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An atomistic vision of the Mass Action Law: Prediction of carbon/oxygen defects in silicon

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We introduce an atomistic description of the kinetic Mass Action Law to predict concentrations of defects and complexes. We demonstrate in this paper that this approach accurately predicts carbon/oxygen related defect concentrations in silicon upon annealing. The model requires binding and migration energies of the impurities and complexes, here obtained from density functional theory (DFT) calculations. Vacancy-oxygen complex kinetics are studied as a model system during both isochronal and isothermal annealing. Results are in good agreement with experimental data, confirming the success of the methodology. More importantly, it gives access to the sequence of chain reactions by which oxygen and carbon related complexes are created in silicon. Beside the case of silicon, the understanding of such intricate reactions is a key to develop point defect engineering strategies to control defects and thus semiconductors properties.

I. INTRODUCTION

Properties in semiconductors are driven by impurity-defect complexes formed during their processing. Several fields of technological interest are impacted by such defect-complexes. In the photovoltaic industry,1,2 in fast responding transistor,3,4 or in semiconductor lasers, defects have to be controlled or suppressed. The stability and the presence of the latter are often analyzed through the Mass Action Law.6 This law states that their binding energy can be used with the Boltzmann statistic to obtain the concentrations. It supposes that the system reaches thermodynamic equilibrium. However, during processing, several interactions compete and thus lead to complex chain reactions. Kinetics controls these kinds of reactions and often leads to metastable situations. The chain reactions cannot be understood only with the defect binding energies. Kinetic Monte Carlo (KMC) simulations could provide such kinetics of formation and dissociation. However, KMC models are computationally expensive and are limited to milliseconds of simulated time at 600 K when fast diffusing species are involved (e.g., mono vacancies in silicon). These limitations can be overcome by working on continuous quantities like concentrations. It corresponds then to a spatial average in the framework of the diffusion-limited reaction theory and of the kinetic Mass Action Law.

In order to ensure the predictability of such a method, we further introduce an atomistic description for the underlying rate constants. The obtained equations are then solved numerically. Such a simulation tool can handle several dozen of defects and allows to reproduce thousand hours of annealing in a calculation time scale of a minute. In this article, we assess the validity of such an approach on a restricted set of experiments. While few parameters are calibrated on a subset, the whole experimental set is reproduced and found in very good agreement. In addition, the method gives access to the underlying chain reactions and opens new ways to prevent or select defect formations.

II. THEORETICAL FRAMEWORK

The model is built on the kinetic Mass Action Law.11 It states that in a system with N reactions described by

$$\sum_i x_i A_i = \sum_j \gamma_j C_j, \quad r \in \{1, \ldots, N\},$$  (1)

the time evolution in concentrations, for one reaction r, is described by

$$\frac{\delta[A_i]}{\delta t} = k_{f} \prod_i [A_i]^{x_i} - k_{r} \prod_j [C_j]^{\gamma_j},$$  (2)

where [A_i] is the concentration of the reactant i, [C_j] is that of the product j, \(x_i\) is the stoichiometric coefficient of reactant i for the forward reaction r, and \(\gamma_j\) is the coefficient of product j in the reaction r. Finally, \(k_{f}\) and \(k_{r}\) are the rate constants and will be detailed later. In the following, we define that a forward reaction corresponds to the formation of a defect complex and a backward reaction its dissociation.

We use the framework of diffusion-limited reactions. This approximation does not induce a measurable error since the travel time for reactants to meet is much bigger than the time for the reaction to take place. Various models have already been applied in this framework. The most widely used defines the rate constant \(k_{r}\) as \(k_{r} = 4\pi R \times \sum_i D_i\), where \(D_i = d_{0} a^{3/2} R^{\nu}

12 where \(R = \sum_i D_i\).

The model requires binding and migration energies of the impurities and complexes, here obtained from density functional theory (DFT) calculations. Vacancy-oxygen complex kinetics are studied as a model system during both isochronal and isothermal annealing. Results are in good agreement with experimental data, confirming the success of the methodology. More importantly, it gives access to the sequence of chain reactions by which oxygen and carbon related complexes are created in silicon. Beside the case of silicon, the understanding of such intricate reactions is a key to develop point defect engineering strategies to control defects and thus semiconductors properties.
through the capture radius $R$. The second and main drawback is that the pre-exponential factor $d_0$ in the diffusivity term $D_i$ is expensive to compute as vibrational properties are to be obtained.

Instead, we are defining the rate constant from an atomistic diffusion of a particle, leading to a predictive and more general model. We consider that particles move on a lattice as shown in Fig. 1. In this model, $k_{r+}$ can be expressed as follows:

$$k_{r+} = p_0 \times s_i \times \nu_0 e^{-\beta E_{ss}^n} = \frac{p_0 \times s_i}{\langle t_{\text{move}} \rangle}, \quad (3a)$$

$$k_{r-} = \nu_0 e^{-\beta (E_{ss}^n + E_{bi})} = \frac{1}{\langle t_{\text{dissociation}} \rangle}, \quad (3b)$$

where $\nu_0 = 10^{-11}s^{-1}$ is the jump frequency, $E_{ss}^n$ is the migration energy of the diffusing particle, and $E_{bi}^n$ is the binding energy between two particles. During a reaction $r$, we consider only the diffusing species with the lowest migration energy. The other migration rate can be neglected due to the exponential decay in $e^{-\beta E_{ss}^n}$. In Fig. 1, the green species has a higher migration energy. It is thus considered as an immobile defect. The parameter $p_0$ is an approximate probability for discovering a distinct site, which was never visited before. It depends on the connectivity of the lattice and its type. In silicon, previous calculations have demonstrated that $p_0 = 0.56$. The factor denoted $s_i$ accounts for the local structure of the defect and its interaction range. A more attractive defect has a higher probability of meeting with other particles. It is defined as the number of neighbor positions for which the binding energy of two separated components is non zero. Thus, this parameter takes into account the capture radius but also the local structure of the defect.

Our approach is based on Eqs. (2) and (3). They give us a system of differential equations. These equations are nonlinear in a general case and cannot be solved analytically.

We have solved them numerically using the so-called “Ordinary Differential Equation” solver of SciLab with, respectively, $10^{-12}$ and $10^{-10}$ as absolute and relative estimated errors on atomic concentrations. This solver automatically selects between non-stiff predictor-corrector “Adams method” and stiff “Backward Differentiation Formula” method. It uses the non-stiff method initially and dynamically monitors data in order to decide which method to use. It results in a calculation cost of the order of a few minutes for a typical thousand hours simulated annealing time as presented in this paper.

### III. CALIBRATION AND ANALYSIS

The model requires three inputs for each reaction: migration and binding energies, and the $s_i$ factor. The binding energies were all calculated using the Density Functional Theory (DFT) approach as in Refs. 16 and 17. We extracted the migration energies also from DFT excepted for the $V$ related complexes. For the latter, we have used experimental values instead due to the known error of the DFT in regard to the excited states at saddle points. The $s_i$ factors, as defined before, are the numbers of neighbors where two separated components have a positive binding energy. They have been calculated by DFT (for the range) and using the connectivity of the silicon lattice. For example, defects on substitutional sites have 4 first neighbors and 12 $s$ neighbors, like carbon related defects; and bond-centered interstitials, like the oxygen, are sitting between 2 substitutional sites, thus having 6 first neighbors. All the corresponding values are listed in Table I.

In the following, we apply our model to reproduce an already published set of experiments where carbon and oxygen react in irradiated silicon. The goal is to first validate our methodology and then analyze the carbon/oxygen-related complexes. These experiments consist in successive

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Binding Energy</th>
<th>Migrating Energy</th>
<th>$E_{bi} + E_{m}$</th>
<th>$s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I + C_i \rightarrow C_{i+}$</td>
<td>1.45</td>
<td>I</td>
<td>0.40</td>
<td>1.85</td>
</tr>
<tr>
<td>$2C_i \rightarrow C_{i+}$</td>
<td>2.28</td>
<td>$C_i$</td>
<td>0.58</td>
<td>2.86</td>
</tr>
<tr>
<td>$C_i + O_i \rightarrow C_{i+}O_i$</td>
<td>1.25</td>
<td>$C_i$</td>
<td>0.58</td>
<td>1.86</td>
</tr>
<tr>
<td>$C_i + O_i \rightarrow C_{i+}I$</td>
<td>1.29</td>
<td>$C_i$</td>
<td>0.58</td>
<td>1.87</td>
</tr>
<tr>
<td>$I + C_iO_i \rightarrow C_{i+}O_i$</td>
<td>1.09</td>
<td>I</td>
<td>0.40</td>
<td>1.49</td>
</tr>
<tr>
<td>$C_iI + O_i \rightarrow C_{i+}O_i$</td>
<td>1.49</td>
<td>$C_i$</td>
<td>0.58</td>
<td>2.52</td>
</tr>
<tr>
<td>$V + O \rightarrow VO$</td>
<td>1.5</td>
<td>$V$</td>
<td>0.45 (Ref. 24)</td>
<td>1.95</td>
</tr>
<tr>
<td>$VO + O_i \rightarrow VO_2$</td>
<td>1.20</td>
<td>$VO$</td>
<td>1.53 (Ref. 23)</td>
<td>2.67</td>
</tr>
<tr>
<td>$VO + C_iO_i \rightarrow C_{i+}O_i$</td>
<td>2.80</td>
<td>$VO$</td>
<td>1.53</td>
<td>4.27</td>
</tr>
<tr>
<td>$VO_2 + C_i \rightarrow C_{i+}O_2$</td>
<td>2.89</td>
<td>$C_i$</td>
<td>0.58</td>
<td>3.47</td>
</tr>
<tr>
<td>$C_i + O_2 \rightarrow C_{i+}O_2$</td>
<td>0.73</td>
<td>$O_2$</td>
<td>1.59</td>
<td>2.32</td>
</tr>
<tr>
<td>$VO_2 + O_i \rightarrow VO_2$ *</td>
<td>1.94 (Ref. 21)</td>
<td>$VO_2$</td>
<td>2.12 (Ref. 21)</td>
<td>12</td>
</tr>
<tr>
<td>$VO_2 + O_i \rightarrow VO_3$ *</td>
<td>2.12 (Ref. 21)</td>
<td>$VO_3$</td>
<td>1.64</td>
<td>12</td>
</tr>
<tr>
<td>$C_i + I \rightarrow C_iI$ *</td>
<td>1.24</td>
<td>I</td>
<td>0.40</td>
<td>1.64</td>
</tr>
<tr>
<td>$C_i + VO \rightarrow C_{i+}O_i$ *</td>
<td>0.58</td>
<td>$C_i$</td>
<td>0.58</td>
<td>12</td>
</tr>
</tbody>
</table>
annealing during 20 min from 50°C to 500°C with a 10°C step. Annealing is done on irradiated silicon with different concentrations of carbon. Concentrations are extracted from infra-red absorption measures. The absorption coefficients are not all known, which leads to some unknown species concentrations. Nevertheless, in the simulations, it is required to follow each species in order to have all the reactions and thus the correct kinetics. KMC is not able to simulate such time-scale with so many complexes. The presented approach should be able to handle such a complexity.

We split the experimental process in two steps. First, to reproduce the irradiation process, a source term has been added in the previously defined model, assuming a constant formation of Frenkel pairs. Including this source term, the evolution of the various complex concentrations is simulated at room temperature during 20 min as in the experiments. The irradiation process leaves the sample in a non-equilibrium state and thus is a good test for our model. During irradiation, self interstitials are trapped by substitutional carbon ($C_s$), creating interstitial carbon ($C_i$). These $C_i$ then diffuse and are further trapped by oxygen ($O$) or carbon substitutional ($C_s$), creating, respectively, $C_sO_i$ or $C_sC_s$. Finally, remaining self interstitials are trapped by $C_sO_i$. On the vacancy side, the chain reaction is much straightforward as vacancies are directly trapped by oxygen to form VO pairs. Other reactions like the ones implying the creation of aggregates of self interstitials or vacancies are not explicitly taken into account in our reaction list. They will appear nonetheless in an effective term that will be detailed later in the article. The goal of this first step is to find the correct concentrations before annealing and validate the chosen set of reactions. The second step is the isochronal annealing itself. Metastable states are reached during this step and are an additional test for our model. It is worth noticing that no cooling steps have been considered between the anneals. Several species interact during such annealing as shown in Fig. 2(b).

We start by a general overview of the experimental reaction chains. Based on experimental measurements, all defects are related to interstitials and vacancies created during irradiation. The first set we consider is made of VO, VO$_2$ for the vacancy related complexes and $C_sC_s$, $C_sO_i$, $C_sC_i$, and $C_iO_i$ for the interstitial related ones. Experimental evidences, see Fig. 2(b), show that $C_sC_s$, $C_sO_i$, and VO are the primary species after irradiation. $C_sC_s$ then dissociates just before $C_sO_i$ around 350°C. These dissociations lead to the release of $C_i$. At the same temperature, VO diffuses and forms VO$_2$. $C_i$ recombine with VO and VO$_2$, forming $C_iO_i$. This is the final product of the reaction sequence, as experimentally observed. However, most of these reactions happen in the same temperature range. Next, we would like to support the relevance of our atomistic description and investigate the physical meaning of the factors $s_i$ and the parameter $p_0$. Thus, we have done a first crude simulation (Fig. 2(a)) that does not take into account the capture radius ($s_i = 1$ for all $i$) and the probability of visiting the same site twice ($p_0 = 1$). This simulation shows that after irradiation, the ratio between $C_sC_s$ and $C_sO_i$ is not consistent with the experimental ratio. This inconsistency is due to the use of the same capture radius and is corrected by assigning to each species a physical structural parameter ($s_i$ in Table I) allowing to reproduce the specific structure of each trap. In addition, a slight error on the temperatures of the reactions is also visible in Fig. 2(a). This can be corrected by using the proper value for $p_0$. Both effects prove the relevance of our atomistic description and that these two parameters are required to predict accurately the concentrations and the transition temperatures. Besides, this first simulation also reveals additional discrepancies with respect to the experimental data due to some missing reactions in the initially considered reaction chain:

1. First, the shape of the VO$_2$ curve in Fig. 2(a) does not reproduce the experimental trend. We observe a complete disappearance above 450°C. This comes from the too high amount of $C_i$ released from the $C_sO_i$ and $C_sC_s$ dissociations. Indeed, the $C_i$ further migrate and recombine with VO and VO$_2$ leading to the mentioned VO$_2$ disappearance and a corresponding overestimation of the $C_iO_i$ concentration. A simple explanation for a limited concentration of $C_i$ in the experimental data lies in the

![FIG. 2. (a) The simulated concentrations of several species at different temperatures during 20 min annealing with lossless equations. (b) The measured concentrations from experimental data. The absorption curve for the 1048 cm$^{-1}$ band corresponding to $C_sO_i$ species, is labeled “a $C_sO_i$”. It has been multiplied by $8 \times 10^{16}$ to be visible. (c) The numerical results, with losses and the same initial conditions as in (a). The lowest temperature represents the concentrations after irradiation.](image-url)
trapping of the latter before they reach VO or VO₂. At the same time, the effect of such sinks is experimentally supported for vacancies. Indeed, in a sample without carbon, there is a 50% loss during the VO → VO₂ reaction. These sinks can represent various defects, which are neither carbon nor oxygen related ones. They can be structural defects like dislocations, other trapping chemical species, or self clustering. In the following, we have thus added sink terms in the model at an atomic concentration of 7.0 × 10⁻⁶ and 5.0 × 10⁻⁸ for V/VO and C₁/I, respectively. These two parameters have been calibrated with another set of experiments on samples of equivalent crystalline qualities. The calibrated values for sinks can thus be used for this experiment.

(2) Second, a quick raise in C₁O₁ concentration is visible on Fig. 2(a) at 200 °C, while this raise is slower in experiment. In our simulation, this excess of C₁O₁ comes from the dissociation of C₁OI into C₁O₁ and silicon self interstitials. Thus, one needs to take into account another possible dissociation reaction (i.e., C₁OI → C₁I + O₁) to limit the C₁OI concentration. Indeed, as C₁I dissociates at higher temperature, it would allow a slower transition between C₁OI and C₁O₁. The existence of C₁I has already been discussed in literature and our analysis of the C₁OI transition further supports its existence.

We now use the full model (Table I) that takes into account all additional reactions and the calibrated sink strength. The simulated kinetics is depicted in Fig. 2(c). It reproduces the main experimental features shown in Fig. 2(b):

1. a low raise in C₁O₁ concentration between 175 °C and 250 °C, due to the concurrent dissociation of C₁OI into C₁O₁ on one hand and C₁I on the other hand.
2. the C₁C₂ dissociation range being in the 270 °C to 320 °C temperature range, as in the experiment. This range comes from a balance between two mechanisms that are required to reproduce the kinetics. The first mechanism is the transfer of C₁ to C₁O₁ as C₁C₂ is less stable than C₁O₁. It is associated with a bump in the C₁O₁ concentration just before the C₁OI dissociation. The second mechanism is the dissociation of both C₁C₂ and C₁OI, which leads to the recombination between V related complexes and C₁ as discuss below.
3. the transition rate from VO to VO₂ is found to be 1/4 as in the experiment. This reduction can be partially assigned to the presence of C₁. They are stored at low temperature in C₁C₂ and C₁OI complexes. When the complexes dissociate, the C₁ are liberated. Then, they recombine with VO and VO₂ to create the C₃O₁ and C₃OI₂ complexes. The latter is experimentally seen (Fig. 2(b)) in the same temperature range as in our simulation with the full model.

IV. PREDICTION OF CARBON/OXYGEN DEFECTS IN SILICON

Next, in order to assess the predictability of our full model (Table I), we have reproduced another set of experiments. The silicon sample used in the latter experiments has a low content of carbon (5 × 10¹⁶ cm⁻³) and the same amount of oxygen (9.5 × 10¹⁷ cm⁻³). As the samples from both high and low carbon concentrations have the same quality, the same sink concentrations can be used. The results are depicted in Fig. 3. A similar behavior to Fig. 2 is observed. The main transition temperatures are well reproduced, and the evolution in the concentrations is in a good quantitative agreement with the experiments. The main difference between the carbon high sample and the carbon low sample is the C₁C₄ concentration. Indeed, due to the low amount of C₃, O₁ are now a more efficient trap than C₃. Thus, the C₃O₁ concentration is higher than the C₃C₄. This result appears clearly in the experiment and is perfectly reproduced in our simulation. The phenomena of C₁ transfer from C₁C₄ and C₃O₁ to C₁O₁ are thus confirmed. Significantly, the raise of C₁O₁ concentration from 2.8 × 10¹⁶ cm⁻³ to 3.3 × 10¹⁶ cm⁻³ is well reproduced without any further calibration.

Up to now, the two simulated kinetics are isochronal annealing. Isothermal annealing is also used in the literature and should be reproducible with our full model and its
calibration. Still we have to introduce the creation of VO\textsubscript{p} complexes up to \(n \leq 4\) (Table I), as they are detected during the two reported isothermal annealing.\textsuperscript{24} The corresponding silicon sample has an oxygen concentration of \(1 \times 10^{19}\) cm\textsuperscript{-3} and no detectable carbon (\(<5 \times 10^{14}\) cm\textsuperscript{-3}). In addition, there is no experimental sign of a decrease in the vacancy concentration,\textsuperscript{21} and thus we can use the full model without any sinks. The results are presented in Fig. 4 and compared to the experimental data as extracted from figure 3 of Ref. 21. While the agreement is excellent at 370°C, we assign this shift to the cooling time of the sample that is not accounted in our simulation. Indeed, the cooling and annealing times are of the same order of magnitude in this latter experiment. Interestingly, the VO behavior is well reproduced despite a difference in the migration energy between our simulation and the experimental analysis.\textsuperscript{21} This difference between the migration energies is indeed balanced by the diffusion constant, which is lower in the simulation. Such effect is well described by the Meyer-Neldel rule.\textsuperscript{22} Nevertheless, the used energy in the full model is in the line of previous experiments.\textsuperscript{23}

Finally, our simulation can also be used to extract infra-red absorption calibration coefficients for uncalibrated complexes as \(C_{i}O_{j}\). Indeed, such a calibration coefficient is required to derive the complex concentrations from the infra-red measurements. Our simulated \(C_{i}O_{j}\) kinetics well reproduce the shape from their experimental counterpart in both Figs. 2 and 3. A value for the calibration coefficient can thus be derived. The values are \(7.8 \times 10^{16}\) cm\textsuperscript{-2} and \(8.6 \times 10^{16}\) cm\textsuperscript{-2} for high and low carbon samples, respectively. Thus, we propose a calibration coefficient of \(8 \pm 1 \times 10^{16}\) cm\textsuperscript{-2} for the \(C_{i}O_{j}\) complex.

V. CONCLUSIONS

Our simulations of the kinetics of the complex reactions between vacancy, interstitial, oxygen, and carbon related defects allow us assessing a set of reactions. The good agreement in the temperature transitions between experiments and simulation validates our approach in general and the reaction paths of the considered experimental set. For the latter, the effect of carbon on the vacancy oxygen related defects is now clearer and can be used to propose defect engineering strategies. In this particular case, the existence of the \(C_{i}\) complex is supported and we have confirmed the ability of carbon to store interstitials at low temperatures (up to 350°C for 20 min annealing).

From a general perspective, the used approach has proven the possibility to simulate thousands hour long annealing for defects in semiconductors. The flexibility of the model allows several types of annealing such as isochronal, isothermal but also aging, and the simulation time-scales easily reach years. This method is built on a DFT energy landscape, and the kinetic Mass Action Law to obtain the concentrations of defects. Experiments are required to calibrate the sink and source terms. Besides, the model described by Eqs. (2) and (3) is general. Through the parameters \(s_{i}\) and \(p_{0_{i}}\), it is adapted for any crystal structures and point defect geometries. We have shown that it can predict quantitatively defect concentrations in the framework of the diffusion limited reaction.

ACKNOWLEDGMENTS

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FIG. 4. The figure (a) represents a simulation of a 350 h annealing at 370°C. The figure (b) represents a simulation of a 16 h annealing at 470°C. Lines represent the simulation results and the dots are the experimental results from Ref. 21.