Heisenberg Uncertainty Inequality

John Michael Rassias

- In 1927, W. Heisenberg (Zeit. Physik 43 (1927) , 172 - ; Univ. Chicago Press, 1930 ; and Dover edit. , New York, 1949) demonstrated the impossibility to specify simultaneously the position and the momentum of an electron within an atom.The following result named , Heisenberg uncertainty inequality, is not actually due to Heisenberg. In 1928, according to H. Weyl (S. Hirzel , Leipzig,1928 ; and Dover edit. , New York , 1950) this result is due to W. Pauli .

Heisenberg Uncertainty Inequality: If $f: R \to C$ is a complex valued function of a random real variable x such that $f \in L^2(R)$, then the product of the second moment of the random real x for $|f|^2$ and the second moment of the random real ξ for $\left|\hat{f}\right|^2$ is at least $\int_R |f(x)|^2 dx \Big/ 4\pi$, where $\hat{f}$ is the Fourier transform of f, such that $\hat{f}(\xi) = \int_R e^{-2i\pi\xi x} f(x)\,dx$ and $f(x) = \int_R e^{2i\pi\xi x} \hat{f}(\xi)\,d\xi$ with $i = \sqrt{-1}$.

According to N. Wiener ("the Fourier integral and certain of its applications" , Cambridge , 1933) a pair of transorms cannot both be very small . This inequality plays an important role in different aspects of Fourier and Time – Frequency Analysis . A huge number of well-written books and overview-papers deals with uncertainty relations .