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An examination of Alternative Multidimensional Scaling Techniques

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Abstract

The purpose of this study is to compare alternative multidimensional scaling (MDS) methods for constraining the stimuli on the circumference of a circle and on the surface of a sphere. Specifically, the existing MDS-T method for plotting the stimuli on the circumference of a circle is applied, and its extension is proposed for constraining the stimuli on the surface of a sphere. The data analyzed come from previous research and concerns Maslach and Jackson's burnout syndrome and Holland's vocational personality types. The configurations for the same data on the circle and the sphere shared similarities but also had differences, that is, the general item-groupings were the same but most of the differences across the two methods resulted in more meaningful interpretations for the three-dimensional configuration. Furthermore, in most cases, items and/or scales could be better discriminated from each other on the sphere.

Keywords

constrained multidimensional scaling, circular and spherical constraints on MDS solutions, extension of MDS-T on the sphere, quadrant-specific arctangent transformations, burnout syndrome, Self-Directed Search

Introduction

Multidimensional scaling (MDS) aims to uncover the underlying structure in a proximity matrix by producing a simple geometrical model resembling a map, such that

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the distances in the resulting configuration fit the raw data as well as possible (Dillon & Goldstein, 1984). The overall purpose of this article is to apply alternative MDS methods in order to depict the same data on a circular continuum and on a spherical surface and describe them in comparison to each other.

Plotting the stimuli on the circumference of a circle and on the surface of a sphere through MDS are types of constrained MDS analysis. The terms ''constrained MDS'' and ''confirmatory MDS'' (Borg & Groenen, 2005; Cox & Cox, 2001; Heiser & Meulman, 1983) refer to situations when additional constraints are imposed on the configuration, except for general constraints like the dimensionality and the type of analysis (metric or nonmetric; Borg & Groenen, 2005; Borg & Lingoes, 1980). These constraints refer to structural hypotheses or conditions that the derived configuration should satisfy so that the researcher may be able to test for these hypotheses. These can be tested by incorporating them as additional constraints in the MDS analysis. For example, in a study by Sidiropoulou-Dimakakou, Mylonas, and Argyropoulou (2008), Holland's (1985) RIASEC types were constrained to circular arrangement. If the configuration with additional structural constraints is almost as good or better in terms of fit in comparison to a configuration without such constraints, then the hypotheses can be considered compatible with the data (Bentler & Weeks, 1978; Borg & Groenen, 2005; Borg & Lingoes, 1980). External constraints on the MDS configuration can be also useful in a more exploratory context. The derived configuration from an unrestricted MDS analysis may have some rather unattractive properties (Borg & Lingoes, 1980). For example, when the stimuli subjected to MDS analysis are designed to differ in terms of certain attributes and the axes of the unconstrained configuration do not correspond exactly to these attributes, the researcher cannot be certain whether these discrepancies are due to random error or due to some nonrandom effect (Bloxom, 1978). Before interpreting the solution, one may prefer to rescale the data with a constrained method (Borg & Lingoes, 1980). The types of constraints addressed in different MDS methods include equality restrictions on coordinates or distances, fixing parameters to a priori values, or estimating the parameters of an MDS solution so as to yield a specific geometrical structure (Bentler & Weeks, 1978; Bloxom, 1978; Borg & Groenen, 2005; Borg & Lingoes, 1980; de Leeuw & Mair, 2009; Lee, 1984; Lee & Bentler, 1980).

Constraining the MDS solution on a circle or on a sphere can be theoretically meaningful and aid the researcher in further stages of his analyses. In some domains the theoretical structure is supposed to be circular or spherical and discrepancies from it may be considered as ''error.'' Bimler and Kirkland (2005) refer to the different distances of items from the origin in an MDS solution as ''specificity,'' which they claim is approximately constant in a well-designed set of items. The theoretically expected structure is a circular or spherical arrangement, for example, when analyzing color-similarity data (the color circle describing perception of colors with differing wavelength) or similarities between nations (the globe is spherical; Borg & Lingoes, 1980; Cox & Cox, 1991, 2001; de Leeuw & Mair, 2009; Lee & Bentler, 1980). Further examples include models related to Prediger's (1982) hypothesis about the structure of Holland's (1985) vocational personality types (e.g., Rounds & Tracey, 1993), where the theoretical presupposition is that the points lie on the circumference of a circle, the spherical model of vocational interests (Tracey & Rounds, 1996) and circular models of the interpersonal domain (Gurtman & Balakrishnan, 1998). Analysis of circumplex models (e.g., the *circulant model*, the *geometric circulant model*, and the *quasi-circumplex model* as described by Tracey, 2000) is an issue related to circular and spherical MDS structures. Such models can be studied through different methods including structural equation modeling and constrained MDS (e.g., Darcy & Tracey, 2007; Rounds & Tracey, 1993). The gain of constraining the MDS configuration on a circle or on a sphere in cases like the ones just mentioned is that the final solution is closer to the original theory, as the hypothesis of circular or spherical arrangement of points is met, and the similarities and differences of the final solution to the theoretically expected can be meaningfully described. In their paradigm of this, Sidiropoulou-Dimakakou et al. (2008) analyzed data with respect to Holland's hexagonal vocational personality model through MDS. In the unconstrained two-dimensional solution, the arrangement of the RIASEC types was approximately circular, but the different radial distances of the points did not aid interpretation, as the theory presupposes that the six types lie on a circular continuum. Consequently, the axes of this configuration could not be easily interpreted as dimensions. The constrained circular solution was closer to the original theory (the circular property was met), and as a result its similarities and differences to Holland's equilateral hexagon could be described for the specific sample (e.g., the main finding was that the Realistic and Investigative types had much smaller distance than expected).

The circular or spherical constraints can also be used as a way to obtain homogeneous groups of items, stimuli, individuals, and so on, for purposes of better interpreting the solution. Kruskal and Wish (1978) refer to the process of interpreting the derived MDS configuration by grouping stimuli that are close to each other, as neighborhood interpretation (they use cluster analysis applied to the proximity matrix in order to find the item-groups). Another example of a technique for obtaining groups of similar stimuli is Latent Class MDS (Vera, Macías, & Heiser, 2009), where the stimuli are partitioned into classes and the cluster centers are represented in a low-dimensional space. Such grouping of stimuli can also be achieved by locating the nearby points on the circumference of the circle (or on the surface of a sphere) in a constrained circular (or spherical) configuration, and describing their common characteristics that differentiate them from other such groups.

According to Guilford's (1954) homogeneity hypothesis, analyzing homogeneous groups of individuals can lead to bringing out the structure of the data more clearly than when these groups are heterogeneous. This kind of homogeneous groups are useful for bias reduction purposes in comparisons between groups. Thus, when homogeneous groups are compared to each other instead of single units, any similarity or difference that exists between these groups can become apparent because of the reduction of error within the homogeneous groups. For example, in Mylonas et al.