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ON THE NUMBER OF REPETITIVE MEASUREMENTS IN A LABORATORY EXPERIMENT

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Abstract

This communication concerns the problems that arose in an attempt to find the correct number N of necessary repetitive measurements in a laboratory experiment. Students usually face difficulties in determining N when a systematic error is also present. Neglecting the correlation between the two kind of errors, could lead to misleading results. Taking it into account usually leads to a more acurate evaluation of the measured quantity. Additionally, in the latter case a more profound understanding of the experimental procedures is attained.

Introduction

The easiest kind if experimental error to estimate and to eliminate is the kind called random ususally caused by experiment disturbances.When we try to determine indirectly a final physical quantity $F= F(x_i)$ $i=1,2,3,.....$ direct measurements of the primary quantites x_i are required. More specifically , in order to find the "best" value of x_i , which will lead to the best value of F, a set of repetitive measurements is carried out. The final purpose is to find the arithmetic mean of these measurements which is considered as an estimate value most closely to the real value. A problem which immediately arise is that of the suitable choise of the Number N of repetitive measurements if a systematic error is present . In what follows we shall discuss the cases where a) the order of magnitude of the systematic error is known and b) the order of magnidutes of the various components of the systematic error are known.

a) Determimation of an unknown ohmic resistance with the help of a Wheatstone bridge.

The wheatstone bridge usually is used to measure an unknown resistance through the relation

$$R = R_s \frac{x}{1-x} \qquad (1)$$

where R_s is a standerd resistance of known value with a toletance $\Delta R_s / R_s$ and x determines the equilibrium position of the bridge. Obviously, the measurement of x is subjected to experimental errors. This fact leads the students to repeat the measurements. However, we strongly believe that the decision for the repetition of the measurements is made almost mechanically by them with the purpose of reducing the experimental errror. In our opinion the following proceduce should be followed. At first the random error Δx of one measurement x is evaluated. Then the corresponding error ΔR is calculated from formula (1). It is obvious that a repetition of the measurements is needed only when $\Delta R/R > \Delta R_s / R_s$. In this case $\Delta R/R$ is calculated for different values of N and the relation $\Delta R/R$ versus N is represented in the form of a graph. $\Delta R/R$ is a diminishing function of N (1). The desired number N is that for which $\Delta R/R = \Delta R_s/R_s$. We should notice however that after a number of measurements the fall of $\Delta R/R$ is practically zero. If that occurs when $\Delta R/R > \Delta R_s/R_s$ then it is evident that the performance of the experimental set up has reach its limits.

β) Determination of the focal legth of a Lens

The focal length f is usually determined by use of the lens formula

$$1/f = 1/a + 1/b \qquad (2)$$

where a and b are the object distance and the image distance correspondingly. However, relation (2) holds only for paraxial rays and thin lenses. In the case of thick lenses or rays which cannot be considered as incident parallely to the axis the above formula is inadequate to give an accurate account of the image detail. Significantly, the calculation of f from rel (2) comprises an unknown systematic error. Thus a proper choise of the number N cannot be made, with all the aforementioned consequences.

Nevertheless, it is possible in our case to resolve partly the problem and remove some of the above consequences. The order of magnitude of the systematic error from the spherical aberration of thin lenses could be found in literature. Rays for example transversing a zone of radious h = 1.5cm give a spherical aberration of about 4 per cent of the paraxial focal length (2). Thus, we could consider that the systematic error is larger than the above value and accordingly choose as previously a proper number N allowing for the quality of the experimental organs. In a recent publication (3), the more realistic case of thick lenses has been investigated. According to this study, the longitudinal aberration alone of a thick lens of h = 1.5 cm introduces an error of about 10% in the focus distance. It is evident that an arbitrary choice of N could lead to an accidental error smaller than the systematic one which is not only a waste of time but a misleading result.

The above thoughts could be easily extended in a case often encountered in laboratory experiments where the physical quantity F depends on more than one primary quantities x_i In that case one has to correlate not only the random error of every x_i with the systematic error but also to make a correlation between the random errors themselves. The desired number N should generally ensure that the random errors are of the same order of magnitude and the total random error is of the same order with the systematic error.

Concluding remarks

A proper error analysis of certain experimental data presupposes a suitable number N of repetitive measurements. When both systematic and random errorsare involved in the measurements and the former error is known then N can be determined by trying to equalize the two errors. Since the systematic error is generally constant this entails a repetition of the measurements so many times as to restrict the random error to the same order of magnitude. The completion of this purpose apparently depends upon the quality of the available operators.

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