th and 5th cohort to dominate the dynamics of our previous simulations, the simulations of Figure 12 offer a poor reduction of \mathcal{R}_0 . Neither in Figure 12a nor Figure 12b can horizontal lockdown scenarios \mathcal{H} and \mathcal{M} be replaced by a combination of measures in the 4th and 5th cohort. Only scenario \mathcal{L} can be replaced, and that is with austere restrictions on the 5th



(a) \mathcal{R}_0 when social distancing is enforced on the 4th cohort and testing is enforced on the 5th cohort ($W_{\beta} = \{4\}$ and $W_{\gamma} = \{5\}$).

(b) \mathcal{R}_0 when social distancing is enforced on the 5th cohort and testing is enforced on the 4th cohort ($W_{\beta} = \{5\}$ and $W_{\gamma} = \{4\}$).

Figure 12: Two density plots of the influence of the 4st cohort and 5nd cohort on the dynamics of \mathcal{R}_0 . Neither case was able to offer a replacement to horizontal lockdown scenario \mathcal{M} and scenario \mathcal{H} . Restrictions on the combination of the 4th cohort and the 5th cohort result in the poorest reduction in \mathcal{R}_0 when compared to the remaining of our simulations. When measures are imposed to the 4th and 5th cohort, the restrictions on the 4th cohort dominate the dynamics of \mathcal{R}_0 .

cohort. In particular, scenario \mathcal{L} can be achieved either when the reduction of the average amount of contacts of the 4th cohort is 80% or when the symptomatic individuals of the 4th cohort are detected and removed from the community at around 2.5 days after symptom onset. Finally, there is a clear domination of the 4th cohort in this particular combination of age-based measures, with the measures enforced on the 5th cohort being irrelevant.

 $S_{2_{13,14,15,16}}$ vs S_1 : Lastly, we present the final combination of measures in Figure 13. This final set of restrictions acts as a viable proposal to a real life situation with the economical impact of the measures in mind, since it targets the 2nd cohort, i.e. school students, whose contact reduction, or in other words school closures, would minimally affect the economy. Additionally, Figure 13, allows us to examine the difference between the grouping of the two older cohorts and their individual contribution to \mathcal{R}_0 , in combination to another, younger, cohort. As can be seen in Figure 13a, for horizontal lockdown scenario \mathcal{M} to be replaced, the contact reduction of the 2nd cohort needs to be at least 85% and the infectious individuals of the 4th cohort need to be found and removed from the community at least before the fourth day after symptom onset. Compared to Figure 9a, there is a 5% increase in the required contact reduction for scenario \mathcal{M} to be replaced, as well as about a 1.5 day decrease between the required detection-and-removal day for the symptomatic individuals of the 4th cohort and the grouping of the 4th and 5th cohort. On the other hand, Figure 13b is identical to Figure 9b, meaning that the 5th cohort's contribution to the dynamics of \mathcal{R}_0 is minimal. This is further proved in Figure 13c and Figure 13d, where we see that the 2nd cohort dominates the dynamics of the simulation. In particular, when the contact reduction of the 2nd cohort is 50%, scenario \mathcal{L} can be replaced, whereas when the infectious individuals of the 2nd cohort are removed from the community at around the 4th day, scenario \mathcal{M} can be replaced.





(a) \mathcal{R}_0 when social distancing is enforced on the 2nd cohort and testing is enforced on the 4th cohort ($W_{\beta} = \{2\}$ and $W_{\gamma} = \{4\}$).

(b) \mathcal{R}_0 when social distancing is enforced on the 4th cohort and testing is enforced on the 2nd cohort ($W_{\beta} = \{4\}$ and $W_{\gamma} = \{2\}$).



(c) \mathcal{R}_0 when social distancing is enforced on the 2nd cohort and testing is enforced on the 5th cohort ($W_{\beta} = \{2\}$ and $W_{\gamma} = \{5\}$).

(d) \mathcal{R}_0 when social distancing is enforced on the 5th cohort and testing is enforced on the 2nd cohort ($W_{\beta} = \{5\}$ and $W_{\gamma} = \{2\}$).

Figure 13: Four density plots of the influence of the interactions of the 2nd and 4th cohort, as well as the 2nd and 5nd cohort on the dynamics of \mathcal{R}_0 . None of the simulations was able to offer a replacement to horizontal lockdown scenario \mathcal{H} . The 5th cohort's contribution to the dynamics of \mathcal{R}_0 is insignificant, since it's removal from the measure-targeted cohorts, minimally affects the dynamics of \mathcal{R}_0 , as can be seen when Figure 13a, Figure 13b and Figure 9 are compared. Additionally, the 2nd cohort completely dominates the dynamics of \mathcal{R}_0 , when the 5th cohort is included in the simulations as can be seen from Figure 13c and Figure 13d.

4.4.2 Epidemiological overview of the results, S_1 vs S_2

Throughout our simulations we let (a, b) of each gradation to take values in the 2D interval $[0, 1)^2$ and illustrate the results in contour plots, where in the *x*-axis and *y*-axis we have $a \cdot 100\%$ and $b \cdot P_{I \to R} = \frac{b}{\gamma_I} = b \cdot 14$ days, respectively.

 \mathcal{H} vs S_2 : We begin by examining how many substrategies of $\{S_{2_i}\}_{i=1}^{16}$ can be considered as an alternative to scenario \mathcal{H} . As can be seen from Figure 14c, five substrategies of $\{S_{2_i}\}_{i=1}^{16}$ admit the same \mathcal{R}_0 as the respective one of scenario \mathcal{H} . Therefore, the epidemiological coverage of scenario \mathcal{H} by the substrategies $\{S_{2_i}\}_{i=1}^{16}$ is 31.25%. We highlight the fact that every one of the five substrategies that can replace scenario \mathcal{M} , regards restrictions on the 1st cohort.

 \mathcal{M} vs S_2 : Next, we examine how many substrategies of $\{S_{2i}\}_{i=1}^{16}$ can be considered as an alternative to scenario \mathcal{M} . As can be seen from Figure 14b, eleven substrategies of $\{S_{2i}\}_{i=1}^{16}$ admit the same \mathcal{R}_0 as the respective one of scenario \mathcal{M} . Therefore, the epidemiological coverage of scenario \mathcal{M} by the substrategies $\{S_{2i}\}_{i=1}^{16}$ is 68.75%. We highlight the fact that every one of the eleven substrategies that can replace scenario \mathcal{M} , regards restrictions on the 1st and 2nd cohort.

 \mathcal{L} vs S_2 : Finally, we examine how many substrategies of $\{S_{2_i}\}_{i=1}^{16}$ can be considered as an alternative to scenario \mathcal{L} . As can be seen from Figure 14a, all substrategies $\{S_{2_i}\}_{i=1}^{16}$ admit the same \mathcal{R}_0 as the respective one of scenario \mathcal{L} . Therefore, the epidemiological coverage of scenario \mathcal{L} by the substrategies $\{S_{2_i}\}_{i=1}^{16}$ is 100%.

A summary of the results of §4 can be seen in Table 6.

5 Conclusions and discussion

In this paper, we introduced a scheme for the comparison of certain interventions for the restriction of an epidemiological phenomenon. This scheme incorporates some novel notions such as "strategy" and "substrategy", "gradable strategy" and its "gradation", "comparison matrix", as well as "epidemiological" and "social coverage", which are also introduced here. Then, we utilized the aforementioned scheme and the age-based epidemiological compartment problem studied in [4] to compare horizontal lockdowns policy with various age-based interventions.

In particular, we distributed the total population into five cohorts, based on the age of each individual (in ascending order) and we defined the graded strategy of horizontal lockdowns, considering three horizontal lockdowns austerity scenarios, Low (\mathcal{L}), Medium (\mathcal{M}) and High (\mathcal{H}). We also defined the strategy of age-based restrictions consisting of 16 substrategies. In general, our results suggest that these two strategies are comparable mainly at low or medium level of austerity. Precisely, throughout our simulations with data from the literature, we deduced that strategies that include the 1st and 2nd cohort had the best epidemiological coverage. Moreover, all substrategies admitted the same \mathcal{R}_0 as the respective one of scenario \mathcal{L} , meaning that the social coverage of \mathcal{L} by the substrategies is 100%, while the social coverage of scenarios \mathcal{M} and \mathcal{H} by the substrategies is 68.75% and 31.25%, respectively.

In terms of the proposed scheme, generalizations may include the notion of strategy, hence the comparison process itself.



(a) Loci of $\left(a \cdot 100\%, \frac{b}{\gamma_I}\right)$ of the respective substrategies of the family $\left\{S_{2_i}\right\}_{i=1}^{16}$ such that their \mathcal{R}_0 equals 0.571, i.e the \mathcal{R}_0 of scenario \mathcal{H} .



(b) Loci of $\left(a \cdot 100\%, \frac{b}{\gamma_I}\right)$ of the respective substrategies of the family $\left\{S_{2_i}\right\}_{i=1}^{16}$ such that their \mathcal{R}_0 equals 1.427, i.e the \mathcal{R}_0 of scenario \mathcal{M} .



(c) Loci of $\left(a \cdot 100\%, \frac{b}{\gamma_I}\right)$ of the respective substrategies of the family $\left\{S_{2_i}\right\}_{i=1}^{16}$ such that their \mathcal{R}_0 equals 2.283, i.e the \mathcal{R}_0 of scenario \mathcal{L} .

Figure 14: Three contour plots illustrating the epidemiological coverage of each substrategy $(\mathcal{L}, \mathcal{M} \text{ and } \mathcal{H})$ of horizontal lockdown strategy. The denser the plot is, the higher the epidemiological coverage.

Horizontal lockdowns Age-based restrictions	$\mathrm{High}\ (\mathcal{H})$	Medium (\mathcal{M})	$\mathrm{Low}\;(\mathcal{L})$	Epidemiological coverage
Contact reduction: 1st, 2nd, 3rd cohorts Testing: 4th, 5th cohorts	80.6% and 3.06 days 88.% and 6.4 days 95.4% and 9.82 days	42.3% and 3.71 days 50.5% and 7.43 days 58.9% and 11.4 days	4.55% and 11.7 days $12.5%$ and 8.19 days $19.9%$ and 4.59 days	100%
Contact reduction: 4th, 5th cohorts Testing: 1st, 2nd, 3rd cohorts	21.1% and 2.92 days 50.4% and 2.9 days 81.7% and 2.89 days	18.4% and 7.43 days 50.% and 7.68 days 78.9% and 7.96 days	17.8% and 13.4 days 42.9% and 12.4 days 67.9% and 11.7 days	100%
Contact reduction: 1st cohort Testing: 4th, 5th cohorts	×	98.8% and 1 day 99.1% and 1.05 days 99.5% and 1.12 days	10.7% and 11.9 days 34.% and 8.18 days 55.4% and 4.31 days	66.66%
Contact reduction: 4th, 5th cohorts Testing: 1st cohort	18.8% and 2.05 days 48.6% and 2.1 days 80.2% and 2.18 days	17.9% and 4.72 days $50.%$ and 5.11 days $79.1%$ and 5.57 days	17.5% and 9.41 days $43.2%$ and 10.8 days $68.6%$ and 12.7 days	100%
Contact reduction: 2nd cohort Testing: 4th, 5th cohorts	×	82.7% and 1.86 days 89.6% and 3.29 days 96.4% and 4.71 days	10.6% and 11.7 days 27.7% and 8.32 days 43.% and 4.73 days	100%
Contact reduction: 4th, 5th cohorts Testing: 2nd cohort	×	21.2% and 4.65 days 50% and 4.4 days 82.1% and 4.22 days	17.9% and 9.82 days 43.1% and 11 days 67.9% and 12.7 days	66.66%
Contact reduction: 3rd cohort Testing: 4th, 5th cohorts	×	×	19.5% and 8.89 days 50.9% and 6.45 days 82.1% and 4.04 days	33.33%
Contact reduction: 4th, 5th cohorts Testing: 3rd cohort	×	×	23.1% and 4.93 days 54.7% and 6.89 days 75.% and 11 days	33.33%
Contact reduction: 1st cohort Testing: 2nd cohort	58.1% and 3.29 days 74.% and 2.52 days 89.5% and 1.59 days	19.6% and 4.95 days 50% and 6.27 days 80.2% and 7.71 days	14% and 10.1 days 34.3% and 11.4 days 54.5% and 13 days	100%
Contact reduction: 2nd cohort Testing: 1st cohort	20.2% and 2.41 days 51.2% and 2.23 days 82.% and 2.09 days	20.1% and 4.88 days 52.5% and 5.64 days 82.1% and 6.77 days	10.7% and 9.36 days 28.2% and 10.8 days 44.4% and 12.6 days	100%
Contact reduction: 4th cohort Testing: 5th cohort	×	×	81.1% and 3.46 days 81.8% and 7.5 days 82.3% and 11.2 days	33.33%
Contact reduction: 5th cohort Testing: 4th cohort	×	×	18.9% and 2.53 days 48.9% and 2.53 days 79.7% and 2.53 days	33.33%
Contact reduction: 2nd cohort Testing: 4th cohort	×	87.5% and 1.62 days 92.% and 2.51 days 96.4% and 3.41 days	11.5% and 2.76 days 33.8% and 7.25 days 52.8% and 11.3 days	66.66%
Contact reduction: 4th cohort Testing: 2nd cohort	×	21.4% and 4.63 days 51.1% and 4.4 days 81.5% and 4.22 days	17.9% and 9.81 days 44.5% and 11 days 68.1% and 12.6 days	66.66%
Contact reduction: 2nd cohort Testing: 5th cohort	×	×	52.1% and 3.52 days 52.5% and 7.36 days 52.9% and 11.3 days	33.33%
Contact reduction: 5th cohort Testing: 2nd cohort	×	20.4% and 4.12 days $51.%$ and 4.12 days $81.2%$ and 4.12 days	18.9% and 9.22 days 49.1% and 9.25 days 79.3% and 9.27 days	66.66%
Social coverage	31.25%	68.75%	100%	66.66%

Table 6: Horizontal lockdowns vs age-based restrictions. The total coverage of horizontal lockdowns from age-based restrictions is 66.66%. Additionally, the table is populated with representative values of $\left(a \cdot 100\%, \frac{b}{\gamma_I}\right)$ of the strategic scale that each age-based strategy needs to have in order for the strategy to have the same \mathcal{R}_0 as each of the three horizontal lockdown scenarios.

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