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The ramification sequence for a fixed point of an automorphism of a curve and the Weierstrass gap sequence

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Abstract For nonsingular projective curves defined over algebraically closed fields of positive characteristic the dependence of the ramification filtration of decomposition groups of the automorphism group with Weierstrass semigroups attached at wild ramification points is studied. A faithful representation of the p-part of the decomposition group at each wild ramified point to a Riemann–Roch space is defined.

1 Introduction

An important difference between the theory of curves in characteristic 0 and in positive characteristic is the existence of wild ramification. Let X be a smooth, complete, projective curve over an algebraically closed field k of characteristic $p > 0$ and let G be a subgroup of the automorphism group. The decomposition group $G(P)$ of a point P is defined as the subgroup of G consisting of elements that fix P. The decomposition group $G(P)$ of a point on X admits the following ramification filtration

$$G(P) = G_0(P) \supset G_1(P) \supset G_2(P) \supset \cdots , \tag{1}$$

where the groups $G_i(P)$ are defined as $G_i(P) = \{\sigma \in G(P) : v_P(\sigma(t)) - t \geq i + 1\}$, and t is a local uniformizer at P and v_P is the corresponding valuation. Note that $G_1(P)$ is the p-part of $G(P)$. The ramification filtration is very well studied in the theory of local fields [12]. Global properties of the curve like the genus do not seem to affect the filtration. However, it is known that several global invariants can affect the ramification filtration at wild ramification points. For example, it is known that if the p-rank of the Jacobian equals the genus then $G_2(P) = \{1\}$ for all P, see [10]. The Riemann Hurwitz formula gives also some connection of the genus to the ramification groups.

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We will define a faithful representation of $G_1(P)$ in $\mathrm{GL}(L(mP))$, where $L(mP) = \{f \in k(X)^* : \mathrm{div}(f) + mP \geq 0\} \cup \{0\}$. This faithful representation takes into account global properties of the curve. Using this representation we are able to relate the jumps in the ramification filtration to the gaps of the Weierstrass semigroup at the point P and to give bounds on the number of jumps and of the highest jump in the ramification filtration in terms of the genus.

The curves X with order of the group G divisible by the characteristic so that $G_2(P) = \{1\}$ for all ramified points, are in some sense the most simple ones. We will call these curves *weakly ramified*. Many intractable problems for the theory of curves with general automorphism group can be solved for weakly ramified curves. For example, the computation of the G-module structure of spaces of holomorphic differentials [6] or the computation of the deformation rings of curves with automorphisms [2].

In our representation perspective it seems natural to consider as the simplest curves those with 2-dimensional representations at all wild ramification points. We provide many examples of such curves and we compare them to weakly ramified curves.

2 Representations

Let X be a nonsingular projective complete curve defined over an algebraically closed field of characteristic $p \neq 2, 3$. Let G be a subgroup of the automorphism group of X, and let P be a wildly ramified point of the cover $X \to X/G$. For every point P of the curve X of genus g we consider the sequence of k-vector spaces

$$k = L(0) = L(P) = \cdots = L((i-1)P) < L(iP) \leq \cdots \leq L((2g-1)P), \tag{2}$$

where

$$L(iP) := \{f \in k(X)^* : \mathrm{div}(f) + iP \geq 0\} \cup \{0\}.$$

We will write $\ell(D) = \dim_k L(D)$. An integer i will be called a pole number if there is a function $f \in k(X)^*$ so that $\mathrm{div}_\infty(f) = iP$ or equivalently $\ell\big((i-1)P\big) + 1 = \ell\big(iP\big)$. The set of pole numbers at P form a semigroup that is called the Weierstrass semigroup at P. It is known that there are exactly g pole numbers that are smaller or equal to $2g-1$ and that every integer $i \geq 2g-1$ is in the Weierstrass semigroup, see [14, I.6.7].

Lemma 2.1 *If $g \geq 2$ and $p \neq 2, 3$ then there is at least one pole number $m \leq 2g-1$ not divisible by the characteristic p.*

Proof The number of integers $0 \leq i \leq 2g-1$ that are divisible by p is given by $\left\lfloor \frac{2g-1}{p} \right\rfloor + 1$. Since $p \geq 5$ we have

$$\left\lfloor \frac{2g-1}{p} \right\rfloor + 1 \leq \left\lfloor \frac{2g-1}{5} \right\rfloor + 1 \leq g - \left\lceil \frac{3g+1}{5} \right\rceil < g;$$

therefore there must be pole numbers in the interval $0 \leq i \leq 2g-1$ not divisible by p. □

Lemma 2.2 *Let $1 \leq m \leq 2g-1$ be the smallest pole number not divisible by the characteristic. There is a faithful representation*

$$\rho : G_1(P) \to \mathrm{GL}\big(L(mP)\big) \tag{3}$$

Proof It is clear that the space $L(mP)$ is preserved by any automorphism in $G_1(P)$. Hence we have the desired representation ρ. We now prove that it is faithful. Let f be a function with pole at P of order m. We can write f as $f = u/t^m$, where u is a unit in the local ring $\mathcal{O}_P$. Since $(m, p) = 1$, Hensel's lemma implies that u is an m-th power so the local uniformizer can be selected so that $f = 1/t^m$. Let $\sigma \in G_1(P)$ be an element that acts trivially on $L(mP)$. Then $\sigma(1/t^m) = 1/t^m$ and $\sigma(t) = \zeta t$, where ζ is an m-th root of unity. Since σ is an element of order p and $(p, m) = 1$ we have that $\zeta = 1$, and σ is the identity element of $G_1(P)$. □

The above lemma makes $G_1(P)$ realizable as a finite algebraic subgroup of the linear group $\mathrm{GL}_{\ell(mP)}(k)$. Moreover the flag of vector spaces $L(iP)$ for $i \leq m$ is preserved, so the representation matrices are upper triangular, or in other words $G_1(P)$ is a subgroup of the Borel group of the flag.

We assume that $m = m_0 > m_1 > \cdots > m_r = 0$, are the pole numbers $\leq m$. Therefore, a basis for the vector space $L(mP)$ is given by

$$\left\{1, \frac{u_i}{t^{m_i}}, \frac{1}{t^m} : \text{ where } 1 < i < r,\ p \mid m_i \text{ and } u_i \text{ are certain units}\right\}$$

With respect to this basis, an element $\sigma \in G_1(P)$ acts on $1/t^m$ by

$$\sigma \frac{1}{t^m} = \frac{1}{t^m} + \sum_{i=1}^{r} c_i(\sigma) \frac{u_i}{t^{m_i}},$$

and then it maps the local uniformizer t to

$$\sigma(t) = \frac{\zeta t}{\left(1 + \sum_{i=1}^{r} c_i(\sigma) u_i t^{m-m_i}\right)^{1/m}}, \tag{4}$$

where ζ is an m-th root of 1.

The above expression can be written in terms of a formal power series as:

$$\sigma(t) = \zeta t \left(1 + \sum_{\nu \geq 1} a_\nu(\sigma) t^\nu\right). \tag{5}$$

Since σ is an automorphism of the power series ring and $\sigma^{|G_1(P)|} = 1$, and $(m, p) = 1$ we obtain that $\zeta = 1$ and (5) can be written as:

$$\sigma(t) = t \left(1 + \sum_{\nu \geq 1} a_\nu(\sigma) t^\nu\right). \tag{6}$$

The above computation allows us to compute the jumps in the filtration of the group $G_1(P)$.

Proposition 2.3 *Let P be a wild ramified point on the curve X and let*

$$\rho : G_1(P) \to \mathrm{GL}_{\ell(mP)}(k)$$

be the corresponding faithful representation we considered in Lemma 2.2. Let $m = m_0 > m_1 > \cdots > m_r = 0$ be the pole numbers at P that are $\leq m$. If $G_i(P) > G_{i+1}(P)$ then $i = m - m_k$, for some pole number m_k.

Proof We use the notation of (4) for σ. If all $c_i(\sigma)$ for $i = 1, \ldots, r$ vanish then $\sigma(1/t^m) = 1/t^m$ and σ is the identity. The valuation of the expression $\sigma(t) - t$ can be explicitly computed. We have

$$\sigma(t) - t = -\frac{1}{m}\sum_{i=1}^{r} c_i(\sigma)u_i t^{m-m_i+1} + \cdots,$$

therefore $v_P\big(\sigma(t) - t\big) = m - m_k + 1$, where $k = \min\{i : c_i(\sigma) \neq 0\}$. Assume that $\sigma \in G_i(P)$ but $\sigma \notin G_{i+1}(P)$, thus $v_P(\sigma(t) - t) = i + 1$ and this equals some $m - m_k + 1$. □

Corollary 2.4 *No jump i in the ramification filtration is divisible by p, i.e., if $G_i(P) < G_{i+1}(P)$ then $p \nmid i$.*

Proof By lemma 2.3 every gap in the ramification filtration is given as $m - m_k$, where m is not divisible by p and m_k are divisible by p, cf. [12, IV. Proposition 11]. □

Corollary 2.5 *We have $\ell(mP) \leq \left\lfloor \frac{g}{p-1} \right\rfloor + 2$.*

Proof Using m as before we let λp be a pole number smaller than m. There are exactly g pole numbers smaller or equal to $2g - 1$. The number of pole numbers smaller or equal to λp is $\leq \lambda + 1$. (Note that 0 is a pole number). Therefore the cardinality of the set $\Sigma := \{\kappa \in \mathbb{Z}_{\geq 0} : \lambda p < \kappa \leq 2g - 1\}$ should be at least $g - (\lambda + 1)$. The set Σ has cardinality $2g - 1 - \lambda p$, so

$$g - \lambda - 1 \leq 2g - 1 - \lambda p \text{ hence } \lambda \leq \left\lfloor \frac{g}{p-1} \right\rfloor. \tag{7}$$

On the other hand we have $\ell(mP) = 1 + \#\{$ number of poles in $[0, \lambda p]\} \leq \lambda + 2$. The desired result follows by using the bound (7). □

Remark 1 The bound in (2.5) is best possible. For example the Artin–Schreier function field F given by $(y^5 - y)x^2(x-1)(x-2)(x-4) = 1$ is of genus 14 and $\left\lfloor \frac{g}{p-1} \right\rfloor + 2 = 5$. The gap sequence at the unique place Q of F above $P_{x=0}$ is 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 18. The pole numbers up to first pole number not divisible by 5 are 0, 5, 10, 15, 17, i.e., $m = 17$, and $\ell(17Q) = 5$, as one can compute using the Magma program [1].

Remark 2 For an estimate of $\ell(kP)$ for $k < 2g - 2$ in terms of the genus one can employ the theorem of Clifford [4, Corollary 4.4.18]

$$\ell(kP) \leq \frac{k}{2} + 1 \leq \left\lfloor \frac{2g-1}{2} \right\rfloor + 1 = g.$$

For wild ramification points Corollary 2.5 is an improvement of the above estimate.

Corollary 2.6 *Let n be the length of the ramification filtration, i.e. $G_n(P) \neq \{1\}$ and $G_{n+1}(P) = \{1\}$ and let r be the number of jumps in the ramification filtration. If the first non zero pole number at P is divisible by p, then*

$$n \leq \left(\left\lfloor \frac{g}{p-1} \right\rfloor\right) p \text{ and } r \leq \left\lfloor \frac{g}{p-1} \right\rfloor + 2.$$

Proof By proposition 2.3 every jump in the ramification filtration corresponds to a pole number $m - m_i + 1$. The number n is a gap in the ramification filtration therefore it is of the form $m - m_{i_0}$ for some m_{i_0}. If there is no pole number $< m$ divisible by the p then $n = m < g + 1$.

Assume now that there are pole numbers $m_i < m$ divisible by p. Let $\lambda p < m$ be the maximum pole number smaller than m. The integer $\lambda p + m_{i_0}$ is also a pole number divisible by p, therefore $m < \lambda p + m_{i_0}$. We have

$$n = m - m_{i_0} \leq \lambda p + m_{i_0} - m_{i_0} \leq \left(\left\lfloor \frac{g}{p-1} \right\rfloor \right) p.$$

For the second inequality observe that every jump in the ramification filtration correspond to some m_i and the number of the m_i's is bounded by $\ell(mP)$. ◻

Let $c_r(\sigma), \ldots, c_1(\sigma)$ be the elements of the last row of the representation matrix of (3). We consider them as functions $G_1(P) \to k$. Some of them can be identically zero. Let $1 \leq t_1 \leq r$ be the first index so that $c_{t_1} : G_1(P) \to k$ is not identically zero. The first jump in the ramification filtration occurs at $i_1 = m - m_{t_1}$, i.e.

$$G_1(P) = G_2(P) = \cdots = G_{i_1}(P) > G_{i_1+1}(P) = \cdots$$

Let $t_2 < t_1$ be the first index so that $c_{t_2} : G_{i_1+1}(P) \to k$ is not identically zero. The second jump in the ramification filtration occurs at $i_2 = m - m_{t_2}$. Proceeding inductively we define a sequence $t_1 < t_2 < \cdots < t_s$ so that the sequence of jumps in the ramification filtration is given by $i_\nu = m - m_{t_\nu}$. Moreover $G_{i_\nu+1}(P) = \ker(c_{t_\nu})$, so c_{t_ν} induces the following injective homomorphism:

$$c_{t_\nu} : \frac{G_{i_\nu}(P)}{G_{i_\nu+1}(P)} \to k.$$

Examples **1.** The Fermat curves $x^n + y^n + 1 = 0$, where $n - 1 = p^h$. The automorphisms of these curves where studied by H. W. Leopoldt in [9]. Leopoldt constructed a basis for the space of holomorphic differentials of the curve and he was able to prove that for the points of the form $P : (x, y) = (\zeta_{2n}, 0)$ where ζ_{2n} is a $2n$th root of 1, we have the following sequence of k-vector spaces [9, Satz 4]:

$$k = L(0P) = L(P) = \cdots = L((n-2)P) < L((n-1)P) < L(nP) \leq \cdots$$

The interesting case for us (Hermitian function fields) appears when $n - 1$ is a power of the characteristic, so in this case the representation of the decomposition subgroup is of the form:

$$\rho : G_0(P) \to \mathrm{GL}\big(L(nP)\big)$$

with

$$\sigma \mapsto \begin{pmatrix} 1 & 0 & 0 \\ \alpha & \chi & 0 \\ \gamma & \beta & \psi \end{pmatrix}.$$

According to Proposition 2.3 the filtration of the decomposition subgroup is given by:

$$G_0(P) > G_1(P) > G_2(P) = \cdots = G_n(P) > G_{n+1}(P) = \{1\},$$

i.e., the gaps of the filtration are in $0, 1, n$.

2. We consider the curves $x^n + x^m + 1 = 0$, where $m \mid n$ and $m - 1 = p^h$. The automorphism group of a nonsingular model of the above curve is studied by the author, in [7]. It is proved that for the points $P : (x, y) = (\zeta_{2n}, 0)$ we have the following sequence of vector spaces [7, Eq. (4)]

$$k = L(0P) = L(P) = \cdots = L((m-1)P) < L(mP) = L((m+1)P) \leq \cdots$$

Since m is not divisible by p we have the following representation

$$G_0(P) \to GL\big(L(mP)\big),$$

sending

$$\sigma \mapsto \begin{pmatrix} 1 & 0 \\ \alpha & \chi \end{pmatrix}.$$

Thus $G_0(P)$ is the semidirect product of an elementary abelian group by a cyclic group of order prime to the characteristic. For the ramification filtration of $G_0(P)$ we have

$$G_0(P) > G_1(P) = G_2(P) = \cdots = G_m(P) > \{1\}.$$

3. Ordinary Curves. A curve is called ordinary if the p-rank of the Jacobian is equal to the genus of the curve. It is known that ordinary curves form a Zariski-open set in the moduli space of curves of genus g in characteristic p. For ordinary curves we have that $G_2(P) = \{1\}$ (see [10]), thus we have the following picture for the faithful representation ρ of the group $G_1(P)$: There is a gap at $G_1(P) > G_2(P) = \{1\}$, thus $1 = m - m_i$ for some i, therefore $m_i = m - 1$ and $i = 1$. This implies that if the genus g of X is $g \geq 1$, then the representation has dimension at least 3, because otherwise, i.e., if the representation is 2-dimensional, we have the following sequence

$$k = L(0P) = L(P) = \cdots L((m-1)P) < L((m)P).$$

But $m - 1$ is a pole number so $m - 1 = 0$ and $m = 1$, *i.e.*, the Weierstrass semigroup is the semigroup of natural numbers, a contradiction, for $g \geq 2$.

4. We will now consider p-cyclic covers of the affine line. In this example we apply our computations to Artin–Schreier curves that are nonsingular models of the function field defined by:

$$C_{t_0,\ldots t_{s-1}} : W^p - W = \sum_{i=0}^{s-1} t_i X^{p^i+1} + X^{p^s+1}.$$

These curves give extreme examples of automorphism groups and were studied by Stichtenoth [13] and Lehr and Matignon [8], Elkies [3], van der Geer and van der Vlugt [15].

There is only one ramified point in the cover $C_{t_0,\ldots t_{s-1}} \to \mathbb{P}^1$, the point P that lies over the point $X = \infty$ of $\mathbb{P}^1$. The Weierstrass semigroup at P is computed by Stichtenoth [13] to be equal to $(p^s + 1)\mathbb{Z}_{\geq 0} + p\mathbb{Z}_{\geq 0}$.

Thus, the smallest pole number that is not divisible by p is $p^s + 1$ and the Weierstrass semigroup up to $p^s + 1$ is computed to be

$$0, p, 2p, \ldots, \left[\frac{1+p^s}{p}\right] p, 1 + p^s.$$

Observe that $\left[\frac{1+p^s}{p}\right] p = p^s$. According to Proposition 2.3 the possible gaps of the ramification filtration are the numbers $p^s + 1 - kp$, with $k = 0, \ldots, p^s$. Notice that by the work of Lehr and Matignon (see [8]) we know that the gaps of the ramification filtration are 1 and $1 + p^s$, therefore a converse of Proposition 2.3 is not true. The dimension $\dim_k L\big((p^s + 1)P\big)$ is $n = \left[\frac{1+p^s}{p}\right] + 2 = p^{s-1} + 2$ and the representation of $G_1(P)$ on $L\big((p^s + 1)P\big)$ is given by an $n \times n$ lower triangular matrix with 1 in the diagonal. More precisely, if we choose the natural basis $\{1, X, X^2, \dots, X^{p^{s-1}}, W\}$ of $L\big((p^s+1)P\big)$ then the representation ρ is given by the matrix

$$\rho(\sigma)_{ij} = \begin{cases} 0 & \text{if } i < j \\ 1 & \text{if } i = j \\ y^j \binom{i}{j} & \text{if } i > j, i \neq p^{s-1} + 1 \\ b_j(y) & \text{if } i = p^{s-1} + 1 > j, \end{cases} \tag{8}$$

where $b_j(y)$ are the coefficients of the polynomial $P_f(X, y)$, and y is a solution of $Ad_f(Y) = 0$ as defined in lemma 4.1 and Definition 4.2 in [8].

3 Two-dimensional representations

One can argue that among wildly ramified covers the simplest are the weakly ramified covers i.e. covers where $G_2(P) = \{1\}$ at all ramified points. However, in our setting it seems that the simplest covers are the ones with 2-dimensional representations attached at wild ramification points. Using Proposition 2.3 we observe that curves with 2-dimensional representations have only one jump in their ramification filtration at P, and that jump occurs at m, where m is the first non zero pole number. Moreover in this case the group $G_1(P)$ has to be elementary abelian.

It is tempting to consider the bound given in Corollary 2.5 in order to give a criterion for a representation to be 2-dimensional. We see that if $g < p - 1$ then $\left\lfloor \frac{g}{p-1} \right\rfloor = 0$ and the dimension is at least 2-dimensional. Roquette in [11] proved that if a curve has a wild ramification point then $p \leq g + 1$ with only one exception, the hyperelliptic curve

$$y^2 = x^p - x.$$

Therefore, the $g < p - 1$ condition is not that useful. We observe that the representation at P is 2-dimensional if and only if the first nonzero pole number of the Weierstrass semigroup at P is not divisible by p.

We can construct many curves that have 2-dimensional representation space. For example the curves that correspond to function fields defined by the equation

$$F : \sum_{\nu=0}^{n} a_n y^{p^\nu} = \sum_{\mu=0}^{m} b_\mu x^\mu, \tag{9}$$

so that $m \not\equiv 0 \bmod p$, $a_n, a_0, b_0 \neq 0$, $n \geq 1$, $m \geq 2$, studied by Stichtenoth in [13]. Let P_∞ be the unique place above the place p_∞ of the function field $k(x)$. Stichtenoth proved that the Weierstrass semigroup at P_∞ is given by $m\mathbb{Z}_{\geq 0} + p^n\mathbb{Z}_{\geq 0}$. Thus, if we select $m < p^n$, we see that the first pole number is 0 and the second is m therefore $d = 2$. Moreover, the ramification filtration of $G = \mathrm{Gal}(F/k(x))$ at P_∞ is given by (see [13, Satz 1.]):

$$G_0(P_\infty) = G_1(P_\infty) = \cdots = G_m(P_\infty) = \mathrm{Gal}(F/k(x)) > G_{m+1}(P_\infty) = \{1\}.$$

Notice that if the right hand side of (9) is generic then $G = \mathrm{Aut}(F)$.

On the other hand, assume that we have a curve that has a 2-dimensional representation attached at a wild ramified point P. The group $G_1(P)$ is elementary abelian $G_1(P) \cong \bigoplus_{\nu=1}^{r} \mathbb{Z}/p\mathbb{Z}$. Let f be a function so that $\langle 1, f\rangle$ is a basis of $L(mP)$. We would like to write down an algebraic equation for the cover $X \to X/G_1(P)$. The representation $c_1 : G_1(P) \to k$ is a faithful homomorphism of additive groups. We consider the action of $G_1(P)$ on f:

$$\prod_{\sigma \in G_1(P)} \sigma(f) = \prod_{\sigma \in G_1(P)} (f + c_1(\sigma)).$$

Let

$$\Delta(w_1, \dots, w_s) = \det \begin{pmatrix} w_1 & \cdots & w_s, \\ w_1^p & \cdots & w_s^p \\ \vdots & & \vdots \\ w_1^{p^{s-1}} & \cdots & w_s^{p^{s-1}} \end{pmatrix}$$

be the *Moore determinant*, see [5, 1.3.2]. Let $F(Y)$ be the additive polynomial with roots $c_1(\sigma)$, where σ is running over $G_1(P)$. The polynomial $F(Y)$ can be computed as follows: Select a basis σ_i, with $1 \le i \le s$ of $G_1(P)$ seen as $\mathbb{F}_p$-vector space. Then

$$F(Y) = \frac{\Delta(\sigma_1, \dots, \sigma_s, Y)}{\Delta(\sigma_1, \dots, \sigma_s)},$$

see [5, Lemma 1.3.6]. Thus, the cover $X \to X/G_1(P)$ is given in terms of the generalized Artin–Schreier equation

$$F(Y) = \prod_{\sigma \in G_1(P)} \sigma f = N_{G_1(P)}(f).$$

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