

Erl. H. Topponja Tur Matematis

Fotw M n -diagramm utsoffigotya cu \mathbb{R}^{n+1} , ofagn.

Dewarfa

Vtobetwfe oti n M ena eftutefan n -utepotefan cu \mathbb{R}^{n+1} , kou vtnoetwfe oti ena exaxistku,

$M \subset B_{p_0}(x_0) = \{x \in \mathbb{R}^{n+1} \mid |x - x_0| \leq p_0\}$. Tote ioxues

$0 < r < s$

$$(70) \quad \frac{d}{dr} \left(\frac{1}{r^n} \int_{M \cap B_r(x)} f \right) = \frac{d}{dr} \int_{M \cap B_r(x)} f \frac{|(x - x_0)|^{\frac{n}{2}}}{|x - x_0|^{n+2}}$$

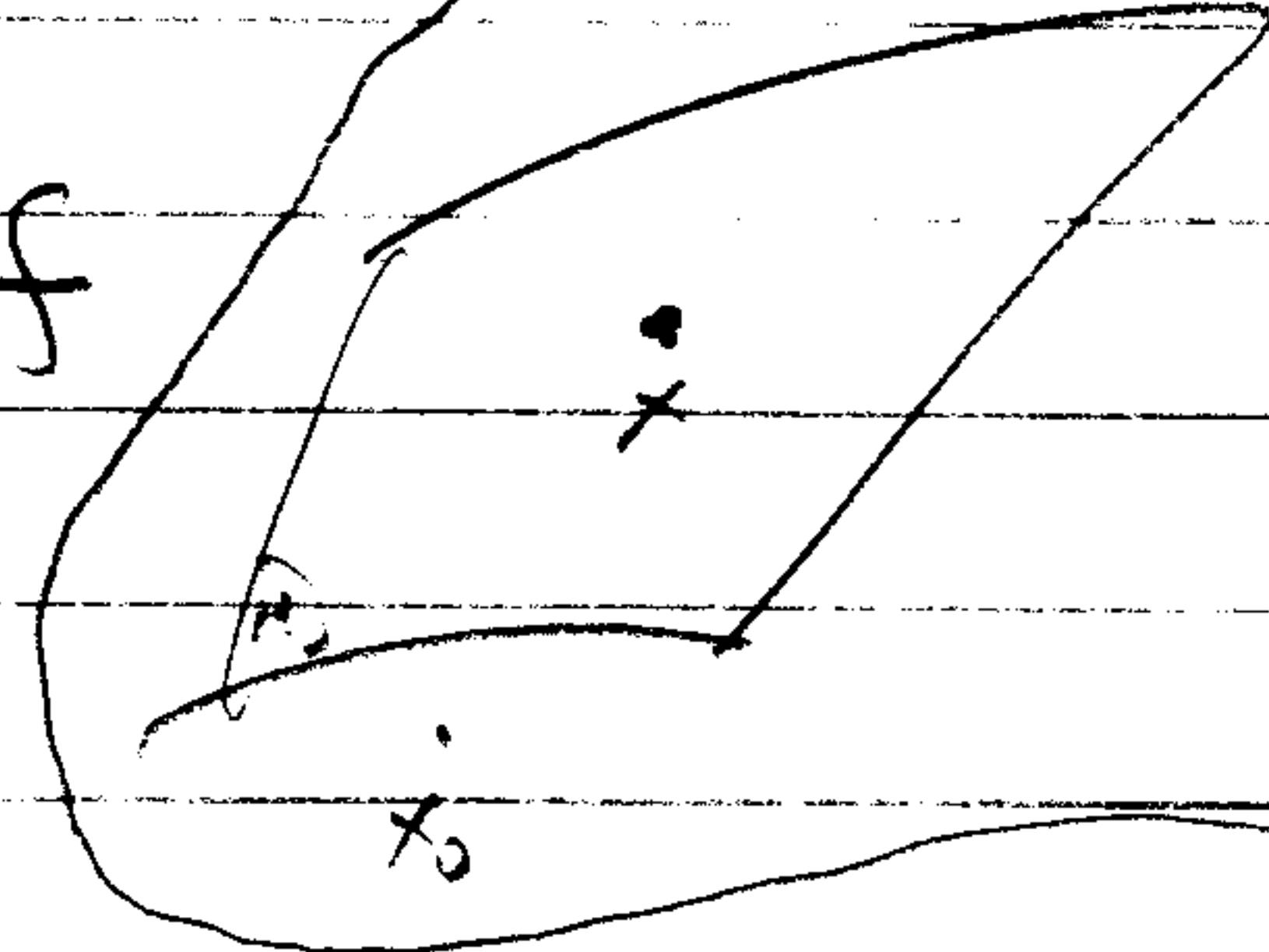
$M \cap B_r(x)$

$M \cap B_r(x_0)$



$$+ \frac{1}{2r^{n+1}} \int_{M \cap B_r(x)} (r^2 - |x - x_0|^2) \Delta_M f$$

$M \cap B_r(x_0)$



$\forall r \in (0, p_0)$, on $f: M \rightarrow \mathbb{R}$, ofagn asptw.

Ena enas $f \geq 0$ cu f utaffunca cu ta M , dyal.

$$(71) \quad \frac{d}{dr} \left(\frac{1}{r^n} \int_{M \cap B_r(x_0)} f \right) \geq 0,$$

TOTE

$$(72) \quad \frac{d}{dr} \left(\frac{1}{r^n} \int_{M \cap B_r(x_0)} f \right) \geq 0.$$

Teps ena $x_0 \in M$ TOTE exafte

$$(73) \quad f(x_0) \leq \frac{1}{\omega_n p^n} \int_{M \cap B_p(x_0)} f \quad \begin{array}{l} (\omega_n = |B(0, 1)| \\ B \subset \mathbb{R}^n) \end{array}$$

□

Topografie

Tic f $\equiv 1$ topografie

$$(74) \frac{d}{dr} \left(\frac{\text{H}^n(M \cap B_r(x))}{\omega_n r^n} \right) = \frac{d}{dr} \int \frac{(x-x_0)^{1/2}}{|x-x_0|^{n+2}}$$

$M \cap B_r(x)$

$$\stackrel{(69)'}{=} \int \frac{(x-x_0)^{1/2}}{|x-x_0|^{n+2}} \frac{1}{|\Delta^M_{x_0}|} \geq 0.$$

$\partial B_r(x_0) \cap M$

□

Flächengeometrie

i) Für $M \subset \mathbb{R}^n$, geod. eingeschränkt konvex, $x_0 \in M$.
Dann $(x-x_0)^{1/2} = 0$ bei x_0 und $\Delta f \geq 0$ in (70) führt

$$\frac{d}{dr} \left(\frac{1}{r^n} \int f \right) \geq 0$$

$B_r(x_0)$

⇒

$$\lim_{r \rightarrow 0} \frac{1}{r^n} \int f \geq \lim_{r \rightarrow 0} \frac{1}{r^n} \int f = f(x_0) \omega_n.$$

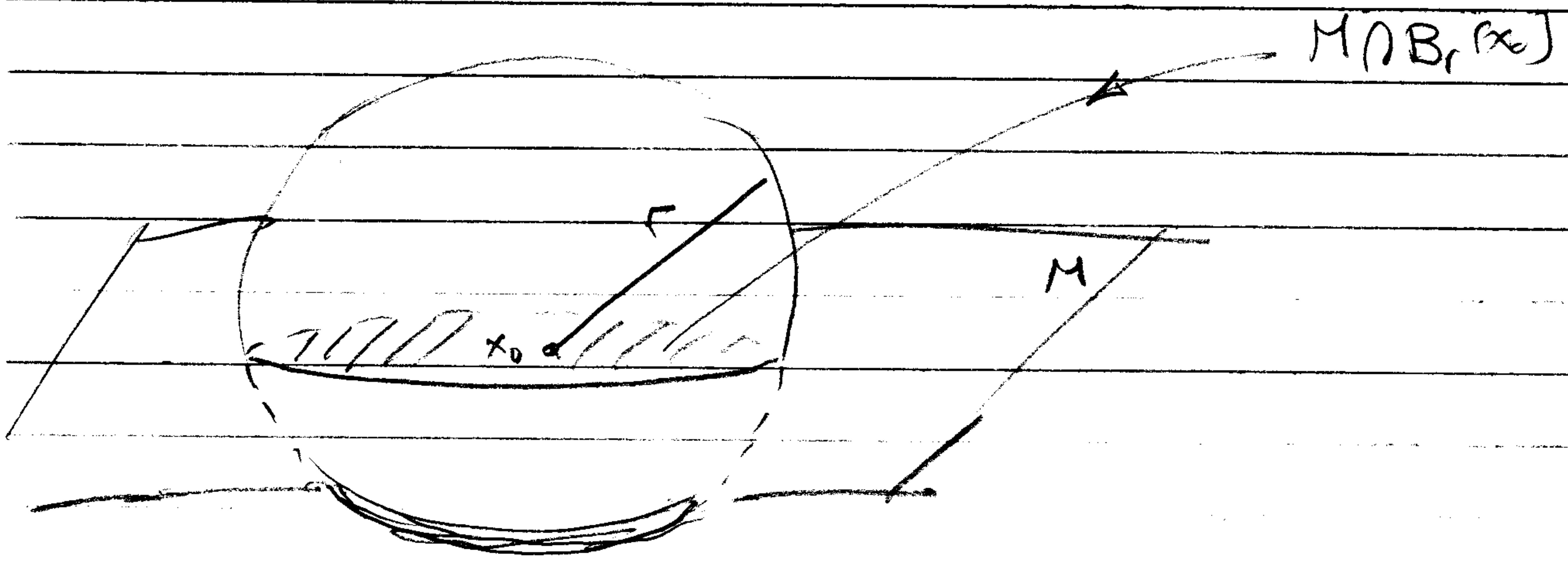
∴

$$f(x_0) \leq \frac{1}{\omega_n r^n} \int f$$

$B_r(x_0)$

⇒ f ist im Intervall um x_0 aufwärts gerichtet (da f konvex)

Summe der ω_n ist positiv in (74) für $r > 0$.



2) $(x - x_0)^\perp$ отваря в опложната гипотеза някои ото
съдълбовите еквивалентни на M ото x_0 .

$$3) (74) \Rightarrow \frac{d}{dr} \left(\frac{\text{H}^n(M \cap B_r(x_0))}{\omega_n r^n} \right) > 0 \Rightarrow$$

$$\frac{\text{H}^n(M \cap B_r(x_0))}{\omega_n r^n} \geq \lim_{r \rightarrow 0} \frac{\text{H}^n(M \cap B_r(x_0))}{\omega_n r^n} = 1 \text{ за } x_0 \in M.$$

$$4) \Delta_M f := \operatorname{div}_M \nabla^M f$$

(β за \mathcal{S}_1).

Аноден Демонстрац

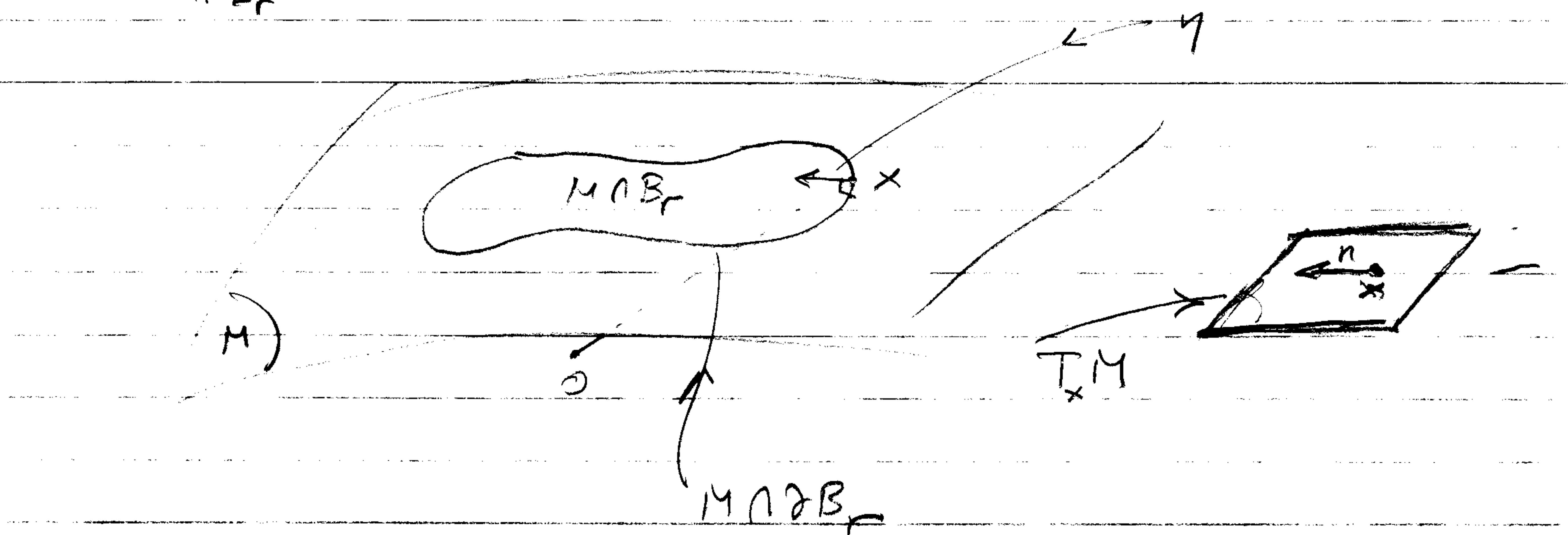
$$\text{Ето } x_0 = 0, B_r = B_r(0)$$

$$(69)' \Leftrightarrow \frac{d}{dr} \int_{M \cap B_r} f = \int_{M \cap B_r} f \frac{1}{|\nabla^M|_{x_1}|}$$

а.з. ща $r \in (0, R)$. Ако Sand нюките
от $M \cap B_r$ отън върху всяка ѝ с радиус r .

\sum τη μεγάλα διακύρωση χαρακτηρίζει την διεύθυνση της απόστασης (5). Επαναστρέψτε $M \Rightarrow H=0$

$$(75) \int_{M \cap B_r} \operatorname{div}_\mu \left(\frac{x}{r} f \right) = \int_{M \cap B_r} f \cdot \frac{x}{r} \cdot \gamma$$



$$\gamma = \frac{x^T}{|x|} = \frac{\nabla^M |x|}{|\nabla^M |x||} \quad (\text{βλ. Αριθμ. 15})$$

$$\frac{x}{r} \cdot \gamma = |\nabla^M |x||, \quad \int \frac{x}{r} \cdot \gamma = \int |\nabla^M |x||$$

$\therefore (75), (69)'$

$$(76) \frac{d}{dr} \int_{M \cap B_r} f = \frac{d}{dr} \int_{M \cap B_r} f \left(1 - |\nabla^M |x||^2 + |\nabla^M |x||^2 \right)$$

$$(69)' = \int_{M \cap B_r} f \frac{1 - |\nabla^M |x||^2}{|\nabla^M |x||} + \int_{M \cap B_r} \operatorname{div} \left(\frac{x}{r} f \right).$$

Harmonische Funktionen

Zusatzaufgabe 16

$$\left\{ \begin{array}{l} 1 - |\nabla^M|_{x_1} |x_1|^2 = |\nabla^M|_{x_1} |x_1|^2 = \frac{|x_1|^2}{|x_1|^2} \\ \operatorname{div}_M \left(\frac{x}{r} f \right) = \frac{n}{r} f + \frac{x}{r} \cdot \nabla^M f. \end{array} \right.$$

Angenommen weiter unten dass div_M existiert
 $(\#6) \Rightarrow$

$$(\#7) \quad \frac{d}{dr} \int_{M \cap B_r} f = \int_{M \cap B_r} f \frac{|x_1|^2}{|x_1|^2} \frac{1}{|\nabla^M|_{x_1} |x_1|} + \int_{M \cap B_r} \operatorname{div}_M \left(\frac{x}{r} f \right)$$

$$(69)' \quad = \frac{d}{dr} \int_{M \cap B_r} f \frac{|x_1|^2}{|x_1|^2} + \frac{n}{r} \int_{M \cap B_r} f + \int_{M \cap B_r} \frac{x}{r} \cdot \nabla^M f.$$

Harmonische OTI

$$\nabla^M (r^2 - |x|^2) = \left(\operatorname{grad}_{\mathbb{R}^{n+1}} (r^2 - |x|^2) \right)^T = -2x^T$$

für OTI

$$r^2 - |x|^2 = 0 \quad \text{OTI } \partial B_r$$

Kontur ausgespart

$$\int_{M \setminus B_r} \frac{x^\perp}{r} \cdot \nabla^M f = -\frac{1}{2r} \int_{M \setminus B_r} \nabla^M (r^2 |x|^2) \cdot \nabla^M f$$

$M \setminus B_r$

$M \setminus B_r$

$$= -\frac{1}{2r} \int_{M \setminus B_r} (r^2 |x|^2) \Delta_M f$$

$M \setminus B_r$

$$(78) \frac{d}{dr} \int_{M \setminus B_r} f = \frac{d}{dr} \int_{M \setminus B_r} f - \frac{|x|^{1/2}}{|x|^2} + \frac{1}{r} \int_{M \setminus B_r} f + \frac{1}{2r} \int_{M \setminus B_r} (r^2 - |x|^2) \Delta_M f$$

\Rightarrow

$$\frac{1}{r^n} \frac{d}{dr} \int_{M \setminus B_r} f - \frac{1}{r^{n+1}} \int_{M \setminus B_r} f$$

$$= \frac{1}{r^n} \frac{d}{dr} \int_{M \setminus B_r} f \frac{|x|^{1/2}}{|x|^2} + \frac{1}{2r^{n+1}} \int_{M \setminus B_r} (r^2 - |x|^2) \Delta_M f$$

$$(79) \frac{d}{dr} \left(\frac{1}{r^n} \int_{M \setminus B_r} f \right) = \frac{1}{r^n} \frac{d}{dr} \int_{M \setminus B_r} f \frac{|x|^{1/2}}{|x|^2}$$

$$+ \frac{1}{2r^{n+1}} \int_{M \setminus B_r} (r^2 - |x|^2) \Delta_M f.$$

Tegos kontras xpm tw

Aksum 17

$$(69)' \Rightarrow$$

$$(80) \quad \frac{1}{r^n} \frac{d}{dr} \int_{M \cap B_r} f \frac{|x|^{1/2}}{|x|^2} = \frac{d}{dr} \int_{M \cap B_r} f \frac{|x|^{1/2}}{|x|^{n+2}}$$

Hauptrufe

$$\frac{d}{dr} \left(\frac{1}{r^n} \int_{M \cap B_r} f \right) = \frac{d}{dr} \int_{M \cap B_r} f \frac{|x|^{1/2}}{|x|^{n+2}} + \frac{1}{2r^{n+1}} \int_{M \cap B_r} (r^2 - |x|^2) \Delta_g f.$$

OED

Aksum 18

Dageste oti

$$\frac{d}{dr} M(M \cap B_r) \geq M(M \cap \partial B_r)$$

Vtrodzen: $(69)'$ koi Aksum 15.

Zinf. H dodevna roxuej via flat apnides.