A Post-TOV Formalism - Motivations and Overview

Despite being in agreement with observations and widely accepted, the strong field regime of the new century central gravity (GR) remains fairly unexplored. Motivations coming from cosmology and high-energy physics suggest that GR might not be the final theory of the gravitational interaction. In this context, various modified gravity theories (MGTs) have been proposed.

Compact objects, such as neutron stars (NSs) and black holes (BHs), are ideal laboratories to study gravity in the strong-field regime. Naturally, a large amount of work has been done studying NSs and BHs for GR. These works show a degeneracy between MGTs, in the sense that, for a set of particular realistic equation of state (EoS), the mass-radius relation deviates from GR in a similar manner for different MGTs. Here, instead of committing ourselves to one particular theory, we propose a unified approach to study modifications of NS structure due to MGTs. This is achieved by formulating a post-TOV formalism [1].

Stellar structure within the parametrized post-Newtonian (PPN) theory was first studied in Refs. [2, 3] in the 70s-80s.

Current Solar System bounds constrain the standard 1PPN parameters to be close to their GR values [4]. Thus, NSs are indistinguishable from GR at this order.

Based on the form of the PPN correction terms we construct second order terms, introducing new, so far unconstrained, free parameters $\sigma_i $ ($i = 1, 2, 3, 4, 5$).

The post-TOV equations can, to some extent, be obtained from a modified energy-momentum tensor $\mathcal{T}_\nu$, and a static spherically symmetric metric $g_{\mu\nu}$, combined with $\nabla_\nu T^{\mu\nu} = 0$ and the GR field equations.

The post-TOV equations

In GR, the structure of static, spherically symmetric stars is determined by the Tolman-Oppenheimer-Volkoff (TOV) equations [5]:

$$\frac{dm}{dr}_{\text{GR}} = 4\pi r^2 \rho,$$  
(1)  

$$\frac{dp}{dr}_{\text{GR}} = -(e + p) \left[\frac{m + 4\pi r^3 \rho}{r(r - 2m)}\right],$$  
(2)  

where $m$ is the mass function, $\rho$ the pressure and $e \equiv \rho(1 + \Pi)$ the total energy density, with $\rho$ being the rest-mass density and $\Pi$ the internal energy per unit baryonic mass.

Based on previous work [2, 3], we can construct PPN corrected TOV equations, as

$$\frac{dm}{dr}_{\text{TOV}} = 4\pi r^2 \rho + 4\pi r^3 \rho \left(\delta_1 - \delta_2 \frac{m}{r}\right),$$  
(3)  

$$\frac{dp}{dr}_{\text{TOV}} = -\frac{\rho m}{r} \left(\delta_0 + \delta_1 \frac{m}{r} + \delta_2 \frac{4\pi r^3 \rho}{m}\right),$$  
(4)  

where $\delta_i \equiv 3 + 3\gamma - 6\beta + \xi_1$, $\delta_0 \equiv 1 + 1\xi_2$, $\delta_1 \equiv (1/2)(11 + \gamma - 12\beta + \xi_2 - 2\xi_3)$ and $\delta_2 \equiv \xi_3$ are constants related with the standard PPN parameters [4]. Current Solar System bounds on these parameters reveal that stellar models built from Eqs. (3) and (4) are indistinguishable from GR.

Thus, we must seek higher-order corrections, to obtain observationally significant deviations from GR. This is achieved by introducing new PPN parameters. These new 2PPN terms, are chosen by creating combinations of the PPN terms. Suppressing the previous PPN terms in Eqs. (3)-(4), we can write

$$\frac{dm}{dr}_{\text{TOV}} = 4\pi r^2 \rho + 4\pi r^3 \rho \left(\sigma_1 \Pi^2 + \sigma_2 \frac{m^2}{r^2} + \sigma_3 \frac{m}{r}\right),$$  
(5)  

$$\frac{dp}{dr}_{\text{TOV}} = -\frac{\rho m}{r} \left(\sigma_1 4\pi r^3 \rho + \sigma_2 \frac{m^2}{r^2} + \sigma_3 \frac{m}{r}\right),$$  
(6)  

where $\sigma_i $ ($i = 1, 2, 3, 4, 5$) are the post-TOV parameters.

The formalism is also complete, in the sense that, modulo a negligible PPN term, the post-TOV equations are derivable from a metric $g_{\mu\nu}$, an energy-momentum tensor $T_{\mu\nu}$ and the GR field equations. Starting from the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + \frac{1}{1 - 2Mr/r} dr^2 + r^2 d\Omega,$$

and an energy-momentum tensor $T_{\mu\nu} = (E + P)u^\mu u^\nu + Pg_{\mu\nu}$, the post-TOV equations follow, provided that $M \approx \sigma_1$, $P \approx P$ and $E \approx E + \rho + \Pi + K$, where $A$ and $K$ are the PPN and 2PPN terms appearing in Eqs. (3) and (5), respectively. The quantity $\xi$ represents an effective energy density. At last, the equation for the potential $\nu$ can be written as

$$\frac{dv}{d\rho} \approx -\frac{2}{p} \frac{dp}{d\rho},$$  
(8)  

Numerical Results

To study the influence of the various post-TOV terms, we integrated numerically the post-TOV equations, setting $\sigma_i = 0$ while keeping $\sigma_j \neq 0$ ($i \neq j$). In order to facilitate the comparison of the effect of the different post-TOV terms on the mass-radius curves, we assume that $\sigma_i \in [-1, +1]$ and the post-TOV equations are integrated with $\delta_0 = 0.2$ increments.

As a representative EoS, we consider SLy4, which in GR yields NSs with a maximum mass of $M \approx 2.05 M_\odot$. The mass-radius relation for SLy4 EoS corresponds to the solid curve in all figures below. Positive (negative) values of $\sigma_i$ are represented by the dashed (dash-dotted) curves.

Conclusions

Summary

Guided by the PPN formalism, we developed a post-TOV formalism to study stellar structure.

Numerical integration of the these equations reveals that the post-TOV terms affect the mass-radius relation of NSs in different ways.

Work in progress / Open questions

Computation of surface redshifts from the post-TOV equations.

Check if softer/stiffer EoSs are more sensitive to the post-TOV parameters, in the sense that the mass-radius relation deviates more from GR, for a fixed set of values of the parameters.

Can this approach be emulated, at least qualitatively, non-perturbative effects known in MGTs, such as spontaneous scalarization in scalar-tensor theories [6]?

Can the knowledge of masses and radii of NSs allows us to put bounds on the post-TOV parameters $\sigma_i$?

Can the formalism be extended to rotating stars? E.g. can we set-up a post-Hartle-Thorne formalism?

References