Testing the Kerr hypothesis with QNMs and ringdowns

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Motivation
- The Kerr solution
- Testing the Kerr hypothesis
- QNMs and ringdown
- Eikonal limit of QNMs and photon orbits
- Stating the problem

post-Kerr QNM spectroscopy
- post-Kerr setup
- Null tests of Kerr

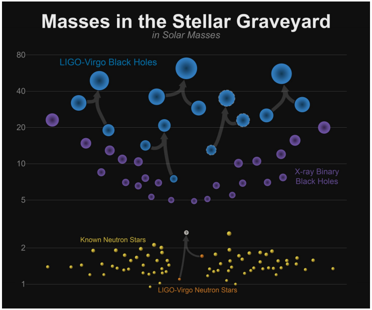
UltraCompact Objects ringdowns
- What is the nature of compact objects?
- UCOs, QNMs and Ringdowns
- Rotation breaks the degeneracy

QNM spectroscopy and Spherical Photon Orbits
- Setup of the problem and Spheroidal condition
- Separable spacetimes: Brief outline
- Separability and spherical photon orbits
- non-Separable spacetimes and loss of spherical photon orbits

Conclusions

The Kerr solution:

- The Kerr solution has been one of the most important results of classical GR.
- It describes the end state of a generic gravitational collapse to a rotating BH (Israel, Carter, Hawking, Robinson).
- There are a lot of indications that it is a stable solution (Dafermos et al.).
- We have observational indications that Kerr BHs exist (X-ray binaries, Supermassive BHs at the centres of galaxies, GW detections by LIGO/VIRGO).[a]



[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL 119, 161101 (2017).

- It is a solution of several modifications to GR.

The significance of the Kerr solution is what makes testing every aspect of it so important.

Testing the Kerr hypothesis:

There are several ways to test for the Kerr solution.

• Tests of the multipolar structure (QNM spectrum/BH spectroscopy, EMRIs, inspiral waveforms, accretion properties).[1]

• BH shadows (raytracing of photon orbits).

• Searching for echos in the post merger GW spectrum (ringdown).

• Testing for the separability of the spacetime[2] (this will be done by connecting the separability of a given spacetime with the properties of spherical photon orbits for the same spacetime and ultimately the presence of QNMs).

[1] K.Glampedakis et al., 2017 PRD 96, 064054; K.Glampedakis, GP, 2018 PRD 97, 041502(R)

[2] GP, K.Glampedakis, arXiv:1806.04091, 1806.09333

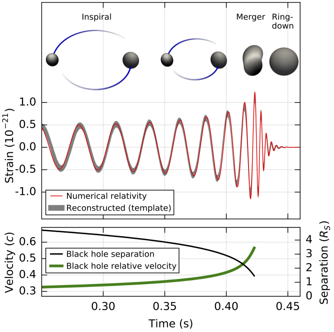
The Gravitational wave signal can be split in three parts, Inspiral, Merger, and Ringdown.

The signal for the first detection, GW150914 clearly shows both the inspiral and the ringdown parts.

In the ringdown signal we can find encoded the QNM spectrum of the merger remnant.

The disturbed spacetime that results after a merger, radiates away the additional features (perturbations) until a smooth stationary configuration remains.

Inspiral/Merger/Ringdown phases for GW150914



B.P. Abbott et al. PRL 116, 061102 (2016).

The QNMs are the modes (perturbations) that are temporarily trapped near the remnant and gradually escape to infinity where we are measuring them. They are described by the perturbed field equations.

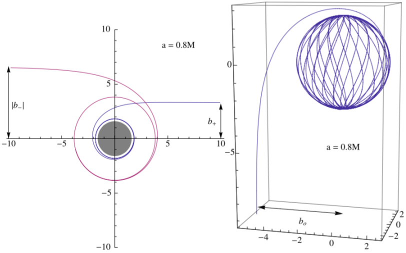
QNMs are characterised by their three quantum numbers, the multipole ℓ, the azimuthal number m, and the overtone number n.

A useful approx. for QNMs is the **eikonal limit**, i.e., $\ell \gg 1$ (works well for $\ell \sim 1$ too). In that limit the modes can be described as null particles.[3]

Specific photon orbits around a Kerr BH correspond to specific QNM modes in the eikonal limit (trapping zoom-whirl orbits).

Left: Equatorial co-rotating ($\ell = m$) and counter-rotating ($\ell = -m$) photon orbits.

Right: Polar spherical photon orbit ($m = 0$).



Dolan, PRD 82, 104003 (2010).

The frequency of the modes is related to the characteristics of the photon orbits.

$$\omega_{QNM}^{\ell,m,n} \approx \ell\Omega - i(n+1/2)|\lambda|,$$

where λ: Lyapunov exp., and in general

$$\Omega = \Omega_\phi + \frac{m}{\ell}\Omega_{nod},$$

Ω_ϕ: orbital freq., Ω_{nod}: precession freq.

[3] Ferrari & Mashhoon, PRD 30, 295 (1984); Yang et al., PRD 86, 104006 (2012)

Testing the Kerr hypothesis with the QNM spectrum:

For a Kerr BH in GR the QNM spectrum is well known. The modes of the spectrum depend on only M and a, and measuring fully a couple of modes can give both M, a as well as test the Kerr hypothesis.

For non-Kerr objects or for theories beyond GR the picture is different. The spectrum in these cases can be modified due to:

- *Dynamical modifications*: These are modifications due to the changes introduced in the perturbed field equations of any given modified theory of gravity w.r.t. what we have in GR.
- *Background modifications*: These are modifications due to the fact that a BH solution might not be a Kerr BH and instead be described by a non-Kerr spacetime (BH mimickers, UCOs).

Cases where both modifications might exist are, Einstein-dilaton-Gauss-Bonnet (EdGB) gravity, and dynamical Chern-Simons (dCS).

Calculating the QNM spectrum for exotic objects or in modified theories of gravity can be hard and can be done only in a case-by-case basis. One 1st needs BH solutions and then one needs to study perturbations in these backgrounds.

A first approach can be a parameterised framework that models deviations from the Kerr QNM spectrum, while using the eikonal limit.

post-Kerr formalism:[4]

• Do not commit to any particular theory of gravity.

• Assume an axisymmetric, stationary spacetime for the remnant which can be expressed as a small deviation from Kerr:

$$g_{\mu\nu} = g^{\mathrm{K}}_{\mu\nu} + \varepsilon h_{\mu\nu} + \mathcal{O}(\varepsilon^2)$$

• Use the eikonal limit to approximate the fundamental $\ell = |m|$ QNM modes by the characteristics of the equatorial light rings that the spacetime might have.

• The post-Kerr QNM frequencies in the eikonal limit will be given by the expression

$$\omega_{QNM} \approx \ell\Omega - i\frac{1}{2}|\lambda|,$$

where $\Omega = \Omega_{\mathrm{K}} + \varepsilon\delta\Omega$, and $\lambda = \lambda_{\mathrm{K}} + \varepsilon\delta\lambda$ are calculated at the light ring.

[4] K.Glampedakis et al., 2017 PRD 96, 064054.

• For a Kerr spacetime in GR the eikonal approximation can be corrected to match the actual QNM values using an offset function:

$$\omega^K_{QNM} = \omega^K_{eik} + \beta_K$$

This offset function corresponds to a small correction.

• For a small deviation from Kerr, the new eikonal frequency will be

$$\omega_{eik} = \omega^K_{eik} + \delta\omega_{eik}$$

This can be used as a null test of Kerrness since for a non Kerr QNM the deviation from the Kerr value will be

$$\delta\omega_{QNM} \simeq \delta\omega_{eik}.$$

This assumes the same offset β_K as we have for Kerr in GR, but for small deviations, one would expect that the error in the offset should be subleading. Nevertheless it will introduce some error.

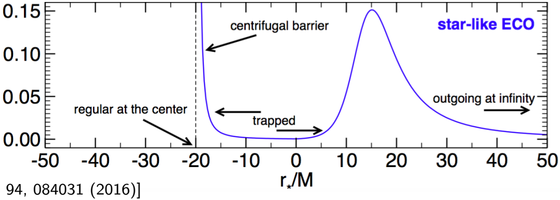
• There are some caveats: 1) geodesic correspondence, 2) no coupling to extra fields, 3) having "Kerr light rings" for non-Kerr spacetimes.

Another alternative to test for (or against) are UltraCompact Objects (UCOs). This is a test of the Kerr hypothesis within GR.

• UCOs are compact enough to pass for BHs, but don't have a horizon. Their surface is at $R < r_{ph}$.

There exist several candidates, such as Gravastars and Boson stars (as well as issues with formation mechanisms and stability).

Wave potential:



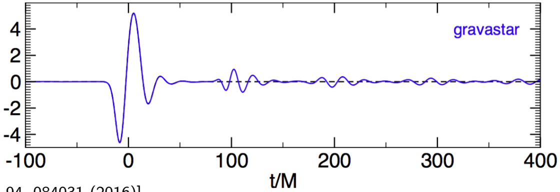
[Cardoso et al. PRD 94, 084031 (2016)]

– A non-rotating UCO has the same wave potential as a BH in it's exterior.

– On the other hand it can have trapped w-modes since there is no horizon.

– The QNM spectrum is different from that of a BH's.

Ringdown:



[Cardoso et al. PRD 94, 084031 (2016)]

– Remarkably, non-rotating UCOs and BHs share identical early ringdown signals, but differ at late times due to the presence of the w-mode echoes.[5]

Intuitive explanation: ringdown is dominated by scattering at the potential peak which is the same for non-rotating UCOs and BHs.

– This was first seen in work on the discretisation of the Schwarzschild BH potential by Nollert back in 1996.[6]

[5] Cardoso et al. PRL 116, 171101 (2016).

[6] H.-P. Nollert, PRD 53, 4397 (1996).

Can we tell UCO ringdowns from Kerr BH ringdowns? Yes. Rotation is the key.
Rotating UCOs have different multipole moments $\to$ different photon ring.

Rotating UCO model: Hartle-Thorne at $\mathcal{O}(\epsilon^3)$

$$ds^2 = -e^{\nu}\left[1+2\epsilon^2\left(h_0+h_2P_2\right)\right]dt^2+\left(1-\frac{2m}{r}\right)^{-1}\left[1+2\epsilon^2\frac{\left(\mu_0+\mu_2P_2\right)}{r-2m}\right]dr^2$$
$$+r^2\left(1+2\epsilon^2k_2P_2\right)\left[d\theta^2+\sin^2\theta\left[d\varphi-\epsilon\left\{\Omega-\omega_1P_1+\epsilon^2\left(w_1P_1'+w_3P_3'\right)\right\}dt\right]^2\right].$$

The multipole moments for this spacetime are:

$$M_2=-\chi^2M^3\left(1-\delta q\right),\ S_3=-\chi^3M^4\left(1-\delta s_3\right),\ \chi=J/M^2,$$

and the light ring is at:

$$r_{\rm ph}=M\left(3-\frac{2\chi}{\sqrt{3}}-\chi^2\left[\frac{1}{27}+\frac{5}{4}\left(13-\frac{45}{4}\log 3\right)\delta q\right]\right.$$
$$\left.-\frac{\sqrt{3}}{2}\chi^3\left[\frac{4}{81}-(553-505\log 3)\,\delta q+\frac{7}{4}(148-135\log 3)\,\delta s_3\right]\right).$$

Eikonal limit ringdown for a rotating UCO:

$\ell = 2$ ringdown frequency and the damping rate,

$$M\omega_{\mathrm{R}} = \frac{2}{3\sqrt{3}} + \frac{4\chi}{27} + \frac{2\chi^2}{3\sqrt{3}}\left[\frac{11}{54} + 5\left(\frac{15}{16}\log 3 - 1\right)\delta q\right] + \frac{4\chi^3}{81}\left[1 + \frac{9}{64}\,(\,5652 - 5125\log 3\,)\,\delta q \right.$$
$$\left. - \frac{21}{128}\,(\,2228 - 2025\log 3\,)\,\delta s_3\right],$$

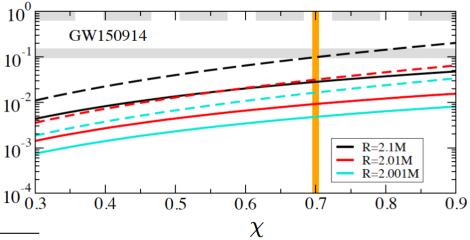
$$M\omega_{\mathrm{I}} = \frac{1}{6\sqrt{3}} - \frac{\chi^2}{6\sqrt{3}}\left[\frac{4}{54} + \frac{5}{8}\,(15\log 3 - 16)\,\delta q\right] - \frac{\chi^3}{93312}\left[640 - 270\,(14595\log 3 - 16076)\,\delta q \right.$$
$$\left. + 945\,(2025\log 3 - 2228)\,\delta s_3\right],$$

Application: rotating Gravastar[7]

Fractional difference with respect to *same spin-order eikonal* Kerr values

ω_{R} ———

ω_{I} - - - - -



[7] K.Glampedakis, GP, 2018 PRD 97, 041502(R).

More general off-equatorial orbits - Speroidal/Spherical photon orbits:

We consider an arbitrary axisymmetric, stationary and equatorially symmetric spacetime $g_{\mu\nu}(r,\theta)$ in a spherical-like coordinate system, that is circular, i.e., that the 2-dimensional surfaces orthogonal to the Killing fields are integrable,

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + 2g_{t\varphi}dtd\varphi + g_{\theta\theta}d\theta^2 + g_{\varphi\varphi}d\varphi^2. \tag{1}$$

Geodesics have a conserved energy $E = -u_t$ and angular momentum $L = u_\varphi$ (along the symmetry axis). For null geodesics in particular, we only need the impact parameter $b = L/E$.

The norm $u^\mu u_\mu = 0$ for photons then becomes

$$g^{rr}u_r^2 + g^{\theta\theta}u_\theta^2 = \frac{1}{\mathcal{D}}\left(g_{tt}b^2 + 2g_{t\varphi}b + g_{\varphi\varphi}\right) \equiv V_{\rm eff}(r,\theta,b), \tag{2}$$

where $\mathcal{D} = g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}$.

The quadratic form of this implies that $V_{\rm eff} = 0$ marks the zero-velocity separatrix between allowed and forbidden regions for geodesic motion.

Furthermore, the four-acceleration is given by (second-order geodesic equation)

$$\alpha_\kappa \equiv \frac{du_\kappa}{d\lambda} = \frac{1}{2}g_{\mu\nu,\kappa}u^\mu u^\nu. \tag{3}$$

Let's consider a general class of non-equatorial orbits where motion is confined on a spheroidal shell $r_0 = r_0(\theta)$.
For these spheroidal orbits, the velocity components u^r, u^θ are related as

$$u^r = r_0' u^\theta \quad \Rightarrow \quad u_r = \frac{g_{rr}}{g_{\theta\theta}} r_0' u_\theta , \tag{4}$$

From this condition we can derive a diff. eq. for the spheroidal orbits $r_0(\theta)$.
Taking the derivative w.r.t. the affine parameter and using the equations of motion, we obtain after some algebra,

$$0 = g_{rr}(r_0')^3 \left(g_{rr} V_{\rm eff}\right)_{,\theta} + (r_0')^2 \left[\left(g_{\theta\theta} g_{rr,r} - 2 g_{rr} g_{\theta\theta,r} \right) V_{\rm eff} - g_{rr} g_{\theta\theta} V_{{\rm eff},r} \right]$$
$$+ r_0' \left[\left(2 g_{\theta\theta} g_{rr,\theta} - g_{rr} g_{\theta\theta,\theta} \right) V_{\rm eff} + g_{rr} g_{\theta\theta} V_{{\rm eff},\theta} \right] + g_{\theta\theta} \left[2 g_{rr} V_{\rm eff} r_0'' - \left(g_{\theta\theta} V_{\rm eff} \right)_{,r} \right] . \tag{5}$$

For a spherical. i.e., $r_0(\theta)$=const$\equiv r_0$ solution, we would only need

$$\left(g_{\theta\theta} V_{\rm eff} \right)_{,r} |_{r_0} = 0, \tag{6}$$

Separability:

For a spacetime of the general form (1) the Hamilton-Jacobi equation for null geodesics becomes,[8]

$$\frac{(S_{,r})^2}{g_{rr}} + \frac{(S_{,\theta})^2}{g_{\theta\theta}} - V_{\text{eff}} = 0, \tag{7}$$

where $S(r,\theta)$ is Hamilton's characteristic function. A separable spacetime entails an additive S, i.e. $S = S_r(r) + S_\theta(\theta)$.[9] Assuming the conditions

$$g_{\theta\theta} V_{\text{eff}} = f_1(r)h(\theta) + g(\theta), \tag{8}$$

$$\frac{g_{\theta\theta}}{g_{rr}} = f_2(r)h(\theta), \tag{9}$$

we can rearrange (7) as,

$$f_2(r)(S_r')^2 - f_1(r) = \frac{1}{h(\theta)}\left[g(\theta) - (S_\theta')^2\right] = \mathcal{C}, \tag{10}$$

and demonstrate the separability of the system, with $\mathcal{C}$ playing the role of the third constant (or 'Carter constant').

[8] MTW, Gravitation (1973).

[9] Landau & Lifshitz, Mechanics (1969).

Putting eqs. (8) & (6) together we can connect separability to spherical orbits,

$$(8) : g_{\theta\theta} V_{\rm eff} = f_1(r)h(\theta)+g(\theta), \quad (6) : \left(g_{\theta\theta} V_{\rm eff}\right)_{,r}|_{r_0} = 0 \quad \left[(9) : \frac{g_{\theta\theta}}{g_{rr}} = f_2(r)h(\theta)\right]$$

The general ansatz (8) satisfies the condition (6) as long as $f_1'(r) = 0$ has roots.

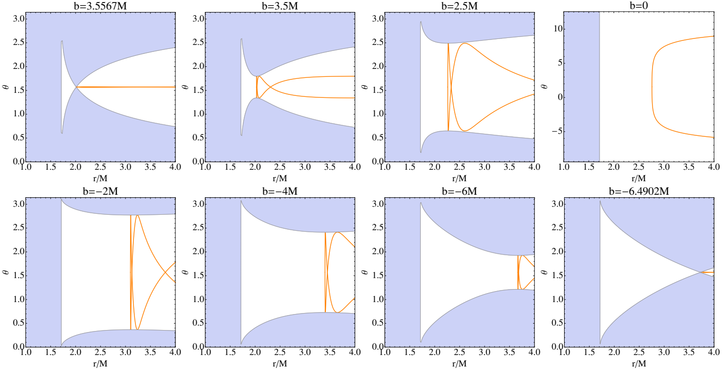
Result:

a) If a spacetime is separable, then (8) holds, which then implies that (6) also holds as long as $f_1'(r) = 0$ has roots. Therefore a separable spacetime can admit spherical photon orbits. If a given separable spacetime is additionally known to have photon rings, then this ensures that the equation $f_1'(r) = 0$ has roots.

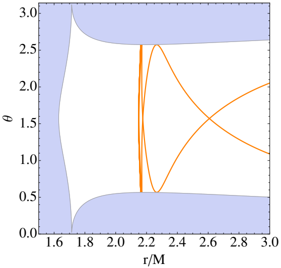
A corollary of this is that a spacetime *cannot be separable* if it does not admit spherical orbits while it admits equatorial photon rings.

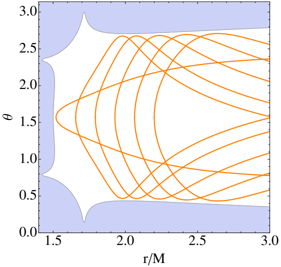
One can further show that only spherical and not spheroidal orbits can exist in a separable spacetime.

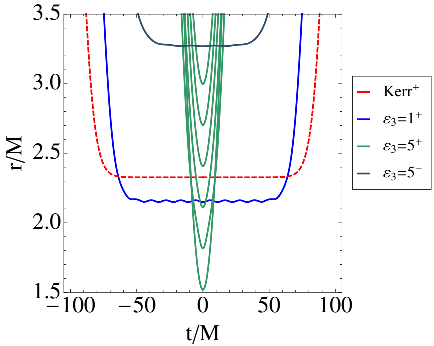
Behaviour of the zoom-whirl orbits in a Kerr spacetime with $a = 0.7M$. Starting from top left we have the co-rotating light ring at $b \simeq 3.5567$, going through to $b = 0$ at bottom left with the spherical polar orbits, and all the way to the counter-rotating light ring for $b \simeq -6.4902$.



Johannsen-Psaltis zoom-whirl orbits.

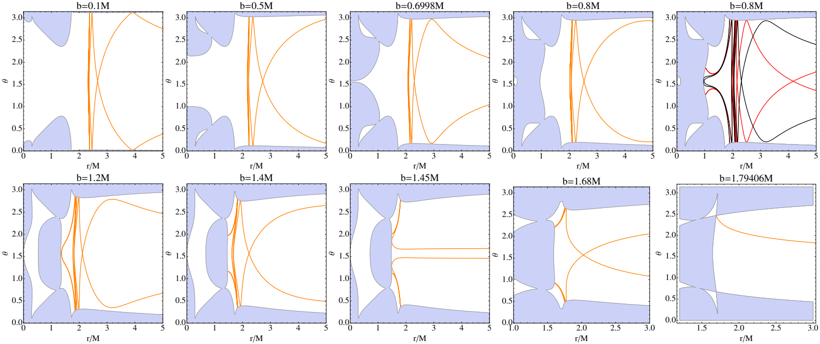






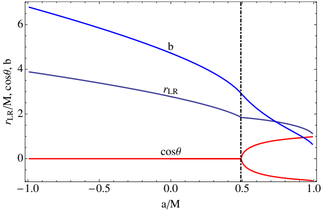
The blue curve is the $r(t)$ profile for a JP with a small deformation. It is of $r_0(\theta)$ form. The light green curves are for a JP with a large deformation.

Behaviour of the zoom-whirl orbits in a JP spacetime with $a = 0.7M$ and $\varepsilon_3 = 5$. Here we show only the positive impact parameter results starting from $b = 0.1$ on the top row.

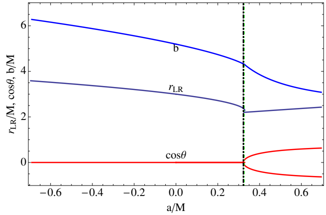


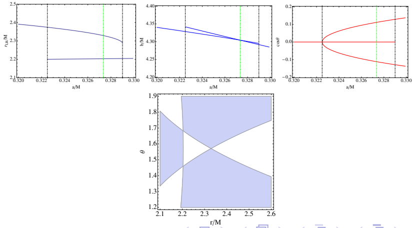
Clearly orbits that can be found in Kerr, and would correspond to QNMs with $\ell = m$ or m/ℓ not very small and close to 1, are lost in the JP spacetime.

An interesting effect related to the previous behaviour, is the behaviour of the light-ring in a JP spacetime.

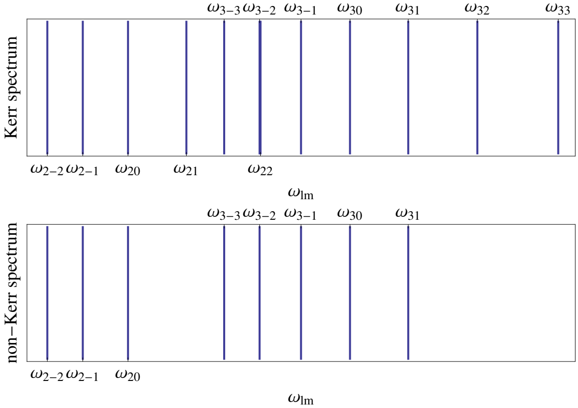


This behaviour is not exclusive to the JP spacetime. A similar behaviour can be seen in the Hartle-Thorne spacetime as well, for a quadrupole deformation of $\delta q = 1$





A cartoon picture of the comparison of a Kerr QNM spectrum[10], against a non-Kerr ringdown spectrum[11] based on the previous results.



Smoking gun non-Kerr behaviour.

[10] Based on arXiv:0905.2975 and gr-qc/0512160; http://www.phy.olemiss.edu/ berti/ringdown/

[11] Frequency shifts are not shown here.

- post-Kerr parameterised formalism can be a useful first tool to test the Kerr hypothesis.
- There are caveats to consider, e.g. coupling with extra d.o.f.
- Rotating UCOs and BH mimickers can be distinguished from Kerr BHs if they are not ε away from Kerr.
- Rotation is what magnifies the difference. Some mimicker models may be excluded in the near future.
- Geodesic properties related to separability could probe Kerrness.
- Some strong statements can be made, such as, if a spacetime doesn't admit spherical photon orbits but admits LRs, then it cannot be separable.
- When a spacetime is deformed away from being separable, spherical photon orbits get deformed becoming spheroidal and eventually can be completely lost.
- This could have interesting implications for the QNMs of non-separable spacetimes. Loss of spherical orbits $\rightarrow$ loss of Kerr QNM modes.
- The loss of trapping photon orbits could also affect BH shadows.

Thank You